Uniformly Loaded Square Plate with Partially Simply Supported at the Middle Edges and Point-Column Supported at the Corners: II – Analytical Expressions

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ABSTRACT

In a companion paper, it has already been treated and derived an integral equation of the Fredholm-type. This equation governs the bending of uniformly loaded square plate in which the plate is supported by corner-point supports together with partial simple supports placed at the middle edges. Therefore, the current paper is aimed to further derive and propose the analytical closed-form expressions involving the plate deformations and stress resultants; namely, deflection, slope, bending moment, shear force in element, and corner force of the plate.

Keywords: *Analytical expression, Fredholm integral equation, Hankel integral transform, Plate Deformation, Square plate, Stress resultants.*

1 INTRODUCTION

It has been shown in the previous paper [1] that static bending problem of uniformly loaded square plate supported by corner-point supports and partial simple supports located at the middle edges as illustrated in Figure 1 is solved by means of the method of finite Hankel integral transforms. The solution of problem can be reduced to and determined from an inhomogeneous Fredholm integral equation of the second kind in terms of an unknown auxiliary function. It is, however, notable that this unknown function has not a meaning in the physical senses, but only introduced mathematically. Nevertheless, it is still relatable interested physical quantities of the plate.

In the present paper, the objective is to provide the analytical closed-form expressions for the deflection, slope, bending moment, shear force in element, and corner force of the plate.

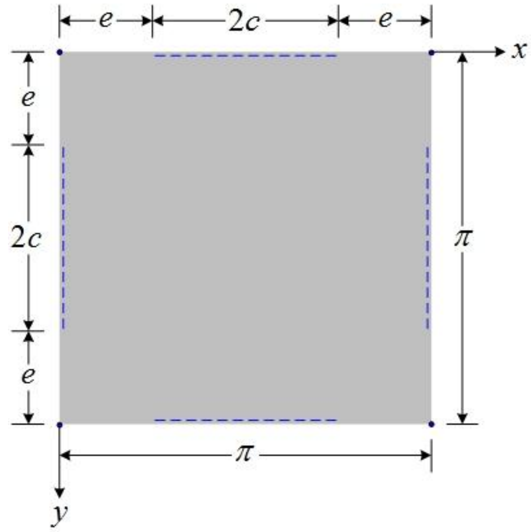

Fig. 1 *Corner-supported square plate with partially simply supported edges.*

Refer to the plate as shown in Figure 1, the lengths involved are scaled by the factor $\pi/\bar{a}$ whereas the actual length of square plate is $\bar{a}$, and e, c are the free edge length and the half-length of partial simple support, respectively.

The total deflection which satisfies the governing equation of the plate [2] is taken as the sum of particular (w_p) and complementary (w_c) solutions as follows:

$$w(x,y) = w_p(x,y) + w_c(x,y)\,, \tag{1}$$

where

$$w_p(x,y)=\frac{q\bar{a}^4}{2D}\sum_{m=1,3,5,...}^{\infty}\frac{4}{\pi^5 m^5}\left[\sin(mx)+\sin(my)\right], \quad (2)$$

and also,

$$w_c(x,y)=\frac{q\bar{a}^4}{2D}\sum_{m=1,3,5,...}^{\infty}\{[A_m\cosh(my)$$
$$+B_m my\sinh(my)+C_m\sinh(my)$$
$$+D_m my\cosh(my)]\sin(mx)$$
$$+[A_m\cosh(mx)+B_m mx\sinh(mx)$$
$$+C_m\sinh(mx)+D_m mx\cosh(mx)]$$
$$\times\sin(my)\}, \quad (3)$$

whileas q is a uniformly distributed load and D is the bending rigidity of the plate.

The unknown constants A_m, B_m, and C_m are related to D_m by the following relations:

$$A_m=\frac{4\nu\eta'}{\pi^5 m^5}+2D_m\eta'\coth\beta\,, \quad (4)$$

$$B_m=-D_m\coth\beta\,, \quad (5)$$

$$C_m=-\frac{4\nu\eta'\tanh\beta}{\pi^5 m^5}-D_m\left[2\eta'+\beta\left(\tanh\beta-\coth\beta\right)\right], \quad (6)$$

with

$$\eta'=\frac{1}{1-\nu}\,, \quad (7)$$

$$\beta=\frac{m\pi}{2}\,, \quad (8)$$

where ν is the Poisson's ratio of the plate.

The unknown constant D_m can be determined from the relation given below whether the unknown function P_m is known,

$$D_m=\left(P_m-\frac{2}{\pi^5 m^5}\right)\tanh\beta\,. \quad (9)$$

Based on the method of Hankel integral transforms, the unknown function P_m can be assumed in the form as

$$m^2 P_m=\int_0^e t\phi(t)\left[J_1(mt)-\frac{t}{e}J_1(me)\right]dt\;;\; m=1,3,5,\ldots\,, \quad (10)$$

in which t is a dummy variable, $\phi(t)$ is an unknown auxiliary function, and $J_n(u)$ is the Bessel function of the first kind and order n with argument u [3], [4].

2 GOVERNING INTEGRAL EQUATION

With the use of Eq.(10), the problem can be reduced to finding the solution of an inhomogeneous Fredholm integral equation of the second kind [1], which is

$$\Phi(\rho)+\int_0^1 K(\rho,r)\Phi(r)dr=f(\rho)\;;\;0\le\rho\le 1\,, \quad (11)$$

where ρ is a dummy variable and

$$\Phi(\rho)=\phi(e\rho)\;;\;\Phi(r)=\phi(er)\,, \quad (12)$$

$$K(\rho,r)=2e^2 r\sum_{m=1,3,5,...}^{\infty}\left[-4\eta m/\pi-\eta m L_1(me\rho)\right.$$
$$-\eta m^2 e\rho L_0(me\rho)+mF_m^{(1)}J_1(me\rho)$$
$$\left.+m(F_m^{(2)}-F_m^{(3)})I_1(me\rho)+m^2F_m^{(3)}e\rho I_0(me\rho)\right]$$
$$\times\left[J_1(mer)-rJ_1(me)\right]-2e^2 r$$
$$\times\int_0^\infty\frac{sI_1(se\rho)\left[I_1(ser)-rI_1(se)\right]}{\exp(\pi s)+1}ds\,, \quad (13)$$

$$f(\rho)=2\sum_{m=1,3,5,...}^{\infty}\left[F_m^{(4)}J_1(me\rho)+(F_m^{(6)}-F_m^{(7)})I_1(me\rho)\right.$$
$$+mF_m^{(7)}e\rho I_0(me\rho)+(F_m^{(8)}-F_m^{(5)})L_1(me\rho)$$
$$\left.-mF_m^{(8)}e\rho L_0(me\rho)\right], \quad (14)$$

in which $L_n(u)$ is the modified Struve function of order n with argument u [3], and

$$F_m^{(1)}=\frac{(3+\nu)\sinh\beta\cosh\beta-(1-\nu)\beta}{(3+\nu)\cosh^2\beta}-1\,, \quad (15)$$

$$F_m^{(2)}=\eta(2\tanh\beta+\beta\mathrm{sech}^2\beta)\,, \quad (16)$$

$$F_m^{(3)}=\eta\tanh\beta\,, \quad (17)$$

$$F_m^{(4)}=\frac{2\left[(3-\nu)\tanh\beta-(1-\nu)\beta\mathrm{sech}^2\beta)\right]}{(3+\nu)\pi^5 m^2}\,, \quad (18)$$

$$F_m^{(5)} = \frac{4}{(3+\nu)\pi^5 m^2}\,, \tag{19}$$

$$F_m^{(6)} = \frac{2\left[2\tanh\beta + (1-\nu)\beta\operatorname{sech}^2\beta\right]}{(3+\nu)\pi^5 m^2}\,, \tag{20}$$

$$F_m^{(7)} = \frac{2\eta\tanh\beta}{\pi^5 m^2}\,, \tag{21}$$

$$F_m^{(8)} = \frac{2\eta}{\pi^5 m^2}\,, \tag{22}$$

$$\eta = \frac{1-\nu}{3+\nu}\,. \tag{23}$$

Since the unknown function $\Phi(\rho)$ is determined from Eq.(11), the function P_m can also be obtained. In view of Eq.(12), P_m as given in Eq.(10) becomes

$$m^2 P_m = e^2 \int_0^1 \rho\,\Phi(\rho)\left[J_1(me\rho) - \rho J_1(me)\right]d\rho$$
$$;\ m = 1,3,5,\ldots \tag{24}$$

Using Eqs.(9) and (24), the unknown constants can be rewritten as follows:

$$A_m = -\frac{4}{\pi^5 m^5} + 2\eta' e^2 \Lambda_m\,, \tag{25}$$

$$B_m = \frac{2}{\pi^5 m^5} - e^2\Lambda_m\,, \tag{26}$$

$$C_m = \frac{2}{\pi^5 m^5}(2\tanh\beta - \beta\operatorname{sech}^2\beta)$$
$$-(2\eta'\tanh\beta - \beta\operatorname{sech}^2\beta)e^2\Lambda_m\,, \tag{27}$$

$$D_m = -\frac{2\tanh\beta}{\pi^5 m^5} + \tanh\beta\, e^2\Lambda_m\,, \tag{28}$$

where

$$\Lambda_m = \frac{1}{m^2}\int_0^1 \rho\Phi(\rho)\left[J_1(me\rho) - \rho J_1(me)\right]d\rho\,. \tag{29}$$

3 DEFLECTIONS OF THE PLATE

Substituting Eq.(25) to Eq.(28) for the unknown constants into Eq.(2), the general total deflection of Eq.(1) becomes

$$\frac{w(x,y)}{q\bar{a}^4/2D} = \sum_{m=1,3,5,\ldots}^{\infty} [\{(4/\pi^5 m^5) - [(4/\pi^5 m^5) - 2\eta' e^2\Lambda_m]$$
$$\times\cosh(my) + [(2/\pi^5 m^5) - e^2\Lambda_m]my\sinh(my)$$
$$+[(2/\pi^5 m^5)(2\tanh\beta - \beta\operatorname{sech}^2\beta)$$
$$-(2\eta'\tanh\beta - \beta\operatorname{sech}^2\beta)e^2\Lambda_m]\sinh(my)$$
$$-[(2\tanh\beta/\pi^5 m^5) - e^2\Lambda_m\tanh\beta]$$
$$my\cosh(my)\}\sin(mx) + \{(4/\pi^5 m^5)$$
$$-[(4/\pi^5 m^5) - 2\eta' e^2\Lambda_m]\cosh(mx)$$
$$+[(2/\pi^5 m^5) - e^2\Lambda_m]mx\sinh(mx)$$
$$+[(2/\pi^5 m^5)(2\tanh\beta - \beta\operatorname{sech}^2\beta)$$
$$-(2\eta'\tanh\beta - \beta\operatorname{sech}^2\beta)e^2\Lambda_m]\sinh(mx)$$
$$-[(2\tanh\beta/\pi^5 m^5) - e^2\Lambda_m\tanh\beta]$$
$$\times mx\cosh(mx)\}\sin(mx)]$$
$$;\ 0 \le x, y \le \pi/2\,. \tag{30}$$

To determine the deflection along the free edge of the plate, first setting $y = 0$ into Eq.(30) and considering the identity as given below [5]-[7],

$$\sum_{m=1,3,5,\ldots}^{\infty} m^{-2} J_1(mt)\sin(mx)$$
$$= \left\{\begin{array}{l} \dfrac{1}{4}\left[\dfrac{x}{t}(t^2 - x^2)^{\frac{1}{2}} + t\sin^{-1}\dfrac{x}{t}\right],\ x < t \\ \dfrac{\pi}{8}t,\ x \ge t \end{array}\right\};\ x + t < \pi\,, \tag{31}$$

and thus, leads to

$$\frac{w(x,0)}{q\bar{a}^4/2D} = (\eta' e^3/2)\{(\pi/2)\int_0^{\xi} \rho^2\Phi(\rho)d\rho$$
$$+\int_{\xi}^{1}[\xi\sqrt{\rho^2 - \xi^2} + \rho^2\sin^{-1}(\xi/\rho)]\Phi(\rho)d\rho$$
$$-\int_0^1 \rho^2[\xi\sqrt{1-\xi^2} + \sin^{-1}\xi]\Phi(\rho)d\rho\}$$
$$:\ \xi = x/e;\ 0 \le \xi, \rho \le 1\,. \tag{32}$$

The deflection along the middle line of the plate can be obtained from Eq.(30) with setting $y = \pi/2$, which is

$$\frac{w(x,\pi/2)}{q\bar{a}^4/2D} = \sum_{m=1,3,5,\ldots}^{\infty} [\{(4/\pi^5 m^5) - [(4/\pi^5 m^5) - 2\eta' e^2 \Lambda_m] \times\cosh\beta + [(2/\pi^5 m^5) - e^2\Lambda_m]\beta\sinh\beta + [(2/\pi^5 m^5)(2\tanh\beta - \beta\text{sech}^2\beta) - (2\eta'\tanh\beta - \beta\text{sech}^2\beta)e^2\Lambda_m]\sinh\beta - [(2\tanh\beta/\pi^5 m^5) - e^2\Lambda_m\tanh\beta]\beta\cosh\beta\} \times\sin(mx) + \{(4/\pi^5 m^5) - [(4/\pi^5 m^5) - 2\eta' e^2\Lambda_m]\cosh(mx) + [(2/\pi^5 m^5) - e^2\Lambda_m] \times mx\sinh(mx) + [(2/\pi^5 m^5)(2\tanh\beta - \beta\text{sech}^2\beta) - (2\eta'\tanh\beta - \beta\text{sech}^2\beta)e^2\Lambda_m] \times\sinh(mx) - [(2\tanh\beta/\pi^5 m^5) - e^2\Lambda_m\tanh\beta]mx\cosh(mx)\} \times(-1)^{(m-1)/2}];\ 0 \le x \le \pi/2. \quad (33)$$

4 SLOPES OF THE PLATE

The slope of the plate can be determined by the derivative,

$$\theta_x = \frac{\pi}{\bar{a}}\left(\frac{\partial w}{\partial x}\right). \quad (34)$$

The slope along the middle line at $y = \pi/2$ is obtained by substituting Eq.(33) into Eq.(34) so that,

$$\frac{\theta_x(x,\pi/2)}{q\bar{a}^3\pi/2D} = \sum_{m=1,3,5,\ldots}^{\infty} [\{(4/\pi^5 m^4) - [(4/\pi^5 m^4) - 2\eta' e^2 m\Lambda_m]\cosh\beta + [(2/\pi^5 m^4) - e^2 m\Lambda_m] \times\beta\sinh\beta + [(2/\pi^5 m^4)(2\tanh\beta - \beta\text{sech}^2\beta) - (2\eta'\tanh\beta - \beta\text{sech}^2\beta)e^2 m\Lambda_m]\sinh\beta - [(2\tanh\beta/\pi^5 m^4) - e^2 m\Lambda_m\tanh\beta] \times\beta\cosh\beta\}\cos(mx) + \{[-(4/\pi^5 m^4) + 2\eta' e^2 m\Lambda_m]\sinh(mx) + [(2/\pi^5 m^4) - e^2 m\Lambda_m][mx\cosh(mx) + \sinh(mx)] + [(2/\pi^5 m^4)(2\tanh\beta - \beta\text{sech}^2\beta) - (2\eta'\tanh\beta - \beta\text{sech}^2\beta)e^2 m\Lambda_m] \times\cosh(mx) - [(2\tanh\beta/\pi^5 m^4) - e^2 m\Lambda_m\tanh\beta][mx\sinh(mx) + \cosh(mx)]\}(-1)^{(m-1)/2}] ;\ 0 \le x \le \pi/2. \quad (35)$$

To determine the slope along the diagonal line of the plate, substitution of Eq.(30) into Eq.(34) yields,

$$\frac{\theta_x(x,x)}{q\bar{a}^3\pi/2D} = \sum_{m=1,3,5,\ldots}^{\infty} [\{(4/\pi^5 m^4) - [(4/\pi^5 m^4) - 2\eta' e^2 m\Lambda_m] \times\cosh(mx) + [(2/\pi^5 m^4) - e^2 m\Lambda_m] \times mx\sinh(mx) + [(2/\pi^5 m^4)(2\tanh\beta - \beta\text{sech}^2\beta) - (2\eta'\tanh\beta - \beta\text{sech}^2\beta) e^2 m\Lambda_m]\sinh(mx) - [(2\tanh\beta/\pi^5 m^4) - e^2 m\Lambda_m\tanh\beta]mx\cosh(mx)\}\cos(mx) + \{[-(4/\pi^5 m^4) + 2\eta' e^2 m\Lambda_m]\sinh(mx) + [(2/\pi^5 m^4) - e^2 m\Lambda_m][mx\cosh(mx) + \sinh(mx)] + [(2/\pi^5 m^4)(2\tanh\beta - \beta\text{sech}^2\beta) - (2\eta'\tanh\beta - \beta\text{sech}^2\beta) \times e^2 m\Lambda_m]\cosh(mx) - [(2\tanh\beta/\pi^5 m^4) - e^2 m\Lambda_m\tanh\beta][mx\sinh(mx) + \cosh(mx)]\}\sin(mx)];\ 0 \le x \le \pi/2, \quad (36)$$

and then, the slope along the diagonal line of the plate can be determined with the use of a following equation,

$$\theta_n(x,x) = \sqrt{2}\theta_x(x,x). \quad (37)$$

5 BENDING MOMENTS OF THE PLATE

The bending moments in both directions of the plate are given as follows: [2]

$$M_x = -D\left(\frac{\pi}{\bar{a}}\right)^2\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right), \tag{38}$$

$$M_y = -D\left(\frac{\pi}{\bar{a}}\right)^2\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right). \tag{39}$$

Substituting Eq.(30) into Eq.(38) and Eq.(39) and also letting $y = \pi/2$, the bending moments along the middle line of the plate can then be expressed in the following equations:

$$\begin{aligned}
\frac{M_x(x,\pi/2)}{q\bar{a}^2\pi^2/2} = \sum_{m=1,3,5,...}^{\infty} & [\{(4/\pi^5 m^3) - [(4(1-\nu)/\pi^5 m^3) \\
& -2e^2m^2\Lambda_m]\cosh\beta + [(2/\pi^5m^3) - e^2m^2\Lambda_m] \\
& \times[(1-\nu)\beta\sinh\beta - 2\nu\cosh\beta] \\
& +[(2/\pi^5m^3)(2\tanh\beta - \beta\mathrm{sech}^2\beta) \\
& -(2\eta'\tanh\beta - \beta\mathrm{sech}^2\beta)e^2m^2\Lambda_m](1-\nu) \\
& \times\sinh\beta - [(2\tanh\beta/\pi^5m^3) - e^2m^2\Lambda_m \\
& \times\tanh\beta][(1-\nu)\beta\cosh\beta - 2\nu\sinh\beta]\} \\
& \times\sin(mx) + \{(4\nu/\pi^5m^3) + [(4(1-\nu) \\
& /\pi^5m^3) - 2e^2m^2\Lambda_m]\cosh(mx) - [(2/\pi^5m^3) \\
& -e^2m^2\Lambda_m][(1-\nu)mx\sinh(mx) \\
& +2\cosh(mx)] - [(2/\pi^5m^3)(2\tanh\beta \\
& -\beta\mathrm{sech}^2\beta) - (2\eta'\tanh\beta - \beta\mathrm{sech}^2\beta) \\
& \times e^2m^2\Lambda_m](1-\nu)\sinh(mx) + [(1-\nu)mx \\
& \times\cosh(mx) + 2\sinh(mx)][(2\tanh\beta/\pi^5m^3) \\
& -e^2m^2\Lambda_m\tanh\beta]\}(-1)^{(m-1)/2}] \\
& ;\ 0 \le x \le \pi/2,
\end{aligned} \tag{40}$$

and

$$\begin{aligned}
\frac{M_y(x,\pi/2)}{q\bar{a}^2\pi^2/2} = \sum_{m=1,3,5,...}^{\infty} & [\{(4\nu/\pi^5m^3) + [(4(1-\nu)/\pi^5m^3 \\
& -2e^2m^2\Lambda_m]\cosh\beta - [(2/\pi^5m^3) - e^2m^2\Lambda_m] \\
& \times[(1-\nu)\beta\sinh\beta + 2\cosh\beta] - [(2/\pi^5m^3) \\
& \times(2\tanh\beta - \beta\mathrm{sech}^2\beta) - (2\eta'\tanh\beta \\
& -\beta\mathrm{sech}^2\beta)e^2m^2\Lambda_m](1-\nu)\sinh\beta \\
& +[(2\tanh\beta/\pi^5m^3) - e^2m^2\Lambda_m\tanh\beta] \\
& \times[(1-\nu)\beta\cosh\beta + 2\sinh\beta]\}\sin(mx) \\
& +\{(4\nu/\pi^5m^3) - [(4(1-\nu)/\pi^5m^3) \\
& -2e^2m^2\Lambda_m]\cosh(mx) + [(2/\pi^5m^3) \\
& -e^2m^2\Lambda_m][(1-\nu)mx\sinh(mx) \\
& +2\nu\cosh(mx)] + [(2/\pi^5m^3)(2\tanh\beta \\
& -\beta\mathrm{sech}^2\beta) - (2\eta'\tanh\beta - \beta\mathrm{sech}^2\beta) \\
& \times e^2m^2\Lambda_m](1-\nu)\sinh(mx) + [(1-\nu)mx \\
& \times\cosh(mx) - 2\nu\sinh(mx)] \\
& \times[(2\tanh\beta/\pi^5m^3) - e^2m^2\Lambda_m\tanh\beta]\} \\
& \times(-1)^{(m-1)/2}];\ 0 \le x \le \pi/2.
\end{aligned} \tag{41}$$

6 SHEAR FORCE IN ELEMENT OF THE PLATE

To determine the shear force in element of the plate, it can be obtained from the relation shown below [2],

$$Q_x = -D\left(\frac{\pi}{\bar{a}}\right)^3\left(\frac{\partial^3 w}{\partial x^2} + \frac{\partial^3 w}{\partial x\partial y^2}\right). \tag{42}$$

Thus, the shear force along the middle line of the plate is determined by substituting Eq.(30) into Eq.(42) and setting $y = \pi/2$, and then,

$$\begin{aligned}
\frac{Q_x(x,\pi/2)}{q\bar{a}\pi^3} = \sum_{m=1,3,5,...}^{\infty} & \{[(2/\pi^5m^2) - [(2/\pi^5m^2) - e^2m^3\Lambda_m] \\
& \times\cosh\beta + [(2/\pi^5m^2) - e^2m^3\Lambda_m]\tanh\beta \\
& \times\sinh\beta]\cos(mx) - [[(2/\pi^5m^2) - e^2m^3\Lambda_m]
\end{aligned}$$

$$\times \sinh(mx) - [(2/\pi^5 m^2) - e^2 m^3 \Lambda_m] \tanh\beta$$

$$\times \cosh(mx)](-1)^{(m-1)/2}\}$$

$$;\ 0 \le x \le \pi/2 \,. \qquad (43)$$

It can also be concluded that the shear force in element $Q_y(x,\pi/2)$ automatically equals zero because of the symmetry of deflection and lateral load.

7 CORNER FORCE OF THE PLATE

The corner force of the plate at the point where $x = 0$ and $y = 0$ can be determined by considering equation [2],

$$R = 2D(1-\nu)\left(\frac{\pi}{\bar{a}}\right)^2 \left(\frac{\partial^2 w}{\partial x \partial y}\right). \qquad (44)$$

In view of Eq.(30) and Eq.(44) leads to, after substituting $x = 0$ and $y = 0$,

$$\frac{R(0,0)}{q\bar{a}^2\pi^2} = 2(1-\nu) \sum_{m=1,3,5,...}^{\infty} [(2/\pi^5 m^3)(\tanh\beta - \beta \,\mathrm{sech}^2\,\beta)$$

$$-e^2(\eta'' \tanh\beta - \beta\,\mathrm{sech}^2\,\beta) m^2 \Lambda_m], \qquad (45)$$

with

$$\eta'' = \frac{1+\nu}{1-\nu}. \qquad (46)$$

8 CONCLUSION

In this paper the physical quantities pertaining to the plate under consideration herein can be analytically derived in closed-form expressions. They are the deflection, slope, bending moment, shear force in element, and corner force. Their results can then be carried out, since the unknown auxiliary function $\Phi(\rho)$ that shown in Eq.(11) is numerically evaluated.

However, all numerical results in correspondence with the quantities of the plate that provided in the present paper will be prepared and dealt with in a subsequent paper.

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