Uniformly Loaded Square Plate with Partially Simply Supported at the Middle Edges and Point-Column Supported at the Corners: III – Numerical Results

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ABSTRACT

In this paper, numerical results concerning deflection, slope, bending moment, shear force in element, and corner force of the plate such as stipulated in the title are graphically presented and also numerically given in tabular form for easy reference by other investigators. It, however, is worth noting that all numerical results carried out in the present paper are possibly viewed to be an exact solution in mathematical senses. Therefore, they can be used as benchmark in comparison with alternative numerical methods.

Keywords: *Exact solution, Point-column support, Partial simple support, Square plate*.

1 INTRODUCTION

In mathematical viewpoints, analytical formulation for physical phenomena often involves either Cauchy-type singular integral equation or Fredholm-type integral equation. Since closed-form solutions to these integral equations in general are not available and then, much attention has been emphasized on the numerical method to determine their solutions implicitly [1]-[3]. Over the past five decades, much work on approximate analytical method has continually developed for solving a large class of integral equations.

However, in order to obtain an accurate approximate solution, fast solution methods have been considered and proposed. Frammartino et al.[4] proposed numerical methods to solve a class of Fredholm integral equations of the second kind on unbounded intervals by using interpolating process that based on the zeros of Hermite polynomials. Rajan [5] studied a convergence analysis for solving Fredholm integral equations of the first kind by the use of Tikhonov regularization under supremum norm and also derived the error estimate by imposing additional conditions under a priori parameter choice strategy for choosing the regularization parameter.

Babolian et al.[6] extended the method of Adomain decomposition with considering convergence analysis for solving the systems of linear and nonlinear Fredholm integral equations of the second kind.

In recent years, Sompornjaroensuk and Kongtong [7] proposed the structure resolvent and asymptotic analysis in the kernels and inhomogeneous parts of Fredholm integral equations of the second kind for the natural receding contact problem between a rectangular plate and unilateral supports. Its objective is to improve and accelerate the convergence of summation of infinite series involving large arguments of the Bessel and Struve functions.

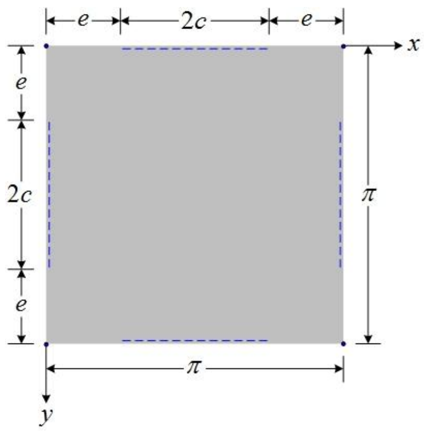

Fig. 1 *Corner-supported square plate with partially simply supported edges*.

In this paper, the technical method as previously proposed by Sompornjaroensuk and Kongtong [7] is then applied in treatment of inhomogeneous Fredholm integral equation of the second kind, which derived in Sompornjaroensuk and Ratchatasakron [8] for the problem of uniformly loaded square plate supported by corner-point supports and partial simple supports at the middle edges. The geometry of the plate and its dimensions and coordinates are demonstrated in Figure 1.

Note that the coordinates and dimensions as shown in Figure 1 are scaled by the factor $\pi/\bar{a}$, where $\bar{a}$ is the actual length of square plate. To determine the actual (barred) and coordinates, they can easily be obtained by the use of relation: $(\bar{x},\bar{y},\bar{c},\bar{e})=(\bar{a}/\pi)(x,y,c,e)$.

2 FREDHOLM INTEGRAL EQUATION

Having shown in the previous work [8] that solution of problem is governed by determining the unknown function $\Phi(\rho)$ of inhomogeneous Fredholm integral equation of the second kind, which is

$$\Phi(\rho)+\int_0^1 K(\rho,r)\Phi(r)dr = f(\rho)\,;\; 0\le\rho\le 1\,, \tag{1}$$

whereas the kernel and inhomogeneous part of integral equation are, respectively, defined by

$$\begin{aligned}K(\rho,r) = 2e^2 r\sum_{m=1,3,5,\ldots}^{\infty} &\big[-4\eta m/\pi - \eta m L_1(me\rho)\\ &-\eta m^2 e\rho L_0(me\rho)+mF_m^{(1)}J_1(me\rho)\\ &+m(F_m^{(2)}-F_m^{(3)})I_1(me\rho)\\ &+m^2F_m^{(3)}e\rho\, I_0(me\rho)\big]\\ &\times\big[J_1(mer)-rJ_1(me)\big]-2e^2r\\ &\times\int_0^\infty \frac{sI_1(se\rho)\big[I_1(ser)-rI_1(se)\big]}{\exp(\pi s)+1}ds\,,\end{aligned} \tag{2}$$

and

$$\begin{aligned}f(\rho)=2\sum_{m=1,3,5,\ldots}^{\infty} &\big[F_m^{(4)}J_1(me\rho)+(F_m^{(6)}-F_m^{(7)})I_1(me\rho)\\ &+mF_m^{(7)}e\rho I_0(me\rho)+(F_m^{(8)}-F_m^{(5)})L_1(me\rho)\\ &-mF_m^{(8)}e\rho L_0(me\rho)\big],\end{aligned} \tag{3}$$

with

$$F_m^{(1)}=\frac{(3+\nu)\sinh\beta\cosh\beta-(1-\nu)\beta}{(3+\nu)\cosh^2\beta}-1\,, \tag{4}$$

$$F_m^{(2)}=\eta(2\tanh\beta+\beta\,\mathrm{sech}^2\beta)\,, \tag{5}$$

$$F_m^{(3)}=\eta\tanh\beta\,, \tag{6}$$

$$F_m^{(4)}=\frac{2\big[(3-\nu)\tanh\beta-(1-\nu)\beta\,\mathrm{sech}^2\beta)\big]}{(3+\nu)\pi^5m^2}\,, \tag{7}$$

$$F_m^{(5)}=\frac{4}{(3+\nu)\pi^5m^2}\,, \tag{8}$$

$$F_m^{(6)}=\frac{2\big[2\tanh\beta+(1-\nu)\beta\,\mathrm{sech}^2\beta\big]}{(3+\nu)\pi^5m^2}\,, \tag{9}$$

$$F_m^{(7)}=\frac{2\eta\tanh\beta}{\pi^5m^2}\,, \tag{10}$$

$$F_m^{(8)}=\frac{2\eta}{\pi^5m^2}\,, \tag{11}$$

and also,

$$\beta=\frac{m\pi}{2}\,, \tag{12}$$

$$\eta=\frac{1-\nu}{3+\nu}\,. \tag{13}$$

The functions $J_n(u)$, $I_n(u)$, and $L_n(u)$ that presented in Eq.(2) and Eq.(3) are the Bessel and modified Bessel functions of the first kind and order n with argument u, and the modified Struve function of order n with argument u, respectively [9], [10].

However, for more details of derivation of Eq.(1), the reader should consult the work of Sompornjaroensuk and Ratchatasakron [8].

3 NUMERICAL PROCEDURE

It is obvious that there are different summation terms in Eq.(2) and Eq.(3) which make the series converged slowly. Utilizing the proposed techniques [7], Eq.(2) and Eq.(3) can then be rewritten in the following forms:

$$\begin{aligned}K(\rho,r) = 2e^2 r\sum_{m=1,3,5,\ldots}^{\infty} &\big[-4\eta m/\pi - \eta m L_1(me\rho)\\ &-\eta m^2 e\rho L_0(me\rho)+mA_m^{(1)}J_1(me\rho)\end{aligned}$$

$$+mA_m^{(2)} I_1(me\rho) + m^2 A_m^{(3)} e\rho I_0(me\rho)$$
$$+\eta m I_1(me\rho) + \eta m^2 e\rho\, I_0(me\rho)\big]$$
$$\times\big[J_1(mer) - rJ_1(me)\big] - 2e^2 r$$
$$\times\int_0^\infty \frac{sI_1(se\rho)\big[I_1(ser) - rI_1(se)\big]}{\exp(\pi s)+1} ds\,, \quad (14)$$

$$f(\rho) = 2\sum_{m=1,3,5,\dots}^{\infty} \Big[B_m^{(1)} J_1(me\rho) + B_m^{(2)} I_1(me\rho)$$
$$+B_m^{(7)} me\rho I_0(me\rho) + B_m^{(4)} L_1(me\rho)$$
$$-B_m^{(5)} me\rho L_0(me\rho) + \frac{2(1+\nu)}{(3+\nu)\pi^5 m^2} I_1(me\rho)$$
$$-\frac{2\eta}{\pi^5 m^2} me\rho I_0(me\rho)\Big]\,, \quad (15)$$

where

$$A_m^{(1)} = F_m^{(1)} - 1\,, \quad (16)$$

$$A_m^{(2)} = F_m^{(2)} - F_m^{(3)} - \eta\,, \quad (17)$$

$$A_m^{(3)} = F_m^{(3)} - \eta\,, \quad (18)$$

$$B_m^{(1)} = F_m^{(4)}\,, \quad (19)$$

$$B_m^{(2)} = F_m^{(6)} - F_m^{(7)} - \frac{2(1+\nu)}{(3+\nu)\pi^5 m^2}\,, \quad (20)$$

$$B_m^{(3)} = F_m^{(7)} - \frac{2\eta}{\pi^5 m^2}\,, \quad (21)$$

$$B_m^{(4)} = F_m^{(8)} - F_m^{(5)}\,, \quad (22)$$

$$B_m^{(5)} = F_m^{(8)}\,. \quad (23)$$

It is worth noting that since m is large, Eq.(16) to Eq.(23) approach some certain values in the limit as follows:

$$A_m^{(1)} = A_m^{(2)} = A_m^{(3)} = 0\,, \quad (24)$$

$$B_m^{(1)} = \frac{2(3-\nu)}{(3+\nu)\pi^5 m^2}\,, \quad (25)$$

$$B_m^{(2)} = B_m^{(3)} = 0\,, \quad (26)$$

$$B_m^{(4)} = -\frac{2(1+\nu)}{(3+\nu)\pi^5 m^2}\,, \quad (27)$$

$$B_m^{(5)} = \frac{2\eta}{\pi^5 m^2}\,. \quad (28)$$

With the use of Eq.(24) to Eq.(28), both equations of Eq.(14) and Eq.(15) becomes

$$K(\rho,r) = 2e^2 r \sum_{m=1,3,5,\dots}^{\infty} \eta m\big\{-(4/\pi) + I_1(me\rho) - L_1(me\rho)$$
$$+me\rho[I_0(me\rho) - L_0(me\rho)]\big\}$$
$$\times\big[J_1(mer) - rJ_1(me)\big] - 2e^2 r$$
$$\times\int_0^\infty \frac{sI_1(se\rho)\big[I_1(ser) - rI_1(se)\big]}{\exp(\pi s)+1} ds\,, \quad (29)$$

and

$$f(\rho) = 2\sum_{m=1,3,5,\dots}^{\infty} \Big\{B_m^{(1)} J_1(me\rho) - B_m^{(4)}\big[I_1(me\rho) - L_1(me\rho)\big]$$
$$+B_m^{(5)} me\rho\big[I_0(me\rho) - L_0(me\rho)\big]\Big\}\,. \quad (30)$$

Further considering an asymptotic expansion that involves the modified Bessel and Struve functions [9], the following equations can be found for the large argument z,

$$I_0(z) - L_0(z) = \frac{2}{\pi z}\,, \quad (31)$$

$$I_1(z) - L_1(z) = \frac{2}{\pi z}\,. \quad (32)$$

Substituting Eq.(31) and Eq.(32) into Eq.(29) and Eq.(30) leads to

$$K(\rho,r) = -2e^2 r\int_0^\infty \frac{sI_1(se\rho)}{\exp(\pi s)+1}\big[I_1(ser) - rI_1(se)\big] ds\,, \quad (33)$$

$$f(\rho) = 2\sum_{m=1,3,5,\dots}^{\infty} \left[\frac{2(3-\nu)}{(3+\nu)\pi^5 m^2} J_1(me\rho) + \frac{8}{(3+\nu)\pi^6 m^2}\right]. \quad (34)$$

Noted that the integrand of improper infinite integral in the kernel of Eq.(2) increases monotonically up to some maximum value and decays exponentially. Thus, to evaluate the infinite integral, it is approximated by

$$\int_0^{\infty} \mathrm{I}(s)ds \cong \int_0^{s=S} \mathrm{I}(s)ds \,, \tag{35}$$

where S is defined to be some large value for the upper limit of integration, I(s) is the integrand of infinite integral. Additionally noted that I(s) becomes zero when s is approached to infinity ($s \to \infty$).

To simplify the development somewhat, but without removing any generality of the result, Eq.(35) can be rewritten in the form as

$$\int_0^{s=S} \mathrm{I}(s)ds = \frac{S}{2}\int_{-1}^{+1} \mathrm{I}(\xi)d\xi \cong \frac{S}{2}\sum_{k=1}^{NG} w_k \tilde{\mathrm{I}}(\xi_k) \,, \tag{36}$$

and

$$s = \frac{S(1+\xi)}{2} \,, \tag{37}$$

where ξ is the new normalized variable, I(ξ) and $\tilde{\mathrm{I}}(\xi_k)$ are the integrand of improper infinite integral in terms of variable ξ and its discrete value according to the Gauss quadrature formula [9], [11]-[13], respectively. NG is the number of Gauss points and w_k, ξ_k are the weight factors and Gauss point locations, respectively.

It can be stated that there is no analytical solution for the unknown auxiliary function $\Phi(\rho)$ in the integral equation of Eq.(1) because of the complexity of the kernel as seen in Eq.(2) and the right side of integral equation presented in Eq.(3). However, its solution can be carried out by means of numerical treatment [2], [3] with using a standard method in which the Simpson's rule [9], [12], [13] is one the simple methods that can be used for this purpose. Therefore, the solution of integral equation can be evaluated approximately in terms of the discrete values of the unknown auxiliary function with required acceptable degree of accuracy and then, the quantities of the plate that have already derived in close-form expressions [14] can be numerically computed.

In the first step, the integral equation of Eq.(1) is approximated by considering a sum over discrete values in each r and ρ with N numbers of discrete points where N must be odd number. After that the Simpson's rule [11]-[13] is further used to create a system of linear simultaneous equations. Thus, Eq.(1) can be written in the matrix notation as [15],

$$[A]_{N\times N}\{\Phi\}_{N\times 1} = \{F\}_{N\times 1} \,, \tag{38}$$

where the augmented matrix [A] is defined by

$$[A]_{N\times N} = [I]_{N\times N} + [K]_{N\times N} \,, \tag{39}$$

in which the identity matrix [I],

$$[I]_{N\times N} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, \tag{40}$$

and the matrices of discrete value for the kernel multiplied with the weight function [K], discrete value for the unknown function {Φ}, and discrete value for the right-hand side function of integral equation {F} are shown, respectively, as follows:

$$[K]_{N\times N} = \begin{bmatrix} W_1K(r_1,r_1) & W_2K(r_1,r_2) & \cdots & W_NK(r_1,r_N) \\ W_1K(r_2,r_1) & W_2K(r_2,r_2) & \cdots & W_NK(r_2,r_N) \\ \vdots & \vdots & \ddots & \vdots \\ W_1K(r_N,r_1) & W_2K(r_N,r_2) & \cdots & W_NK(r_N,r_N) \end{bmatrix}, \tag{41}$$

$$\{\Phi\}_{N\times 1} = \left[\Phi(r_1) \quad \Phi(r_2) \quad \dots \quad \Phi(r_N)\right]^T , \tag{42}$$

$$\{F\}_{N\times 1} = \left[f(r_1) \quad f(r_2) \quad \dots \quad f(r_N)\right]^T , \tag{43}$$

together with the weight functions W_i , according to the Simpson's rule of numerical integration [9]-[11],

$$W_1 = W_N = \frac{1}{3(N-1)} , \tag{44a}$$

$$W_2 = W_4 = W_6 = \dots = \frac{4}{3(N-1)} , \tag{44b}$$

$$W_3 = W_5 = W_7 = \dots = \frac{2}{3(N-1)} . \tag{44c}$$

4 NUMERICAL RESULTS

In the calculation of the elements in matrices [K] and {F} of Eq.(41) and Eq.(43), respectively, the infinite series of the kernel $K(\rho,r)$ and function $f(\rho)$ were evaluated to a relative error criterion of 0.00001, i.e., the series evaluation was terminated when the ratio of the absolute value of the last term calculated to the absolute value of the sum of all previous terms became less than 0.00001 [16]-[19]. Also noted that the 16-point Gauss-Legrendre quadrature formula [9], [11], [13] was used in the evaluation of the infinite integral.

Table 1 *Deflections along the free edge at $0 \le x \le e$, $y = 0$ and $\nu = 0.3$.*

x/e	$w(x,0)/(q\bar{a}^4/10^3D)$		
	$e = 0.10\pi$	$e = 0.20\pi$	$e = 0.30\pi$
0.0	0.0000	0.0000	0.0000
0.2	0.0021	0.0263	0.1121
0.4	0.0033	0.0408	0.1744
0.6	0.0031	0.0393	0.1655
0.8	0.0016	0.0206	0.0891
1.0	0.0000	0.0000	0.0000
x/e	$w(x,0)/(q\bar{a}^4/10^3D)$		
	$e = 0.40\pi$	$e = 0.45\pi$	$e = 0.495\pi$
0.0	0.0000	0.0000	0.0000
0.2	0.3076	0.4566	0.5995
0.4	0.4769	0.7053	0.9165
0.6	0.4496	0.6609	0.8415
0.8	0.2401	0.3485	0.4242
1.0	0.0000	0.0000	0.0000

Table 2 *Deflections along the middle line at $0 \le x \le \pi/2$, $y = \pi/2$ and $\nu = 0.3$.*

x/π	$w(x,\pi/2)/(q\bar{a}^4/10^3D)$		
	$e = 0.10\pi$	$e = 0.20\pi$	$e = 0.30\pi$
0.0	0.0000	0.0000	0.0000
0.1	1.3156	1.3183	1.3413
0.2	2.4628	2.4675	2.5079
0.3	3.3365	3.3421	3.3914
0.4	3.8789	3.8848	3.9368
0.5	4.0625	4.0684	4.1208
x/π	$w(x,\pi/2)/(q\bar{a}^4/10^3D)$		
	$e = 0.40\pi$	$e = 0.45\pi$	$e = 0.495\pi$
0.0	0.0000	0.0000	0.0000
0.1	1.4447	1.5751	1.7574
0.2	2.6847	2.8848	3.1228
0.3	3.6035	3.8342	4.0972
0.4	4.1607	4.4026	4.6766
0.5	4.3466	4.5910	4.8680

Table 3 *Slopes along the middle line at $0 \le x \le \pi/2$, $y = \pi/2$ and $\nu = 0.3$.*

x/π	$\theta_x(x,\pi/2)/(q\bar{a}^3/10^3D)$		
	$e = 0.10\pi$	$e = 0.20\pi$	$e = 0.30\pi$
0.0	13.4824	13.5110	13.7495
0.1	12.5398	12.5642	12.7761
0.2	10.2329	10.2473	10.3788
0.3	7.1473	7.1525	7.2033
0.4	3.6577	3.6586	3.6685
0.5	0.0000	0.0000	0.0000
x/π	$\theta_x(x,\pi/2)/(q\bar{a}^3/10^3D)$		
	$e = 0.40\pi$	$e = 0.45\pi$	$e = 0.495\pi$
0.0	14.8213	16.2687	19.4747
0.1	13.7096	14.7086	15.5743
0.2	10.9054	11.3613	11.7173
0.3	7.4110	7.5973	7.7633
0.4	3.7191	3.7756	3.8388
0.5	0.0000	0.0000	0.0000

Table 4 *Slopes along the diagonal line at $0 \le x \le \pi/2$, $y = x$ and $\nu = 0.3$*

x/π	$\theta_n(x,x)/(q\bar{a}^3/10^3D)$		
	$e = 0.10\pi$	$e = 0.20\pi$	$e = 0.30\pi$
0.0	1.0168	1.0168	2.9084
0.1	5.6255	5.6255	5.6837
0.2	8.6718	8.6718	8.3352
0.3	8.2451	8.2451	8.1357
0.4	4.9289	4.9289	4.9188
0.5	0.0000	0.0000	0.0000
x/π	$\theta_n(x,x)/(q\bar{a}^3/10^3D)$		
	$e = 0.40\pi$	$e = 0.45\pi$	$e = 0.495\pi$
0.0	5.9954	7.9264	9.4948
0.1	6.5588	7.3962	8.2294
0.2	7.9788	8.0455	8.3157
0.3	7.8808	7.8082	7.8776
0.4	4.9085	4.9334	4.9934
0.5	0.0000	0.0000	0.0000

All numerical results carried out and demonstrated in this paper were obtained from computer programming on FORTRAN language [20], [21] and computing with the highest accuracy attainable using double precision.

Since Eq.(38) is formed, the matrix {Φ} for the discrete value of the unknown auxiliary function in integral equation can be numerically solved using the Gaussian elimination with partial pivoting [15]. After that the interested physical quantities of the plate derived in closed-form expression [14] can be computed numerically.

The numerical evaluation was carried out for square plate with the Poisson's ratio taken as 0.1, 0.3, and 0.5, and e/π - ratio varied from 0.10π to 0.495π. Numerical results are graphically presented in Figures 2 to 11, and also prepared in tabular form in Tables 1 to 8.

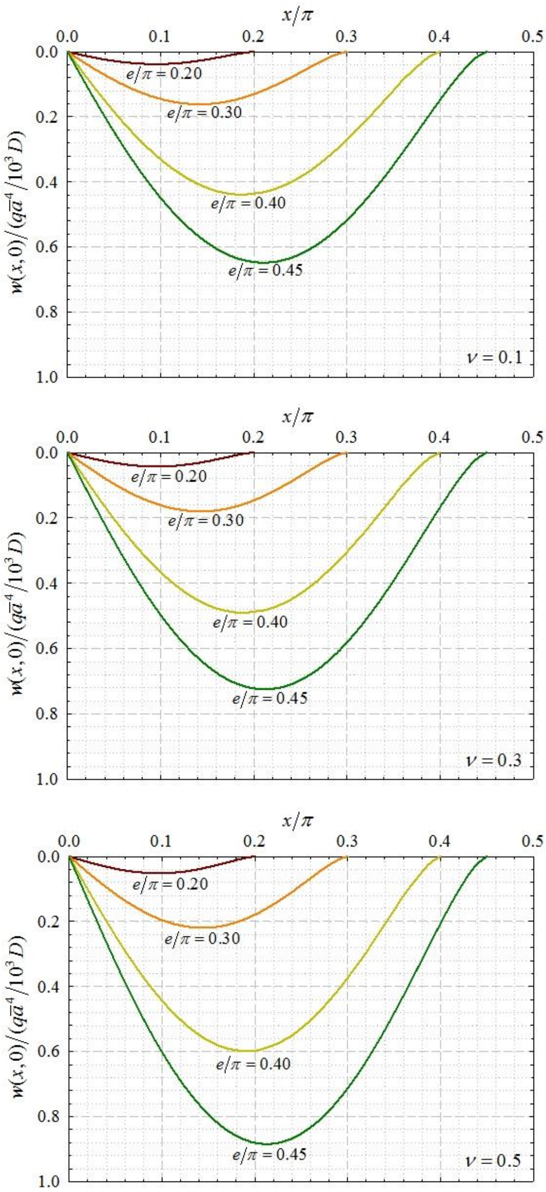

Fig. 2 *Deflections along the free edge*.

5 CONCLUSION

The method presented in the first paper [8] was found to be efficient for solving the problem of uniformly loaded square plate with partially simply supported at the middle edges and point-column supported at the corners. Since the plate has the mixed boundary conditions between the simple support and free edge, the inverse-square-root moment singularities were taken into account in their analysis at the ends of partial simple support.

Closed-form expressions for the deformations and stress resultants of the plate have derived and provided in the second paper [14] and their numerical results were given in the current paper. In the authors' opinion, these results are possibly viewed to be an exact solution in mathematical senses. Therefore, they can be used as benchmark in comparison with other methods.

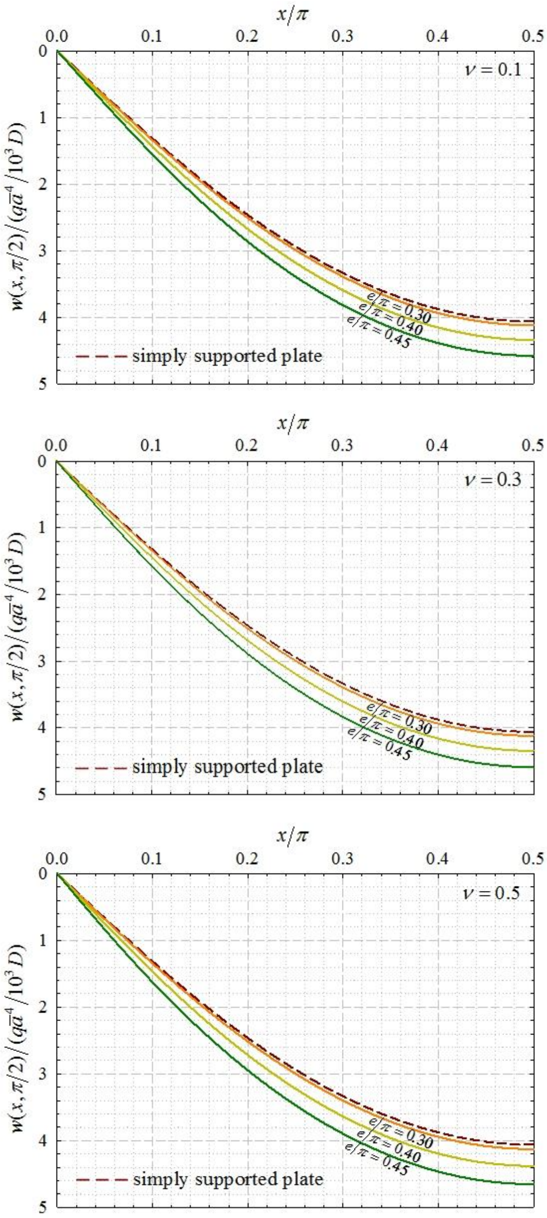

Fig. 3 *Deflections along the middle line*.

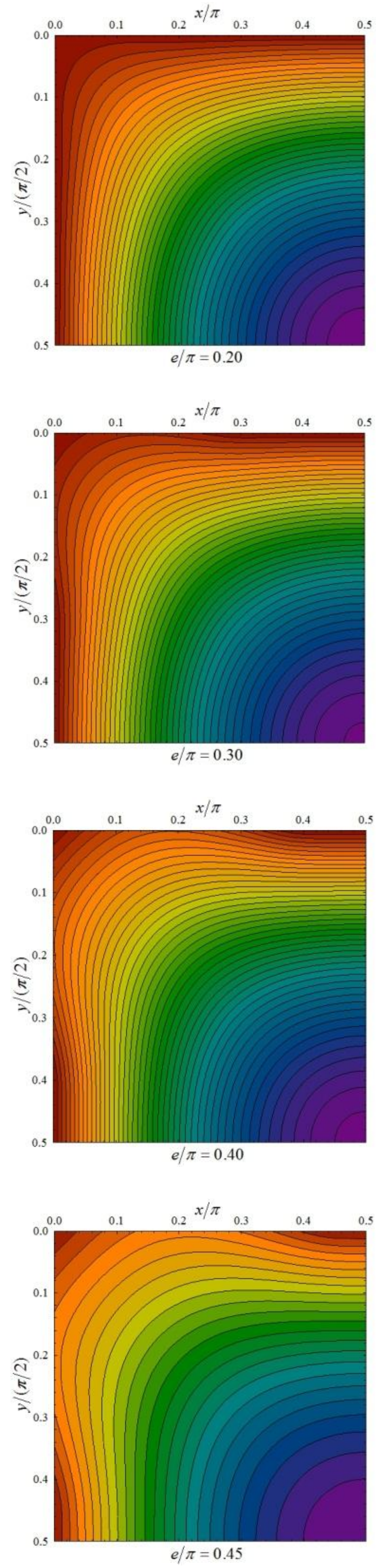

Fig. 4 *Deflection contours with* ν = 0.3.

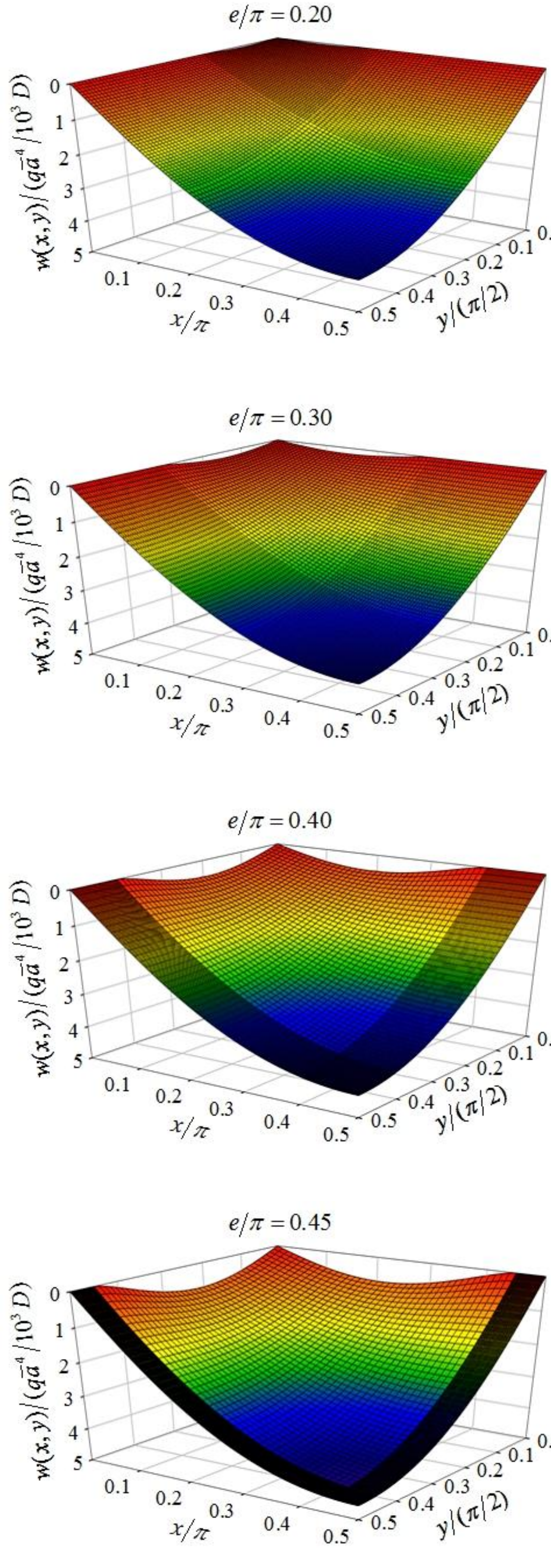

Fig. 5 *Deflection surfaces with* ν = 0.3.

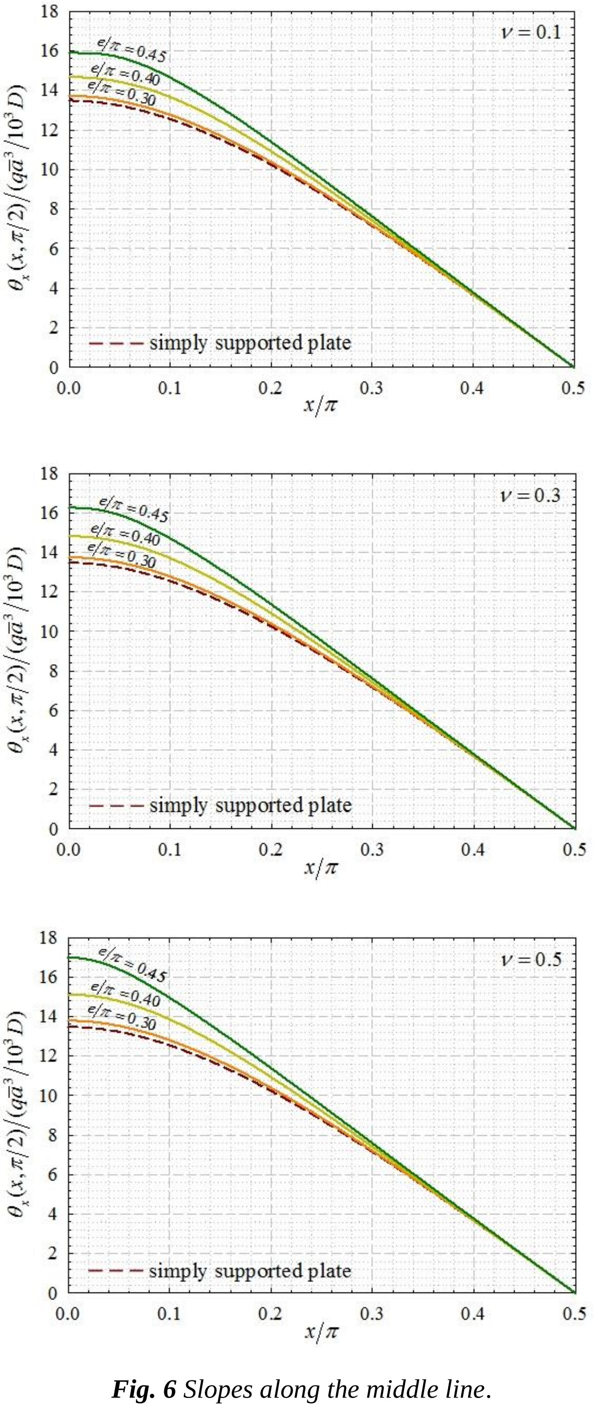

Fig. 6 *Slopes along the middle line*.

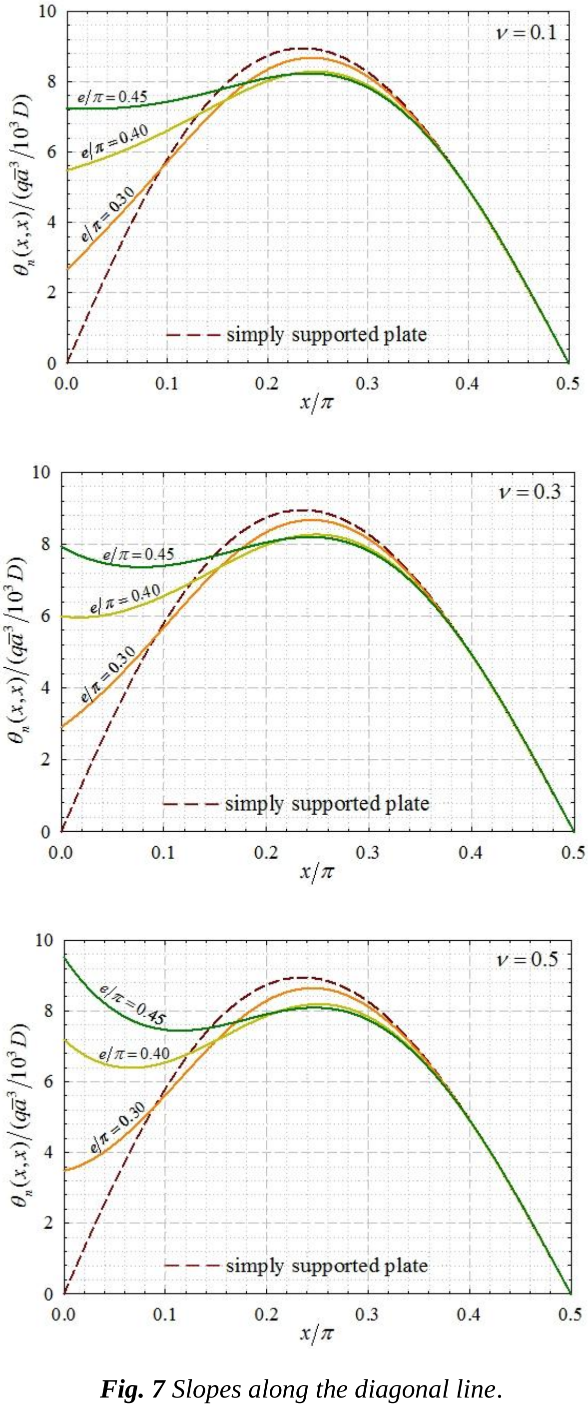

Fig. 7 *Slopes along the diagonal line*.

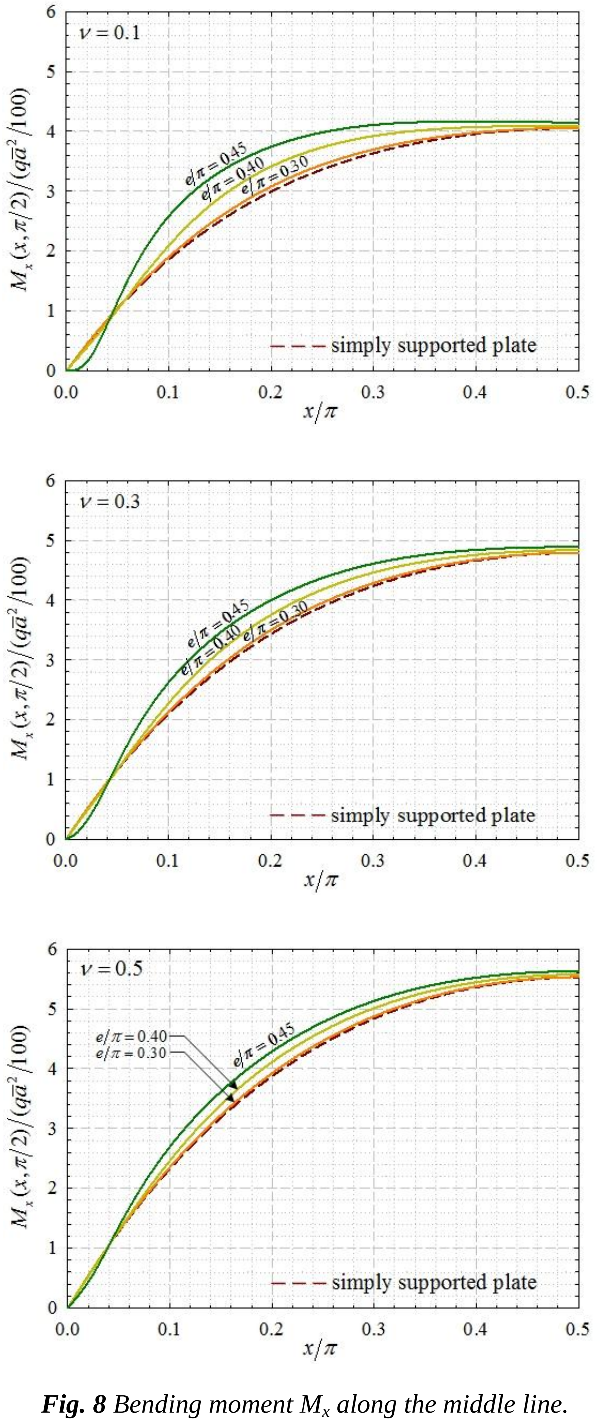

__Fig. 8__ Bending moment M_x along the middle line.

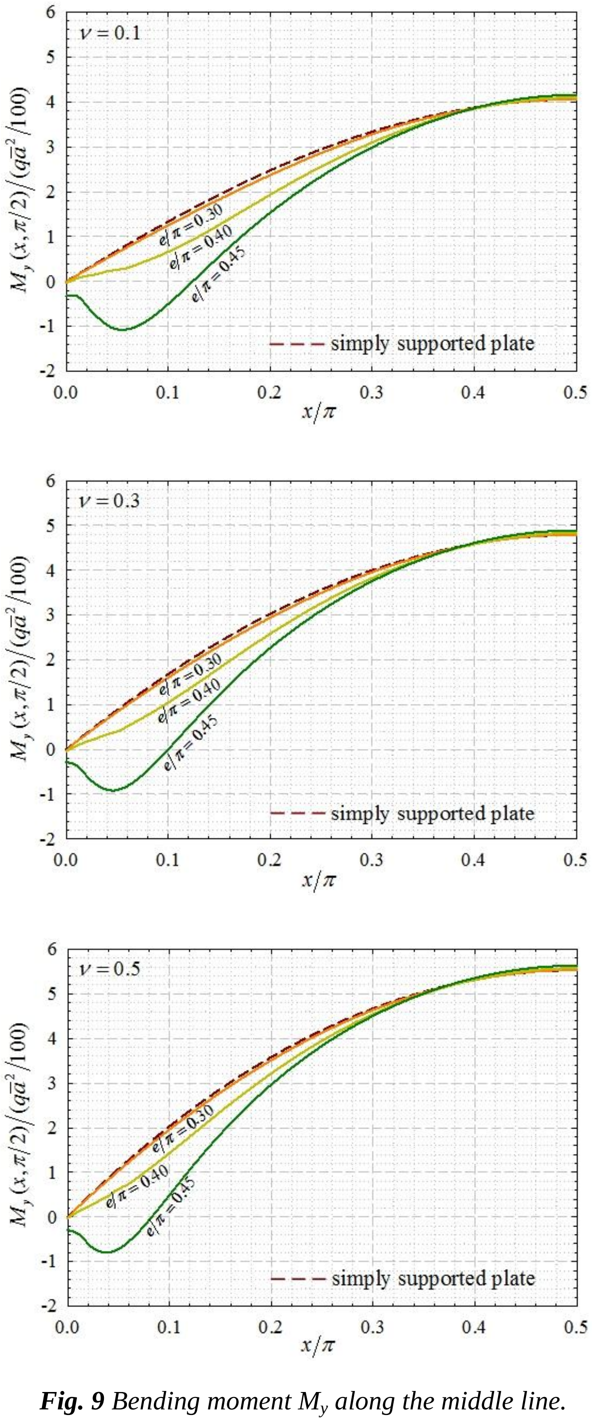

__Fig. 9__ Bending moment M_y along the middle line.

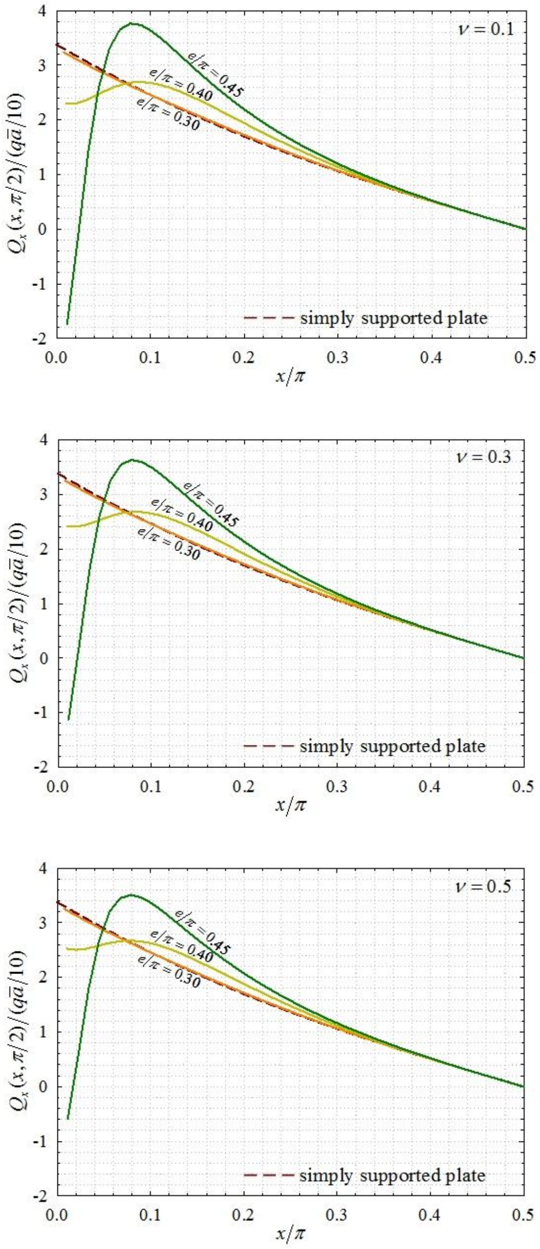

Fig. 10 *Shear forces along the middle line.*

Table 5 *Moments (M_x) along the middle line at* $0 \le x \le \pi/2$, $y = \pi/2$ *and* $\nu = 0.3$.

x/π	$M_x(x,\pi/2)/(q\bar{a}^2/100)$		
	$e = 0.10\pi$	$e = 0.20\pi$	$e = 0.30\pi$
0.0	0.0000	0.0000	0.0000
0.1	2.0916	2.0969	2.1271
0.2	3.4328	3.4400	3.4998
0.3	4.2366	4.2415	4.2859
0.4	4.6581	4.6597	4.6769
0.5	4.7886	4.7889	4.7944
x/π	$M_x(x,\pi/2)/(q\bar{a}^2/100)$		
	$e = 0.40\pi$	$e = 0.45\pi$	$e = 0.495\pi$
0.0	0.0000	0.0000	0.0000
0.1	2.2725	2.6198	3.1318
0.2	3.7516	3.9969	4.1908
0.3	4.4608	4.6101	4.7331
0.4	4.7572	4.8404	4.9276
0.5	4.8345	4.8906	4.9634

Table 6 *Moments (M_y) along the middle line at* $0 \le x \le \pi/2$, $y = \pi/2$ *and* $\nu = 0.3$.

x/π	$M_y(x,\pi/2)/(q\bar{a}^2/100)$		
	$e = 0.10\pi$	$e = 0.20\pi$	$e = 0.30\pi$
0.0	0.0010	0.0092	-0.0341
0.1	1.6839	1.6766	1.6053
0.2	3.0278	3.0197	2.9407
0.3	4.0028	3.9980	3.9576
0.4	4.5918	4.5907	4.5840
0.5	4.7886	4.7889	4.7944
x/π	$M_y(x,\pi/2)/(q\bar{a}^2/100)$		
	$e = 0.40\pi$	$e = 0.45\pi$	$e = 0.495\pi$
0.0	-0.0303	-0.2886	-2.8654
0.1	1.0458	0.0017	-1.1829
0.2	2.5820	2.2639	2.0860
0.3	3.8256	3.7568	3.7588
0.4	4.5817	4.6088	4.6659
0.5	4.8345	4.8906	4.9634

Table 7 *Shear forces (Q_x) along the middle line at $0 \le x \le \pi/2$, $y = \pi/2$ and $\nu = 0.3$.*

x/π	$Q_x(x,\pi/2)/(q\bar{a}/10)$		
	$e = 0.10\pi$	$e = 0.20\pi$	$e = 0.30\pi$
0.0	5.3235	-0.4592	28.8531
0.1	2.4591	2.4588	2.4578
0.2	1.6959	1.6970	1.7206
0.3	1.0583	1.0589	1.0693
0.4	0.5073	0.5074	0.5088
0.5	0.0000	0.0000	0.0000
x/π	$Q_x(x,\pi/2)/(q\bar{a}/10)$		
	$e = 0.40\pi$	$e = 0.45\pi$	$e = 0.495\pi$
0.0	-30.648	186.36	1747.8
0.1	2.6509	3.4951	4.8380
0.2	1.9077	2.1303	2.3012
0.3	1.1268	1.1773	1.2105
0.4	0.5160	0.5219	0.5256
0.5	0.0000	0.0000	0.0000

Table 8 *Corner forces (R) at x = 0, y = 0 and ν = 0.1, 0.3, 0.5.*

e/π	$R(0,0)/(q\bar{a}^2/100)$		
	$\nu = 0.1$	$\nu = 0.3$	$\nu = 0.5$
0.00*	8.3516	6.4943	4.6388
0.10	7.5848	5.7402	3.9288
0.20	5.8838	4.1404	2.4186
0.30	3.6351	2.0164	0.3851
0.40	0.9542	-0.4779	-2.0110
0.45	-0.2610	-1.6200	-3.0738
0.49	-0.8910	-2.3945	-3.8066
0.495	-0.8828	-2.3883	-3.8306

* Simply supported square plate

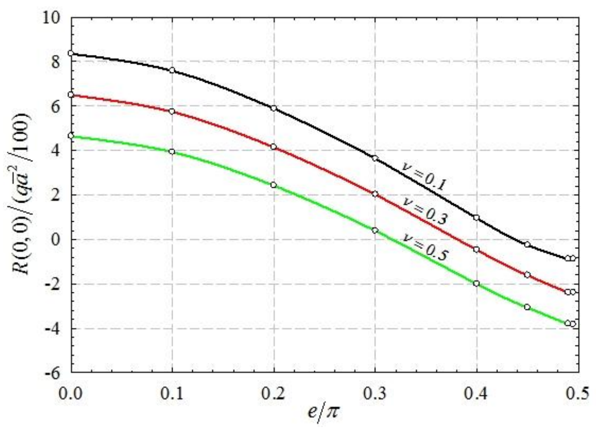

Fig. 11 *Corner forces.*

REFERENCES

[1] L.V. Kantorovich and V.I. Krylov, Approximate Methods of Higher Analysis, Groningen, The Netherlands, P. Noordhoff, Ltd., 1964.

[2] L.M. Delves and J. Walsh, Numerical Solution of Integral Equations, Oxford, Clarendon Press, 1974.

[3] C.T.H. Baker, The Numerical Treatment of Integral Equations, Oxford, Clarendon Press, 1978.

[4] C. Frammartino, C. Laurita and G. Mastroianni, "On the numerical solution of Fredholm integral equations on unbounded intervals," Journal of Computational and Applied Mathematics, vol. 158, pp. 355 - 378, 2003.

[5] M.P. Rajan, "Convergence analysis of a regularized approximation for solving Fredholm integral equations of the first kind," Journal of Mathematical Analysis and Applications, vol. 279, pp. 522 - 530, 2003.

[6] E. Babolian, J. Biazar and A.R. Vahidi, "The decomposition method applied to systems of Fredholm integral equations of the second kind," Applied Mathematics and Computation, vol. 148, pp. 443 - 452, 2004.

[7] Y. Sompornjaroensuk and P. Kongtong, "On a treatment of coupled integral equations in nonlinear unilateral contact problem of plates," Applied Mathematical Sciences, vol. 7, no. 43, pp. 2103 - 2120, 2013.

[8] Y. Sompornjaroensuk and P. Ratchatasakron, "Uniformly loaded square plate with partially simply supported at the middle edges and point-column supported at the corners: I - Theoretical formulation," Engineering Transactions (accepted for publication).

[9] M. Abramowitz and I.A. Stegun, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, U.S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series 55, New York, Dover Publications, 1964.

[10] J.J. Tuma, Engineering Mathematics Handbook, 2nd enlarged and revised ed., New York, McGraw-Hill, 1979.

[11] B. Carnahan, H.A. Luther and J.O. Wilkes, Applied Numerical Methods, New York, John Wiley & Sons, Inc., 1969.

[12] J.J. Tuma, Handbook of Numerical Calculations in Engineering, New York, McGraw-Hill Publishing Company, 1989.

[13] H.M. Antia, Numerical Methods for Scientists and Engineers, New Dehli, Tata McGraw-Hill Publishing Company Limited, 1991.

[14] P. Ratchatasakron and Y. Sompornjaroensuk, "Uniformly loaded square plate with partially simply supported at the middle edges and point-column supported at the corners: II - Analytical expressions," Engineering Transactions (accepted for publication).

[15] G.H. Golub and C.F. Van Loan, Matrix Computations, 3rd ed., Baltimore, The Johns Hopkins, University Press, 1996.

[16] Y. Sompornjaroensuk and D. Dy, "Bending of a uniformly loaded, corner-supported square plate with partially simply supported edges," in Proceedings of 15th National Convention on Civil Engineering (NCCE15), Ubonratchatani, Thailand, May 12-14, paper STR022, 2010.

[17] Y. Sompornjaroensuk and K. Kiattikomol, "Dual-series equations formulation for static deformation of plates with a partial internal line support," Theoretical and Applied Mechanics, vol. 34, no. 3, pp. 221 - 248, 2007.

[18] Y. Sompornjaroensuk and K. Kiattikomol, "Exact analytical solutions for bending of rectangular plates with a partial internal line support," Journal of Engineering Mathematics, vol. 62, pp. 261 - 276, 2008.

[19] Y. Sompornjaroensuk and K. Kiattikomol, "Inverse square root moment singularities in rectangular plates containing an internal line support," International Journal of Computational Methods, vol. 6, no. 1, pp. 1 - 21, 2009.

[20] S.J. Chapman, Fortran 90/95 for Scientists and Engineers, Singapore, WCB/ McGraw-Hill, 1998.

[21] W.H. Press, S.A. Teukolsky, W.T. Vetterling and B.P. Flannery, Numerical Recipes in Fortran: The Art of Scientific Computing, 2nd ed., New York, Cambridge University Press, 1992.

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