

Analysis of new variable step-size NLMS-based algorithm for OFDM systems

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ABSTRACT—

New variable step-size normalised least mean square (NLMS) algorithm is proposed for frequency domain equalisation in the orthogonal frequency division multiplexing (OFDM) systems. An optimal variable step-size approach is introduced how to derive with the method of mean square deviation scheme based on NLMS algorithm. An adaptive frequency domain equalisation approach based on the variable step-size NLMS algorithm is presented with regard to the proposed optimal variable step-size algorithm for OFDM-based systems. We present simulation results to compare the proposed algorithm on the basis of their performance and rate of convergence. Simulation results show that the bit error rate as well as the mean square error can be reduced by the proposed algorithm. The proposed algorithm can be achieved in terms of fast convergence and robustness in comparison with the existing algorithm.

Keywords—Optimal variable step-size algorithm, normalized least mean square (NLMS), frequency domain equalisation (FEQ), orthogonal frequency division multiplexing (OFDM).

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an efficient multicarrier modulation in order to fight against delay spread or frequency-selective fading of both wireless and wireline channels. This approach has been adopted in standards for several high-speed wireless and wireline data applications, including digital audio and video broadcasting [1]. For broadband channels, the conventional time-domain equalisation is impractical, according to long channel impulse response

in time domain. The approach for frequency domain equalisation (FEQ) is based on the discrete Fourier transform (DFT) and its inverse DFT (IDFT) between both time and frequency domains. The cyclic prefix (CP) is added between OFDM symbols in order to avoid intersymbol interference (ISI) and intercarrier interference (ICI) at the transmitter. An OFDM receiver transforms the received signal to frequency domain by applying a DFT. It performs a separate FEQ for each subcarrier.

A class of adaptive algorithms based on least mean square (LMS) algorithm employing order statistic filtering of the sampled gradient estimates has been presented in [2], [3] and [4], which can provide in terms of low computational and robust adaptive filters across a wide range of input environments.

The purpose of this paper is as follows. First, we introduce a criterion that indicates how to derive the optimal variable step-size NLMS algorithm by defining the mean square deviation (MSD) method. Second, we apply an adaptive implementation using this criterion to present how the solution of L -tap complex-valued FEQ based on the variable step-size NLMS (VS-NLMS) algorithm in OFDM systems can be achieved compared with the existing algorithm.

2. SYSTEM AND NOTATION

In this section, we explain shortly the baseband of OFDM system model. At the transmitter, the input binary bit stream is sent into a serial-to-parallel converter. Then, each data stream modulates the corresponding subcarrier by binary phase shift keying (BPSK), and quadrature phase shift keying (QPSK) or quadrature amplitude modulation (QAM). The modulated data symbols are then transformed by the inverse fast Fourier transform (IFFT).

The output symbols $x(k)$ are defined by

$$x(k) = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} X(m) \cdot e^{j2\pi(\frac{km}{M})}, 0 \leq k \leq M-1. \quad (1)$$

where M denotes as the number of subcarriers in the OFDM systems. The cyclic prefix (CP) symbols are added in front of each frame of the IFFT output symbols in order to avoid ISI. After that, the parallel data are converted back to a serial data stream and transmitted over the frequency-selective channel with additive white Gaussian noise (AWGN).

The channel model can be described by

$$y(k) = \sum_{l=0}^{L-1} h_l \cdot x(k-l) + \eta(k), \quad 0 \leq k \leq M-1. \quad (2)$$

where h_l denotes as the channel impulse response (CIR), which represents a frequency-selective Rayleigh fading channel. The parameter L is the length of the CIR, where $0 \leq l \leq L-1$. The i.i.d. complex-valued Gaussian random variables $\eta(k)$ is included with zero mean and variance σ^2 for both real and imaginary components, where $0 \leq k \leq M-1$.

The received data after removing the CP symbols are converted by applying FFT at the receiver. In the frequency domain, the received data are obtained by

$$Y(m) = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} y(k) e^{j2\pi(\frac{km}{M})}, 0 \leq m \leq M-1. \quad (3)$$

where M is the number of subcarriers in the OFDM system. Some notation will be used throughout this paper as follows: the operator $(\cdot)^H$ and $(\cdot)^*$ denote as the Hermitian and complex conjugate operators, respectively. A tilde over the variable indicates the frequency-domain. The vectors are in bold lowercase and matrices are in bold uppercase.

3. NORMALISED LEAST MEAN SQUARE (NLMS) ALGORITHM

To determine the updated L-tap complex-valued tap-weight estimated vector $\hat{\mathbf{p}}(k)$, the solution following optimization criterion to minimise the squared Euclidean norm of the change is as

$$\min \|\delta \hat{\mathbf{p}}(k)\|^2 = \|\hat{\mathbf{p}}(k) - \hat{\mathbf{p}}(k-1)\|^2, \quad (4)$$

subject to the constraint

$$\hat{\mathbf{p}}(k) \tilde{\mathbf{y}}(k) = \tilde{x}(k), \quad (5)$$

where $\tilde{x}(k)$ and $\tilde{\mathbf{y}}(k)$ are the k -th transmitted OFDM-symbol and received signal vector at symbol k after applying a DFT. The operator $\|\cdot\|^2$ denotes as the squared Euclidean norm.

To solve the optimisation problem with the method of Lagrange multipliers, the update of L-tap complex-valued FEQ vector $\hat{\mathbf{p}}(k)$ can be formulated as [6]

$$\hat{\mathbf{p}}(k) = \hat{\mathbf{p}}(k-1) - \frac{\mu}{\|\tilde{\mathbf{y}}(k)\|^2} \tilde{\mathbf{y}}(k) e^*(k), \quad (6)$$

And

$$e(k) = \tilde{x}(k) - \hat{\mathbf{p}}^H(k-1) \tilde{\mathbf{y}}(k), \quad (7)$$

where $e(k)$ is the *a priori* estimation error and μ is the step-size parameter.

4. OPTIMAL VARIABLE STEP-SIZE NLMS ALGORITHM FOR FREQUENCY DOMAIN EQUALIZATION

We define the deviation between the optimal and tap-weight estimated FEQ vector as

$$\boldsymbol{\varsigma}(k) = \mathbf{p}_{opt} - \hat{\mathbf{p}}(k). \quad (8)$$

We consider the desired output $\tilde{x}(k)$ that arise from this model as

$$\tilde{x}_o(k) = \mathbf{p}_{opt}^H \tilde{\mathbf{y}}(k) + \eta(k). \quad (9)$$

Where $\eta(k)$ is the noise signal.

Then, we define the excess error $\xi(k)$ as

$$\xi(k) = e(k) - \eta(k). \quad (10)$$

By substituting (6)-(9) into (10), we get

$$\begin{aligned} \xi(k) &= e(k) - \eta(k) \\ &= \tilde{x}_o(k) - \hat{\mathbf{p}}^H(k-1) \tilde{\mathbf{y}}(k) - \eta(k) \\ &= \mathbf{p}_{opt}^H \tilde{\mathbf{y}}(k) + \eta(k) - \hat{\mathbf{p}}^H(k-1) \tilde{\mathbf{y}}(k) - \eta(k) \\ &= (\mathbf{p}_{opt} - \hat{\mathbf{p}}(k))^H \tilde{\mathbf{y}}(k) = \boldsymbol{\varsigma}^H(k-1) \tilde{\mathbf{y}}(k) \end{aligned} \quad (11)$$

In the proposed approach, the weight deviation vector $\boldsymbol{\varsigma}(k)$ can be expressed with the recursion method as [7]

$$\boldsymbol{\varsigma}(k) = \boldsymbol{\varsigma}(k-1) - \frac{\tilde{\mu}(k)}{\|\tilde{\mathbf{y}}(k)\|^2} \tilde{\mathbf{y}}(k) e^*(k). \quad (12)$$

Squaring both sides of (12) and taking expectations, the mean square deviation (MSD) can be expressed as

$$\begin{aligned} & E\{\|\zeta(k)\|^2\} \\ &= E\{\|\zeta(k-1)\|^2\} \\ &- 2\hat{\mu}(k)\Re\left\{E\left\{\frac{\zeta^H(k-1)\tilde{\mathbf{y}}(k)e^*(k)}{\|\tilde{\mathbf{y}}(k)\|^2}\right\}\right\} \\ &+ \hat{\mu}^2(k)E\left\{\left(\frac{\tilde{\mathbf{y}}(k)e^*(k)}{\|\tilde{\mathbf{y}}(k)\|^2}\right)^H\left(\frac{\tilde{\mathbf{y}}(k)e^*(k)}{\|\tilde{\mathbf{y}}(k)\|^2}\right)\right\}. \end{aligned} \quad (13)$$

Where $\Re\{\cdot\}$ and $E\{\cdot\}$ denote as the real operator and the expectation operator, respectively. The notation $\|\cdot\|$ is the Euclidean norm of a vector.

We find that the MSD in (13) satisfies as

$$E\{\|\zeta(k)\|^2\} = E\{\|\zeta(k-1)\|^2\} - \Delta\hat{\mu}(k), \quad (14)$$

Where

$$\begin{aligned} \Delta\hat{\mu}(k) &= 2\hat{\mu}(k)\Re\left\{E\left\{\frac{\zeta^H(k-1)\tilde{\mathbf{y}}(k)e^*(k)}{\|\tilde{\mathbf{y}}(k)\|^2}\right\}\right\} + \\ &\hat{\mu}^2(k)E\left\{\left(\frac{\tilde{\mathbf{y}}(k)e^*(k)}{\|\tilde{\mathbf{y}}(k)\|^2}\right)^H\left(\frac{\tilde{\mathbf{y}}(k)e^*(k)}{\|\tilde{\mathbf{y}}(k)\|^2}\right)\right\} \end{aligned} \quad (15)$$

The idea is that if we choose $\hat{\mu}(k)$ such that $\Delta\hat{\mu}(k)$ is maximised, then this choice guarantees that the MSD will undergo with the largest of decrease from each previous symbol $(k-1)$ to symbol k .

By differentiating $\Delta\hat{\mu}(k)$ in (15) with respect to $\hat{\mu}(k)$, and setting to zero. Then, the optimal variable step-size $\hat{\mu}(k)$ based on NLMS algorithm can be formulated as

$$\hat{\mu}_{opt}(k) = \frac{\Re\left\{E\left\{\frac{\zeta^H(k-1)\tilde{\mathbf{y}}(k)e^*(k)}{\|\tilde{\mathbf{y}}(k)\|^2}\right\}\right\}}{E\left\{\left(\frac{\tilde{\mathbf{y}}(k)e^*(k)}{\|\tilde{\mathbf{y}}(k)\|^2}\right)^H\left(\frac{\tilde{\mathbf{y}}(k)e^*(k)}{\|\tilde{\mathbf{y}}(k)\|^2}\right)\right\}} \quad (16)$$

Substituting (11) into (16), the optimal variable step-size NLMS $\hat{\mu}_{opt}(k)$ can be approximated as

$$\hat{\mu}_{opt}(k) = \frac{E\{\xi(k)e^*(k)\}}{E\{|e(k)|^2\}}. \quad (17)$$

To facilitate the derivation, the proposed optimal variable step-size NLMS $\hat{\mu}_{opt}(k)$ is under a few assumptions. The following assumptions are as follows [5].

Assumption (i): We assume that the noise sequence $\eta(k)$ is identically and independently distributed (i.i.d.) and statistically independent of input signal $\tilde{\mathbf{y}}(k)$.

Assumption (ii): We neglect the dependency of the previous deviation vector on past noise.

By applying assumption (i) and (10) to this expression $E\{|e(k)|^2\}$ in (17), we define

$$E\{|e(k)|^2\} = E\{|\xi(k)|^2\} + E\{|\eta(k)|^2\} \quad (18)$$

Using this definition in (18), we may rewrite (17) as

$$\hat{\mu}_{opt}(k) = \frac{E\{\xi(k)e^*(k)\}}{E\{|\xi(k)|^2\} + E\{|\eta(k)|^2\}} \quad (19)$$

Motivating by assumption (i) and using (10) into (9), the optimal variable step-size NLMS $\hat{\mu}(k)$ in (27) can be obtained as

$$\hat{\mu}_{opt}(k) = \frac{E\{\xi(k)\xi^*(k)\}}{E\{|\xi(k)|^2\} + E\{|\eta(k)|^2\}} \quad (20)$$

Following this assumption (ii), the optimal variable step-size NLMS $\hat{\mu}_{opt}(k)$ in (20) becomes

$$\hat{\mu}_{opt}(k) = \frac{E\left\{\frac{\xi(k)\tilde{\mathbf{y}}(k)^H \xi^*(k)\tilde{\mathbf{y}}(k)}{\|\tilde{\mathbf{y}}(k)\|^2 \|\tilde{\mathbf{y}}(k)\|^2}\right\}}{E\left\{\frac{\xi(k)\tilde{\mathbf{y}}(k)^H \xi^*(k)\tilde{\mathbf{y}}(k)}{\|\tilde{\mathbf{y}}(k)\|^2 \|\tilde{\mathbf{y}}(k)\|^2}\right\} + E\left\{\frac{|\eta(k)|^2}{\|\tilde{\mathbf{y}}(k)\|^2}\right\}} \quad (21)$$

Let define that

$$\mathbf{g}(k) = \frac{\xi^*(k)\tilde{\mathbf{y}}(k)}{\|\tilde{\mathbf{y}}(k)\|^2}. \quad (22)$$

And its hold under expectation that

$$E\{\mathbf{g}(k)\} = E\left\{\frac{\xi^*(k)\tilde{\mathbf{y}}(k)}{\|\tilde{\mathbf{y}}(k)\|^2}\right\} \quad (23)$$

And

$$\|\mathbf{g}(k)\|^2 = \frac{\xi(k)\tilde{\mathbf{y}}(k)}{\|\tilde{\mathbf{y}}(k)\|^2} \cdot \frac{\xi^*(k)\tilde{\mathbf{y}}(k)}{\|\tilde{\mathbf{y}}(k)\|^2} \quad (24)$$

Using (24) into (21), an approximately optimal variable step-size NLMS $\hat{\mu}_{opt}(k)$ can then be calculated as

$$\hat{\mu}_{opt}(k) = \frac{E\{\|\mathbf{g}(k)\|^2\}}{E\{\|\mathbf{g}(k)\|^2\} + \mathbb{C}}, \quad (25)$$

And \mathbb{C} is a positive constant.

It is noted that \mathbb{C} in (25) is related to (21) as

$$\mathbb{C} = E\left\{\frac{|\eta(k)|^2}{\|\tilde{\mathbf{y}}(k)\|^2}\right\} = \sigma_\eta^2 \text{Tr}\left[E\left\{\left(\tilde{\mathbf{y}}(k)^H \tilde{\mathbf{y}}(k)\right)^{-1}\right\}\right], \quad (26)$$

Where $\text{Tr}[\cdot]$ is the trace of a matrix and σ_η^2 is the variance of the noise signal $\eta(k)$ at symbol k .

5. ANALYSIS OF NEW VARIABLE STEP-SIZE NLMS ALGORITHM

We propose a new variable step-size NLMS-FEQ for OFDM systems. Following (25), the variable step-size mechanism based on NLMS (VS-NLMS) algorithm can be calculated by

$$\hat{\mu}(k) = \mu_{\max} \frac{\|\mathbf{g}(k)\|^2}{\|\mathbf{g}(k)\|^2 + \mathbb{C}} \quad (27)$$

Where $\|\mathbf{g}(k)\|^2$ is introduced in (24). The parameter \mathbb{C} is a positive constant.

It is observed that if $\|\mathbf{g}(k)\|^2$ in (27) is large, the proposed variable step-size $\hat{\mu}(k)$ leads to μ_{\max} . Besides, if $\|\mathbf{g}(k)\|^2$ is small, $\hat{\mu}(k)$ is small.

Depending on $\|\mathbf{g}(k)\|^2$, then $\hat{\mu}(k)$ varies between 0 and μ_{\max} . As suggested in [5], μ_{\max} should be less than 2.

For convenience, the previous deviation vector $\hat{\xi}(k-1)$ is approximated as

$$\hat{\xi}(k-1) \approx \frac{\tilde{\mathbf{y}}(k-1)\hat{\mathbf{e}}^*(k)}{\|\tilde{\mathbf{y}}(k-1)\|^2} \quad (28)$$

Substituting $\hat{\xi}(k-1)$ into (11), the excess error $\xi(k)$ becomes

$$\xi(k) \approx \frac{\tilde{\mathbf{y}}(k-1)\hat{\mathbf{e}}^*(k)}{\|\tilde{\mathbf{y}}(k-1)\|^2} \quad (29)$$

Using (29) into (22), we have

$$\hat{\mathbf{g}}(k) = \frac{\tilde{\mathbf{y}}(k-1)\tilde{\mathbf{y}}^H(k)\tilde{\mathbf{y}}(k)\hat{\mathbf{e}}^*(k)}{\|\tilde{\mathbf{y}}(k)\|^2\|\tilde{\mathbf{y}}(k-1)\|^2}, \quad (30)$$

and

$$\|\hat{\mathbf{g}}(k)\|^2 = \frac{\hat{\mathbf{e}}(k)\tilde{\mathbf{y}}^H(k-1)\tilde{\mathbf{y}}(k-1)\tilde{\mathbf{y}}^H(k)\tilde{\mathbf{y}}(k)\hat{\mathbf{e}}^*(k)}{\|\tilde{\mathbf{y}}(k)\|^2\|\tilde{\mathbf{y}}(k-1)\|^2}. \quad (31)$$

Therefore, the proposed variable step-size NLMS (VS-NLMS) $\hat{\mu}(k)$ by modified averaging as

$$\hat{\mu}(k) = \alpha \hat{\mu}(k-1) + (1 - \alpha) \frac{\|\hat{\mathbf{g}}(k)\|^2}{\|\hat{\mathbf{g}}(k)\|^2 + \mathbb{C}} \quad (32)$$

Where $\|\hat{\mathbf{g}}(k)\|^2$ is given in (31). The parameter α is a smoothing factor, where $0.90 \leq \alpha < 1$. The constant \mathbb{C} is a small positive constant referred to (26).

6. SIMULATION RESULTS

In this section, we consider the performance of the proposed VS-NLMS algorithm compared with the minimum mean square error (MMSE) approach in terms of bit error rate (BER) performance. In all simulations, the mean square error (MSE) performance is firstly used to evaluate in comparison with the proposed VS-NLMS algorithm in the corrupted channel for multipath channel following the ITU-Pedestrian A [7] with additive white Gaussian noise (AWGN). The bit error rate (BER) performance is taken into account, and at last we focus on the tracking and convergence speed of proposed algorithms.

We constructed an OFDM simulation model with the 16-QAM modulation, which are similar to the 802.11a specification in order to demonstrate the effectiveness of the proposed algorithms, including with two experiments. The entire channel bandwidth is 20MHz, and is divided into 64 subcarriers. The symbol duration is chosen as 3.2 μs . The total OFDM frame length is $T_s = 100\mu\text{s}$. The receiver processing consists of MMSE equalisation, hard symbol decision and decoding. The fading gains are randomly generated by complex Gaussian distributed random variables with zero mean and unit variance.

The initial parameters of proposed VS-NLMS are as follows: $L = 5$, $\hat{\mathbf{p}}(0) = [0 \ 0 \ 0]^T$, $\hat{\mu}(0) = 0.01, 0.1, 0.5, 1.5$. The other parameter of conventional complex fixed step-size algorithms have been optimised with $\alpha = 0.925$. The optimized parameters are chosen based on simulation results in order to achieve the good performance. The experiment investigated the performance of proposed VS-NLMS and fixed step-size NLMS algorithms compared with the MMSE approach for multipath channel. The subcarrier at 30 and 50 were the representatives of simulations.

Fig.1 and Fig.2 illustrate the trajectories of variable step-size $\mu(k)$ of proposed VS-NLMS with the samples of subcarrier at 30 and 50, respectively, at four different values of initial step-size $\mu(0) = 0.01, 0.1, 0.5$ and 1.5. They are shown to converge to their equilibrium despite large variations in initial settings.

Fig.3 shows the bit error rate (BER) performance of OFDM systems with different types of proposed VS-NLMS compared with the method of MMSE equalisation and conventional fixed step-size NLMS FEQ. It is seen that the BER of VS-NLMS are able to enhance performance close to the MMSE equalisation.

Fig.4 depicts the learning curves of mean square errors of proposed VS-NLMS FEQ with using $\mu(0) = 0.95$ and 0 of NLMS using the fixed step-size at $\mu = 0.15$. The proposed VS-NLMS algorithm can converge rapidly to steady-state condition with the high initial

step-size parameter with the samples of subcarriers at 30, when SNR is of 24dB.

7. CONCLUSION

In this paper, we have introduced the new VS-NLMS algorithm for frequency domain equalization in OFDM

systems. We have described how to investigate the optimal VS-NLMS algorithm as a solution of MSD approach. The trajectories of variable step-size parameters of VS-NLMS are also shown to converge to their own equilibrium despite 100-fold initial variations. Our results indicate that the performance is acceptable comparison with the several existing NLMS algorithm.

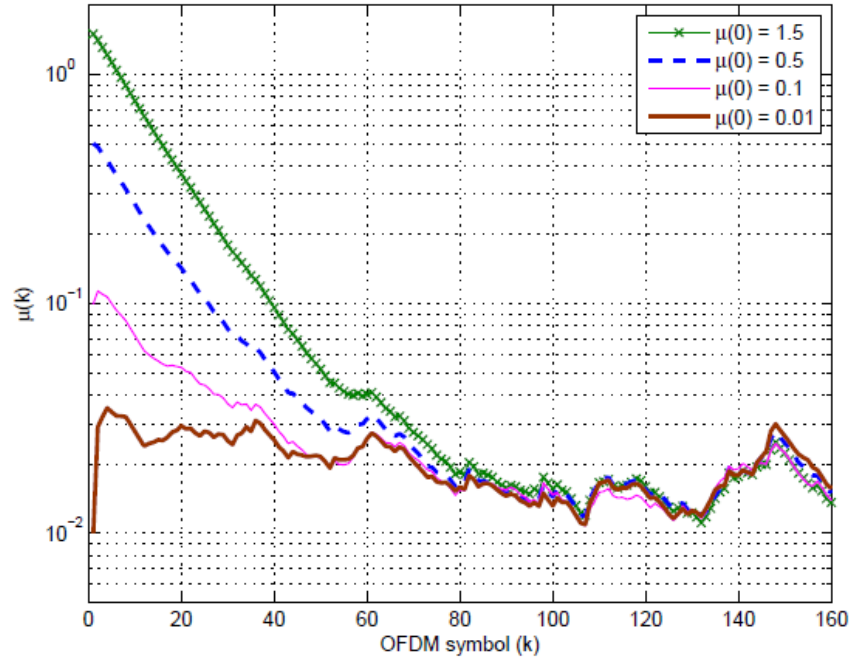


Fig 1 Trajectories of variable step-size $\mu(k)$ of proposed VS-NLMS with the samples of subcarrier at 30 and different settings of initial step-size $\mu(0) = 0.01, 0.1, 0.5$ and 1.5 , when $\alpha = 0.925$ and $\text{SNR} = 24\text{dB}$.

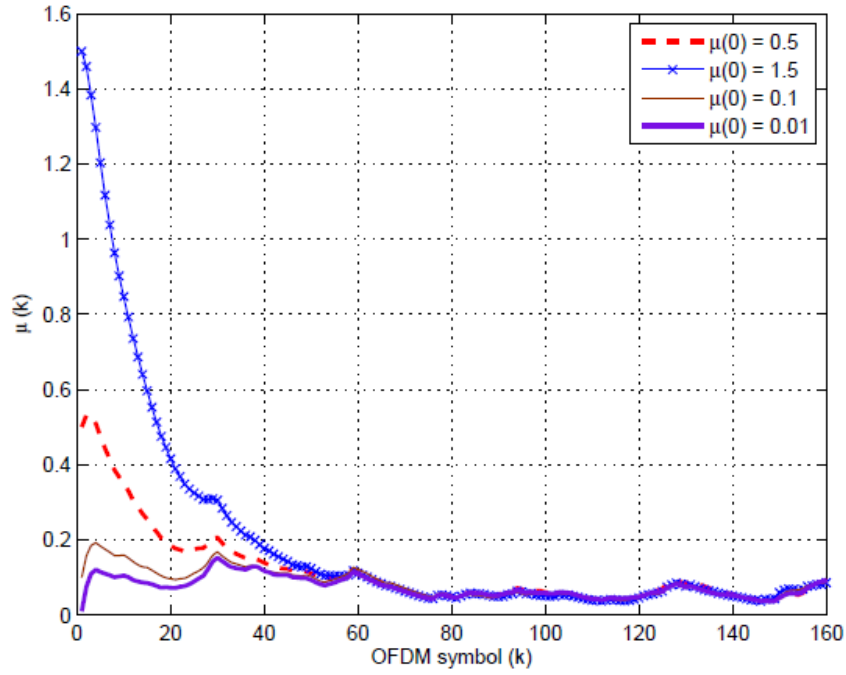


Fig 2 Trajectories of variable step-size $\mu(k)$ of proposed VS-NLMS with the samples of subcarrier at 50 and different settings of initial step-size $\mu(0) = 0.01, 0.1, 0.5$ and 1.5 , when $\alpha = 0.925$ and $SNR = 24dB$.

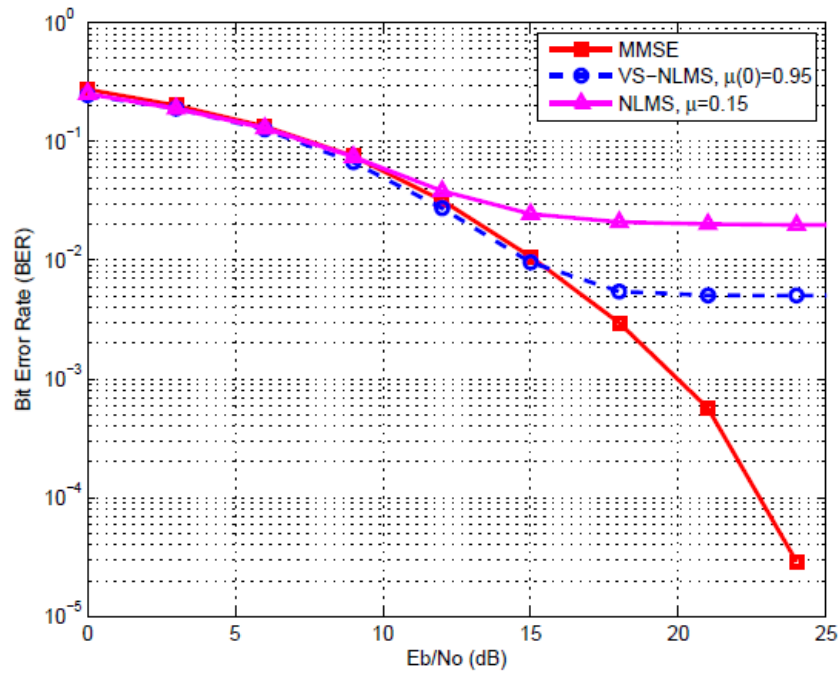


Fig 3 Bit error rate performance with the different types of MMSE, proposed VS-NLMS and conventional fixed step-size NLMS frequency domain equalisation for OFDM systems.

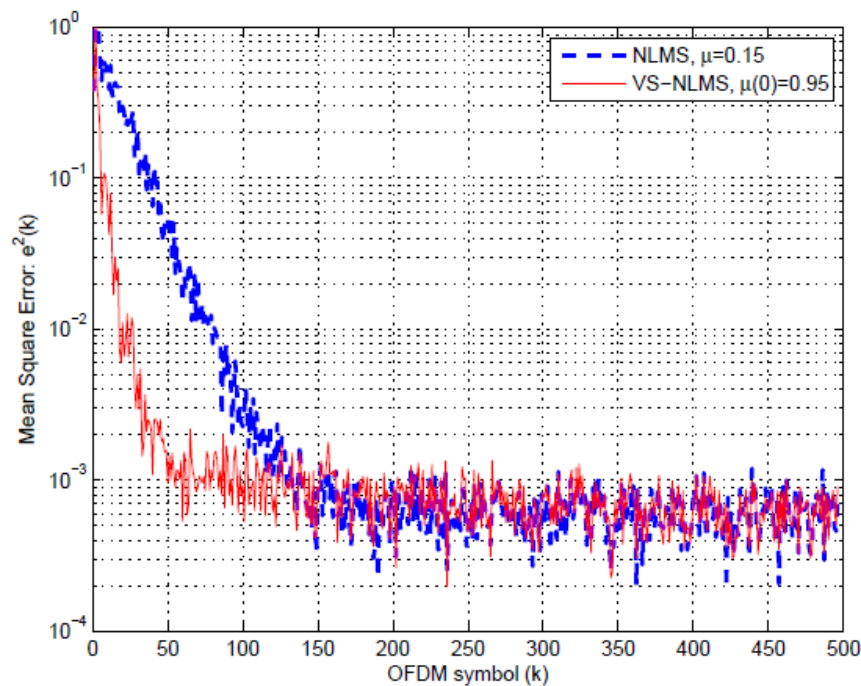


Fig 4 Learning curves of mean square errors of proposed VS-NLMS compared with conventional fixed step-size NLMS FEQ with the samples of subcarrier at 30, when SNR = 24dB.

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