# Analysis of Adaptive Kronecker Sampled-Function Weighted Order Filters

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## ABSTRACT

***This paper introduces a data-adaptive Kronecker filtering framework based on the data-reusing sampled-function weighted order (KSFWO) and switching KSFWO filters by means of data-reusing least mean square (DR-LMS) algorithm. The data-reusing algorithm is introduced and parameterized by the number of reuses of each weight update per data sample. We propose the adaptive KSFWO and switching KSFWO filters based on DR-LMS algorithm with the smoothing and robust characteristics. The coefficients of proposed filters are the samples of bounded real-valued function. These filters can be designed in form of a stochastic gradient filter. The proposed filters can be performed the robust smoothing filtering in some applications.***

**Keywords:** *Estimation, nonlinear filters, weighted order filter, switching filter, data-reusing approach, adaptive algorithm.*

## 1. INTRODUCTION

Nonlinear filters based on order filters have been useful particularly in many applications [1], [2]. The properties of removal non-Gaussian noise of order filters, as Sampled-Function Weight Order (*SFWO) filter*, which presents the ease of design and implementation [1]. The SFWO filter is a non-linear filter to reduce the additive white noise. Although, the $\alpha$-trimmed mean filters are easy to design, but the optimal L-filters are flexible with difficult to design. Another advantage of SFWO filters is that does not totally reject or accept the effects of samples if designed as a smoothly trimmed mean filter. The design of SFWO filters has been presented a good compromising between alpha-trimmed mean filters and optimal L-filters in the form of a smoothly-trimmed mean filter with more parameters performing as well as the optimal L-filter [3].

According to linear filters are able to design for tuning specific frequency property as low-pass, band-pass and high-pass properties, even it cannot remove totally the impulse noise. Accordingly, the L*l* filters consists of the structure of both finite impulse response (FIR) and order-statistics filters [4]. This leads to both properties of frequency selection within linear filters and performance of high de-noising within nonlinear filters. Consequently, the switching method of two sub-L*l* filters has been presented in [5]. This is suitable for characteristics of both edge preserving and noise smoothing filters by tuning the value of $K$ parameter for each sub-L*l* filters with the method of mean square error criterion.

In order to achieve the good convergence, the data-reusing (DR) method is based on *a posteriori* error adaptation that presents the better convergence than standard *a priori* error [6]. This leads to achieve in fixed point iteration while performing *a posteriori* weight update iterations by reusing the external input data. In order to refine the estimate coefficients filter, the available desired response and input vector are used in [7]. The reuse of each received symbol allows faster convergence between the data-reusing least mean square and least mean square algorithms when compared on the requirement of training sequences [8].

In this paper, we introduce the adaptive Kronecker SFWO and switching Kronecker SFWO filters based on data-reusing least mean square algorithm as a flexible image processing filter with the properties of smoothing and robust characteristics.

## 2. DATA-REUSING LEAST MEAN SQUARE ALGORITHM

In [6], data-reusing (DR) algorithms effectively operate between the symbol instants $n$ and $n + 1$ by combining the recursive mode of learning based on the *a priori* output error and iterative mode of learning based on the *a posteriori* errors. Following [8], the update weight vector $\mathbf{w}_{t+1}(n)$ in the adaptive data-reusing least mean square (DR-LMS) algorithm is given as

$$\mathrm{w}_{t+1}(n) = \mathrm{w}_t(n) + \mu\, \mathrm{x}(n)\, e_t(n)\,, \tag{1}$$

$$e_t(n) = d(n) - \mathbf{x}^T(n)\, \mathbf{w}_t(n)\,, \tag{2}$$

where $\mathbf{w}_1(n) = \mathbf{w}(n)$, $\mathbf{w}_{L+1}(n) = \mathbf{w}(n+1)$, $t = 1, \ldots, L$ and $t$ denotes as the order of data-reuse iteration. The parameter $\mu$ is a step-size. The vector $\mathbf{x}(n)$ is the input signal vector and $d(n)$ is the desired signal. The error $e_t(n)$ is the $t^{th}$ data-reusing estimated error. For the *a priori* and *a posteriori* mode of operation shown in Fig. 1, the relationship at $t = 2$ is as

$$\begin{aligned} e_2(n) &= d(n) - \mathbf{x}^T(n)\, \mathbf{w}_2(n) \\ &= d(n) - \mathbf{x}^T(n)\left(\mathbf{w}_1(n) + \mu\, \mathbf{x}(n)\, e_1(n)\right) \\ &= e_1(n)[1 - \mu \mathbf{x}^T(n)\, \mathbf{x}(n)]\,, \end{aligned} \tag{3}$$

And the $t^{\text{th}}$ data-reusing error can be expressed as

$$e_t(n) = e(n)[1 - \mu\, \mathbf{x}^T(n)\, \mathbf{x}(n)]^{t-1}, t = 1, \ldots, L\,. \tag{4}$$

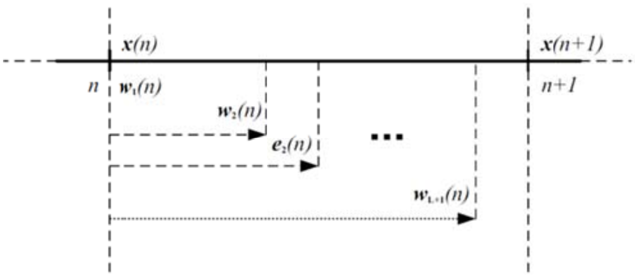


***Fig. 1*** *Symbol alignment within the a posterior data-reusing approach*

Then, the final estimated error with $L$ data-reusing iterations $\sum_{t=1}^{L} e_t(n)$ is defined as [7]

$$\sum_{t=1}^{L} e_t(n) = \sum_{t=1}^{L} e(n)[1 - \mu\, \mathbf{x}^T(n)\, \mathbf{x}(n)]^{t-1}$$

$$= \frac{e(n)\left\{1 - \left(1 - \mu\, \mathbf{x}^T(n)\, \mathbf{x}(n)\right)^L\right\}}{\mu\, \mathbf{x}^T(n)\, \mathbf{x}(n)} \tag{5}$$

Therefore, the update tap-weight DR-LMS vector $\mathbf{w}(n+1)$ can be given as

$$\begin{aligned} \mathbf{w}_{L+1}(n) &= \mathbf{w}_L(n) + \mu\, \mathbf{x}(n)\, e_L(n) \\ &= \mathbf{w}_L(n) + \mu\, \mathbf{x}(n)\left(e_{L-1}(n) + e_L(n)\right) \\ &= \mathbf{w}(n) + \mu \textstyle\sum_{t=1}^{L} e_t(n)\, \mathbf{x}(n) = \mathbf{w}(n+1), \end{aligned} \tag{5}$$

where $\sum_{t=1}^{L} e_t(n)$ is given in (7).

## 3. SAMPLED-FUNCTION WEIGHTED ORDER FILTER

As in [1], the classical problem of estimating a constant signal $\mathcal{D}(n)$ from the additively corrupted observed samples $\mathbb{X}(n)$ within a filter window is considered as

$$\mathbb{X}(n) = \mathcal{D}(n) + \eta\,, \tag{6}$$

Where $\eta$ is assumed to be a zero-mean noise. It is also assumed that the uncorrupted signal $\mathcal{D}(n)$ and the noise $\eta$ are uncorrelated to each other.

All the noise elements within the filter window are assumed to be independent identically distributed (i.i.d) random values and thus, the observed corrupted samples will also be i.i.d samples.

Let the filter window be size of $m \times m$. A sampled-function weighted order (SFWO) filter is a type of order class of L-filters. The output of filter $y_{i,j}(n)$ is defined as [1]

$$\mathbb{Y}(n) = \mathbf{a}^T \mathbf{x}_{i.j}(n) = \frac{\sum_{s=1}^{m} h(t_s)\, \mathbb{X}_s(n)}{\sum_{s=1}^{m} h(t_s)}\,, \tag{7}$$

where $h(t_s)$ is a bounded of sampled-function and real-valued function defined in $[t_1, t_m]$ and $\mathbf{x}_{i.j}(n)$ occupies the ranked sample.

### A. Kronecker SFWO Filter

Following [1] and [4], the coefficients of SFWO filter in the Kronecker product form, called *Kronecker SFWO*, can be expressed by the product of $\alpha_i$ and $\beta_j$ which are coefficients related to the position ($i$) of input signal in the window and the order ($j$) in the local window, respectively. The output $\tilde{y}_{i,j}(n)$ of Kronecker SFWO filter is introduced as

$$\tilde{y}_{i,j}(n) = \frac{\sum_{j=1}^{m} \alpha_i \beta_j\, x_{i,j}(n)}{\sum_{j=1}^{m} \alpha_i \beta_j}\,. \tag{8}$$

### B. Switching Kronecker SFWO Filter

The idea is to switch the output of sub-filters using a signal activity information, so called *a local information* as presented in [5]. So, the output of switching Kronecker SFWO (SK-SFWO) filter $\hat{y}_{i,j}(n)$ can be written as

$$\hat{y}_{\mathrm{i,j}}(n) = K_{T,S}(i,j)\,\tilde{y}_{p_{i,j}}(n) + \left(1 - K_{T,S}(i,j)\right)\tilde{y}_{s_{i,j}}(n)\,, \quad (9)$$

where $\tilde{y}_{p_{i,j}}(n)$ and $\tilde{y}_{s_{i,j}}(n)$ are the outputs of Kronecker SFWO filters referring to a smooth and preserving signal as

$$\tilde{y}_{p_{i,j}}(n) = \frac{\sum_{j=1}^{m} \alpha_i\beta_j\, x_{i,j}(n)}{\sum_{j=1}^{m} \alpha_i\beta_j}\,, \quad (10)$$

$$\tilde{y}_{s_{i,j}}(n) = \frac{\sum_{j=1}^{m} \tilde{\alpha}_i\tilde{\beta}_j\, x_{i,j}(n)}{\sum_{j=1}^{m} \tilde{\alpha}_i\tilde{\beta}_j}\,, \quad (11)$$

where the product of $\alpha_i$ and $\beta_j$ are the coefficients of Kronecker SFWO filter for preserving and the product of $\tilde{\alpha}_i$ and $\tilde{\beta}_j$ are the coefficients of Kronecker SFWO filter for smooth signal.

The parameter $K_{T,S}(i,j)$ is local information which is calculated by [8]

$$K_{T,S}(i,j) = \frac{\sigma_{T,S}^2(i,j)}{\sigma_{T,S}^2(i,j) + \sigma_\eta^2}\,, \quad (12)$$

where $S$ and $T$ denote as the higher rank and lower rank. The estimated local variance $\sigma_{T,S}^2(i,j)$ of original signal within the window defined as

$$\sigma_{T,S}^2(i,j) = \max\{\mathrm{Var}_{T,S}(i,j) - \sigma_\eta^2\}\,, \quad (13)$$

where $\max\{\cdot\}$ represents the maximum value and $\sigma_\eta^2$ is the variance of additive Gaussian noise. $\mathrm{Var}_{T,S}(i,j)$ is the local variance within the window.

## 4. ADAPTIVE DATA-REUSING KRONECKER SFWO ALGORITHM

This section presents adaptive Kronecker SFWO (KSFWO) filter based on data-reusing least mean square (DR-LMS).

Adaptive Kronecker SFWO filter can be given as

$$\hat{y}_{i,j}(n) = \frac{\alpha_i(n)\beta_j(n)x_{i,j}(n)}{\alpha_i(n-1)\beta_j(n-1)}\,, \quad (14)$$

where the product of $\alpha_i(n)$ and $\beta_j(n)$ are the coefficients of filters on the sample-by-sample basis.

The estimated error is given as

$$\xi(n) = d(n) - \hat{y}_{i,j}(n) = d(n) - \frac{\alpha_i(n)\beta_j(n)x_{i,j}(n)}{\alpha_i(n-1)\beta_j(n-1)}. \quad (15)$$

The cost function is minimized by mean square error (MSE) criterion. The data-reusing least mean square (DR-LMS) for $\alpha_{t,i}(n)$ can be expressed as [6]

$$\alpha_{t+1,i}(n+1) = \alpha_{t,i}(n) - \nabla_{\alpha_t(n)} J(n), \quad (16)$$

where $J(n)$ is the cost function as $J(n) = \frac{1}{2}\{\xi^2(n)\}$. The parameters: $t = 1, \ldots, L$ and $\alpha_{1,i}(n) = \alpha_i(n)$.

By differentiating the squared estimation error, the gradient $\nabla_{\alpha_t(n)}\mathrm{J}(n)$ with respect to $\alpha_{t,i}(n)$ can be given as

$$\nabla_{\alpha_t(n)}\mathrm{J}(n) = -\mu\,\beta_j(n)x_{i,j}(n)\xi_{\alpha_{t,i}}(n)\,. \quad (17)$$

Consequently, the DRLMS-KSFWO for $\alpha_{t,i}(n)$ can be expressed recursively as

$$\alpha_{t+1,i}(n+1) = \alpha_{t,i}(n) + \mu\,\beta_j(n)x_{i,j}(n)\xi_{\alpha_{t,i}}(n)\,, \quad (18)$$

$$\xi_{\alpha_{t,i}}(n) = \xi(n)\{1 - \mu\,\gamma(n)\beta_j^2(n)x_{i,j}^2(n)\}^{t-1}\,, \quad (19)$$

$$\gamma(n) = \{\alpha_i(n-1)\beta_j(n-1)\}^{-1}\,. \quad (20)$$

Therefore, the summary of adaptive data-reusing Kronecker Sampled-Function Weighted Order (DRLMS-KSFWO) algorithm for $\alpha_i(n+1)$ can be defined as [9]

$$\alpha_i(n+1) = \alpha_i(n) + \mu\,\beta_j(n)x_{i,j}(n)\sum_{t=1}^{L}\xi_{\alpha_{t,i}}(n)\,, \quad (21)$$

and the total of $L$ data-reusing estimated error is defined by

$$\sum_{t=1}^{L}\xi_{\alpha_{t,i}}(n) = \sum_{t=1}^{L}\xi(n)\left[1 - \mu\,\gamma(n)\beta_j^2(n)x_{i,j}^2(n)\right]^{t-1} \approx \frac{\xi(n)\left\{1 - \left[1 - \mu\,\gamma(n)\beta_j^2(n)x_{i,j}^2(n)\right]^L\right\}}{\mu\,\gamma(n)\beta_j^2(n)x_{i,j}^2(n)}\,. \quad (22)$$

where $\xi(n)$ and $\gamma(n)$ are given in (16) and (21), respectively. The constant $L$ is the number of data-reusing iterations. The parameter $\mu$ is a step-size.

In the similar fashion, the summary of adaptive data-reusing Kronecker Sampled-Function Weighted Order

(DRLMS-KSFWO) algorithm for $\beta_j(n+1)$ can be stated as

$$\beta_j(n+1) = \beta_j(n) + \mu\, \alpha_i(n) x_{i,j}(n) \sum_{t=1}^{L} \xi_{\beta_{t,i}}(n)\,, \tag{23}$$

$$\sum_{t=1}^{L} \xi_{\beta_{t,i}}(n) = \sum_{t=1}^{L} \xi(n)\left[1 - \mu\, \gamma(n)\alpha_j^2(n) x_{i,j}^2(n)\right]^{t-1} \approx \frac{\xi(n)\left\{1-\left[1-\mu\, \gamma(n)\alpha_i^2(n) x_{i,j}^2(n)\right]^L\right\}}{\mu\, \gamma(n)\alpha_i^2(n) x_{i,j}^2(n)}\,. \tag{24}$$

where $\xi(n)$ and $\gamma(n)$ are given in (16) and (21), respectively. The constant $L$ is the number of data-reusing iterations. The parameter $\mu$ is a step-size.

## 5. ADAPTIVE DATA-REUSING SWITCHING KRONECKER SFWO ALGORITHM

Following [10], this section presents adaptive switching Kronecker SFWO (SK-SFWO) filter based on data-reusing least mean square (DR-LMS).

Adaptive switching Kronecker SFWO filter can be given as

$$\tilde{y}_{i,j}(n) = K \frac{\alpha_i(n)\beta_j(n)x_{i,j}(n)}{\alpha_i(n-1)\beta_j(n-1)} + (1-K)\, \frac{\tilde{\alpha}_i(n)\tilde{\beta}_j(n)x_{i,j}(n)}{\tilde{\alpha}_i(n-1)\tilde{\beta}_j(n-1)} \tag{25}$$

where $K$ is a robust information of $K_{T,S(i,j)}$ , $0 < K < 1$ . The parameter $T$ and $S$ are defined as the lower rank and higher rank of signals, respectively.

The product of $\alpha_i(n)$ and $\beta_j(n)$ are the coefficients of Kronecker SFWO filter for preserving and the product of $\tilde{\alpha}_i(n)$ and $\tilde{\beta}_j(n)$ are the coefficients of Kronecker SFWO filter for smooth signal.

The estimated error $\tilde{\xi}(n)$ using the robust information $K$ in (13) is given as

$$\begin{aligned}\tilde{\xi}(n) &= d(n) - \tilde{y}_{i,j}(n) \\ &= d(n) - K \frac{\alpha_i(n)\beta_j(n)x_{i,j}(n)}{\alpha_i(n-1)\beta_j(n-1)} + (1-K) \frac{\tilde{\alpha}_i(n)\tilde{\beta}_j(n)x_{i,j}(n)}{\tilde{\alpha}_i(n-1)\tilde{\beta}_j(n-1)}\,.\end{aligned} \tag{26}$$

The cost function is minimized by mean square error (MSE) criterion. The data-reusing least mean square (DR-LMS) for $\alpha_{t,i}(n)$ can be expressed as [6]

$$\alpha_{t+1,i}(n+1) = \alpha_{t,i}(n) - \nabla_{\alpha_t(n)} \mathrm{J}(n), \tag{27}$$

where J($n$) is the cost function as $J(n) = \frac{1}{2}\sum_{n=1}^{N} \tilde{\xi}^2(n)$. The parameters: $t = 1, \ldots, L$ and $\alpha_{1,i}(n) = \alpha_i(n)$.

By differentiating the squared estimation error, the gradient $\nabla_{\alpha_t(n)}\mathrm{J}(n)$ with respect to $\alpha_{t,i}(n)$ can be given as

$$\nabla_{\alpha_t(n)} J(n) = -\mu\, K\beta_j(n) x_{i,j}(n) \tilde{\xi}_{\alpha_{t,i}}(n)\,. \tag{28}$$

Consequently, the update DRLMS-SKSFWO for $\alpha_{t,i}(n)$ can be expressed recursively as

$$\alpha_{t+1,i}(n+1) = \alpha_{t,i}(n) + \mu\, K\beta_j(n) x_{i,j}(n) \tilde{\xi}_{\alpha_{t,i}}(n)\,, \tag{29}$$

$$\tilde{\xi}_{\alpha_{t,i}}(n) = \tilde{\xi}(n)\left\{1 - \mu\, K\, \beta_j(n) x_{i,j}^2(n)\right\}^{t-1}. \tag{30}$$

Therefore, the summary of adaptive data-reusing switching Kronecker Sampled-Function Weighted Order (DRLMS-KSFWO) algorithm for $\alpha_i(n+1)$ can be defined as [10]

$$\alpha_i(n+1) = \alpha_i(n) + \mu\, K\, \beta_j(n) x_{i,j}(n) \sum_{t=1}^{L} \tilde{\xi}_{\alpha_{t,i}}(n)\,, \tag{31}$$

and the total of $L$ data-reusing estimated error is defined by

$$\sum_{t=1}^{L} \tilde{\xi}_{\alpha_{t,i}}(n) = \sum_{t=1}^{L} \xi(n)\left[1 - \mu\, K\beta_j^2(n) x_{i,j}^2(n)\right]^{t-1} \approx \frac{\tilde{\xi}(n)\left\{1-\left[1-\mu\, K\beta_j(n) x_{i,j}^2(n)\right]^L\right\}}{\mu\, K\beta_j(n) x_{i,j}^2(n)}\,. \tag{32}$$

In the similar fashion, the update coefficient $\tilde{\alpha}_i(n+1)$ of DRLMS-KSFWO filter can be expressed as

$$\tilde{\alpha}_i(n+1) = \tilde{\alpha}_i(n) + \mu\, K\, \tilde{\beta}_i(n) x_{i,j}(n) \sum_{t=1}^{L} \tilde{\xi}_{\tilde{\alpha}_{t,i}}(n)\,, \tag{33}$$

$$\sum_{t=1}^{L} \tilde{\xi}_{\tilde{\alpha}_{t,i}}(n) = \sum_{t=1}^{L} \xi(n)\left[1 - \mu\, K\tilde{\beta}_j^2(n) x_{i,j}^2(n)\right]^{t-1} \approx \frac{\tilde{\xi}(n)\left\{1-\left[1-\mu\, K\tilde{\beta}_i(n) x_{i,j}^2(n)\right]^L\right\}}{\mu\, K\tilde{\beta}_i(n) x_{i,j}^2(n)} \tag{34}$$

where $K$ and $\tilde{\xi}(n)$ are given in (13) and (27), respectively.

For the coefficient $\beta_j(n+1)$ of DRLMS-SKSFWO filter can be defined adaptively by

$$\beta_j(n+1) = \beta_j(n) + \mu\, K\, \alpha_i(n) x_{i,j}(n) \sum_{t=1}^{L} \tilde{\xi}_{\beta_{t,i}}(n)\,, \tag{35}$$

$$\sum_{t=1}^{L} \tilde{\xi}_{\beta_{t,i}}(n) = \sum_{t=1}^{L} \xi(n)\left[1 - \mu\, K\alpha_j^2(n) x_{i,j}^2(n)\right]^{t-1} \approx \frac{\tilde{\xi}(n)\left\{1-\left[1-\mu\, K\, \alpha_i(n) x_{i,j}^2(n)\right]^L\right\}}{\mu\, K\, \alpha_i(n) x_{i,j}^2(n)}\,. \tag{36}$$

Therefore, the update coefficient $\tilde{\beta}_j(n+1)$ of DRLMS-SKSFWO filter can be expressed adaptively as

$$\tilde{\beta}_j(n+1) = \tilde{\beta}_j(n) + \mu\, K\, \tilde{\alpha}_i(n) x_{i,j}(n) \sum_{t=1}^{L} \tilde{\xi}_{\beta_{t,i}}(n)\ , \quad (37)$$

$$\sum_{t=1}^{L} \tilde{\xi}_{\tilde{\beta}_{t,i}}(n) = \sum_{t=1}^{L} \xi(n)\left[1-\mu\, K \tilde{\alpha}_i^2(n) x_{i,j}^2(n)\right]^{t-1} \approx \frac{\tilde{\xi}(n)\left\{1-\left[1-\mu\, K\, \tilde{\alpha}_i(n) x_{i,j}^2(n)\right]^L\right\}}{\mu\, K\, \tilde{\alpha}_i(n) x_{i,j}^2(n)}\ . \quad (38)$$

where $K$ and $\tilde{\xi}(n)$ are given in (13) and (27), respectively. The constant $L$ is the number of data-reusing iterations. The parameter $\mu$ is a step-size.

## 6. SIMULATION RESULTS

Simulation of the test signal are using gray-scale of 256×256 of 'Peppers' for original image to assess the performance of proposed DRLMS-KSWFO and DRLMS-SKSWFO filters that we have discussed. This image is corrupted by multiplicative noise with the speckle noise with mean zero and variance ($\sigma_\eta^2$ = 0.1) in Fig.2 and Fig.3. Multiplicative noise, also known as '*speckle noise'*, is common beside additive noise [11].

The window size ($M$×$M$) is of 3×3 and 5×5. The initial parameters of Kronecker SFWO filters based on the DRLMS-KSFWO algorithm are as follows: $\mu$= 0.125 and 0.150, $L$ = 3, 4, 5 and $\alpha_i(0) = \beta_j(0) = 1/\sqrt{M}$, where $M$ is the window number and of Kronecker SFWO filters based on the DRLMS-SKSFWO algorithm are: $\mu$=0.125 and 0.150, $L$ =3, 4, 5 and $\alpha_i(0) = \beta_j(0) = 1/\sqrt{M}$, where $M$ is the window number.

The summary of proposed DRLMS-KSFWO and DRLMS-SKSFWO filters are shown in Table I and Table II for the computation.

The criteria have been used in quantitative comparison of proposed filters as follows [11].

a) Mean square error (MSE) is calculated as:

$$\mathrm{MSE} = \frac{1}{N} \sum_{n=1}^{N} \{y(n) - \hat{y}(n)\}^2\ . \quad (39)$$

b) Root mean square error (RMSE) is computed by:

$$\mathrm{RMSE} = \sqrt{\frac{1}{N} \sum_{n=1}^{N} \{y(n) - \hat{y}(n)\}^2}\ . \quad (40)$$

c) Signal to noise ratio (SNR) is defined in unit of dB as:

$$\mathrm{SNR} = 10 \log_{10}\left(\frac{\sum_{n=1}^{N} y^2(n)}{\sum_{n=1}^{N}\{y(n)-\hat{y}(n)\}^2}\right). \quad (41)$$

d) Improvement in signal-to-noise ratio (ISNR) is given in unit of dB as:

$$\mathrm{ISNR} = 20 \log_{10}\left(\frac{\sigma_\eta}{\mathrm{RMSE}}\right)\ . \quad (42)$$

e) Peak signal-to-noise-ratio (PSNR) is described in unit of dB as:

$$\mathrm{PSNR} = 20 \log_{10}\left(\frac{y_{\max}(n)}{\mathrm{RMSE}}\right)\ . \quad (43)$$

Table III provides the data illustrating of 256×256 of 'Peppers' an improvement of MSE, RMSE and SNR as computed in (40)-(42) which can be obtained by the proposed DRLMS-KSFWO and DRLMS-SKSFWO filters compared with the conventional LMS algorithm for order statistic (LMS-OS) and KSFWO filter (LMS-KSFWO) as given in APPENDIX i and ii, respectively.

The summary of accuracy of the estimation of proposed filters with the methods of ISNR and PSNR in unit of dB as detailed in (43)-(44) using different step-size at various window size and different $L$ is presented in Table III.

In Fig. 2, the results of proposed DRLMS-KSFWO filter are shown in suppressing noise in original image. The original gray-scale image and image corrupted by speckle noise are shown in Fig. 2(a) and Fig.2(b). The filtered image by proposed DRLMS-KSFWO filter with $\mu = 0.150$ and $L = 5$ for the different window size are presented in Fig. 2(c) and Fig. 2(d). According to these results, the accuracy achieved by this proposed filter gives an improvement in terms of SNR improvement.

**TABLE I** Summary of proposed adaptive DR-LMS Kronecker Sampled-Function Weighted Order (DRLMS-KSFWO) algorithm

Starting with $\alpha_i(0) = \beta_j(0) = 1/\sqrt{M}$ where M is the window size. $\xi(0) = \sigma_\eta^2$ and $\mu$ is a small constant.
Do for $n \in N$ ; $n$ = 1, 2, . . . , compute.
for $i$ = 1, 2, . . . , $m$.
for $j$ = 1, 2, . . . , $m$.
1) To compute $\alpha_i(n)$ and $\beta_j(n)$ as:

$$\alpha_i(n+1) = \alpha_i(n) + \mu\, K\, \beta_j(n) x_{i,j}(n) \sum_{t=1}^{L} \xi_{\alpha_{t,i}}(n)\ ,$$
$$\beta_j(n+1) = \beta_j(n) + \mu\, \alpha_i(n) x_{i,j}(n) \sum_{t=1}^{L} \xi_{\beta_{t,i}}(n)\ ,$$

2) To compute $\gamma(n)$ and $\xi(n)$ as:

$$\gamma(n) = \{\alpha_i(n-1)\beta_j(n-1)\}^{-1}\ ,$$
$$\xi(n) = d(n) - \frac{\alpha_i(n)\beta_j(n)x_{i,j}(n)}{\alpha_i(n-1)\beta_j(n-1)}\ .$$

3) To compute $\sum_{t=1}^{L} \xi_{\alpha_{t,i}}(n)$ and $\sum_{t=1}^{L} \xi_{\beta_{t,i}}(n)$

$$\sum_{t=1}^{L} \xi_{\alpha_{t,i}}(n) \approx \frac{\xi(n)\left\{1-\left[1-\mu\,\gamma(n)\beta_j^2(n)x_{i,j}^2(n)\right]^L\right\}}{\mu\,\gamma(n)\beta_j^2(n)x_{i,j}^2(n)}\ ,$$
$$\sum_{t=1}^{L} \xi_{\beta_{t,i}}(n) \approx \frac{\xi(n)\left\{1-\left[1-\mu\,\gamma(n)\alpha_i^2(n)x_{i,j}^2(n)\right]^L\right\}}{\mu\,\gamma(n)\alpha_i^2(n)x_{i,j}^2(n)}\ .$$

end
end
end

**TABLE II** Summary of proposed adaptive DR-LMS Switching Kronecker SFWO (DRLMS-SKSFWO) algorithm

Starting with $\alpha_i(0) = \beta_j(0) = 1/\sqrt{M}$ where M is the window size. $\xi(0) = \sigma_\eta^2$ and $\mu$ is a small constant.
Do for $n \in N$ ; $n$ = 1, 2, . . . , compute.
for $i$ = 1, 2, . . . , $m$.
for $j$ = 1, 2, . . . , $m$.

1) To compute $\alpha_i(n)$ and $\beta_j(n)$ as:

$$\alpha_i(n+1) = \alpha_i(n) + \mu\, K\, \beta_j(n) x_{i,j}(n) \sum_{t=1}^{L} \tilde{\xi}_{\alpha_{t,i}}(n) ,$$
$$\beta_j(n+1) = \beta_j(n) + \mu\, K\, \alpha_i(n) x_{i,j}(n) \sum_{t=1}^{L} \tilde{\xi}_{\beta_{t,i}}(n) ,$$

2) To compute $\sum_{t=1}^{L} \xi_{\alpha_{t,i}}(n)$ and $\sum_{t=1}^{L} \xi_{\beta_{t,i}}(n)$

$$\sum_{t=1}^{L} \tilde{\xi}_{\alpha_{t,i}}(n) \approx \frac{\tilde{\xi}(n)\left\{1-\left[1-\mu\, K \beta_j(n) x_{i,j}^2(n)\right]^L\right\}}{\mu\, K \beta_j(n) x_{i,j}^2(n)} .$$
$$\sum_{t=1}^{L} \tilde{\xi}_{\beta_{t,i}}(n) \approx \frac{\tilde{\xi}(n)\left\{1-\left[1-\mu\, K\, \alpha_i(n) x_{i,j}^2(n)\right]^L\right\}}{\mu\, K\, \alpha_i(n) x_{i,j}^2(n)} .$$

3) To compute $\tilde{\alpha}_i(n)$ and $\tilde{\beta}_j(n)$ as:

$$\tilde{\alpha}_i(n+1) = \tilde{\alpha}_i(n) + \mu\, K\, \tilde{\beta}_i(n) x_{i,j}(n) \sum_{t=1}^{L} \tilde{\xi}_{\tilde{\alpha}_{t,i}}(n) ,$$
$$\tilde{\beta}_j(n+1) = \tilde{\beta}_j(n) + \mu\, K\, \tilde{\alpha}_i(n) x_{i,j}(n) \sum_{t=1}^{L} \tilde{\xi}_{\beta_{t,i}}(n) ,$$

4) To compute $\sum_{t=1}^{L} \tilde{\xi}_{\tilde{\alpha}_{t,i}}(n)$ and $\sum_{t=1}^{L} \tilde{\xi}_{\tilde{\beta}_{t,i}}(n)$ as:

$$\sum_{t=1}^{L} \tilde{\xi}_{\tilde{\alpha}_{t,i}}(n) = \frac{\tilde{\xi}(n)\left\{1-\left[1-\mu\, K \tilde{\beta}_i(n) x_{i,j}^2(n)\right]^L\right\}}{\mu\, K \tilde{\beta}_i(n) x_{i,j}^2(n)} .$$
$$\sum_{t=1}^{L} \tilde{\xi}_{\tilde{\beta}_{t,i}}(n) = \frac{\tilde{\xi}(n)\left\{1-\left[1-\mu\, K\, \tilde{\alpha}_i(n) x_{i,j}^2(n)\right]^L\right\}}{\mu\, K\, \tilde{\alpha}_i(n) x_{i,j}^2(n)} .$$

5) To compute $\tilde{\xi}(n)$ as:

$$\tilde{\xi}(n) = d(n) - K \frac{\alpha_i(n)\beta_j(n)x_{i,j}(n)}{\alpha_i(n-1)\beta_j(n-1)} + (1-K)\frac{\tilde{\alpha}_i(n)\tilde{\beta}_j(n)x_{i,j}(n)}{\tilde{\alpha}_i(n-1)\tilde{\beta}_j(n-1)} .$$

end
end
end

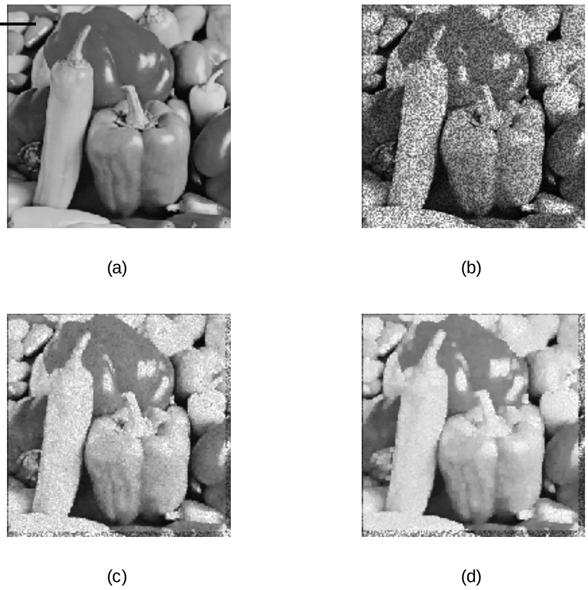


***Fig. 2*** *Proposed DRLMS-KSFWO filter with L = 5, μ = 0.150; (a) Original image; (b) Corrupted image by speckle noise ($\sigma_\eta^2$ = 0.1); (c) Filtered image using window size of 3×3; and (d) Filtered image using window size of 5×5.*

Fig.3 shows the results of proposed DRLMS-SKSFWO filters are shown in suppressing noise in original image. The image corrupted by speckle noise is shown in Fig. 3(b). The filtered image using DRLMS-KSFWO filter with $\mu = 0.150$ and $L = 5$ for the different window size are presented in Fig. 3(c) and Fig. 3(d). According to these results, the accuracy achieved by this proposed filter gives an improvement in terms of SNR improvement.

Another simulation of the test signal is using gray-scale of 512×512 of 'Tree' for original image to assess the performance of proposed DRLMS-SKSWFO filters that we have discussed. This image is corrupted by the speckle noise with mean zero and variance ($\sigma_\eta^2$ = 0.1) in Fig. 4. The window size ($M$×$M$) is of 3×3 and 5×5. The initial parameters of proposed DRLMS-SKSFWO algorithm are as follows: $\mu$= 0.125 and $L$ = 5.

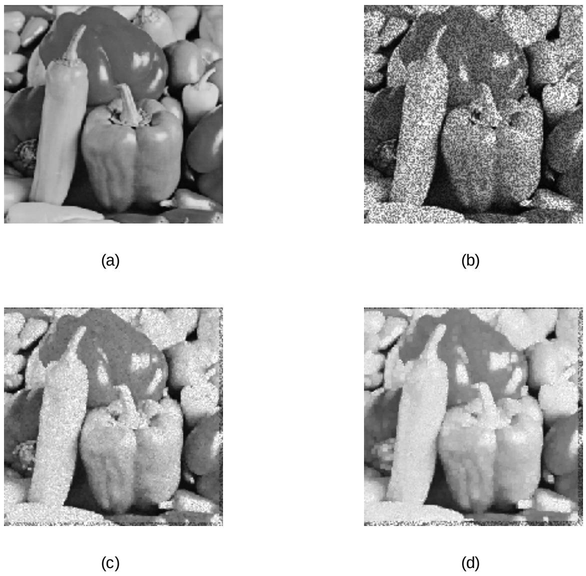


***Fig. 3*** *Proposed DRLMS-SKSFWO filter with L = 5, μ = 0.150; (a) Original image; (b) Corrupted image by speckle noise ($\sigma_\eta^2$ = 0.1); (c) Filtered image using window size of 3×3; and (d) Filtered image using window size of 5×5.*

***TABLE III*** *Accuracy of the estimation of proposed filters using 256×256 of 'Peppers' image, L = 3, 4, 5 and μ = 0.125, 0.150 at window size of 3×3 and 5×5.*

| | *μ* | L | size | MSE | RMSE | SNR (dB) | ISNR (dB) | PSNR (dB) |
|---|---|---|---|---|---|---|---|---|
| SFWO | 0.125 | - | 3×3 | 0.0277 | 0.1665 | 9.8019 | 5.5705 | 14.5219 |
| | | | 5×5 | 0.0076 | 0.0872 | 15.4180 | 11.1867 | 20.1380 |
| | 0.150 | - | 3×3 | 0.0264 | 0.1625 | 10.0147 | 5.7833 | 14.7347 |
| | | | 5×5 | 0.0073 | 0.0857 | 15.5759 | 11.3445 | 20.2959 |
| LMS-OS | 0.125 | - | 3×3 | 0.0175 | 0.1324 | 11.7948 | 7.5635 | 16.5148 |
| | | | 5×5 | 0.0111 | 0.1056 | 13.7610 | 9.5296 | 18.4810 |
| | 0.150 | - | 3×3 | 0.0178 | 0.1334 | 11.7259 | 7.4945 | 16.4459 |
| | | | 5×5 | 0.0104 | 0.1022 | 14.0406 | 9.8093 | 18.7606 |
| LMS-KSFWO | 0.125 | - | 3×3 | 0.0238 | 0.1542 | 10.4723 | 6.2409 | 15.1923 |
| | | | 5×5 | 0.0086 | 0.0930 | 14.8656 | 10.6343 | 19.5856 |
| | 0.150 | | 3×3 | 0.0232 | 0.1523 | 10.5776 | 6.3462 | 15.2976 |
| | | | 5×5 | 0.0080 | 0.0894 | 15.2025 | 10.9711 | 19.9225 |
| DRLMS-KSFWO | 0.125 | 3 | 3×3 | 0.0190 | 0.1380 | 11.4329 | 7.2015 | 16.1529 |
| | | | 5×5 | 0.0058 | 0.0761 | 16.6002 | 12.3689 | 21.3202 |
| | | 4 | 3×3 | 0.0170 | 0.1305 | 11.9163 | 7.6850 | 16.6363 |
| | | | 5×5 | 0.0056 | 0.0745 | 16.7839 | 12.5525 | 21.5039 |
| | | 5 | 3×3 | 0.0160 | 0.1264 | 12.1960 | 7.9646 | 16.9160 |
| | | | 5×5 | 0.0049 | 0.0698 | 17.3543 | 13.1230 | 22.0703 |
| | 0.150 | 3 | 3×3 | 0.0177 | 0.1331 | 11.7446 | 7.5133 | 16.4646 |
| | | | 5×5 | 0.0057 | 0.0753 | 16.6960 | 12.4647 | 21.4160 |
| | | 4 | 3×3 | 0.0161 | 0.1269 | 12.6077 | 7.9293 | 16.8807 |
| | | | 5×5 | 0.0050 | 0.0704 | 17.2798 | 13.0484 | 21.9998 |
| | | 5 | 3×3 | 0.0152 | 0.1231 | 12.4255 | 8.1941 | 17.1455 |
| | | | 5×5 | 0.0049 | 0.0701 | 17.3148 | 13.0835 | 22.0348 |
| DRLMS-SKSFWO | 0.125 | 3 | 3×3 | 0.0188 | 0.1371 | 11.4937 | 7.2623 | 16.2137 |
| | | | 5×5 | 0.0057 | 0.0757 | 16.6449 | 12.4135 | 21.3649 |
| | | 4 | 3×3 | 0.0169 | 0.1301 | 11.9487 | 7.7174 | 16.6687 |
| | | | 5×5 | 0.0052 | 0.0723 | 17.0506 | 12.8193 | 21.7706 |
| | | 5 | 3×3 | 0.0155 | 0.1244 | 12.3364 | 8.1051 | 17.0564 |
| | | | 5×5 | 0.0050 | 0.0705 | 17.2621 | 13.0307 | 21.9821 |
| | 0.150 | 3 | 3×3 | 0.0174 | 0.1320 | 11.8206 | 7.5893 | 16.5406 |
| | | | 5×5 | 0.0056 | 0.0750 | 16.7326 | 12.5012 | 21.4526 |
| | | 4 | 3×3 | 0.0157 | 0.1255 | 12.2610 | 8.0296 | 16.9810 |
| | | | 5×5 | 0.0050 | 0.0709 | 17.2210 | 12.9896 | 21.9410 |
| | | 5 | 3×3 | 0.0151 | 0.1228 | 12.4475 | 8.2161 | 17.1675 |
| | | | 5×5 | 0.0046 | 0.0675 | 17.6395 | 13.4081 | 22.3595 |

***TABLE IV*** *Accuracy of the estimation of proposed filters using 512×512 of 'Tree' image, L = 5 and μ = 0.125 at window size of 3×3 and 5×5.*

| | *μ* | L | size | MSE | RMSE | SNR (dB) | ISNR (dB) | PSNR (dB) |
|---|---|---|---|---|---|---|---|---|
| DRLMS-SKSFWO | 0.125 | 5 | 3×3 | 0.0027 | 0.0527 | 14.1511 | 15.5579 | 25.5579 |
| | | | 5×5 | 0.0012 | 0.0342 | 17.9224 | 19.3293 | 29.3293 |

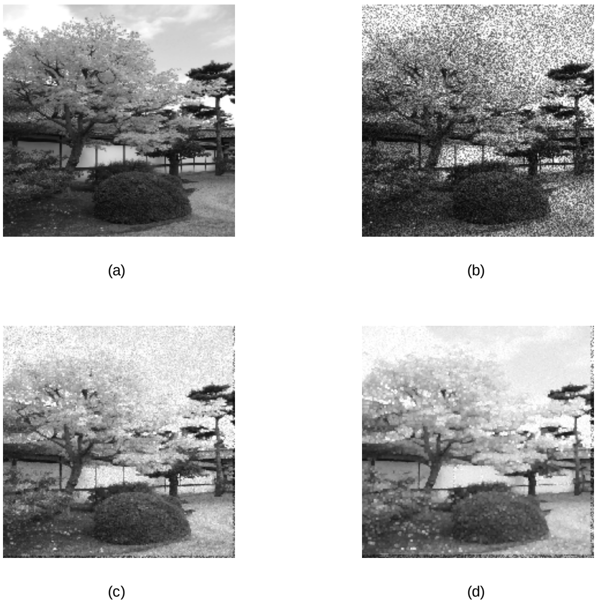


***Fig. 4*** *Proposed DRLMS-SKSFWO filter with L = 5, μ = 0.125; (a) Original image; (b) Corrupted image by speckle noise ($\sigma_\eta^2$ = 0.1); (c) Filtered image using window size of 3×3; and (d) Filtered image using window size of 5×5.*

It can be seen that the proposed DRLMS-SKSWFO filter effectively reduces the multiplicative ‘speckle’ noise as shown in Fig.4. Table IV provides the data illustrating of 512×512 of ‘Tree’, which gives an improvement of MSE, RMSE, SNR, ISNR and PSNR. Results show that the accuracy achieved using proposed DRLMS-SKSWFO filters gives an improvement in terms of SNR improvement.

## 7. CONCLUSION

We introduce the adaptive Kronecker SFWO filters and switching Kronecker SFWO based on the data-reusing least mean square (DR-LMS) algorithm whose coefficients are samples of a bounded real valued function with the properties of robust by means of MSE criterion. The proposed DRLMS-KSFWO filters achieve the improvement in terms of signal-to-noise ratio compared with the KSFWO filters based on the conventional LMS algorithm. The proposed DRLMS-SKSFWO filters obtain the improvement in terms of SNR compared with the KSFWO filters based on the conventional LMS algorithm.

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## APPENDIX

[1] LMS ORDER STATISTIC (LMS-OS) FILTER Following [5], [12], the coefficient $a_i(n)$ of order statistic fitler based on the least mean square (LMS-OS) algorithm is given by

$$a_i(n+1) = a_i(n) + \mu\, x_{i,j}(n)\, \varepsilon(n), \qquad \text{(A. 1)}$$

where $\varepsilon(n)$ is the estimation error on the sample-by-sample basis as

$$\varepsilon(n) = d(n) - a_i(n)\, x_{i,j}(n), \qquad \text{(A. 2)}$$

where $\mu$ is the step-size.

[2] LMS KRONECKER SFWO (LMS-KSFWO) FILTER Following [12], the coefficient $\acute{\alpha}_i(n)$and $\acute{\beta}_i(n)$ of KSFWO fitler based on the least mean square (LMS-KSFWO) algorithm is given by

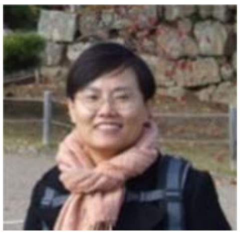
$$\acute{\alpha}_i(n+1) = \acute{\alpha}_i(n) + \mu\, \acute{\beta}_i(n)\, x_{i,j}(n)\, \zeta(n), \qquad \text{(A. 3)}$$

$$\acute{\beta}_j(n+1) = \acute{\beta}_j(n) + \mu\, \acute{\alpha}_i(n)\, x_{i,j}(n)\, \zeta(n), \qquad \text{(A. 4)}$$

where $\zeta(n)$ is the estimation error on the sample-by-sample basis as

$$\zeta(n) = d(n) - \frac{\acute{\alpha}_i(n)\acute{\beta}_j(n)x_{i,j}(n)}{\acute{\alpha}_i(n-1)\acute{\beta}_j(n-1)}\ . \qquad \text{(A. 5)}$$

where $d(n)$ is desired signal.



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