Advanced Frequency-domain Linear Equalisation in SC-FDMA

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ABSTRACT

Single carrier frequency domain equalisation (SCFDE) has been used for broadband data transmission. In this paper, we propose a well-known low complexity mechanism, called adaptive set-membership normalised complex least mean square (SM-NCLMS) algorithm, considering frequency domain linear equalisation (FDLE). Simulation results show that the proposed SM-NCLMS algorithm can achieve the same performance of standard technique as compared with the existing algorithm.

Keyword: *frequency domain linear equalisation (FDLE), Single carrier frequency domain equalisation (SCFDE), adaptive algorithm*

1. INTRODUCTION

For linear distortion, an equalisation has been introduced in order to compensate by the multipath propagation channel. Because of the long channel impulse response, the time domain equalisation is impractical for broadband channel. Moreover, the frequency-domain equalisation is used to be more practical than time-domain equalisation, while the complexity of frequency-domain equaliser is much lower than that of the equivalent time domain equaliser for broadband channels [1], [2].

Single carrier modulation with frequency domain equalization (SC-FDE) scheme is used for mitigating the effects of frequency selective fading [3], [4]. It has the similar spectral efficiency in order to anti-multipath fading properties and low complexity. In [5], the SC-FDE has been proposed for space time block coding scheme that can obtain the transmit diversity with only linear processing at the receiver based on multiple blocks modulation.

Because of broadband channels experience frequency selective fading, the frequency division multiple access (FDMA) schemes are able to employ channel dependent scheduling to obtain multi-user diversity. The scheduling techniques can assign for each terminal into subcarriers with transmission characteristics at the location of the terminal. Single carrier frequency division multiple access (SC-FDMA) is modified of SC-FDE schemes for the multiple access techniques which has been used as the uplink transmission in the 3GPP long term evolution standard, which is a form of SC-FDE with a flexibility in resource allocation and lower peak-to-average power ratio compared to orthogonal frequency division multiple access (OFDMA) [1], [6].

Frequency domain linear equalisation (FDLE) techniques has been presented to combat the frequency selective of the transmission channel. In [7], the optimum power allocation is used with FDLE by means of a minimum mean square error (MMSE) at the receiver for the SC-FDMA transmission in order to maximise the achievable transmission rate. In [8], the optimisation of the power coefficients for MMSE SC-FDE corresponds to the maximisation of the signal to interference and noise ratio. Based on a well-known low complexity adaptive algorithm, the most commonly normalised least mean square (NLMS) algorithm is used to recovery the signal with the robust and good performance [9]. In order to achieve fast convergence, the set-membership algorithm has been presented with low misadjustment [10].

In this paper, we describe firstly about the SC-FDMA systems in Section II. Secondly, we explain briefly how to apply the proposed adaptive set-membership normalised complex least mean square (SM-NCLMS) algorithm for FDLE in Section III. The simulation result also presents using the localised FDMA (LFDMA) subcarrier mapping of assigning the *M* frequency domain modulation symbols to perform the symbol error rate (SER) performance compared with the conventional MMSE-FDLE in Section IV and the conclusion of the algorithm performance is also included in Section V.

Some notation will be used throughout this paper as follows: the operators $(.)^{T}$), $(.)^{H}$ and (.) denote as the transpose, Hermitian and complex conjugate operators, respectively. The vectors are in bold lowercase and matrices are in bold uppercase.

2. SINGLE-CARRIER FDMA

Following [1] and [7], the transmitter and receiver structure of single carrier frequency division multiple access (SC-FDMA) system is shown in Fig 1. The data block consists of *M* complex modulation symbols, using binary phase shift keying in weak channels and up to 64 quadrature amplitude modulation when strong channels. The *M* frequency domain symbols to subcarrier mapping process are produced by the *M*-point discrete Fourier transform (DFT).

The subcarrier mapping block is to assign frequency domain modulation symbols to all subcarriers. Two methods of subcarrier mapping of assigning the *M* frequency domain modulation symbols to subcarriers are called distributed subcarrier mapping and localised subcarrier mapping. For the distributed subcarrier mapping method, the modulation symbols are assigned to *M* connected subcarriers. The modulation symbols for the localised subcarrier mapping method are equally spaced across the entire channel bandwidth. The localised and distributed subcarrier mapping method of SC-FDMA are referred to as localised FDMA (LFDMA) and distributed FDMA (DFDMA), respectively. The LFDMA is the current implementatin in 3GPP LTE.

After that, the inverse discrete Fourier transform (IDFT) creates a time domain of the N subcarrier symbols. Then, the parallel to serial converts the N subcarrier symbols in a time sequence and the cyclic prefix is inserted by providing a guard time in order to prevent interblock interference. The output is suitable for modulating a radio frequency carrier and transmission to the receiver.

The channel equalisation is approximately an inverse filtering of the linear distortion introduced to the channel by the multipath propagation. A linear filtering is a convolution operation in the time-domain and a point-wise multiplication operation in the frequency-domain. Then, the time-domain signal is converted by the discrete Fourier transform (DFT) to frequency-domain signals, which can be equalised by dividing it point-by-point. By applying the N-point DFT, the received signal is transformed to the frequency domain and then is performed the frequency domain equalisation at the receiver. The well-known time domain equalisation techniques, such as minimum mean-square error (MMSE) equalisation can be applied through the subcarrier de-mapping block into this frequency domain equalisation. After the equalisation, an inverse discrete Fourier transform (IDFT) transforms the single carrier signal back to the time domain and a detector recovers the original modulation symbols.

3. SC-FDMA with FDLE

The operation of single carrier frequency division multiple access (SC-FDMA) receiver is depicted from a multiple user access with Q terminals in the uplink as shown in Fig 2. At the base station, the users are separated in the frequency domain during the subcarrier de-mapping process. In the receiver, the received signals are transformed by DFT to the frequency domain to recover N subcarriers. The subcarrier demapping operation isolates the M frequency domain samples of each source signal. Using the single carrier modulation for SC-FDMA, the advantage is to encounter the intersymbol interference (ISI) that the frequency domain linear equalization (FDLE) is able to fight the ISI [1] and [7].

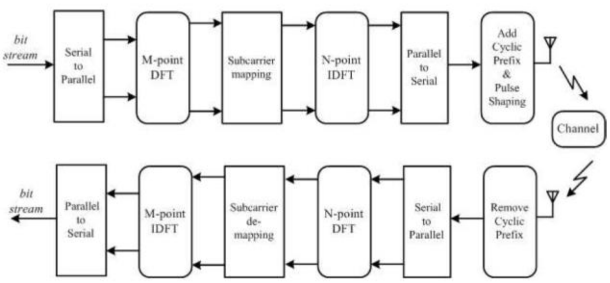

Fig. 1 *Transmitter and receiver structure of SC-FDMA System [1].*

4. PROPOSED NCLMS ALGORITHM

In this section, we present the proposed adaptive normalised complex least mean square (NCLMS) algorithm based on the FDLE, which has several advantages as robust and low complexity.

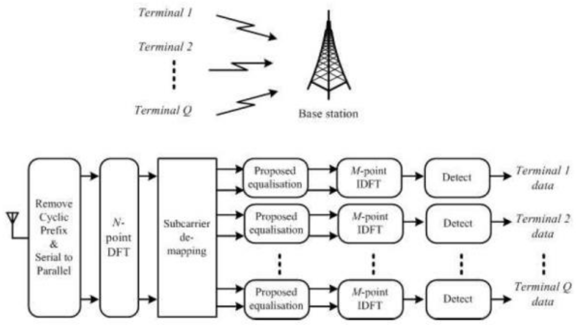

Fig. 2 *SC-FDMA receiver structure from a multiple user access with Q terminals in the uplink. [1]*

Following [2], we introduce the complex constraints for output estimate of tap-weight vector ***p(n)*** as

$$\boldsymbol{p}^H(n+1)\,\boldsymbol{y}(n) = g(n), \quad (1)$$

where ***g(n)*** is the scalar quantity and ***y(n)*** represents the frequency domain samples after subcarrier de-mapping. The parameter *e(n)* is a priori estimated error as

$$\mathrm{e}(n) = \boldsymbol{p}^H(n+1)\,\boldsymbol{y}(n)\,, \quad (2)$$

where *d(n)* is the desired signal.

The square Euclidean norm of $\delta\boldsymbol{p}(n+1)$ can be expressed as

$$\|\delta\boldsymbol{p}(n+1)\|^2 = \sum_{k=0}^{N-1}|\boldsymbol{p}_k(n+1) - \boldsymbol{p}_k(n)|^2 \quad (3)$$

Where

$$\delta p(n+1)=p(n+1)-p(n) \quad . \quad (4)$$

The cost function *J(n)* by formulating using the Lagrange multiplier [9] with respect to the complex constraints given in (1) , we arrive at

$$J(n) = \|\delta\boldsymbol{p}(n+1)\|^2 + \lambda_1\mathcal{R}[\boldsymbol{p}^H(n+1)\boldsymbol{y}(n) - g(n)] + \lambda_2\Im[\boldsymbol{p}^H(n+1)\boldsymbol{y}(n) - g(n)]\,, \quad (5)$$

where $\lambda = \lambda_1 + j\lambda_2$ that is the complex valued of Lagrange multiplier. The operators $\mathcal{R}[\cdot]$ and $\Im[\cdot]$ are real and imaginary operators, respectively.

By differentiating *J(n)* and *min E{|e(n)|²}* with respect to ***p(n)***, then both results are set to zero. We arrive at

$$2\,\Delta\boldsymbol{p}(n+1) + \lambda * \boldsymbol{y}(n) = 0\,, \quad (6)$$

$$\lambda * = \frac{2}{\|\boldsymbol{y}(n)\|^2}\, e * (n)\,, \quad (7)$$

where $\lambda *$ is the complex conjugate of Lagrange multiplier and *e*(n)* is the complex conjugate of estimated error.

The tap-weight error vector $\Delta\boldsymbol{p}(n+1)$ can be expressed by

$$\Delta\boldsymbol{p}(n+1) = \frac{-\mu}{\|\boldsymbol{y}(n)\|^2}\, y(n)e * (n)\,, \quad (8)$$

Therefore, the tap-weight vector ***p(n)*** based on the normalized complex least mean square (NCLMS) algorithm as

$$\boldsymbol{p}(n+1) = \boldsymbol{p}(n) - \frac{\mu}{\|\boldsymbol{y}(n)\|^2}\boldsymbol{y}(n)e * (n)\,, \quad (9)$$

where μ is the step-size parameter.

5. PROPOSED SM-NCLMS ALGORITHM

Following [11], the tap-weight estimate vector ***p(n)*** is designed to achieve the estimated error bound γ_m on the magnitude of the estimated error *e(n)*. The set-membership filtering criterion requires that the output error should be less than the error bound γ_m. We follow this solution belongs to the constraint sets at *n*+1 and *n* for each update.

The proposed set-membership NCLMS algorithm (SM-NCLMS) is based on the frequency domain linear equalisation (FDLE) as defined in [10]. The SM-NCLMS algorithm is similar to the conventional normalised LMS algorithm. The basic idea is that if the previous estimate lies outside is that: If the previous estimated lies outside the constraint set $\mathcal{H}_n$,

$$|d(n) - \boldsymbol{p}^H(n+1)| > \gamma,$$

the new estimate will lie on the closest boundary of $\mathcal{H}_n$ at a minimum distance. So, the minimized SM-NCLMS algorithm of the square Euclidean norm of $\Delta\boldsymbol{p}(n+1)$ is subjected to $p(n+1) \in \mathcal{H}_n$.

Therefore, the tap-weight vector ***p(n+1)*** is estimated as

$$\boldsymbol{p}(n+1) = \boldsymbol{p}(n) - \frac{\xi(n)}{\|\boldsymbol{y}(n)\|^2}\boldsymbol{y}(n)e * (n)\,, \quad (10)$$

$$\xi(n) = \frac{\gamma}{|e(n)|}, \quad (11)$$

where γ is the upper bound on the estimated error. The constant $\xi(n)$ is a sufficient condition to converge,

$$1 < \xi(n) < \frac{1}{\gamma_{max}} \quad (12)$$

where γ_{max} is the maximum eigenvalue of the autocorrelation matrix of input signal. The parameter γ is chosen about $\sqrt{5}\ \sigma_\eta$, where σ_η^2 is the variance of the additional noise.

6. SIMULATION RESULTS

6.1 Simulation for SC-FDMA

We performed a link level simulation for SC-FDMA using the ITU channel delay profile as Pedestrian A and Vehicular A channels [6] as shown in Tab. I. The simulation assumptions and parameters are as follows: the system bandwidth = 5MHz, sampling rate = 5Mega-samples/s, CP length = 20 and location of subband =0 and 15.

The data modulation format is as follows: quadrature phase shift keying (QPSK), the DFT and IDFT size is equal to 512, the number of iterations = 10^4, signal to noise ratio (SNR) = 0dB to 20dB and SC-FDMA input block size = 16 symbols and SC-FDMA input DFT size =16. For the channel delay profile with additive white Gaussian noise (AWGN), we refer the readers to [6] for more details of the ITU Pedestrian A and Vehicle A channels. This link level simulation also presents a symbol error rate (SER) calculation to measure the performance. The initial parameters of the proposed frequency domain linear equalisation based on NCLMS algorithm are in the same condition as follows: $\mu(0)$ = 0.0125, tap of weight vector (L) = 3, $\boldsymbol{p}(0) = [1\ 0\ \ldots\ 0\ ]^T$.

Figs. 3 and 4 compares the symbol error rate (SER) performance between localised subcarrier mapping (LFDMA) with the different equalisations. In Fig. 3, the symbol error rate (SER) performance of SC-FDMA based on the localized subcarrier mapping (LFDMA) with the proposed NCLMS and MMSE equalisations, where the locations of subband 0 and 15 are used for ITU Pedestrian A channel. According to the higher channel gain of the localized subband 0 than the average, the SER performance of proposed NCLMS and MMSE equalisations in localised subband 0 is much better than the lower channel gain than the average of localized subband 15 of ITU Pedestrian A channel. It is noticed that SER curves of proposed low complexity NCLMS algorithm in both subbands can obtain performance.

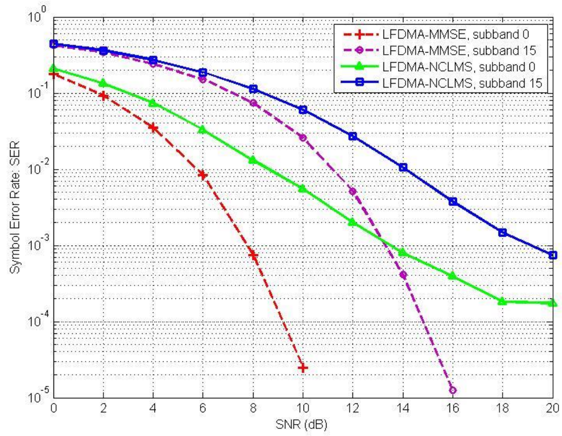

Fig. 3 *Performance of symbol error rate of SC-FDMA with the localized subcarrier mapping (LFDMA) and different equalisations, when the locations of subband are at subband 0 and 15 for ITU Pedestrian A channel. The SCFDMA input block size is of 16 symbols.*

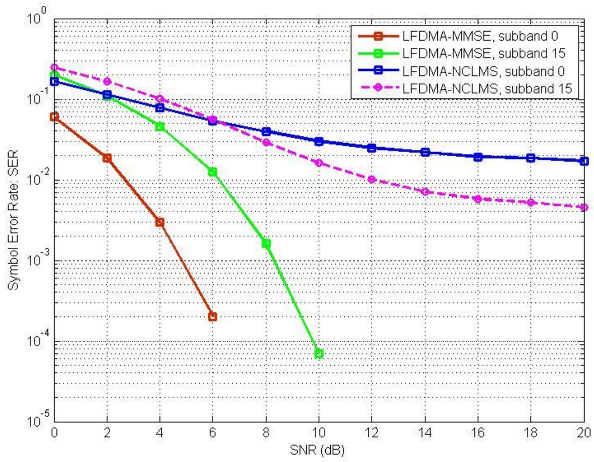

Fig. 4 *Performance of symbol error rate of SC-FDMA with the localized subcarrier mapping (LFDMA) and different equalisations, when the locations of subband aare at subband 0 and 15 for ITU Vehicular A channel. The SCFDMA input block size is of 16 symbols.*

The SER performance of SC-FDMA based on the LFDMA with the proposed NCLMS and MMSE equalisations, where the locations of subband 0 and 15 are used for ITU Vehicular A channel in Fig. 4. According to the higher channel gain of the localised subband 0 than the average, the SER performance of proposed NCLMS and MMSE equalisations in localized subband 0 is much better than the lower channel gain than the average of localised subband 15 of ITU

Vehicular A channel. It is noted that SER curves of proposed low complexity NCLMS algorithm can obtain performance, especially in the localized subband 0.

6.2. Simulation for SC-FDE

We performed a link level simulation for SC-FDE using the ITU channel delay profile as Pedestrian A channel [6] as shown in Tab. I. The simulation assumptions and parameters are as follows: the system bandwidth = 5MHz, sampling rate = 5Mega-samples/s, CP length = 20. The data modulation format is as follows: quadrature phase shift keying (QPSK), the DFT and IDFT size is equal to 512, the number of iterations = 10^4, signal to noise ratio (SNR) = 0dB to 16dB and equalisation are as follows: SM-NCLMS, OFDM-MMSE and SCFDE-MMSE, respectively. This link level simulation also presents a symbol error rate (SER) calculation to measure the performance. The initial parameters of the proposed frequency domain linear equalisation based on SM-NCLMS algorithm are in the same condition as follows: $\mu(0) = 0.0055$, tap of weight vector (L) = 3 and 7, $\boldsymbol{p}(0) = [1\ 0\ \ldots\ 0]^T$.

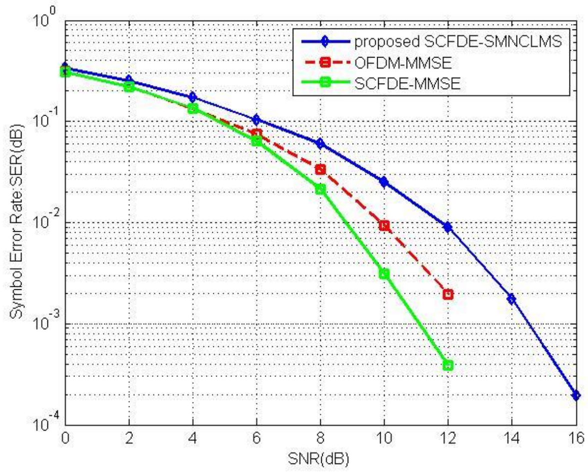

Fig. 5 *Performance of symbol error rate of SC-FDMA with the OFDM-MMSE and SCFDE-MMSE compared to proposed SCFDE-SMNCLMS for ITU Pedestrian A channel, where $\mu(0)=0.0055$ and $L = 5$.*

Figs. 5 and 6 compares the symbol error rate (SER) performance of proposed algorithm between with the orthogonal frequency division multiplexing based on the minimum mean square error (OFDM-MMSE) and single-carrier frequency-domain equaliser based on the minimum mean square error (SCFDE-MMSE) equalisations, when tap of weight vector (L) = 5 and 9, respectively. It is noticed that both of SER curves of proposed SM-NCLMS algorithm for SC-FDE with different number of tap of weight vector can obtain performance.

7. CONCLUSION

In this paper, we have proposed equalisation based on the adaptive set-membership normalised complex least mean square (SM-NCLMS) algorithm for SC-FDMA systems. We have also described concisely how to formulate the tap-weight estimated vector for FDLE with the method of Lagrange multiplier. According to the simulation results of the proposed algorithm, they have provided the good simulation result based on the low complexity algorithm.

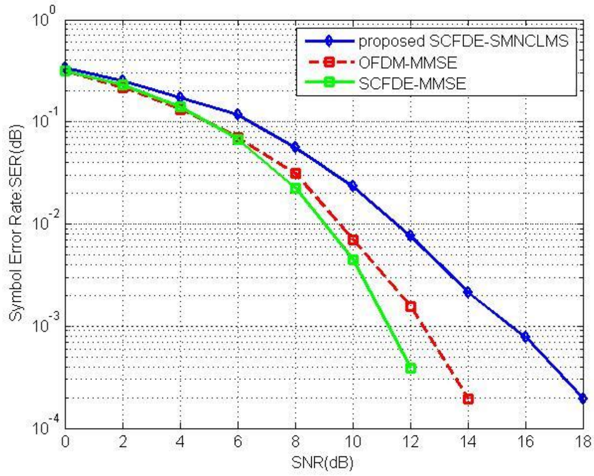

Fig. 6 *Performance of symbol error rate of SC-FDMA with the OFDM-MMSE and SCFDE-MMSE compared to proposed SCFDE-SMNCLMS for ITU Pedestrian A channel, where $\mu(0)=0.0055$ and $L = 9$.*

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