# Simplified Strut and Tie Model for Deep Reinforce Concrete Beam

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**ABSTRACT**

***Experimental program comprised the testing to failure of 9 deep reinforced concrete (RC) beams designed by strut and tie model (STM) according to Appendix A of ACI 318-11. The tested specimens were designed with various shear span-to-effective depth ratios (a/d ratio) and the ratios of horizontal and vertical crack controlled reinforcement ($\rho_h / \rho_v$ ratio). The results indicated that failure mode of deep RC beam changed from diagonal splitting to flexural shear failure when the a/d ratio was increased. Furthermore, the $\rho_h / \rho_v$ ratio was found to have significant influence on the deformation of the horizontal and vertical reinforcements which could change the failure mode of the beam with the same a/d ratio. Based on the test results, the empirical relationships were introduced to predict the strains in horizontal and vertical reinforcements at maximum applied load. The simplified STM was proposed to separate the effect of reinforcing steels from concrete STM to estimate the shear strength of the test specimens. Finally, the validity of proposed modified STM was verified against the experimental results of 408 samples from various researchers.***

***Keywords:*** *deep reinforced concrete, strut and tie model, crack controlled reinforcement, deep beam*

## 1. INTRODUCTION

Behavior of deep RC beam is among one of the interested research topics in structural engineering. Due to the complexity of the flow of stresses in the disturbed region (D–region), Bernoulli's assumption cannot be applied, thus, the well-known flexural theory is invalid. This problem leads to the requirement of the reliable method for predicting the strength of such structures.

In order to investigate the behavior of the D–region in deep RC beam, the strut and tie model (STM) is the simplest analytical method since it was introduced in ACI 318-11[1] and AASHTO LRFD.[2] By considering the flow of compressive and tensile stresses in the D–region, STM replaced the complicated stress flow with the desired truss structure which consists of uni-axial elements namely compression strut and tension tie.[3] It should be noted that only the equilibrium conditions was satisfied under the concept of STM. Compatibility condition was not considered in the STM.[3] Thus, several of STM configurations can be applied to the specimen under consideration.[4]

However, discussions on the adequacy of strength of concrete strut have been triggered by significant discrepancy between experimental results and the prediction strength.[4] – [7] In addition, the strength of concrete strut is assumed to be dependent on the cracking direction and reinforcement provided, but independent of concrete strength and angle of strut.[7] Therefore, significant improvement is required for improving the reliability of STM.

The aim of this research is to investigate the influence of the ratio of crack control reinforcement in term of the horizontal to vertical reinforcement ratio ($\rho_h / \rho_v$ ratio) on the mode of failure of the deep RC beam. The influence of concrete strut angle on the overall behaviors of the specimen is also investigated through the variation of the shear span to effective depth ratio (a/d ratio). Based on the empirical relationship from experimental results together with the original STM, the modified STM is proposed for predicting the shear strength of the deep RC beams.

## 2. EXPERIMENTAL PROGRAM

The experimental program was conducted at the Structural Engineering Laboratory of Khon Kaen University. The program comprised the testing of 9 deep RC beams designed according to Appendix A of the ACI 318-11. All girders were 2000 mm long (L), 200 mm wide (b) and 450 mm deep (h). To investigate the influence of concrete strut angle, test specimens were grouped into 3 series, series A, series B and series C. Specimens in series A, series B and series C were designed for a/d ratio equal to 1.0, 1.5 and 2.0, respectively. In each series the amount of horizontal to vertical reinforcement ratio, $\rho_h/\rho_v$ , was varied from 1.0, 3.0 and 0.3.

Details on the horizontal and vertical reinforcement ratios together and the requirement according to the provision in Appendix A of ACI 318-11 are given in Table 1. From Table 1, $s_h$ and $s_v$ denote the spacing between horizontal reinforcements and vertical reinforcements, respectively. The sound determination of the value in the last column is given elsewhere.1 Tested specimens were loaded at the top surface with a mid-span single point load. Dial gauge was placed at the bottom surface for measuring the mid-span deflection. Fig. 1(a) shows the load and support configuration of the tested specimens.

Deformations of the flexural and crack controlled reinforcement were measured from electrical strain gauges. The theoretical direction of the diagonal concrete struts and the location of all strain gauges are shown in Fig. 1(b). It should be noted that the exact locations of the strain gages were depended on the a/d ratio of the specimens. Reinforcement configurations designed for this experiment and the details are shown in Fig. 2.

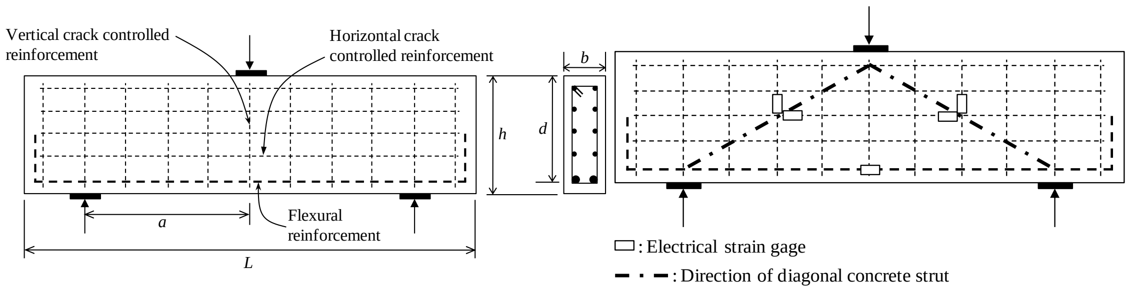



***Fig. 1*** *(a) Load and support configuration of the tested specimens; (b) Direction of diagonal concrete strut and the location of measured strain gauges.*

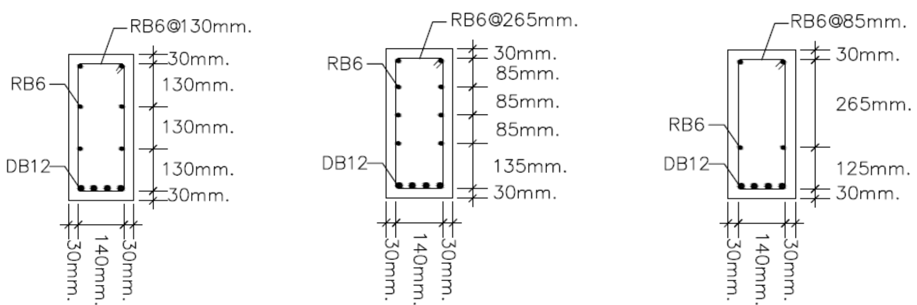



***Fig. 2*** *Typical beam cross section.*

***Table 1*** *Details of horizontal and vertical crack controlled reinforcements.*

| Test series | Specimen Reference | $s_h$ (mm.) | $s_v$ (mm.) | Reinforcement Ratio $\rho_h$ | $\rho_v$ | $\sum \frac{A_{si}}{b_s i_s} \sin \alpha_i$ |
|---|---|---|---|---|---|---|
| A | A1 | 130 | 130 | 0.00218 | 0.00218 | 0.00308 |
| | A2 | 85 | 265 | 0.00333 | 0.00107 | 0.00311 |
| | A3 | 265 | 85 | 0.00107 | 0.00333 | 0.00311 |
| B | B1 | 130 | 130 | 0.00218 | 0.00218 | 0.00302 |
| | B2 | 85 | 265 | 0.00333 | 0.00107 | 0.00274 |
| | B3 | 285 | 85 | 0.00107 | 0.00333 | 0.00336 |
| C | C1 | 130 | 130 | 0.00218 | 0.00218 | 0.00300 |
| | C2 | 85 | 265 | 0.00333 | 0.00107 | 0.00245 |
| | C3 | 265 | 85 | 0.00107 | 0.00333 | 0.00346 |

## 3. MATERIAL PROPERTIES

The concrete used for casting all specimens in this experimental program was mixed in the laboratory. Portland cement type I with local fined and coarse aggregates was used for each concrete batch. River sands with specific gravity of 2.64 and fineness modulus of 2.40 were used as fined aggregates. Crushed limestone with specific gravity of 2.75, maximum size was about 20 mm, was used as coarse aggregates. Each batch was mixed under the water-cement ratio equal to 0.5. The slumps of each fresh concrete batch were found to be in the range of 75 to 125 mm.

Compressive strength of concrete at the day of testing, $f_c'$, was obtained from three standard cylinder tests. All flexural reinforcements were made by deformed bar with diameter 12 mm class SD-30 $\left(F_{yf} \geq 294\ MPa\right)$. Round bar with diameter 6 mm class SR-24 $\left(F_{yw} \geq 235\ MPa\right)$ was used for horizontal and vertical crack controlled reinforcements. Measured concrete compressive strength and the actual yield strength of all reinforcements are given in Table 2.

## 4. TEST RESULTS AND DISCUSSIONS

Behavior of all test specimens were evaluated through the relationships between load and mod-span deflection, failure mode and measured strains in the reinforcements at the specify locations.

***Table 2*** *Material properties of tested specimens.*

| **Test** Series | **Specimen** Reference | $f_c'$ (Mpa) | $f_{yf}$ (Mpa) | $f_{yw}$ (Mpa) |
|---|---|---|---|---|
| A | A1 | 25.4 | 530 | 405 |
| | A2 | 27.7 | 530 | 405 |
| | A3 | 28.0 | 530 | 405 |
| B | B1 | 21.4 | 440 | 420 |
| | B2 | 23.9 | 440 | 420 |
| | B3 | 22.4 | 440 | 420 |
| C | C1 | 21.6 | 440 | 420 |
| | C2 | 22.8 | 440 | 420 |
| | C3 | 23.2 | 440 | 420 |

## 5. LOAD AND DISPLACEMENT RELATIOSHIP

All tested specimens behave as an elastic material until they reached about approximately 90% of maximum load, as shown by the normalized load ($P/f_c'bh$) versus mid-span deflection curves in Fig. 3. The change in member stiffness after first flexural crack occurs increase with an increasing of a/d ratio. In addition, from Fig. 3, it is evident that the ductility of the tested specimens under the applied loads is increased with an increasing of a/d ratio. The failure mode the specimens were changed from diagonal splitting failure to flexural shear failure when the a/d ratio was increased.

## 6. MODE OF FAILURES

The failure modes of all specimens were different. However, the failure modes of specimens with the same a/d ratio were similar. This observation leads to the first conclusion that the amount of horizontal to vertical reinforcement ratio has no effect on the failure mode. Mode of failure could change from diagonal splitting of diagonal concrete strut under compression to diagonal tension for specimens with a/d ratio equal to 1.0 to 1.5. It should be noted that the specimen series A could not be loaded further from this point since the capacity of the self-equilibrated frame was reached (about 690 kN). For specimens with a/d ratio equal to 2.0, it was observed that those were failed under the flexural shear failures. The pictures of some specimens at the end of testing are shown in Fig. 4.

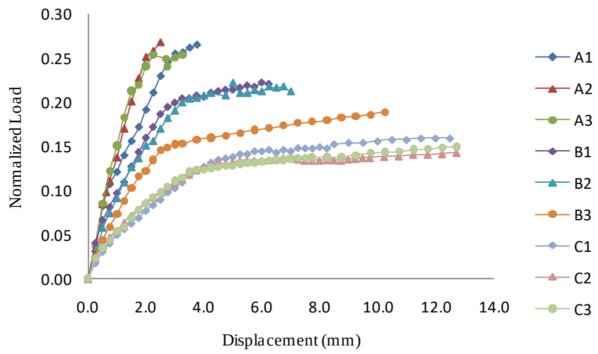



***Fig. 3*** *Load and mid-span deflection curves of all test specimens.*

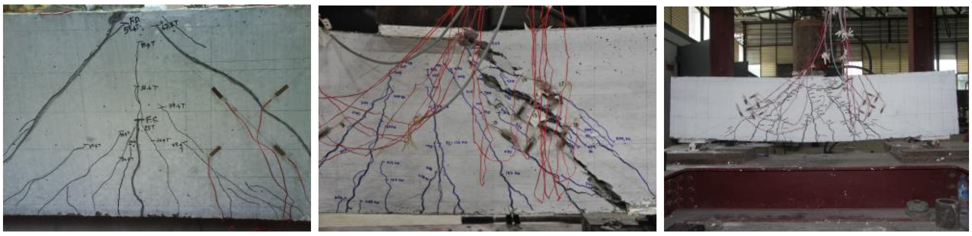


***Fig. 4*** *(a) Typical failure of specimen series A; (b) Typical failure of specimen series B; (c) Typical failure of specimen series C.*

## 7. MODIFIED STRUT AND TIE MODEL

Supposing that the method of superposition can be applied to the deep RC beam, the simple STM for concrete strut and crack controlled reinforcement for half of the beam is shown in Fig. 5.

The vertical shear strength of the beam, $F_{Ry}$, can be determined from the following exhibition

$$F_{Ry} = F_c \sin\theta + F_v + F_h \tan\theta \qquad (1)$$

Where $F_c$ is the concrete compressive strength of the diagonal strut; $F_v$ and $F_h$ are tension in vertical and horizontal reinforcements, respectively. According to Appendix A in ACI 318-11, the compressive strength of the diagonal concrete strut can be approximated from the following equation

$$F_c = 0.85 f_c' \beta_s b w_{strut} \qquad (2)$$

In eq.(2), $b$ is the width of the beam and $w_{strut}$ is the average width of the compressive strut. Since the concrete and crack controlled reinforcements are fully separated, $\beta_s$ is equal to 0.6.

Let $A_v$ be the area of vertical reinforcement and $A_h$ be the area of horizontal reinforcements while $f_v$ and $f_h$ represent the tensile stresses in vertical and horizontal reinforcements, respectively. $F_v$ and $F_h$ can be written as

$$F_v = A_v f_v \quad (3)$$

$$F_h = A_h f_h \quad (4)$$

The vertical shear strength given in eq.(1) is derived from half of the beam span, so, the total shear strength of the deep RC beam, $P_{max}$, is equal to $2F_{Ry}$. In view of eq.(1) through eq.(4), the total shear strength can be expressed as

$$P_{max} = 2F_{Ry} = 2\left[\left(0.51 f_c' b w_{strut}\right)\sin\theta + A_v f_v + \left(A_h f_h\right)\tan\theta\right] \quad (5)$$

Using the empirical relationships given in Fig. 5, the strains and the corresponding stresses in both horizontal and vertical reinforcements could be determined. Representing those stresses into eq.(5) leads to the shear strength of the deep beam.

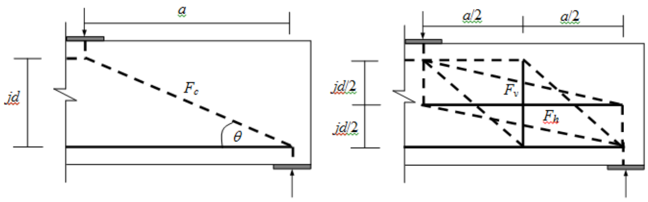
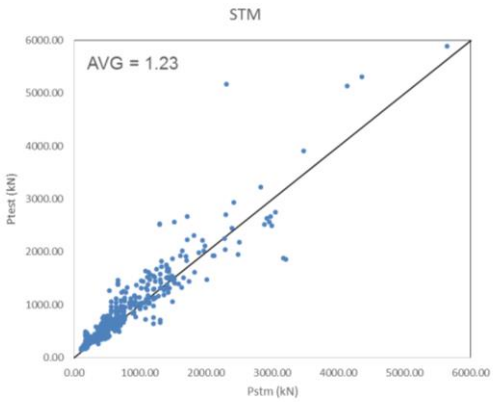



***Fig. 5*** *(a) Strut and tie model for concrete (b) Strut and tie model for crack controlled reinforcement.*

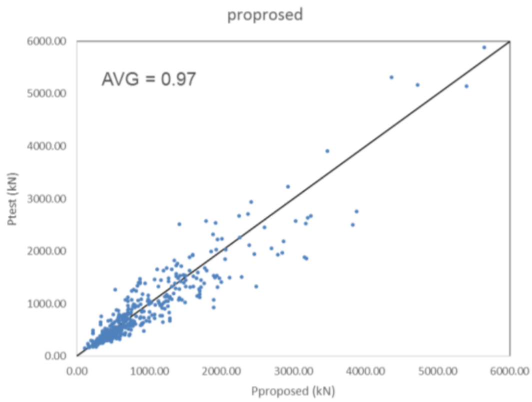
## 8. VALIDATION OF PROPOSED STRUT AND TIE MODEL

The original strut and tie model given in Appendix A of ACI 318-11 and the proposed strut and tie model from the previous section were applied to predict the maximum load of the beams from current test and from other literatures. The actual strength and the predicted strength computed from Appendix A of ACI 318-11 are shown in Fig.6 (a) while the comparison between actual and predicted strength from the proposed STM are given in Fig.6 (b). The strength ($P_{max}$) of 408 samples are compared.[8] – [13]




(a)

***Fig. 6*** *(a) Comparison of actual strength and predicted strength from ACI 318-11 (b) Comparison of actual strength and predicted strength from proposed STM. (cont.)*




(b)

***Fig. 6*** *(a) Comparison of actual strength and predicted strength from ACI 318-11 (b) Comparison of actual strength and predicted strength from proposed STM.*

## 9. CONCLUSIONS

Nine deep beams designed according to the Appendix A of ACI 318-11 were monotonically tested to failure. All specimens were subjected to single point load at mid-span. Based on all findings described in the previous part, the following conclusions could be drawn:

1. With the increasing in the a/d ratio, modes of failure of the tested specimens were changed from splitting of diagonal concrete strut to flexural shear failure regardless of the horizontal to vertical reinforcement ratio.

2. The amount of crack controlled reinforcement and the inclined angle to the direction of the diagonal

concrete strut has profound influence on the magnitude of the corresponding strains.

3. Based on the measured strains, it was evident that the strains in both horizontal and vertical crack controlled reinforcements were not always reached their yield strains.

4. Comparison between actual strength and predicted strength of the reference beams indicates that the predicted beam strength computed from Appendix A of ACI 318-11 seems to be conservative.

5. The results obtained from comparison of appendix A standard ACI 318-11 and proposed model are satisfactory by averages of the data being 1.23 and 0.97, respectively.

## 10. ACKNOWLEDGMENT

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