Numerical Experiments on Unsymmetrical Bending of Uniformly Strip Loaded Square Plates

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ABSTRACT

The objective of the present study deals with the numerical determination of the deflection responses for unsymmetrical bending problem of square plates that subjected to a uniformly distributed strip load by means of the finite element method. Four cases of the plate under consideration can be obtained from the combinations of simply supported (S), clamped (C) and free (F) edges, which are: (1) S-S-S-F plate, (2) C-C-C-S plate, (3) S-S-S-C plate, and (4) C-C-C-F plate. Numerical results are carried out for the deflection distributions along two middle lines, along the diagonal line, and along the free edge of the plates in the form of graph and table. Additionally, the deflection surface and its contour are also prepared and given here to more understand the plate deformation behaviors.

Keywords : *Square plate, Unsymmetrical bending, Finite element method*.

1. INTRODUCTION

To determine the solution of differential equation of plates [1], exact analytical solutions are the most desirable, but not always easily attainable. This is due to the difficulty lied in the requirements to satisfy both, the governing differential equation and all of the boundary conditions exactly. Therefore, many research works on plate-bending analysis have been conducted using a wide range of approximate mathematical techniques.

For the exact analytical methods, Sompornjaroensuk and Kiattikomol [2] used the method of finite Hankel integral transforms to solve the bending problem of uniformly loaded rectangular plates with a partial internal line support located at the center, simply supported on two opposite edges, and the remaining edges having the same type of support condition either simple, clamped or free supports. The solution was derived analytically in the closed-form expressions and results were numerically evaluated for representing deflection surfaces and slopes. Further analysis was made by Sompornjaroensuk and Kiattikomol [3] to find the bending moment distributions, support reactions, corner forces, and the bending stress intensity factors at the ends of partial internal line support.

For the numerical treatments, the finite element method is one of the powerful numerical approximate methods and has advanced with the development of computers. It has been very successful in finding the solution of various problems, particularly its applicability to problems having irregularly shaped or complex boundaries, which involved in discretizing the numerical solution domain.

Papanikolaou and Doudoumis [4] considered and analyzed the uplift potential developed at corners of at least two adjacent unilaterally supported edges of rectangular plates for various combinations of support conditions at the remaining edges. A numerical methodology for analysis of these plates, which taking consistently into account any unilateral support conditions was proposed. Thus, a total of nine different plate configurations were analyzed using the finite element software named MSC /NASTRAN. Numerical results were given in the proper application tables for the stress state of the plates.

Recently, Niamnin et al. [5] investigated the problem of symmetrical bending of uniformly strip loaded square plates with five different support conditions using ANSYS computer finite element software for Windows [6]. Results concerning the deflection surface and deflection contour and the distributions of deflection along the middle line and diagonal line of the plates are presented numerically and graphically.

In the present study, four cases of plate configuration loaded by uniformly distributed strip load that shown in Fig. 1 are numerically analyzed using ANSYS computer finite element program [6] to study the unsymmetrical bending behavior of plates representing in terms of the deflection responses. The analysis procedure is similar to the previous work that made by Niamnin et al. [5], who investigated the symmetrical bending of uniformly strip loaded square plates.

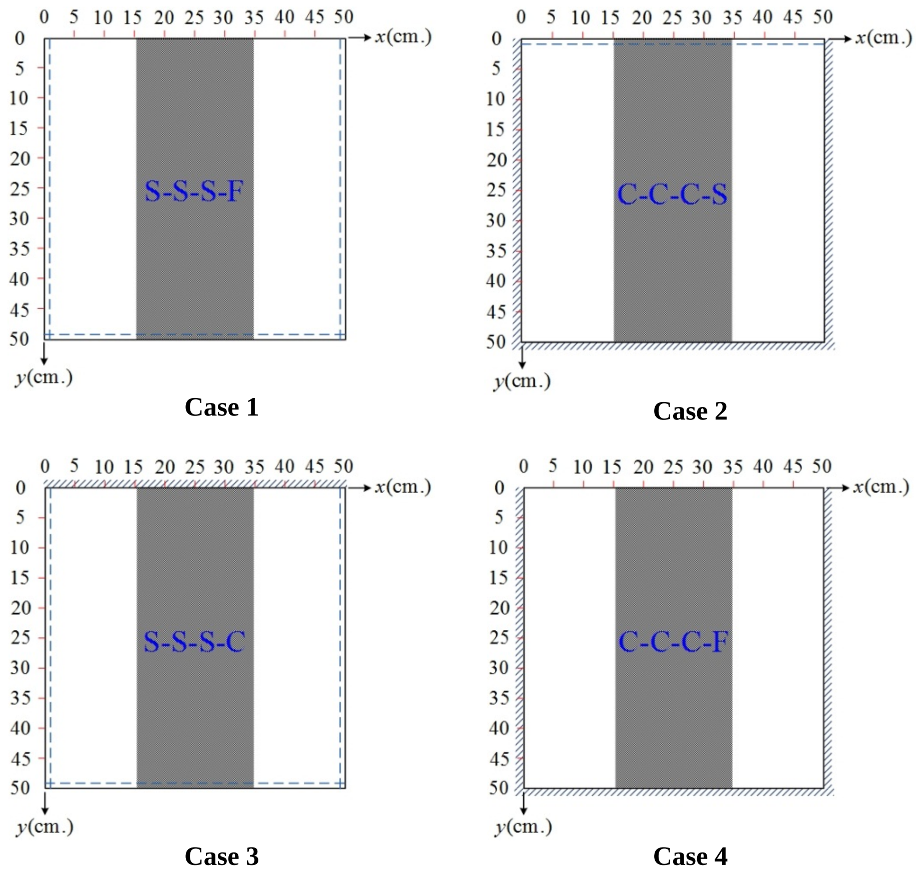

Fig. 1 *Uniformly strip loaded square plates with four different support conditions.*

Before proceeding further to analyze the problems under consideration, it is convenient to represent each edge boundary condition of the plates using the letter symbols that correspond with the designation as given in Khwakhong et al. [8]. Three different types of classical boundary conditions, namely, the simple, clamped, and free supports are designated by the letters S, C, and F, respectively. The letters designate the type of support starting at the left edge and proceeding in a counter-clockwise direction around the other three remaining edges. For example, consider the plate configuration of case 1 as shown in Fig. 1 which has three simply supported edges on the left, right, and lower sides of the geometry plan of the plate, and the remaining upper side is free. Therefore, the designation for this mentioned plate can be given by the letter symbols as S-S-S-F.

2. FINITE ELEMENT DETERMINATIONS

ANSYS finite element code [6] has been used in the present investigation in order to obtain the deflection responses of the plates. SHELL181 element type for the quadrilateral shape of the ANSYS Library [7] as depicted in Fig. 2 has been used to model the plate geometry. This shell element is suitable for analyzing thin to moderately-thick shell or plate structures. It is a four-node element with six degrees of freedom at each node (translations in the *x*, *y*, and *z* directions, and rotations about the *x*, *y*, and *z*-axis). For more details about this element, the reader should consult the ANSYS Theory Reference [6].

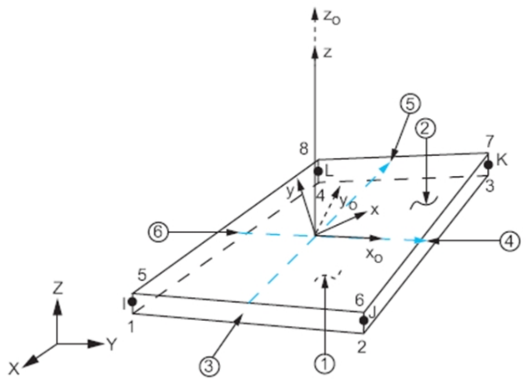

Fig. 2 *ANSYS shell element: SHELL181 [7].*

In the investigation, single square plates with plane dimensions of 50 cm. by 50 cm. and having uniform thickness of 3 mm. are used here. For the plate domain discretization, a uniform mesh for the square plate is shown in Fig. 3. It has 100 elements and 726 degrees of freedom.

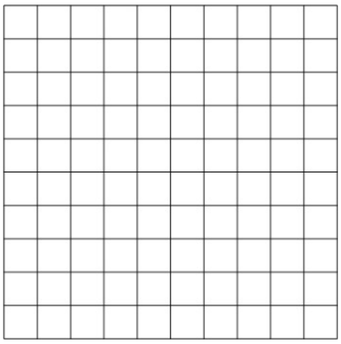

Fig. 3 *Mesh of plate discretization.*

3. NUMERICAL RESULTS

The deflection responses of the plates have been analyzed for: (1) isotropic plate with the Young's modulus (E) and Poisson's ratio (υ) taken as 55.43 GPa and 0.27, respectively [8], and (2) uniformly distributed strip load with intensity of 100 kg/m^2 (distributed load will be transformed to an equivalent point load applied to the node of elements that bounded in the shaded area as indicated in Fig. 1).

When implementing the ANSYS computer program, the numerical deflection values at each node of finite element plate model can be first obtained. They will be listed and imported into the SigmaPlot program [9] for preparation of graphical presentation. Figs. 4 to 7 illustrate the distribution of deflections along two middle lines, along the diagonal line and along the free edge of the plates. Their numerical deflection values that correspond with the plates shown in Figs. 4 to 7 are prepared and given in Table 1 to Table 4, respectively.

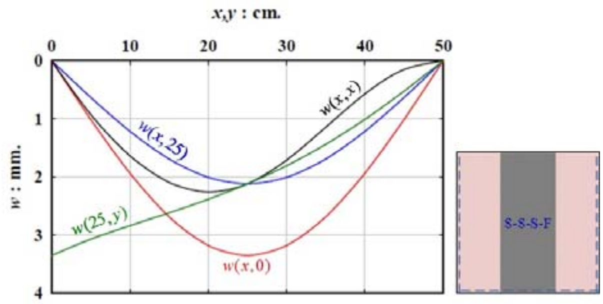

Fig. 4 *Deflection curves (w) for the plate of case 1.*

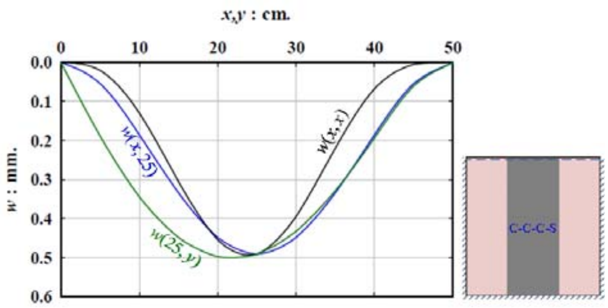

Fig. 5 *Deflection curves (w) for the plate of case 2.*

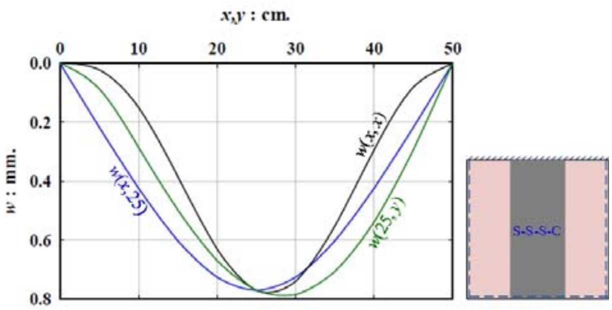

Fig. 6 *Deflection curves (w) for the plate of case 3.*

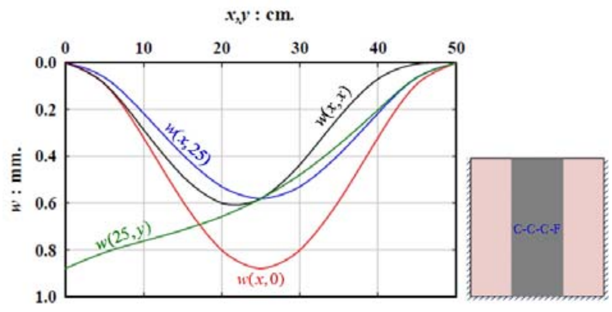

Fig. 7 *Deflection curves (w) for the plate of case 4.*

As demonstrated in Fig. 8 to Fig. 11, they present the comparative deflection distribution curves along two middle lines at $y = 25$ cm. and at $x = 25$ cm., along the free edge at $y = 0$ cm. for the plates of case 1 and case 4, and along the diagonal line at $y = x$ of the plates, respectively. In Fig. 8 and Fig. 10, they show the deflection distribution curves for a half plate. This is due to the symmetry of deflection with respect to the middle line of the plate at $x = 25$ cm. To clearly understand the overall plate deformation behaviors, Figs. 12 to 15 intentionally illustrate the deflection surfaces (3D) and deflection contours (2D) for the plates of case 1 to case 8, respectively.

Table 1 *Deflection values for the plate of case 1: S-S-S-F.*

***x,y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**
0	0.00000	3.35610	0.00000	0.00000
5	0.63387	3.07430	0.92771	1.01210
10	1.21690	2.84110	1.63960	1.93810
15	1.69450	2.62050	2.09810	2.69010
20	2.01060	2.38690	2.26310	3.18390
25	2.12140	2.12140	2.12140	3.35610
30	2.01060	1.80930	1.71400	3.18390
35	1.69450	1.44070	1.14600	2.69010
40	1.21690	1.01100	0.57170	1.93810
45	0.63387	0.52416	0.15226	1.01210
50	0.00000	0.00000	0.00000	0.00000

Table 2 *Deflection values for the plate of case 2: C-C-C-S.*

***x,y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**
0	0.00000	0.00000	0.00000
5	0.05440	0.18879	0.02060
10	0.18561	0.34425	0.12967
15	0.33415	0.44878	0.30555
20	0.44858	0.49765	0.45459
25	0.49128	0.49128	0.49128
30	0.44858	0.43272	0.39462
35	0.33415	0.32869	0.22013
40	0.18561	0.19388	0.06750
45	0.05440	0.06070	0.00517
50	0.00000	0.00000	0.00000

Table 3 *Deflection values for the plate of case 3: S-S-S-C.*

***x,y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**
0	0.00000	0.00000	0.00000
5	0.21717	0.08700	0.02210
10	0.42393	0.28525	0.14968
15	0.60247	0.49746	0.38460
20	0.72619	0.66941	0.63021
25	0.77061	0.77061	0.77061
30	0.72619	0.78504	0.73998
35	0.60247	0.70673	0.55229
40	0.42393	0.53836	0.29394
45	0.21717	0.29289	0.08080
50	0.00000	0.00000	0.00000

Table 4 *Deflection values for the plate of case 4: C-C-C-F.*

x,y : cm.	$w(x,25)$: mm.	$w(25,y)$: mm.	$w(x,x)$: mm.	$w(x,0)$: mm.
0	0.00000	0.87968	0.00000	0.00000
5	0.06270	0.81144	0.08980	0.09030
10	0.21606	0.76266	0.28567	0.32034
15	0.39255	0.71524	0.48436	0.59115
20	0.52967	0.65777	0.59983	0.80103
25	0.58102	0.58102	0.58102	0.87968
30	0.52967	0.47873	0.43596	0.80103
35	0.39255	0.34942	0.23280	0.59115
40	0.21606	0.20135	0.06920	0.32034
45	0.06270	0.06230	0.00519	0.09030
50	0.00000	0.00000	0.00000	0.00000

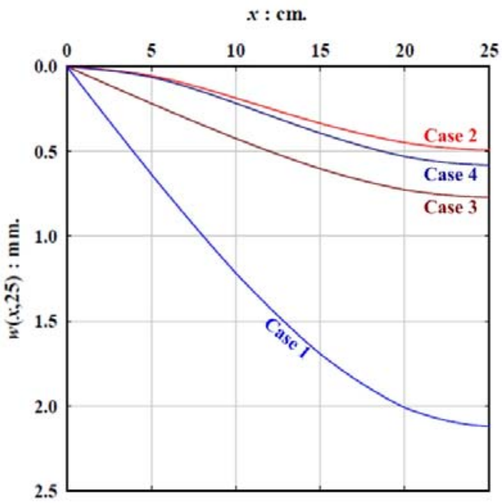

Fig. 8 *Comparative deflections along the middle line w(x,25) of the plates.*

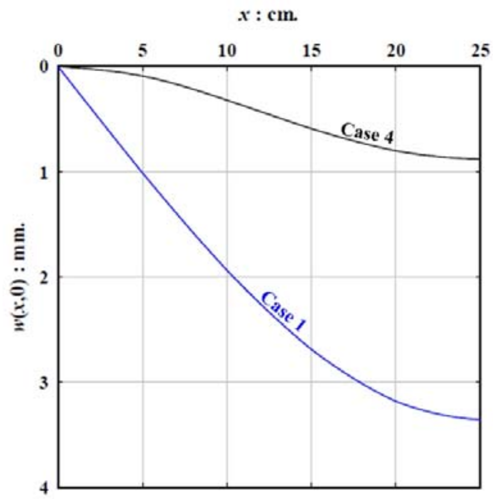

Fig. 10 *Comparative deflections along the free edge w(x,0) of the plates.*

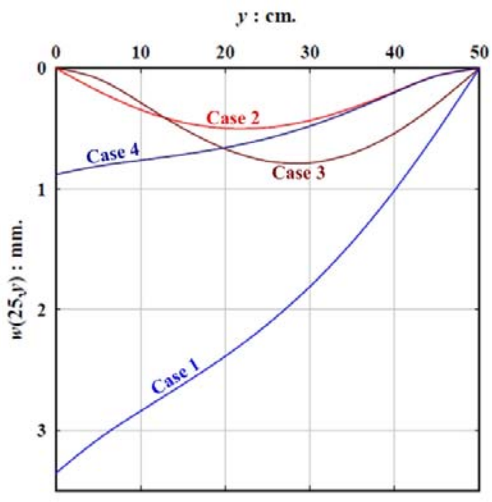

Fig. 9 *Comparative deflections along the middle line w(25,y) of the plates.*

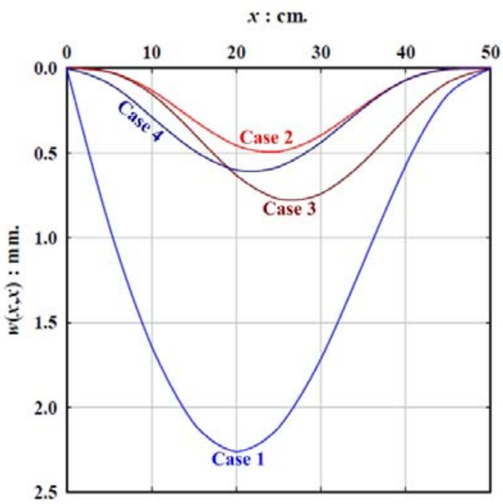

Fig. 11 *Comparative deflections along the diagonal line w(x,x) of the plates.*

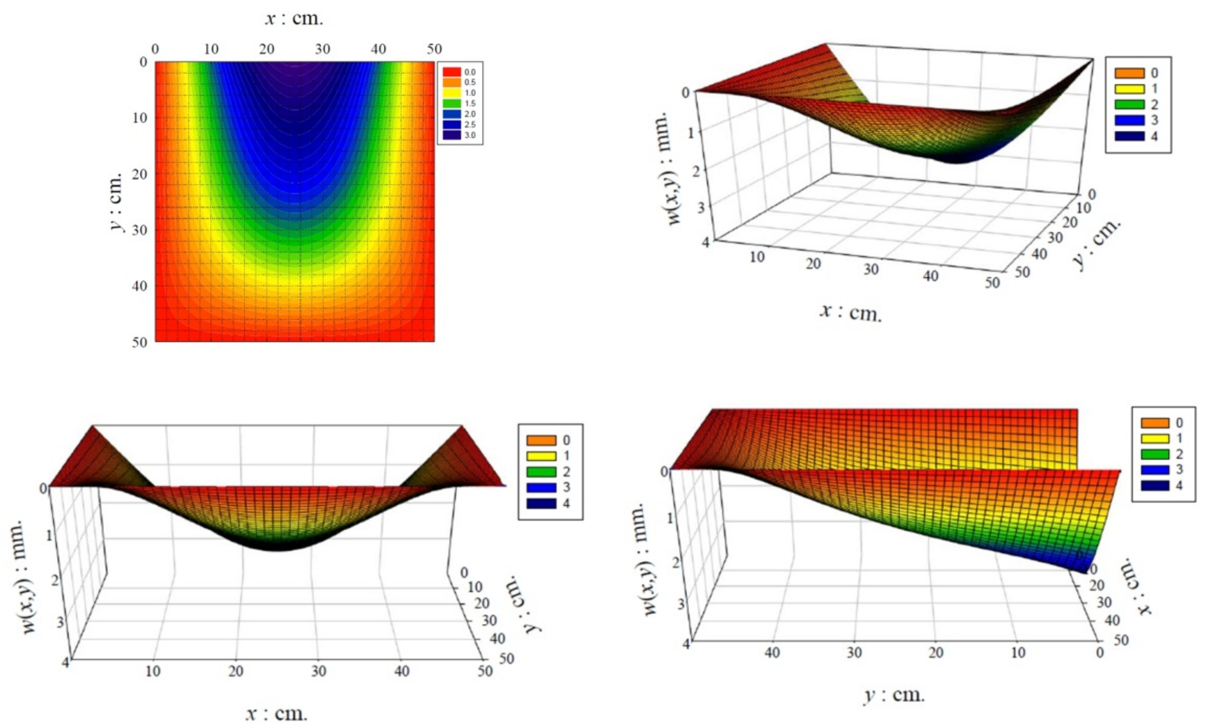

Fig. 12 *Deflection surfaces w(x,y) and contour for the plate of case 1: S-S-S-F.*

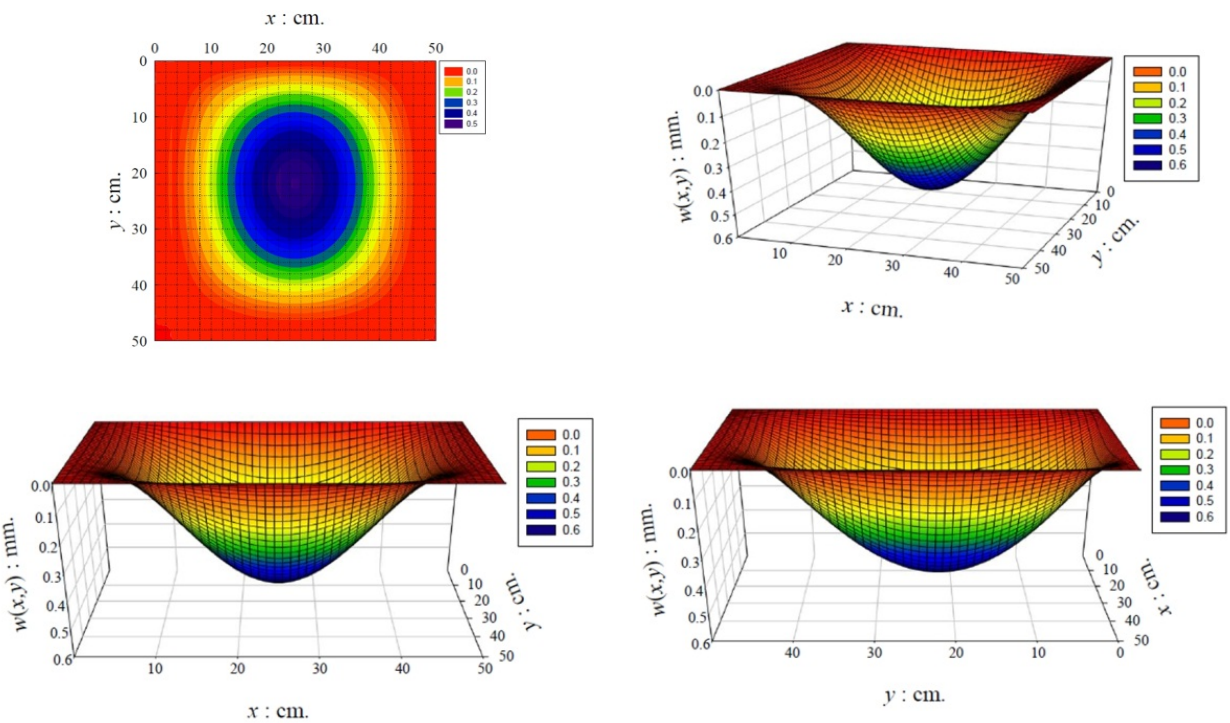

Fig. 13 *Deflection surfaces w(x,y) and contour for the plate of case 2: C-C-C-S.*

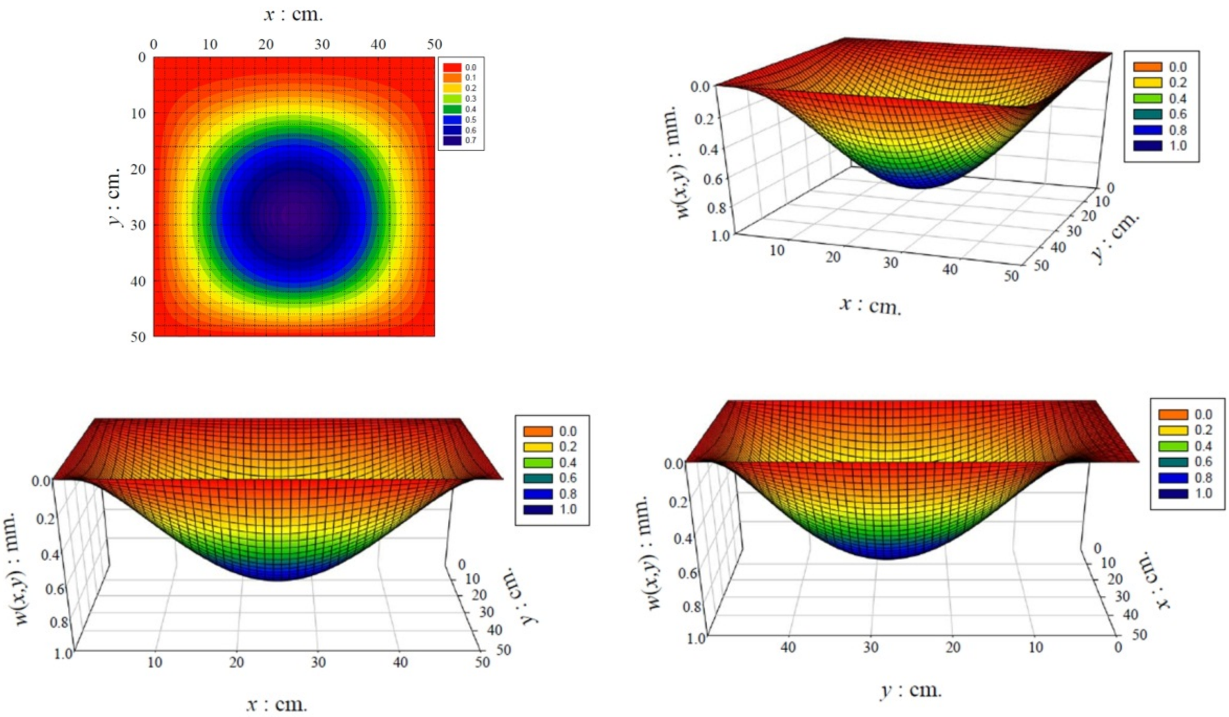

Fig. 14 *Deflection surfaces w(x,y) and contour for the plate of case 3: S-S-S-C.*

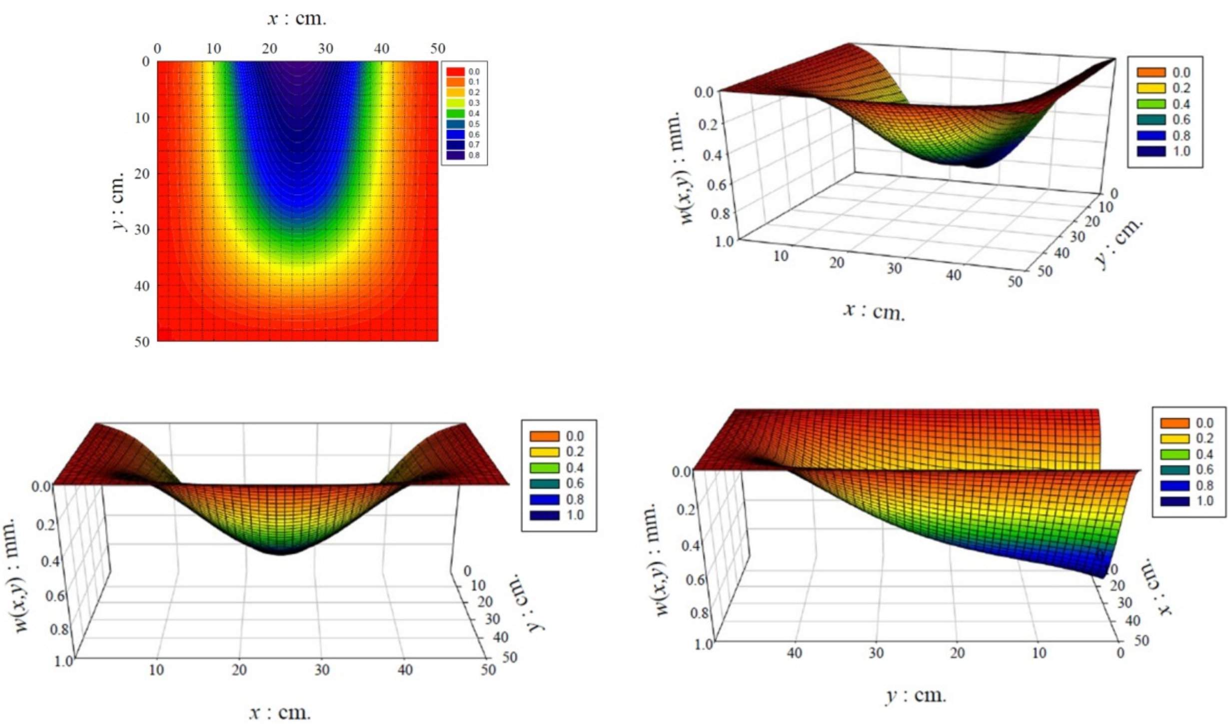

Fig. 15 *Deflection surfaces w(x,y) and contour for the plate of case 4: C-C-C-F.*

4. CONCLUSIONS

Numerical experiments for the unsymmetrical bending analysis of uniformly strip loaded square plates are presented in this study. The ANSYS computer finite element software for windows has been used to model and analyze for determining deflection responses of the plates. Results concerning the deflection surface and its contour and the distribution of deflections along the middle line, along the diagonal line, and along the free edge of the plates are graphically presented. Their numerical deflection values are also tabulated in the form of table for easy use by other investigators.

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