Bending of Square Plates with Mixed Conditions between Simply Supported and Clamped Edges

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ABSTRACT

Numerical experiments for the elastic bending analysis of single square plates having one edge mixed between simple and clamped supports, loaded by uniformly distributed load are studied in this work. Nine different plate configurations have been analyzed using the ANSYS computer finite element code to determine the response of the plates in terms of the deflection. The obtained results are presented for the deflection surface and deflection contour of the plate. Also, the distributions of deflection along two middle lines, along the diagonal line, and along the free edge of the plates are demonstrated graphically. Numerical values for the plate deflection are given in tabular form for easy use by other researchers.

Keywords : *Square plate, Mixed support conditions, Finite element method.*

1. INTRODUCTION

Within the framework of classical plate theory [1], [2], the problems of plate with mixed edge boundary conditions are of considerable academic and practical interest. The important problem of bending of uniformly loaded rectangular plate having clamped portions along arbitrary sections of the edges had been worked out analytically by Kurata [3] using the determination of resisting moments introduced along the clamped portions of the edge. In his work, there were three problem cases of the plate analyzed, which were: (1) symmetrical bending of a simply supported plate clamped along the middle portion of one edge, (2) symmetrical bending of a simply supported plate clamped along the middle portion of four edges, and (3) symmetrical bending of a simply supported plate clamped in the neighborhood of every corner.

Kiattikomol et al. [4] analytically considered the problem of uniformly loaded rectangular plates simply supported on two opposite edges, but only partially constrained along the other two edges. Two specific problems were formulated in their works. In the first case, the plate had symmetrical partial simple supports at the other two edges, while the rest of these edges were free. The second case was that the plate was simply supported on three edges, but partially constrained by a simple support on the fourth edge. The results were shown for the deflection of the free edges and the change in strain energy due to the presence of partial simple supports. Kiattikomol et al. [5] dealt with the analytical solution of a square plate simply supported at all corners by an equal-leg angle under uniformly distributed load. By choosing the proper finite Hankel integral transform, the solution of problem can be reduced to finding the solution of inhomogeneous Fredholm integral equation of the second kind. Numerical results were prepared for the moments and deflections along the middle line of the square plate with various lengths of simple support legs.

In the recent years, a comprehensive study of uniformly loaded rectangular plate with a partial internal line support located at the center, simply supported on two opposite edges, and the remaining edges having the same type of support conditions either simple, clamped or free supports was solved analytically. The results were numerically evaluated for representing deflection surfaces, bending moments, support reactions, corner forces, the change in strain energy, and the bending stress intensity factors [6], [7].

For the numerical treatments, the problem of plate supported by the unilateral supports with no restrained corners is involved to the natural receding contact problem. This mentioned problem had been numerically investigated by Salamon et al. [8], who modeled the unilateral supports by discrete elastic springs using a finite element method where the characteristic of discrete springs as well as the spring stiffnesses could be varied from the elastic support to nearly rigid support. Another numerical method was done by Hu and Hartley [9], who utilized a direct boundary element method to study the problem of general polygonal shape of plates in which the support system consisted of discrete elastic springs.

Tohsaman et al. [10] experimentally studied bending behavior of uniformly loaded, aluminum square plates with mixed boundary support conditions between simple and free supports on two opposite edges. The plate deflection responses along two middle lines and diagonal line of the plate were determined and expressed in the form of polynomial regression equations within the acceptable coefficient of determination not less than 0.98. At the same time, problems of square plates having the same type of support; either simply supported or clamped, at all corners by an equal-leg angle under uniformly distributed load were experimentally investigated by Boonchareon et al. [11]. The influences of the support constraints, support lengths, and plate's flexural rigidities in terms of plate thickness were taken into account.

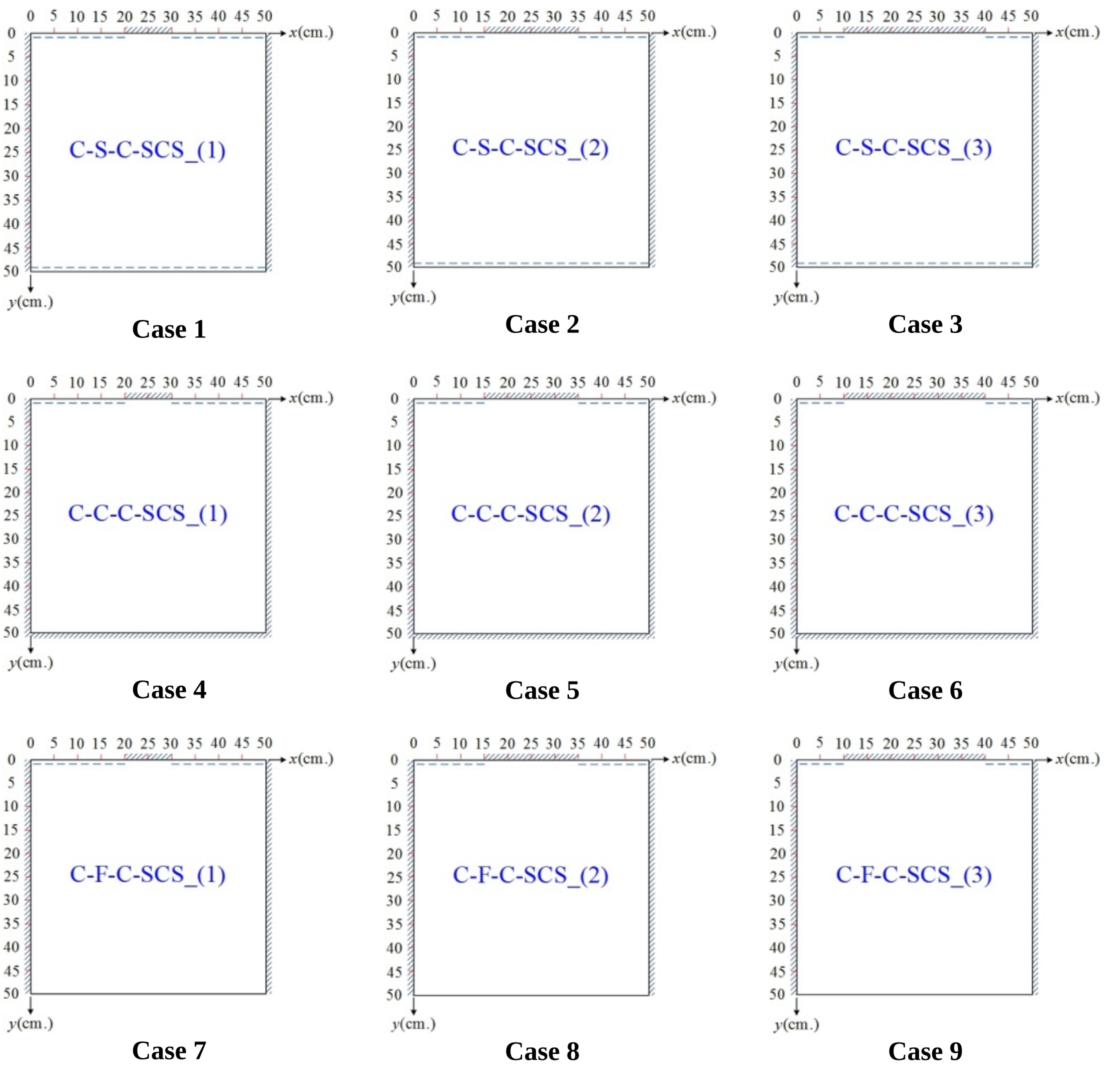

Fig. 1 *Square plates having partial clamped support at the center of one edge.*

2. STATEMETN OF PROBLEM

In the present work, the configuration of plates with mixed edge conditions between simple and clamped supports that shown in Fig. 1 are considered. It can be seen that there are two opposite clamped edges placed at the left and the right edges. In all cases of the plate, the upper edge is the mixed support conditions, while the lower edge can be either simple, clamped or free supports.

However, it is expedient to use the letter symbols designation that depicted in Fig. 1 to separate each case of the plates. Thus, the designation as given in Tohsaman et al. [10] and Boonchareon et al. [11] are used for this purpose. Noted that there are three different types of classical boundary conditions, different letters designating the simple, clamped, and free supports are S, C, and F, respectively. Since the plate has more than one type of boundary support conditions (mixed boundary support conditions) along the edge, this edge will be designated by a sequence of successive combinatorial letters.

Additionally, the numbers (1), (2), and (3) are used in order to separate the plates when the boundary conditions have the same type but different in the lengths of clamped support portion. For all cases of the plate under consideration, the plate has the plane dimensions of 50 cm. by 50 cm. and 3 mm. in thickness, and they are subjected to a uniformly distributed load with intensity of 100 kg/m^2.

The analysis is based upon the finite element method with implementation of the academic well-known ANSYS computer finite element software package for Windows [12]. The details of ANSYS finite element program will be described in the next section.

3. ANALYSIS PROCEDURE

Before proceeding further to analyze the plate-bending problems, it is necessary to suppose the material properties of the plate, which are similar to material properties of aluminum plate that have been used in Tohsaman et al. [10] and Boonchareon et al. [11]. Therefore, the Young's modulus (E) and Poisson's ratio (υ) are taken to be 55.43 GPa and 0.27, respectively.

ANSYS finite element program [12] has been used in the analysis of the present study in order to determine the deflection response of the plates. Shell element of quadrilateral shape as shown in Fig. 2 (SHELL181 element type of the ANSYS Library [13]) is selected to model and analyze the plates. It is a four-node element with six degrees of freedom at each node (translations in the x, y, and z directions, and rotations about the x, y, and z-axis). This element is suitable for analyzing thin to moderately-thick shell or plate structures.

Based on the finite element concept and procedure [14], the plate is discretized with a uniform mesh of 100 square elements as seen in Fig. 3. Since the plates are bounded with different edge support conditions corresponding to each case of the plates that shown in Fig. 1, the proposed criteria to be used in the finite element model for applying the boundary conditions at the transition points of mixed boundary supports (point having two different boundary support types) can be described that stronger boundary support conditions will be chosen to represent the boundary support conditions at that transition point. For example, consider the plate of case 1 that depicted in Fig. 1, the point of the plate at $x = 20$ cm. and $y = 0$ cm. is one of the transition points where the simple support changes to the clamped support. Thus, the clamped support condition will be chosen to represent the conditions at that point in the finite element support model.

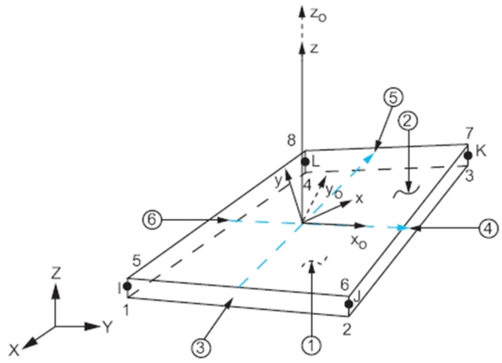

Fig. 2 *ANSYS shell element: SHELL181 [13].*

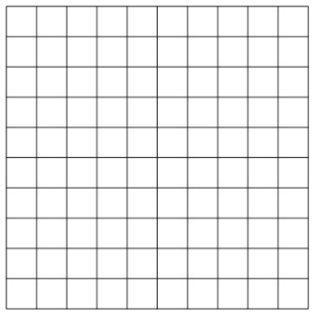

Fig. 3 *Mesh of plate discretization.*

4. NUMERICAL RESULTS

After performing analysis using the ANSYS computer program, the deflection results at each node of finite element plate model can be determined. They are listed and imported into the SigmaPlot program [15]. This program can plot a smooth sharp variation in dependent discretized values within 2D and 3D data sets.

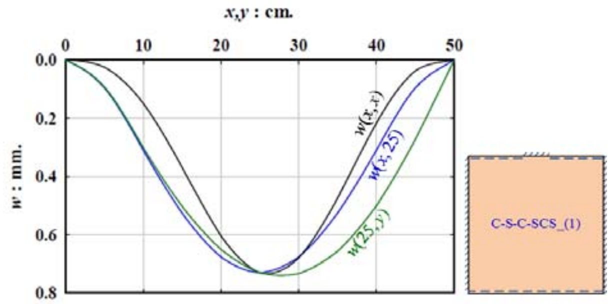

__Fig. 4__ Deflection curves (w) for the plate of case 1.

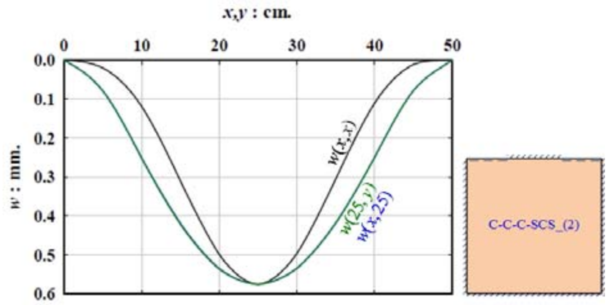

__Fig. 8__ Deflection curves (w) for the plate of case 5.

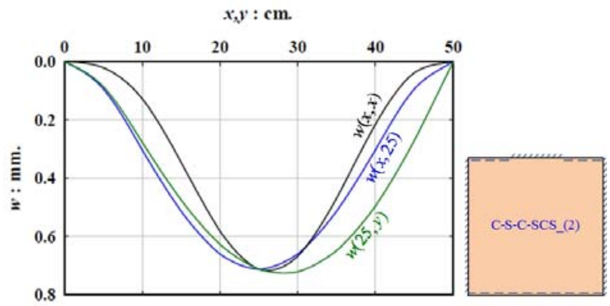

__Fig. 5__ Deflection curves (w) for the plate of case 2.

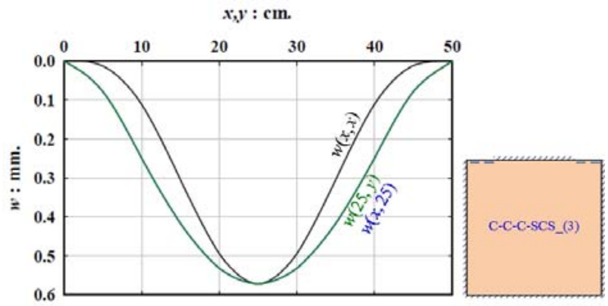

__Fig. 9__ Deflection curves (w) for the plate of case 6.

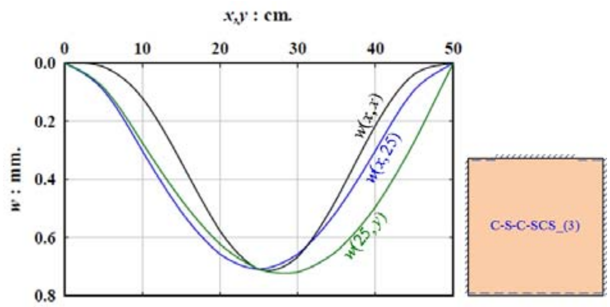

__Fig. 6__ Deflection curves (w) for the plate of case 3.

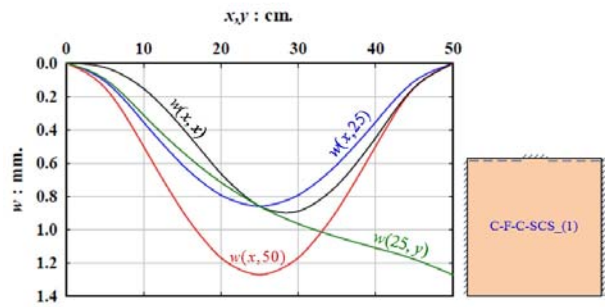

__Fig. 10__ Deflection curves (w) for the plate of case 7.

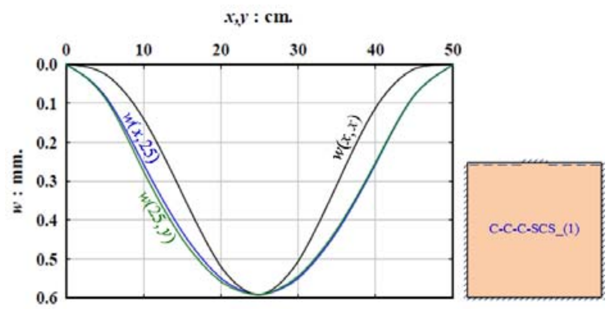

__Fig. 7__ Deflection curves (w) for the plate of case 4.

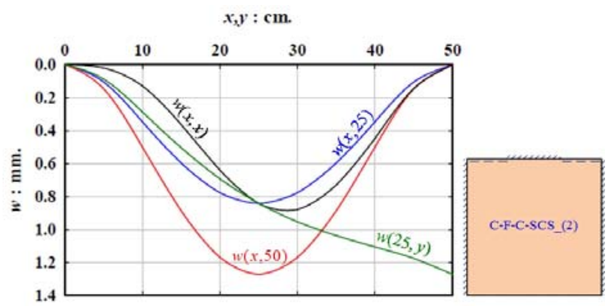

__Fig. 11__ Deflection curves (w) for the plate of case 8.

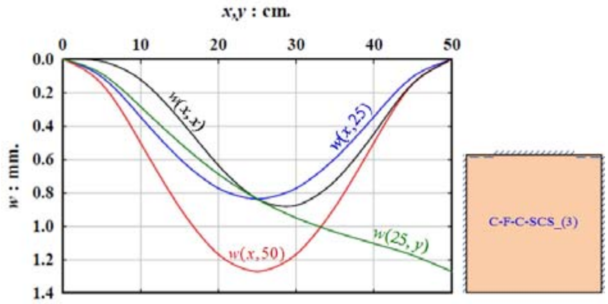

Fig. 12 *Deflection curves (w) for the plate of case 9.*

As shown in Figs. 4 to 12, they simultaneously present the distribution of deflections along two middle lines at $y = 25$ cm. and at $x = 25$ cm., along the diagonal line at $y = x$ and along the free edge at $y = 50$ cm. of the plates with different boundary support conditions (the latter is found in the plates of case 7 to case 9 only).

Tables 1 to 9 provide, respectively, the numerical values for the deflections corresponding to each case of the plates that previously shown in Figs. 4 to 12. It can be noted that deflection distributions along two middle lines for the plates of case 4, 5, and 6 are nearly closed together. The differences can be seen in Tables 4 to 6.

To clearly understand the deformation behaviors of the plates with different boundary support conditions, the deflection contours and deflection surfaces for each case of the plates are, respectively, prepared in Figs. 13 and 14. Figs. 15 to 18 are intentionally presented for showing the effects of different plate support constraint conditions. It can be noted that when the length of clamped support at the upper edge of the plates is increased, the difference in deflection is decreased within the same boundary support conditions.

Table 1 *Deflection values for the plate of case 1: C-S-C-SCS_(1).*

***x,y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**
0	0.00000	0.00000	0.00000
5	0.09580	0.09230	0.02530
10	0.30977	0.30031	0.14904
15	0.52577	0.50264	0.36930
20	0.67687	0.65065	0.60413
25	0.73084	0.73084	0.73084
30	0.67687	0.73375	0.67999
35	0.52576	0.65698	0.47433
40	0.30977	0.50031	0.21684
45	0.09580	0.27270	0.03810
50	0.00000	0.00000	0.00000

Table 2 *Deflection values for the plate of case 2: C-S-C-SCS_(2).*

***x,y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**
0	0.00000	0.00000	0.00000
5	0.09370	0.08590	0.02060
10	0.30278	0.27665	0.12718
15	0.51322	0.47478	0.34589
20	0.66069	0.62755	0.58186
25	0.71291	0.71291	0.71291
30	0.66069	0.72153	0.66863
35	0.51322	0.64896	0.46895
40	0.30278	0.49571	0.21517
45	0.09370	0.27057	0.03790
50	0.00000	0.00000	0.00000

Table 3 *Deflection values for the plate of case 3: C-S-C-SCS_(3).*

***x,y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**
0	0.00000	0.00000	0.00000
5	0.09330	0.08430	0.01280
10	0.30119	0.27344	0.11938
15	0.51059	0.46931	0.33999
20	0.65715	0.62252	0.57708
25	0.70917	0.70917	0.70917
30	0.65715	0.71894	0.66632
35	0.51059	0.64738	0.46789
40	0.30118	0.49478	0.21484
45	0.09330	0.27016	0.03780
50	0.00000	0.00000	0.00000

Table 4 *Deflection values for the plate of case 4: C-C-C-SCS_(1).*

***x,y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**
0	0.00000	0.00000	0.00000
5	0.08010	0.08530	0.02420
10	0.25634	0.27407	0.13918
15	0.43032	0.44792	0.33236
20	0.54989	0.55825	0.51970
25	0.59229	0.59229	0.59229
30	0.54989	0.54469	0.50618
35	0.43032	0.42390	0.30933
40	0.25634	0.25190	0.11144
45	0.08010	0.07880	0.01110
50	0.00000	0.00000	0.00000

Table 5 *Deflection values for the plate of case 5: C-C-C-SCS_(2).*

***x,y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**
0	0.00000	0.00000	0.00000
5	0.07820	0.07920	0.01980
10	0.25006	0.25182	0.11842
15	0.41898	0.42176	0.31031
20	0.53534	0.53692	0.49904
25	0.57611	0.57611	0.57611
30	0.53534	0.53431	0.49645
35	0.41898	0.41773	0.30520
40	0.25006	0.24917	0.11043
45	0.07820	0.07800	0.01100
50	0.00000	0.00000	0.00000

Table 6 *Deflection values for the plate of case 6: C-C-C-SCS_(3).*

x,y **: cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**
0	0.00000	0.00000	0.00000
5	0.07790	0.07780	0.01230
10	0.24858	0.24876	0.11093
15	0.41659	0.41660	0.30470
20	0.53208	0.53218	0.49457
25	0.57270	0.57270	0.57270
30	0.53208	0.53204	0.49447
35	0.41659	0.41652	0.30443
40	0.24858	0.24856	0.11029
45	0.07790	0.07780	0.01100
50	0.00000	0.00000	0.00000

Table 7 *Deflection values for the plate of case 7: C-F-C-SCS_(1).*

x,y **: cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,50) : mm.**
0	0.00000	0.00000	0.00000	0.00000
5	0.10766	0.09450	0.02520	0.14632
10	0.35361	0.31073	0.15145	0.49973
15	0.61003	0.53204	0.38717	0.88690
20	0.79400	0.71662	0.66336	1.17050
25	0.86051	0.86051	0.86051	1.27380
30	0.79400	0.96642	0.89106	1.17050
35	0.61003	1.04550	0.73598	0.88690
40	0.35361	1.11090	0.44881	0.49973
45	0.10766	1.17910	0.14501	0.14632
50	0.00000	1.27380	0.00000	0.00000

Table 8 *Deflection values for the plate of case 8: C-F-C-SCS_(2).*

x,y **: cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,50) : mm.**
0	0.00000	0.00000	0.00000	0.00000
5	0.10557	0.08810	0.02060	0.14656
10	0.34662	0.28709	0.12960	0.50010
15	0.59747	0.50419	0.36377	0.88712
20	0.77780	0.69353	0.64109	1.17040
25	0.84257	0.84257	0.84257	1.27370
30	0.77780	0.95419	0.87967	1.17040
35	0.59747	1.03750	0.73057	0.88712
40	0.34662	1.10630	0.44715	0.50010
45	0.10557	1.17690	0.14486	0.14656
50	0.00000	1.27370	0.00000	0.00000

***Table 9** Deflection values for the plate of case 9: C-F-C-SCS_(3).*

x,y : cm.	$w(x,25)$: mm.	$w(25,y)$: mm.	$w(x,x)$: mm.	$w(x,50)$: mm.
0	0.00000	0.00000	0.00000	0.00000
5	0.10519	0.08650	0.01280	0.14653
10	0.34502	0.28387	0.12177	0.50008
15	0.59483	0.49871	0.35785	0.88708
20	0.77426	0.68847	0.63629	1.17040
25	0.83882	0.83882	0.83882	1.27360
30	0.77426	0.95158	0.87736	1.17040
35	0.59483	1.03590	0.72952	0.88708
40	0.34502	1.10530	0.44684	0.50008
45	0.10519	1.17640	0.14482	0.14653
50	0.00000	1.27360	0.00000	0.00000

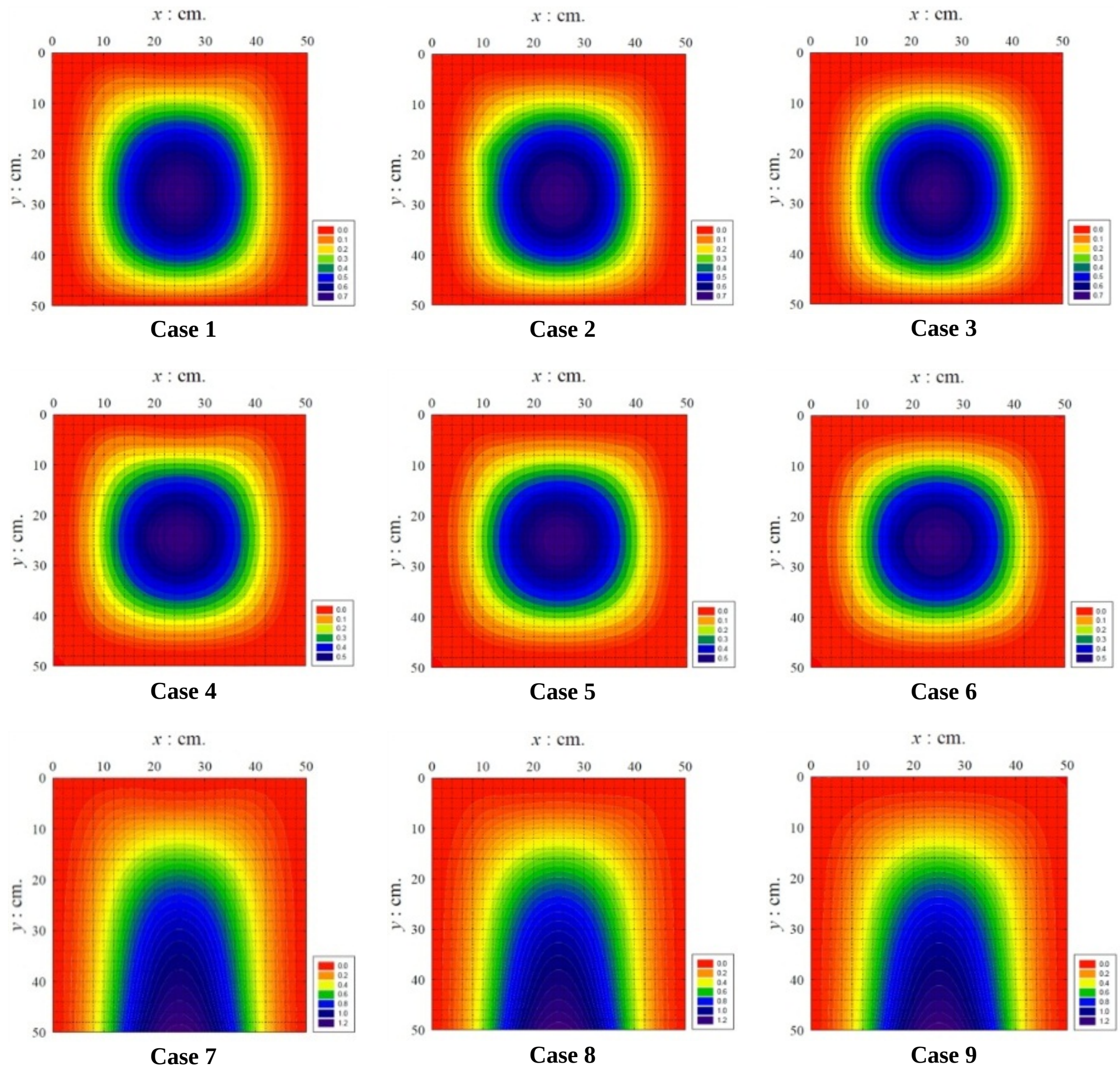

***Fig. 13** Deflection contours w(x,y) for nine different square plates.*

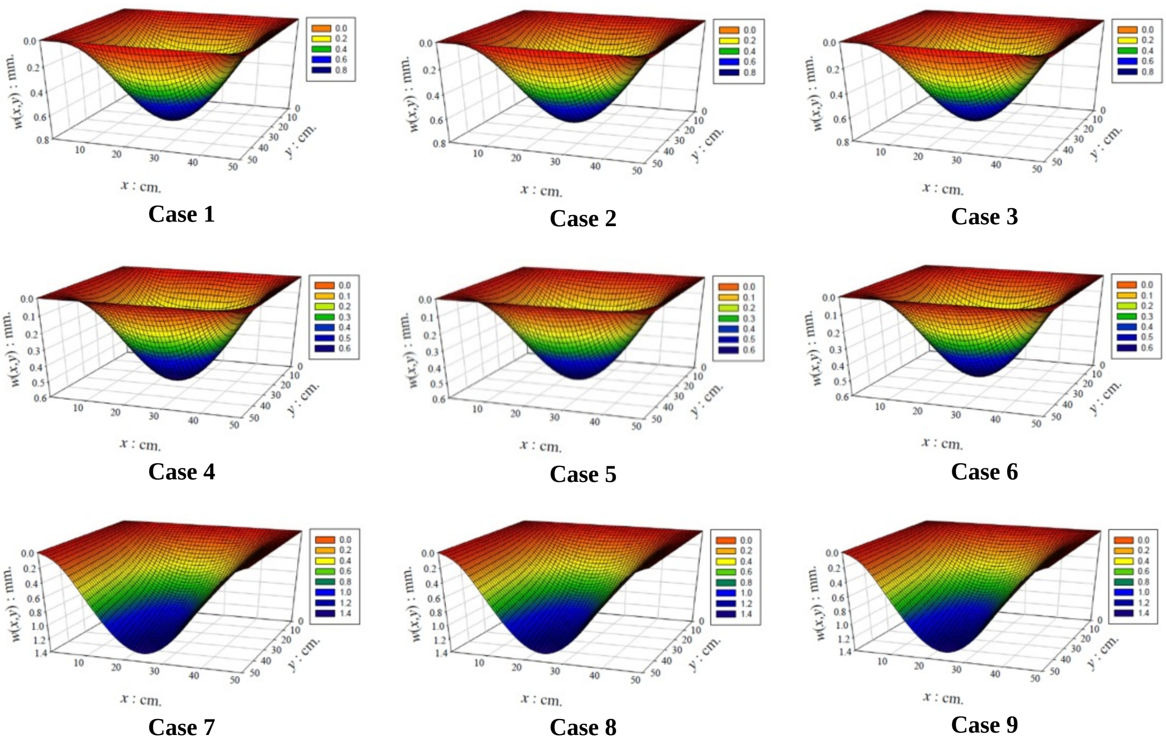

Fig. 14 *Deflection surfaces w(x,y) for nine different square plates.*

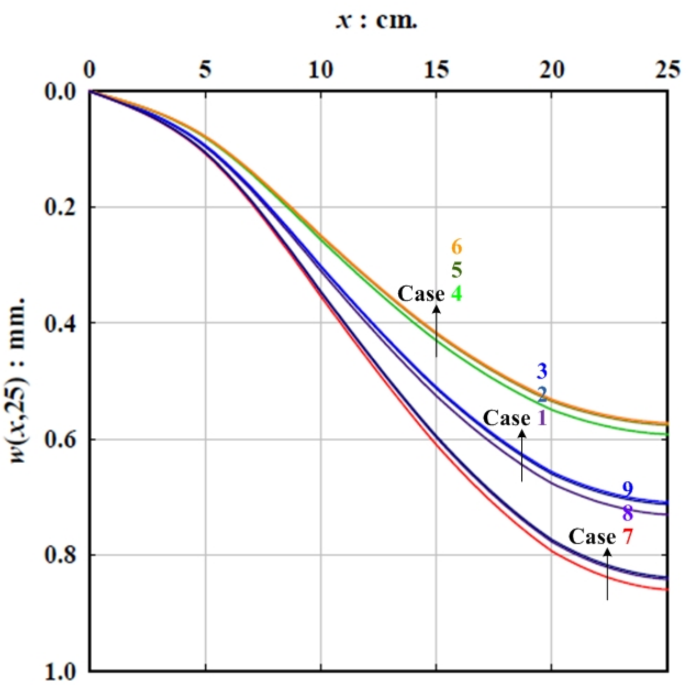

Fig. 15 *Comparative deflections along the middle line w(x,25) of the plates.*

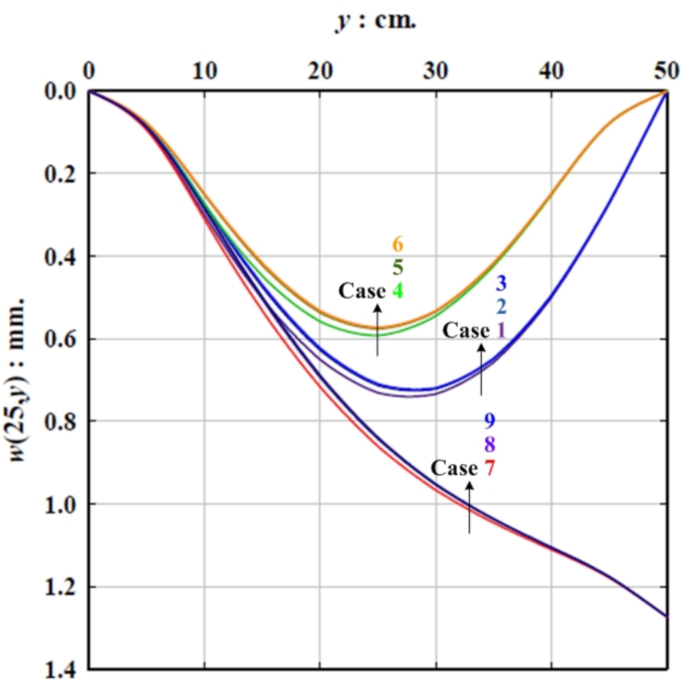

Fig. 16 *Comparative deflections along the middle line w(25,y) of the plates.*

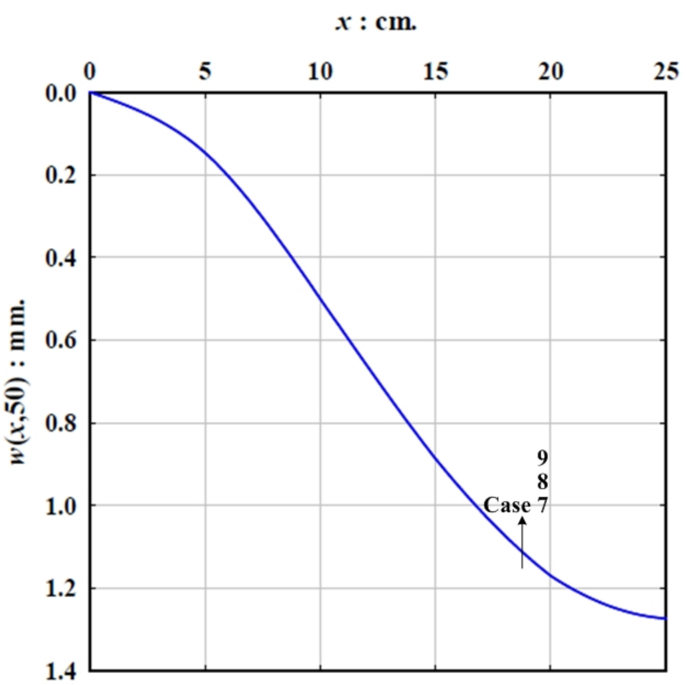

Fig. 17 *Comparative deflections along the free edge w(x,50) of the plates.*

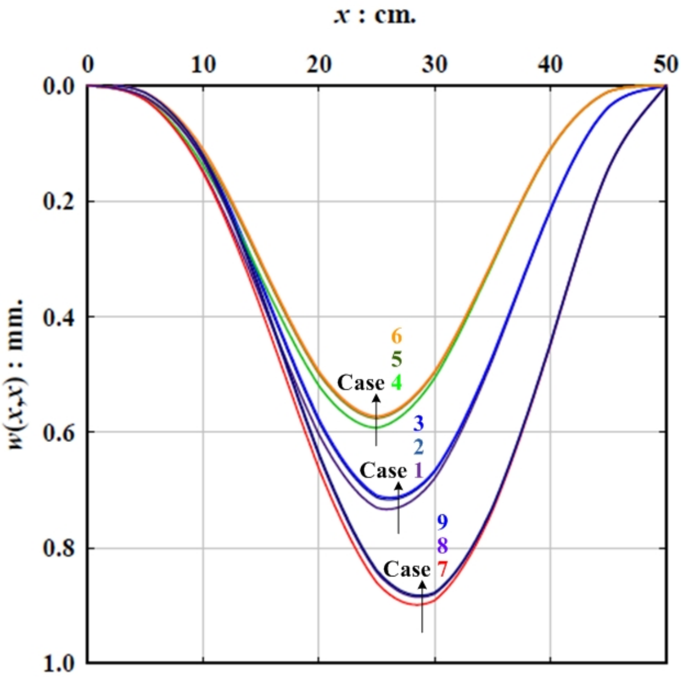

Fig. 18 *Comparative deflections along the diagonal line w(x,x) of the plates.*

5. CONCLUSIONS

The present work deals with a numerical experiment for modeling and analyzing single square plate with mixed boundary conditions between simple and clamped supports placed on one edge in which the plate is subjected to a uniformly distributed load. With the implementation of the ANSYS computer finite element software package, the deflection responses can be determined numerically. The obtained results are graphically demonstrated and then numerically tabulated in the form of table for the deflection distributions along the middle line, along the diagonal line, and along the free edge of the plates.

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