On the Free-Vibration Frequencies of Square Plates with Different Edge Conditions

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ABSTRACT

The present paper deals with a comprehensive study of frequency coefficients for natural free vibrations of twenty-one square plates with different edge support conditions. The ANSYS finite element computer program is used for modeling and analyzing the plate vibration problems with a dense net of 10000 elements in order to determine the frequencies numerically. Results concerning the frequency coefficients and their corresponding mode shapes are given in the form of graph and table for easy reference, which can be used as benchmarks for other alternative methods.

Keywords : *Square plate, Free vibration, Frequency Analysis, Finite element method.*

1. INTRODUCTION

Plates are fundamental components in structural engineering design and application. They are proved to be useful models for more complex structures. A significant contribution and extensive study in the area of plate-bending analysis together with its application have been collected and also summarized in a fundamental monograph of Timoshenko and Woinowsky-Krieger [1] that represented a profound analysis of various plate-bending problems. Additionally, their vibrational behavior is also of great interest, especially the free vibration characteristics (natural frequencies and their corresponding mode shapes). The analysis of free vibration (eigenvalue) problems is of basic and applied interest in several fields of science and technology. An exhaustive summary of the published literature on the free vibrations of various shaped plates is available in Leissa [2].

With the advent of very efficient high speed computers that allowed solution of a large number of algebraic equations in a relatively short time, Cheung and Kong [3] applied a finite strip method to the vibration problem of rectangular plates with complicated boundary and internal support conditions. Avalos et al. [4] proposed an approximate solution to the problem of free vibrations of annular plates of stepped thickness by means of a superposition of simple polynomial coordinates functions. A comparison with frequency coefficients obtained using the finite element code SAMCEF was also presented and good agreement was shown to exist.

Bambill et al. [5] considered the effect of different coordinate functions in vibrating rectangular plate with a free edge. Three approaches, namely, the Rayleigh's optimization concept in cooperating with trigonometric functions, the classical Rayleigh-Ritz method, and the finite element method were used to determine frequency coefficients numerically.

Laura et al. [6] dealt with the determination of the fundamental frequency of transverse vibration of orthotropic, circular annular plates. Two independent methods were applied: the optimized Rayleigh-Ritz and the efficient finite element code named ALGOR. They concluded that the finite element results were extremely accurate in view of the high number of elements used and the analytical results were in very good engineering agreement. Laura et al. [7] used the SAMCEF finite element code to numerically analyze the vibrating circular plates of rectangular orthotropy and carrying a concentrated mass at the plate center. The mesh for the circular plate has 1513 elements and 7275 degrees of freedom. In addition, Laura et al. [8] determined the fundamental frequency coefficients of transverse vibrations of an annular plate with free edges and two intermediate concentric circular supports. The ALGOR finite element code was used to model the plate having 5835 elements for one-quarter of the plate due to symmetry of geometry.

Wei et al. [9] introduced the discrete singular convolution algorithm for vibration analysis of rectangular plates with mixed boundary conditions. A unified scheme was proposed for the treatment of simply supported, clamped, and transversely supported with nonuniform elastic rotational restraint boundary conditions. An extensive work was made by Ng et al. [10], who presented a comprehensive comparison study between the discrete singular convolution and the well-known global method of generalized differential quadrature for vibration analysis of rectangular plates. Xing and Liu [11] proposed the differential quadrature finite element method for the free vibration analysis of thin plates with curvilinear domain. Various shaped plates with different types of regular and irregular planforms, namely, eccentric sectorial plate, elliptic plate, triangular plate, pentagonal plate, symmetric trapezoidal plate and rhombic plate were analyzed numerically.

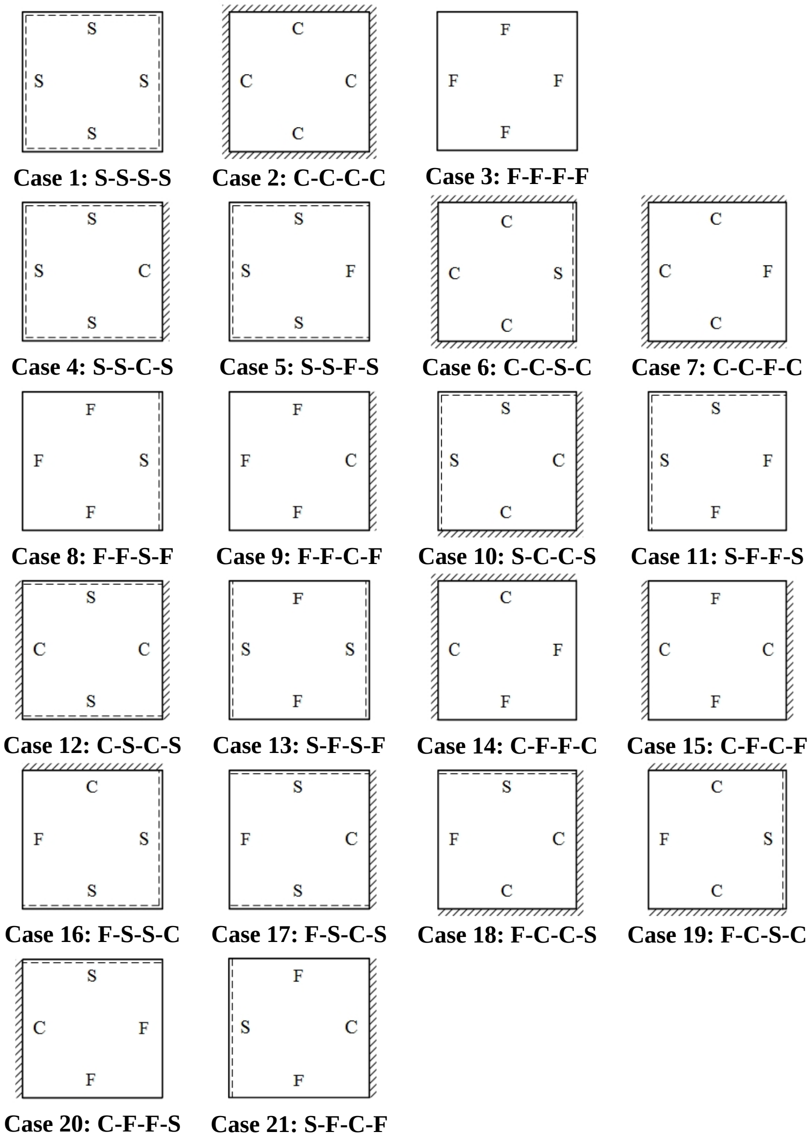

***Fig. 1** Configuration of 21 square plates with different edge conditions.*

2. FINITE ELEMENT DETERMINATION

In the present investigation, twenty-one square plates with combinations of simply supported (S), clamped (C), and free (F) edges that demonstrated in Fig. 1 are numerically analyzed by means of the ANSYS finite element program [12].

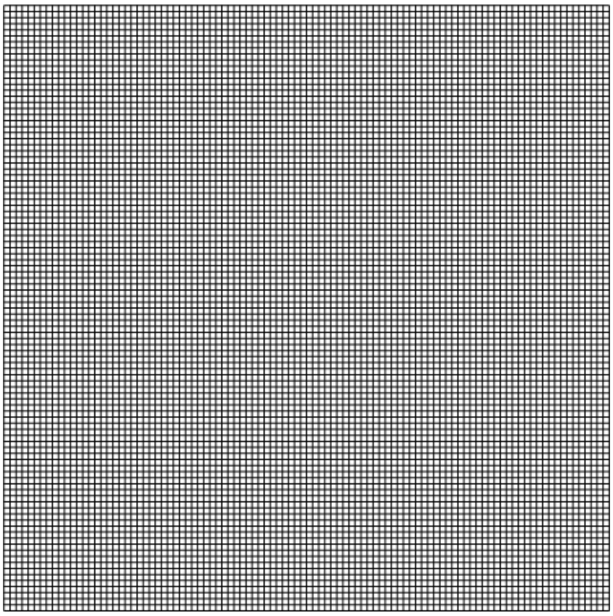

Fig. 2 *Mesh of square plate discretization.*

SHELL181 element type of ANSYS Library [13] is chosen to model the plate. This element is a four-node element with six degrees of freedom at each node (translations in the *x*, *y*, and *z* directions, and rotations about the *x*, *y*, and *z*-axis), and suitable for analyzing thin to moderately-thick plates. In order to obtain the results accurately, the plate discretization with a uniform mesh of 10000 square elements is then used and shown in Fig. 2. All calculations are performed for an isotropic plate having the Poisson's ratio taken as 0.3.

3. NUMERICAL RESULTS

After performing the analysis by ANSYS computer program, results are given in terms of natural frequencies (*f*). They can conveniently be expressed in dimensionless form of frequency coefficients as $\lambda^2 = 2\pi fa^2\sqrt{\rho h / D}$ where *a*, *h*, ρ and *D* are length dimension, plate thickness, mass density per unit area of the plate, and plate's flexural rigidity, respectively. Thus, frequency coefficients are given in Tables A1 to A6 for the first 20^{th} mode of vibrations.

It is interesting to note that the first six modes of F-F-F-F plate and the first mode of F-F-S-F plate as shown in Tables A1 and A2, respectively, are all zero values for frequency coefficients. This is due to rigid body motions of the plates. Figs. A1 to A5 show the frequency coefficients versus the mode numbers of plate vibrations that correspond with values given in Tables A1 to A6.

Finally, Figs. A6 to 26 present the first ten modal patterns for the free vibrations of plates under investigation.

4. CONCLUDING REMARKS

The free vibrations of square plates with different edge conditions are numerically analyzed for finding frequency coefficients. Using ANSYS finite element code, a dense net of plate discretization is used to model the plates in order to accurately determine the frequency coefficients. Their numerical results are given for the first twenty modes of plate vibrations. Additionally, the first ten vibration mode patterns are also graphically demonstrated.

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APPENDIX

Table A1 *Frequency coefficients for the first 20 modes of the plates (case 1-case 4).*

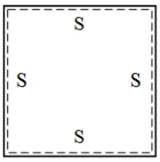
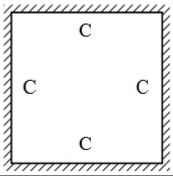
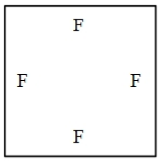
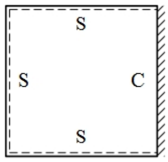
Mode	Case 1: S-S-S-S	Case 2: C-C-C-C	Case 3: F-F-F-F	Case 4: S-S-C-S
				
1	19.739	35.985	0.000	23.647
2	49.347	73.391	0.000	51.674
3	49.347	73.391	0.000	58.646
4	78.957	108.214	0.000	86.134
5	98.693	131.576	0.000	100.268
6	98.693	132.199	0.000	113.226
7	128.301	164.994	13.463	133.787
8	128.301	164.994	19.596	140.840
9	167.779	210.510	24.270	168.954
10	167.779	210.510	34.788	187.429
11	177.648	220.025	34.788	188.106
12	197.385	242.130	61.090	201.717
13	197.385	243.135	61.090	215.288
14	246.723	296.306	63.655	255.458
15	246.723	296.306	69.241	257.529
16	256.607	308.875	77.154	262.512
17	256.607	309.142	105.409	281.316
18	286.196	340.557	105.409	289.744
19	286.196	340.557	117.101	309.388
20	315.806	371.315	122.439	329.545

Table A2 *Frequency coefficients for the first 20 modes of the plates (case 5-case 8).*

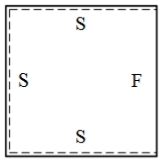
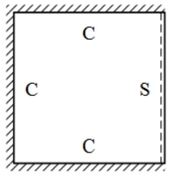
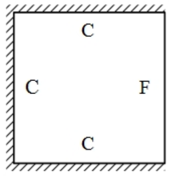
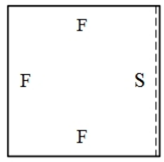
Mode	Case 5: S-S-F-S	Case 6: C-C-S-C	Case 7: C-C-F-C	Case 8: F-F-S-F
				
1	11.684	31.825	23.918	0.000
2	27.752	63.329	39.990	6.642
3	41.194	71.074	63.213	14.901
4	59.058	100.789	76.699	25.367
5	61.853	116.355	80.556	25.997
6	90.288	130.348	116.632	48.438
7	94.469	151.887	122.215	50.569
8	108.909	159.470	134.414	58.728
9	115.676	189.759	140.217	65.173
10	145.616	209.321	172.811	87.928
11	148.485	209.362	176.752	89.066
12	159.068	223.941	200.881	103.135
13	178.081	238.336	212.499	113.150
14	189.377	280.065	219.307	123.044
15	200.631	283.161	232.677	131.455
16	215.575	287.509	249.471	142.598
17	221.952	308.096	256.299	147.907
18	247.543	316.442	299.238	177.006
19	266.819	337.050	308.486	185.635
20	271.576	357.330	310.721	189.543

Table A3 *Frequency coefficients for the first 20 modes of the plates (case 9-case 12).*

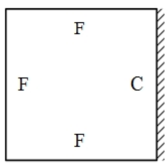
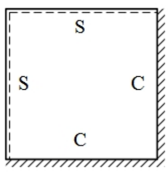
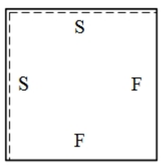
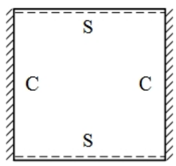
Mode	Case 9: F-F-C-F	Case 10: S-C-C-S	Case 11: S-F-F-S	Case 12: C-S-C-S
				
1	3.471	27.053	3.366	28.950
2	8.504	60.539	17.313	54.742
3	21.283	60.785	19.291	69.325
4	27.194	92.835	38.202	94.584
5	30.945	114.554	51.030	102.214
6	54.163	114.700	53.483	129.091
7	61.250	145.776	72.947	140.201
8	64.133	146.076	74.614	154.770
9	70.947	188.452	104.710	170.342
10	92.894	188.545	107.228	199.802
11	97.039	198.098	112.518	206.696
12	119.047	219.205	126.790	208.378
13	124.204	219.430	128.730	234.563
14	128.771	270.653	167.307	258.596
15	139.428	270.961	168.771	265.178
16	149.859	282.136	178.204	279.634
17	157.696	282.198	181.210	293.742
18	196.977	312.607	200.182	307.297
19	199.720	312.792	202.224	333.933
20	199.806	342.833	225.992	344.514

Table A4 *Frequency coefficients for the first 20 modes of the plates (case 13-case 16).*

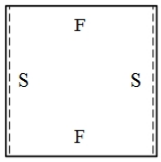
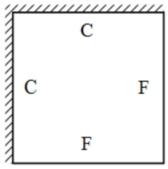
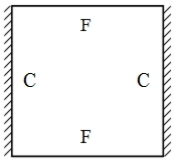
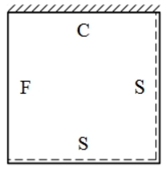
Mode	**Case 13: S-F-S-F**	**Case 14: C-F-F-C**	**Case 15: C-F-C-F**	**Case 16: F-S-S-C**
				
1	9.631	6.918	22.164	16.790
2	16.131	23.897	26.399	31.109
3	36.715	26.581	43.580	51.391
4	38.944	47.636	61.166	64.012
5	46.732	62.698	67.158	67.531
6	70.722	65.526	79.797	101.097
7	75.270	85.674	87.567	105.501
8	87.981	88.320	120.085	117.195
9	96.030	121.327	124.416	122.646
10	110.991	124.034	126.685	153.689
11	122.014	128.420	136.869	157.376
12	133.686	144.392	149.270	179.260
13	156.740	146.471	180.176	190.530
14	164.647	186.390	188.077	197.034
15	164.766	188.944	198.709	210.531
16	169.489	199.716	205.548	226.156
17	191.834	202.919	214.509	232.636
18	212.069	222.280	229.539	272.704
19	224.680	224.721	244.652	282.895
20	236.204	248.425	256.012	283.859

Table A5 *Frequency coefficients for the first 20 modes of the plates (case 17-case 20).*

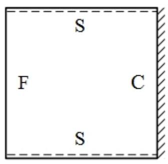
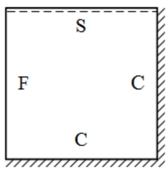
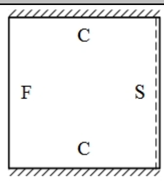
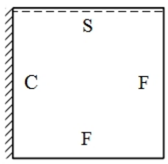
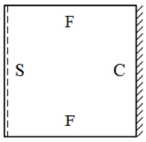
Mode	**Case 17: F-S-C-S**	**Case 18: F-C-C-S**	**Case 19: F-C-S-C**	**Case 20: C-F-F-S**
				
1	12.686	17.535	23.368	5.350
2	33.059	36.016	35.565	19.072
3	41.700	51.805	62.866	24.666
4	63.005	71.064	66.752	43.074
5	72.389	74.316	77.361	52.703
6	90.606	105.774	108.846	63.750
7	103.143	109.340	119.018	77.470
8	111.885	125.353	121.977	83.644
9	131.418	132.784	137.761	106.303
10	152.748	164.049	159.712	120.337
11	159.289	167.235	170.444	122.629
12	162.345	179.459	191.864	130.813
13	180.410	199.177	200.704	142.709
14	210.244	211.289	217.339	174.826
15	212.622	222.096	221.583	179.336
16	221.501	238.213	230.913	181.868
17	241.084	245.021	251.050	200.672
18	247.707	272.847	284.863	203.973
19	268.726	292.676	292.225	221.317
20	281.931	296.490	299.115	237.414

Table A6 *Frequency coefficients for the first 20 modes of the plate of case 21.*

Case 21: S-F-C-F								
	**Mode**		**Mode**		**Mode**		**Mode**	
	1	15.191	**6**	77.308	**11**	134.857	**16**	184.511
	2	20.579	**7**	78.508	**12**	135.249	**17**	209.998
	3	39.723	**8**	103.416	**13**	174.486	**18**	213.196
	4	49.445	**9**	110.695	**14**	175.745	**19**	234.153
	5	56.267	**10**	117.218	**15**	177.109	**20**	251.398

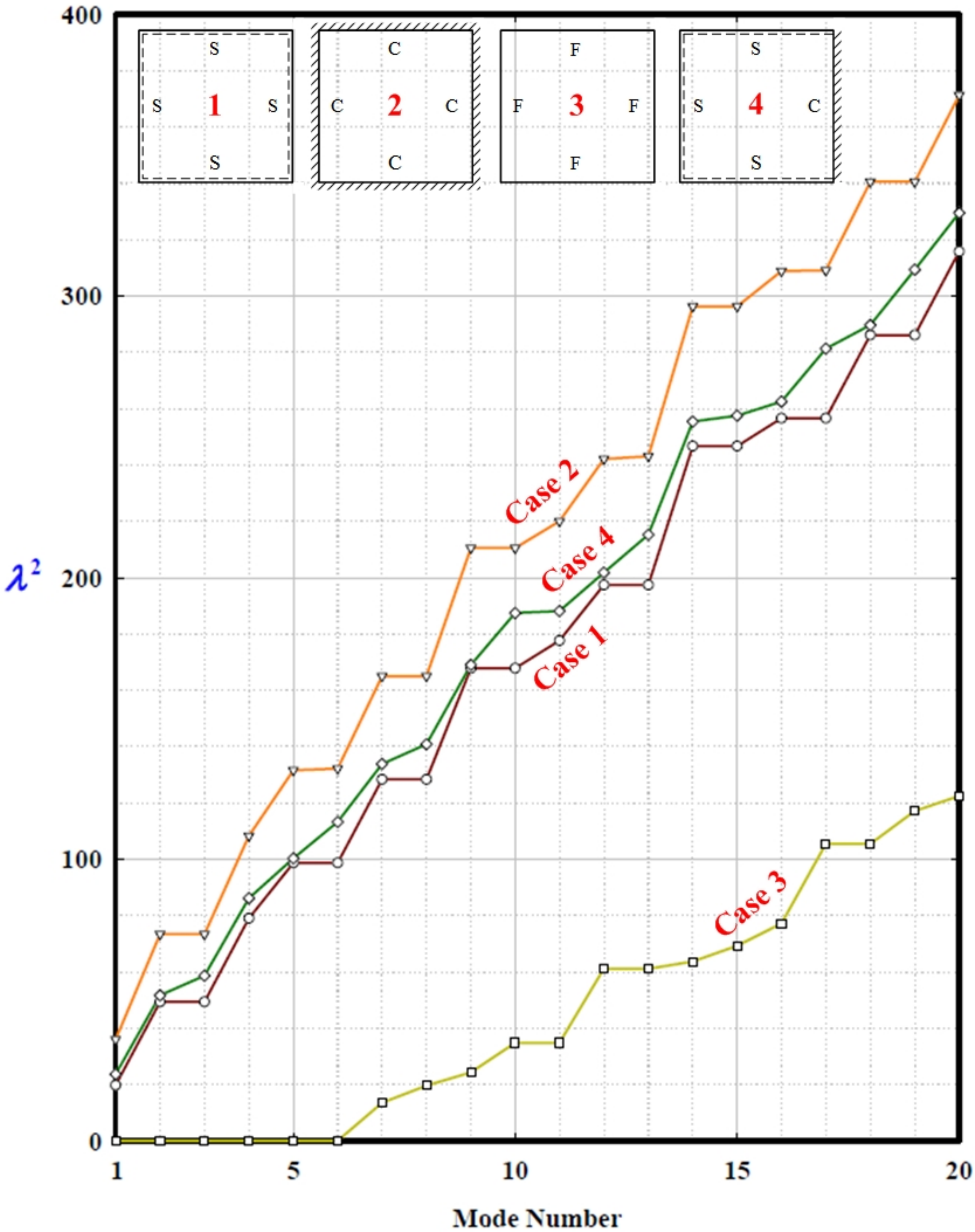


Fig. A1 *Frequency coefficients for the plates of case 1 to case 4.*

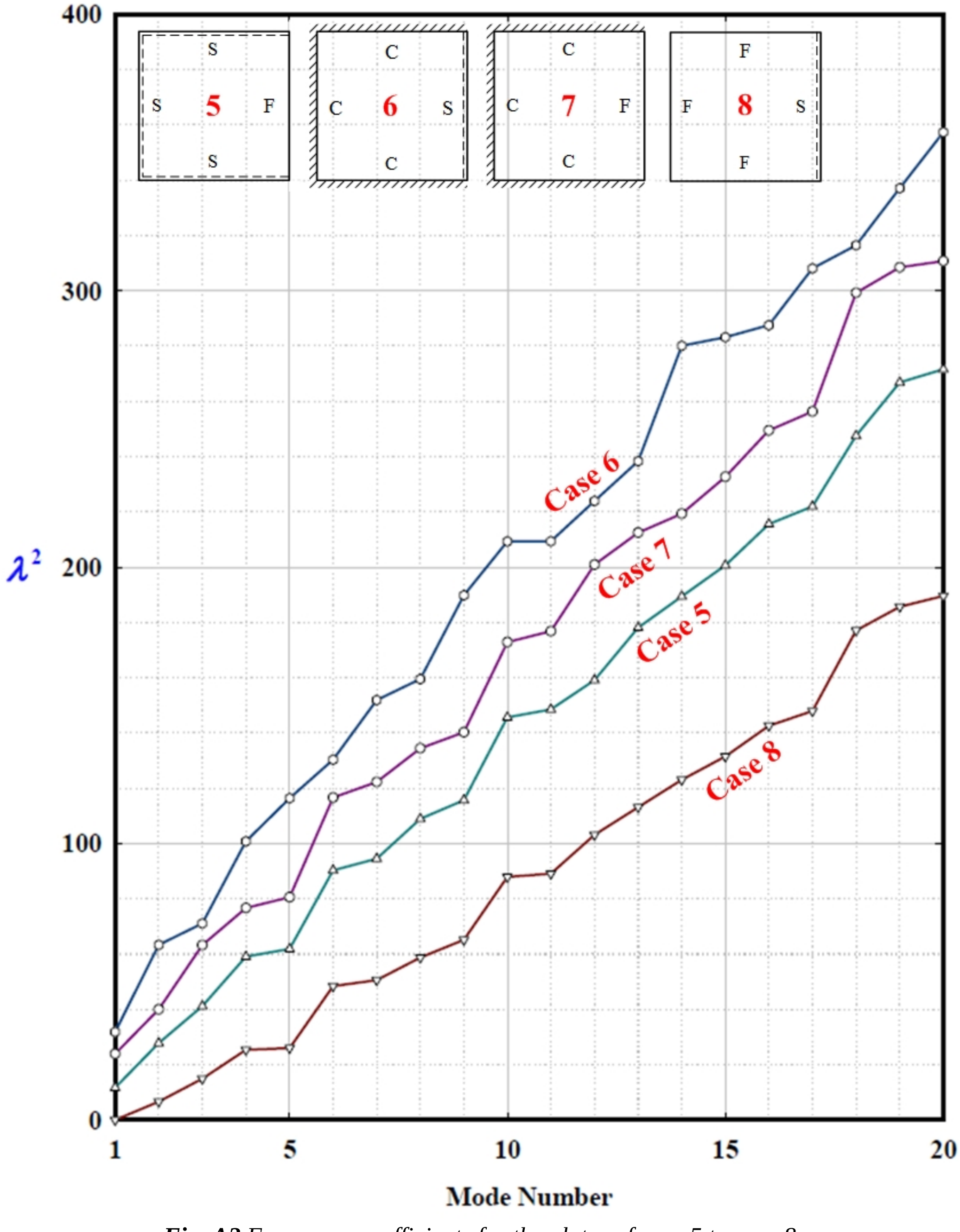


Fig. A2 *Frequency coefficients for the plates of case 5 to case 8.*

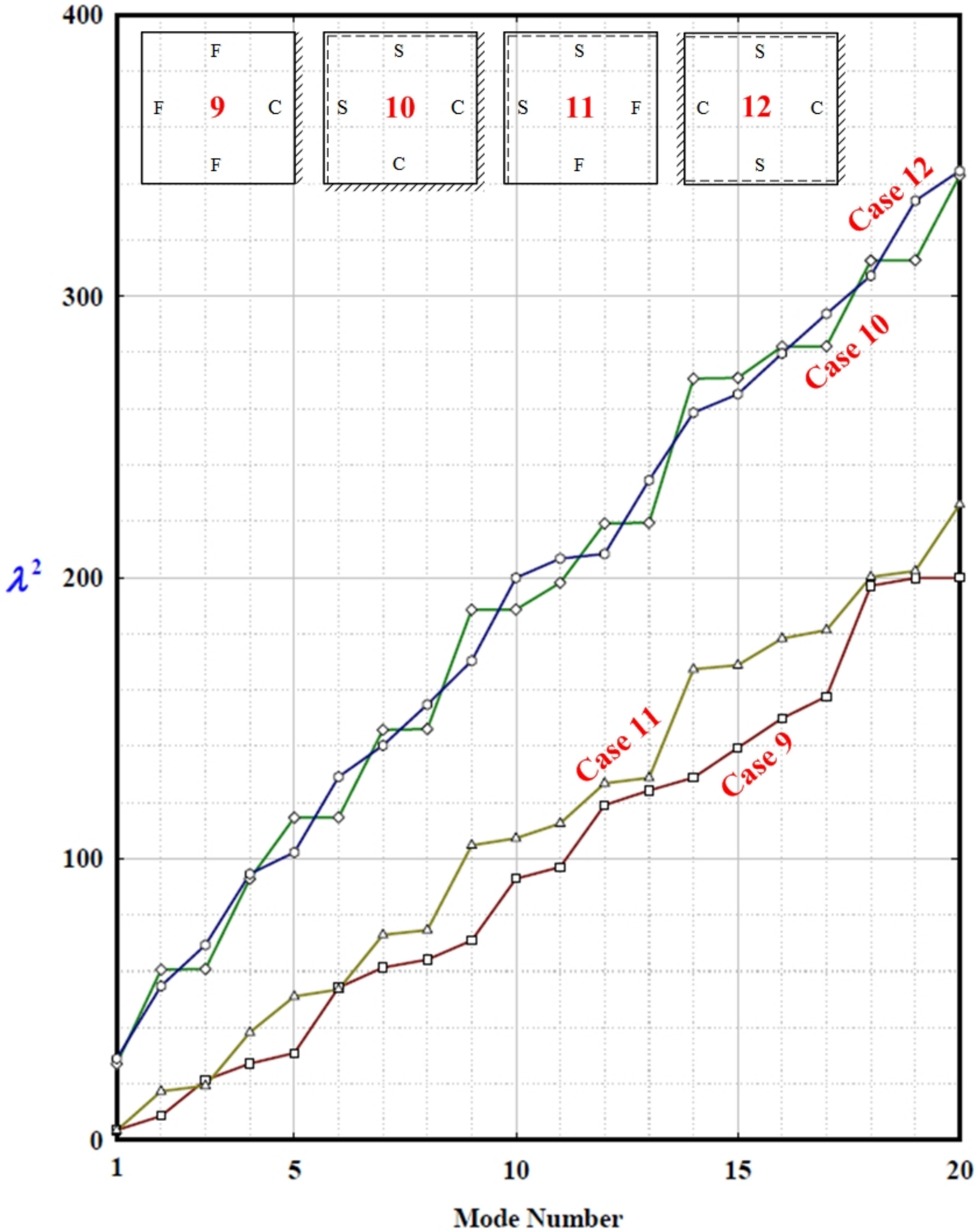


Fig. A3 *Frequency coefficients for the plates of case 9 to case 12.*

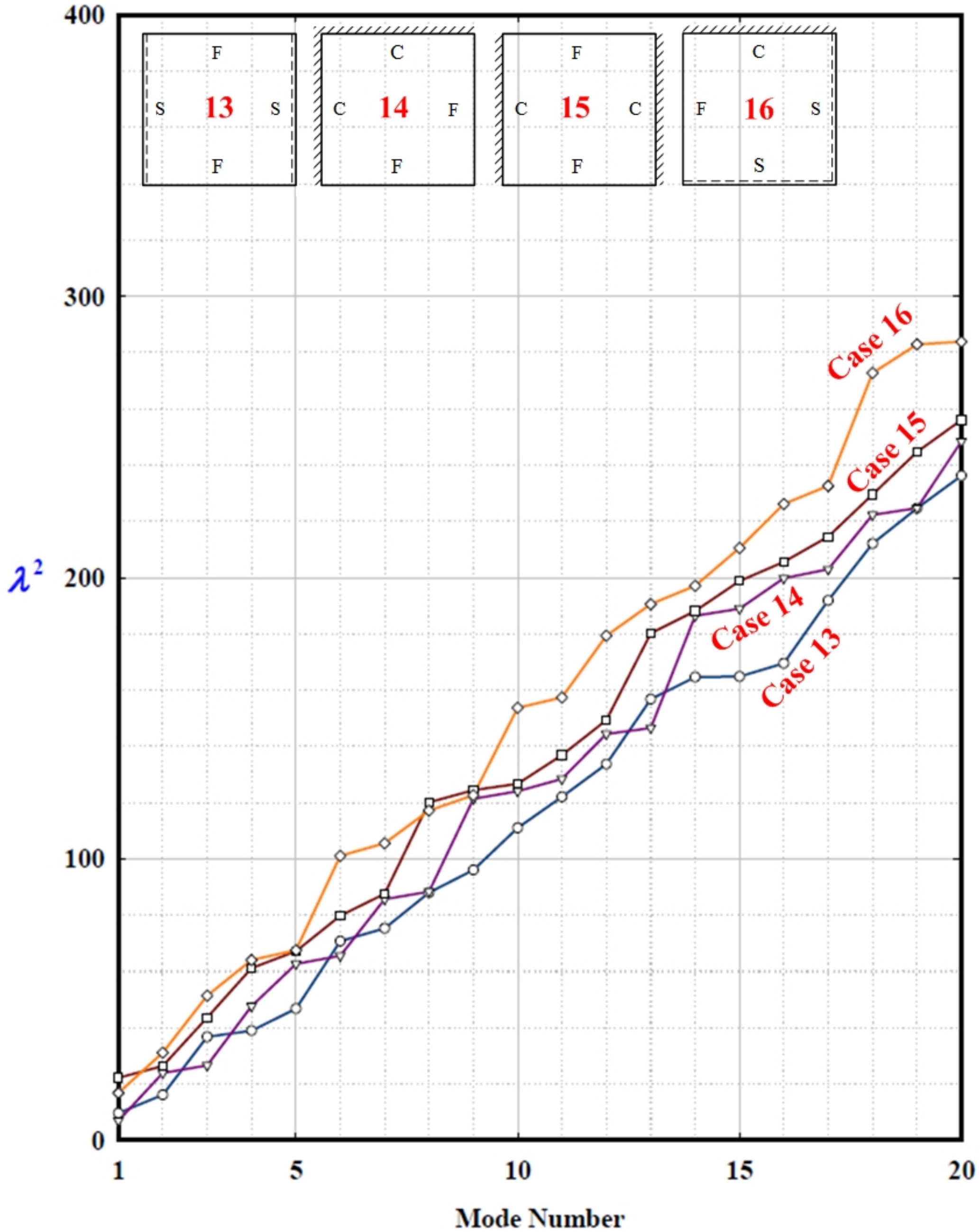


Fig. A4 *Frequency coefficients for the plates of case 13 to case 16.*

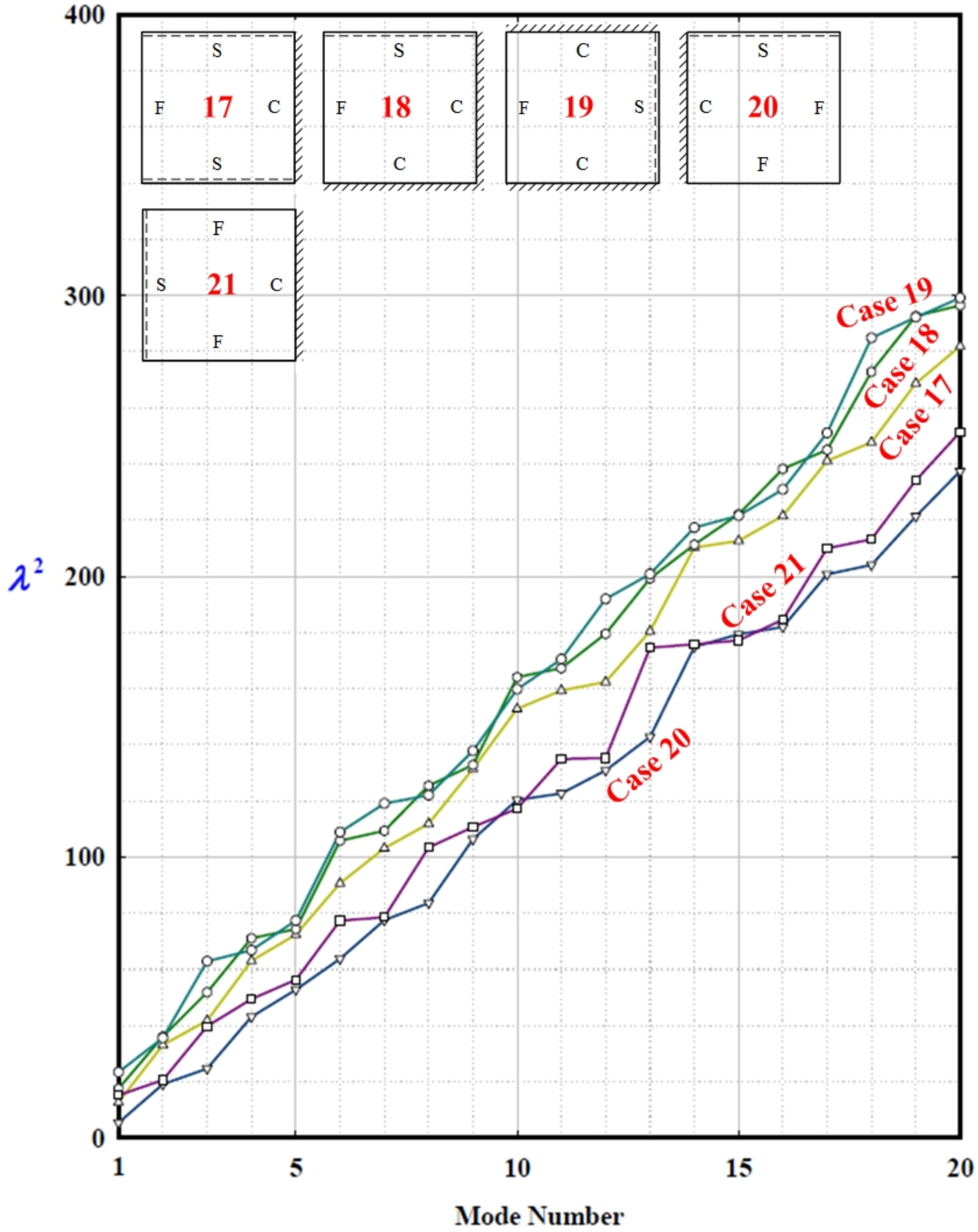


Fig. A5 *Frequency coefficients for the plates of case 17 to case 21.*

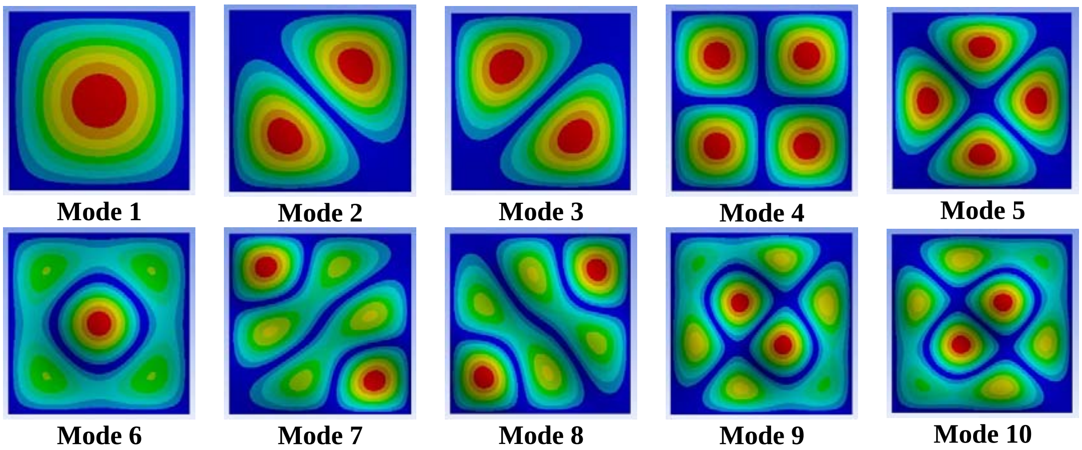


Fig. A6 *First 10 vibration mode contours for square plate of case 1: S-S-S-S.*

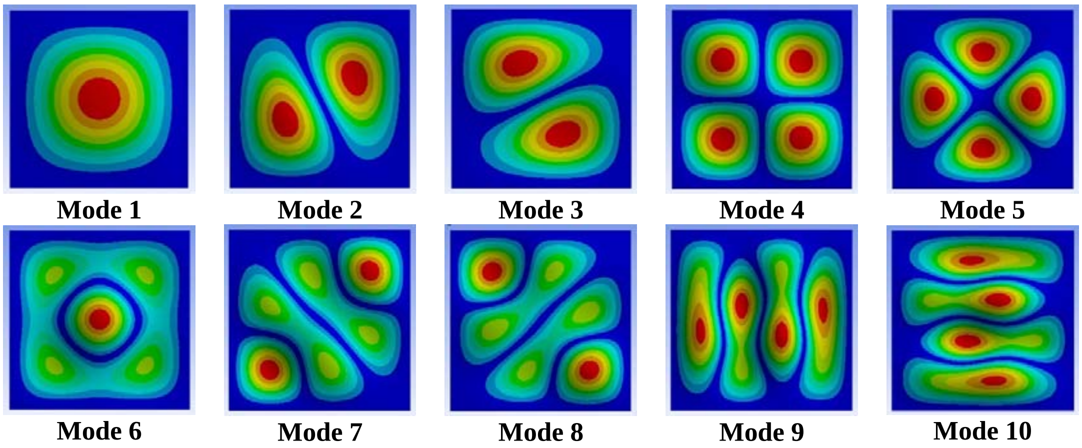


Fig. A7 *First 10 vibration mode contours for square plate of case 2: C-C-C-C.*

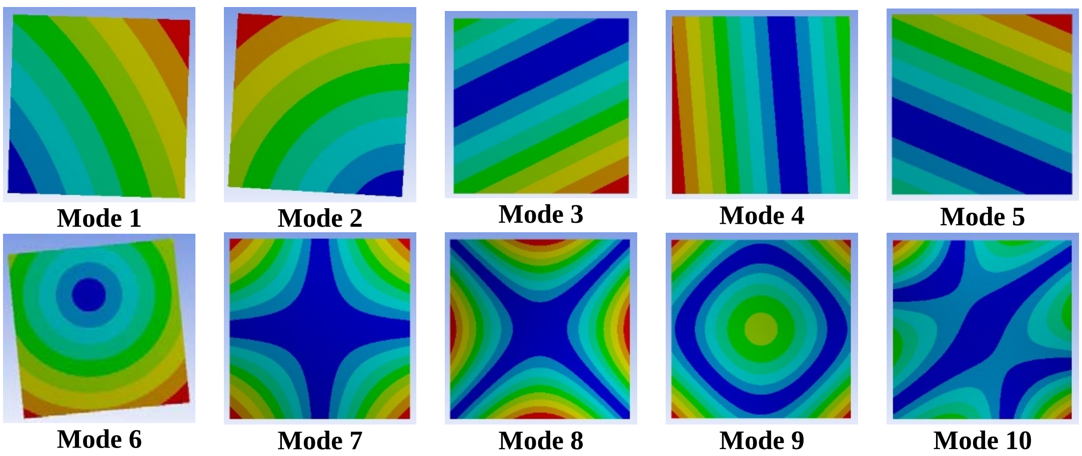


Fig. A8 *First 10 vibration mode contours for square plate of case 3: F-F-F-F.*

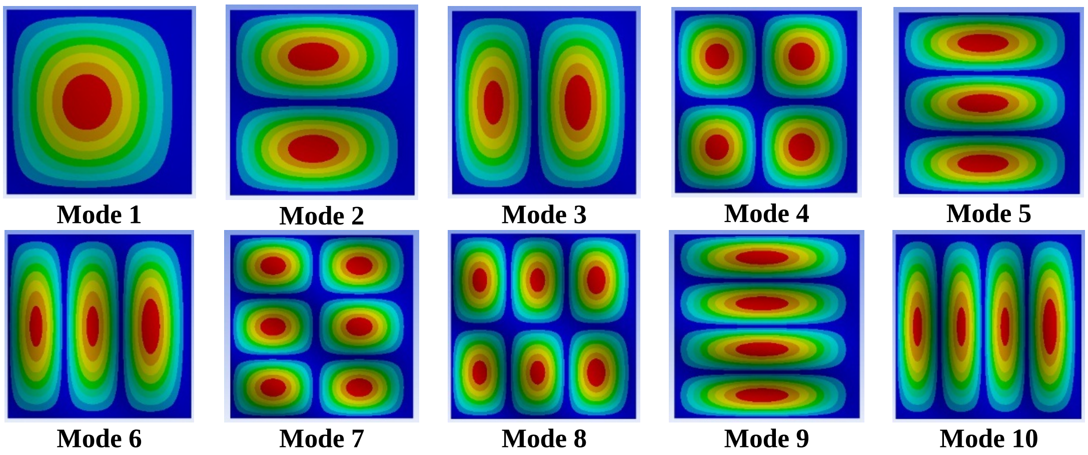


Fig. A9 *First 10 vibration mode contours for square plate of case 4: S-S-C-S.*

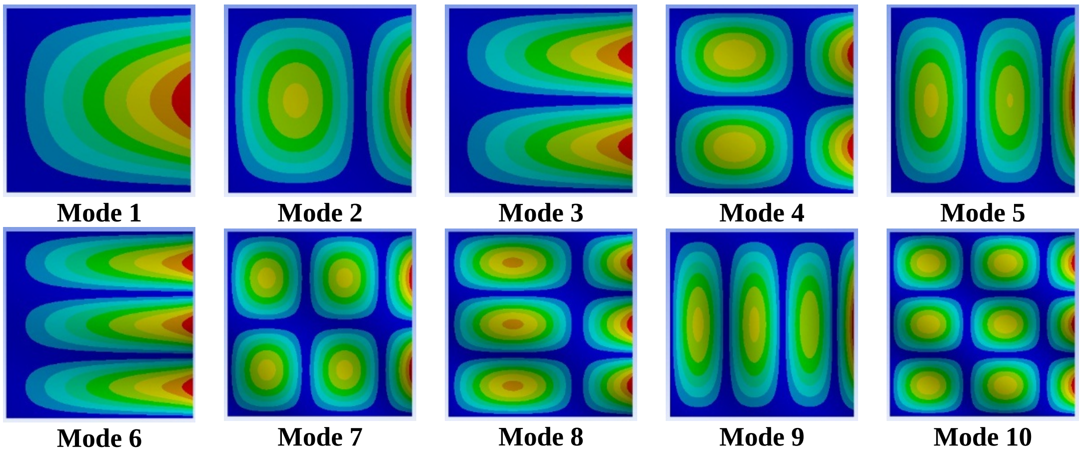


Fig. A10 *First 10 vibration mode contours for square plate of case 5: S-S-F-S.*

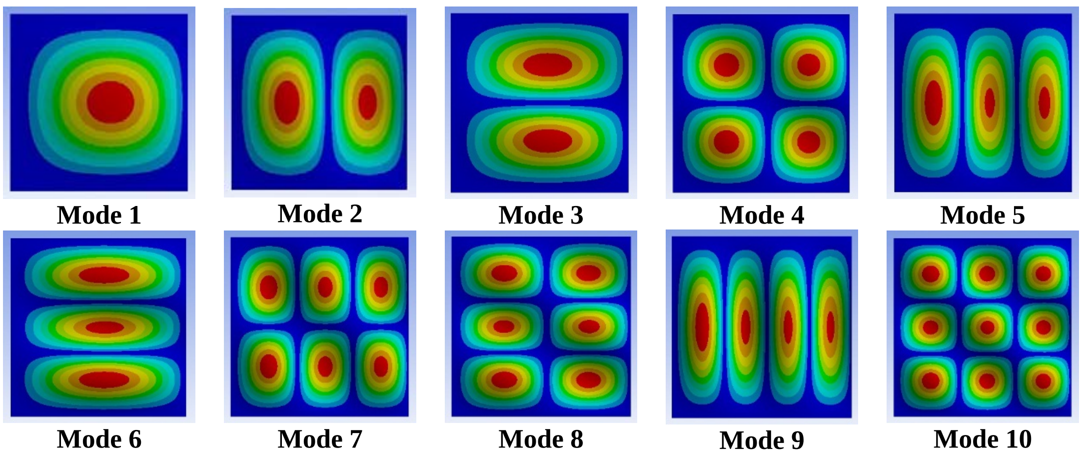


Fig. A11 *First 10 vibration mode contours for square plate of case 6: C-C-S-C.*

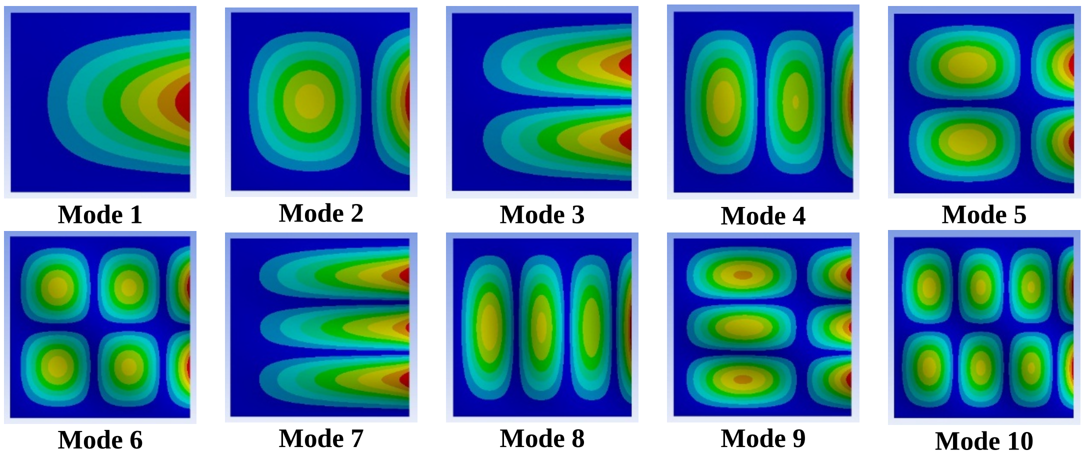


Fig. A12 *First 10 vibration mode contours for square plate of case 7: C-C-F-C.*

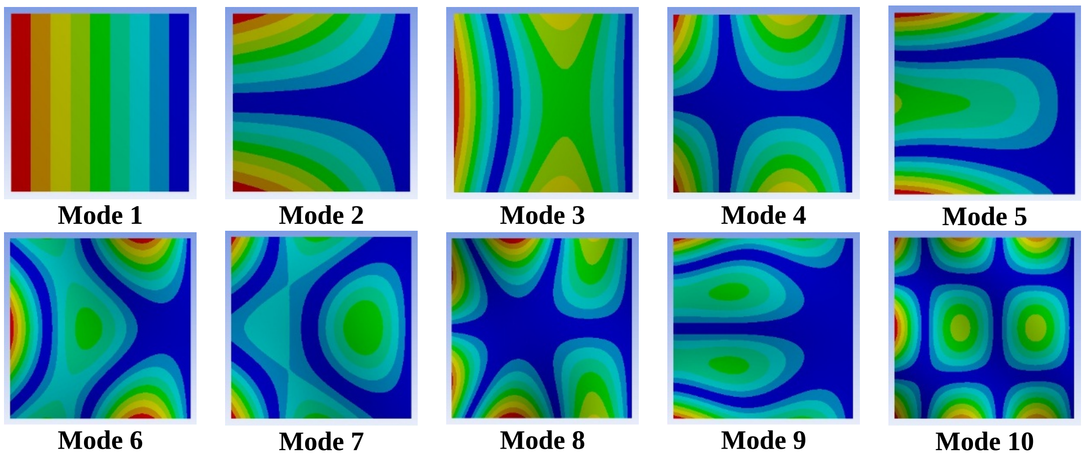


Fig. A13 *First 10 vibration mode contours for square plate of case 8: F-F-S-F.*

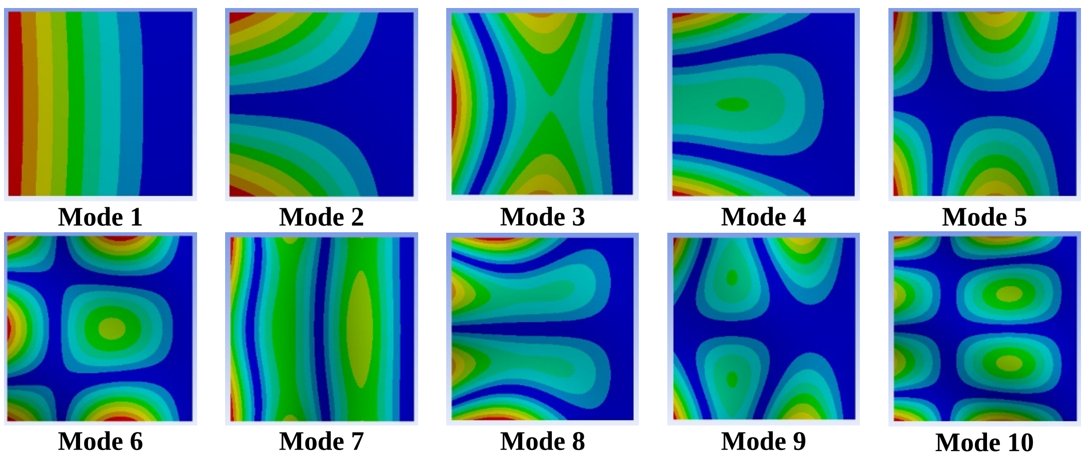


Fig. A14 *First 10 vibration mode contours for square plate of case 9: F-F-C-F.*

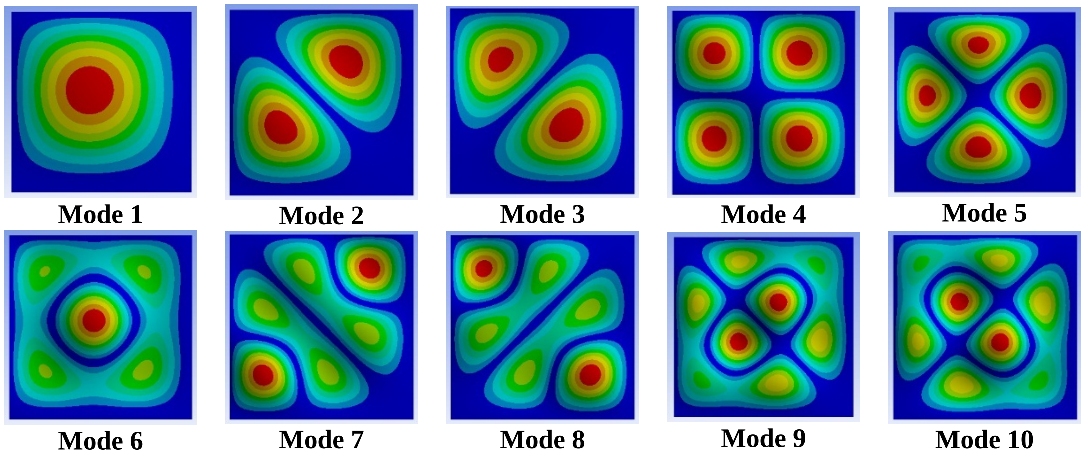


Fig. A15 *First 10 vibration mode contours for square plate of case 10: S-C-C-S.*

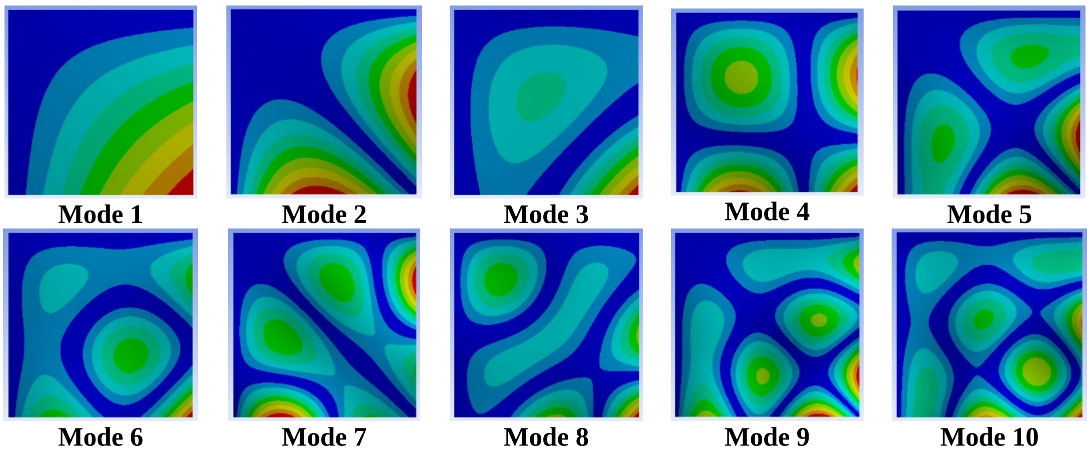


Fig. A16 *First 10 vibration mode contours for square plate of case 11: S-F-F-S.*

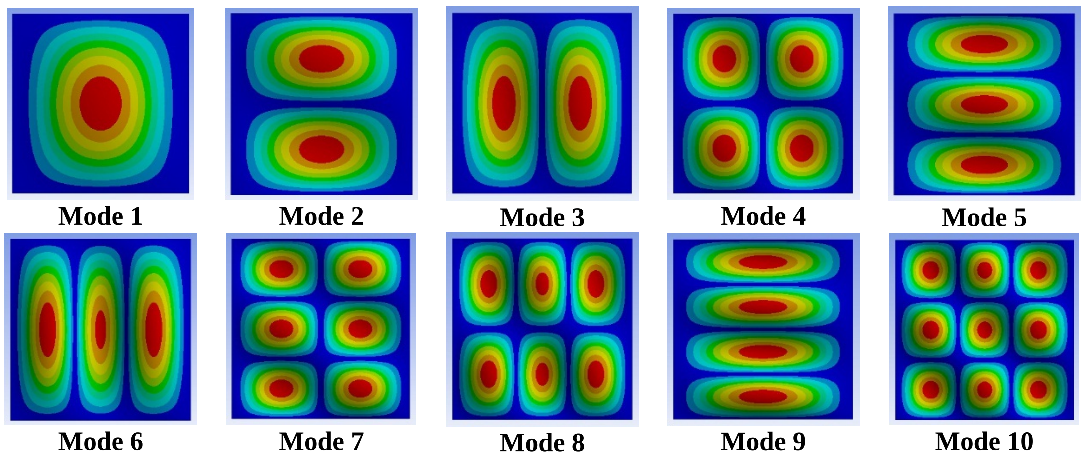


Fig. A17 *First 10 vibration mode contours for square plate of case 12: C-S-C-S.*

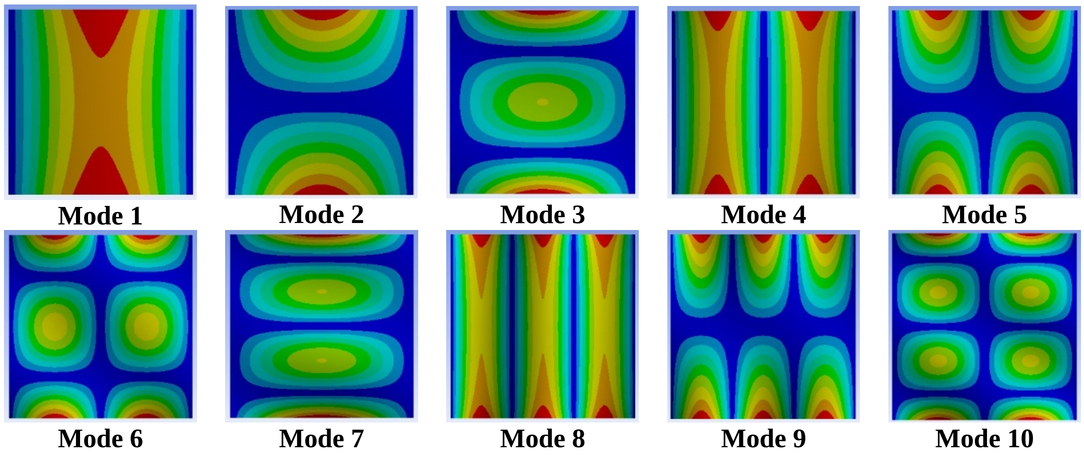


Fig. A18 *First 10 vibration mode contours for square plate of case 13: S-F-S-F.*

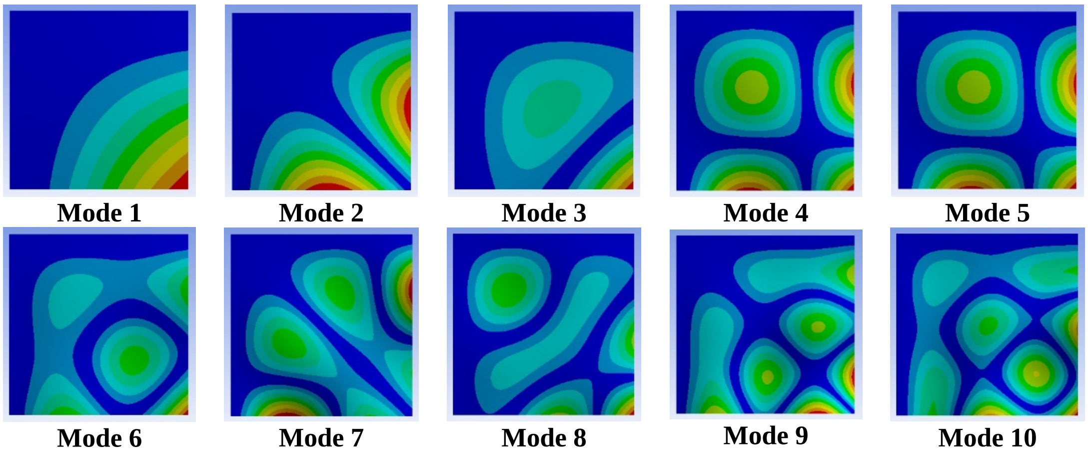


Fig. A19 *First 10 vibration mode contours for square plate of case 14: C-F-F-C.*

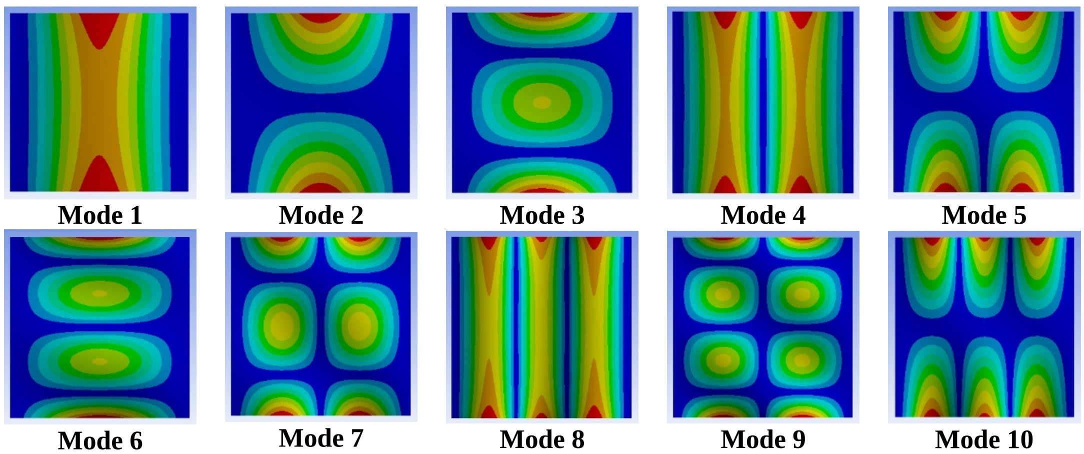


Fig. A20 *First 10 vibration mode contours for square plate of case 15: C-F-C-F.*

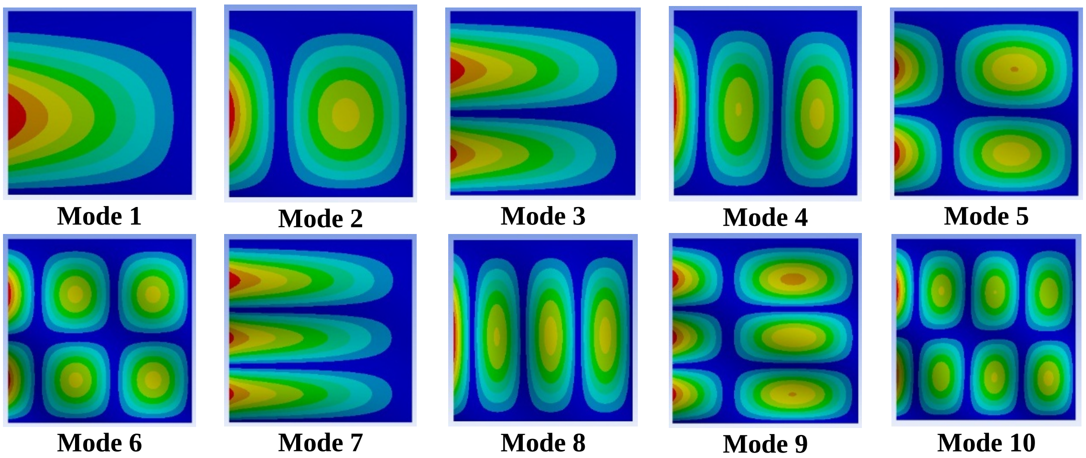


Fig. A21 *First 10 vibration mode contours for square plate of case 16: F-S-S-C.*

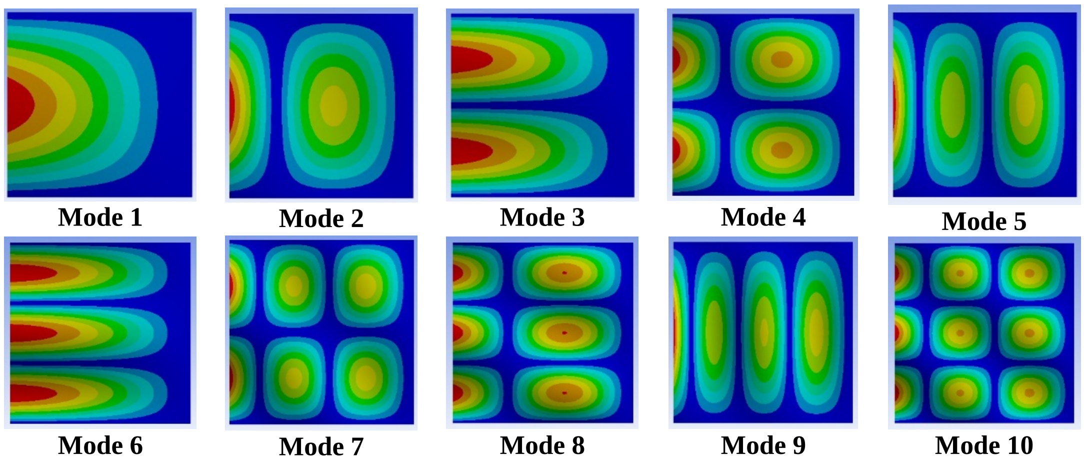


Fig. A22 *First 10 vibration mode contours for square plate of case 17: F-S-C-S.*

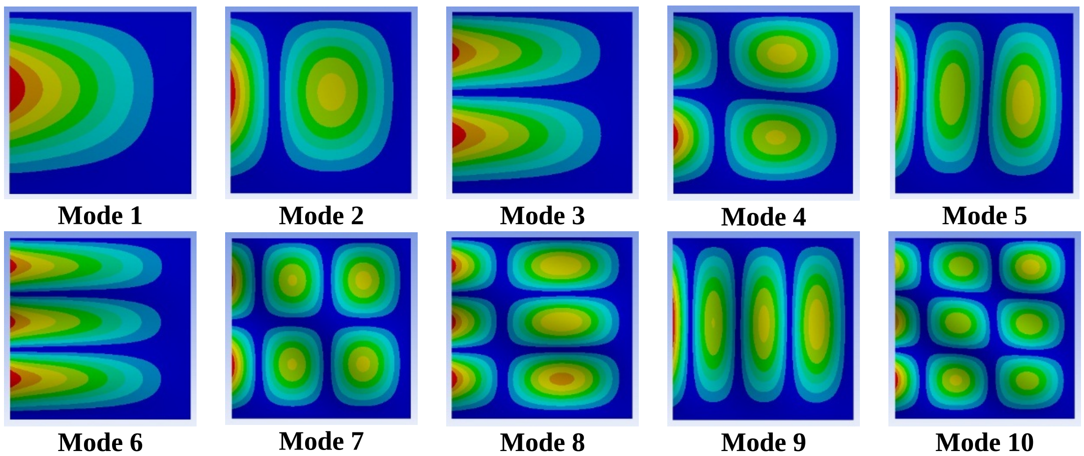


Fig. A23 *First 10 vibration mode contours for square plate of case 18: F-C-C-S.*

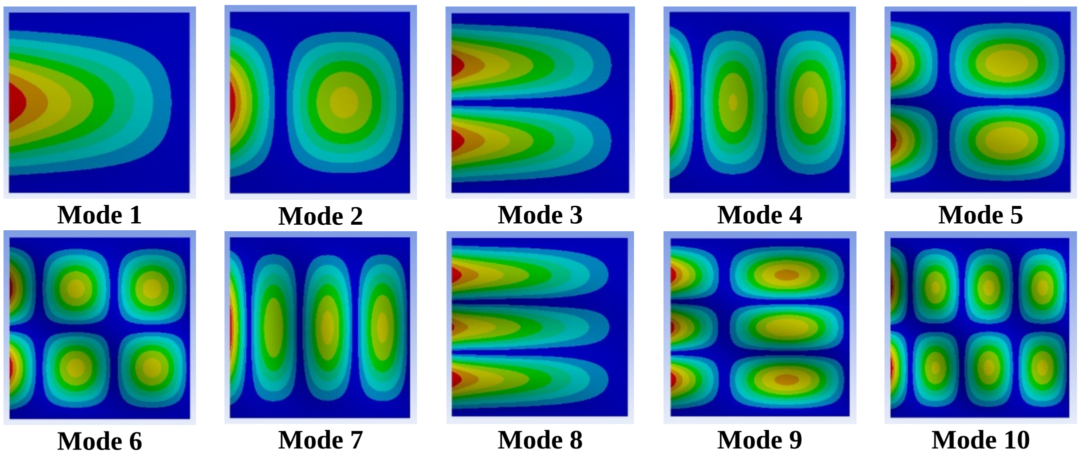


Fig. A24 *First 10 vibration mode contours for square plate of case 19: F-C-S-C.*

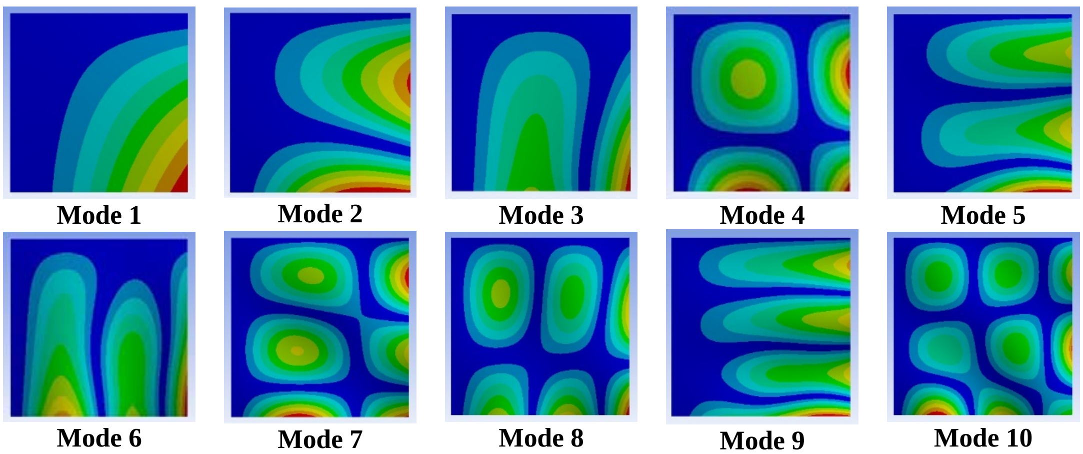


Fig. A25 *First 10 vibration mode contours for square plate of case 20: C-F-F-S.*

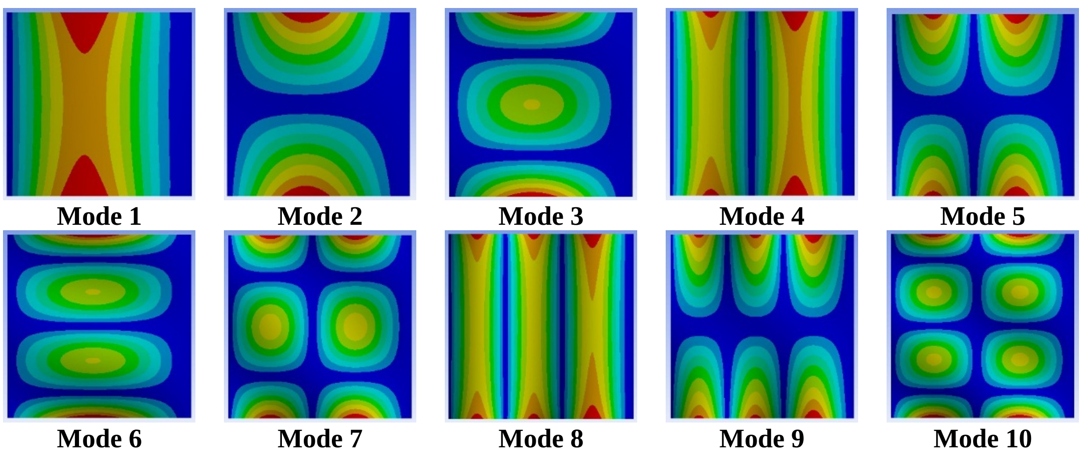


Fig. A26 *First 10 vibration mode contours for square plate of case 21: S-F-C-F.*