Bending of Square Plates with Mixed Conditions between Clamped and Free Edges

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ABSTRACT

A rather limited amount of analytical solutions is available in the case of plates with mixed boundary conditions between clamped and free edges, because the clamped and free edge conditions do not have a common boundary condition. Therefore, the aim of the present study is to deal with the numerical finite element determination for the deflection responses of square plates having mixed conditions between clamped and free edges under a uniformly distributed load. The obtained results are provided for the deflection distributions along the middle line, along the diagonal line, and along the free edge of the plates. In addition, the deflection surface and its contour are also graphically demonstrated herein.

Keywords : *Square plate, Mixed support conditions, Finite element method.*

1. INTRODUCTION

In problems concerning the static bending of thin plates [1], few analytical solutions are found in the scientific or technical literature on the subject of plate having mixed boundary conditions. The situation is more critical in the case of plates with mixed conditions between clamped and free edges, because the clamped and free edges do not have a common boundary condition.

Kurata [2] considered the mixed boundary conditions of rectangular plates and dealt with an analytical series solution method for solving the bending of simply supported rectangular plates having clamped portions along arbitrary sections of the edges. The solution could be obtained by means of determining the resisting moments introduced along the clamped portions to keep zero slopes along the relevant sections. Moreover, experimental works were also made to verify the theoretical results in which the aluminum plates with varying the Young's modulus between 690000 and 720000 kg/cm^2 and the Poisson's ratio taken as 0.3 were used. Another plate-bending problem involving the mixed boundary conditions were analytically treated by Kiattikomol et al. [3], who analyzed the problem of uniformly loaded rectangular plates simply supported on two opposite edges, but only partially constrained along the other two edges. Thus, the boundary conditions of the plates were mixed between simply supported and free edges. Sompornjaroensuk and Kiattikomol [4] presented a comprehensive study of a uniformly loaded, two opposite simply supported rectangular plate with a partial internal line support placed at the center of the plate. Three cases of the plate with different types of support on two remaining edges were considered.

Because the analytical solutions are not always easily attainable as in the case of the plates having mixed boundary conditions, approximate mathematical techniques have been used to remedy the situation. The finite element method is one of the powerful numerical methods. It has been very successful in finding the solution of various problems.

Salamon et al. [5] examined a square plate unilaterally supported on discrete springs around the plate edges by applying the finite element method and found out that the plate lifted off the supports, except for low support stiffnesses. Papanikolaou and Doudoumis [6] proposed a numerical methodology for analysis of rectangular plates incorporated with the implementation of finite element software named MSC/NASTRAN, which taking consistently into account any unilateral support conditions. Numerical results were prepared and given in the proper application tables for the stress state of the plates, in a form similar to that of the well-known tables complied by Czerny.

In the present investigation, numerical experiments for the bending analysis of single square plates with mixed boundary conditions that shown in Fig. 1 are considered. The method of analysis is based on the application of ANSYS computer finite element software [7] similar to that previously treated by Niamnin et al. [8]. Results concerning the deflection surface and its contour and the deflection distributions along two middle lines, diagonal line, and the free edge of the plates are graphically presented. Their numerical values are prepared and given in tabular form for easy use by other investigators.

As seen in Fig. 1, the letter symbols are given for convenience to specify the different plate configurations. Therefore, the descriptive designation is similar to that of Boonchareon et al. [9].

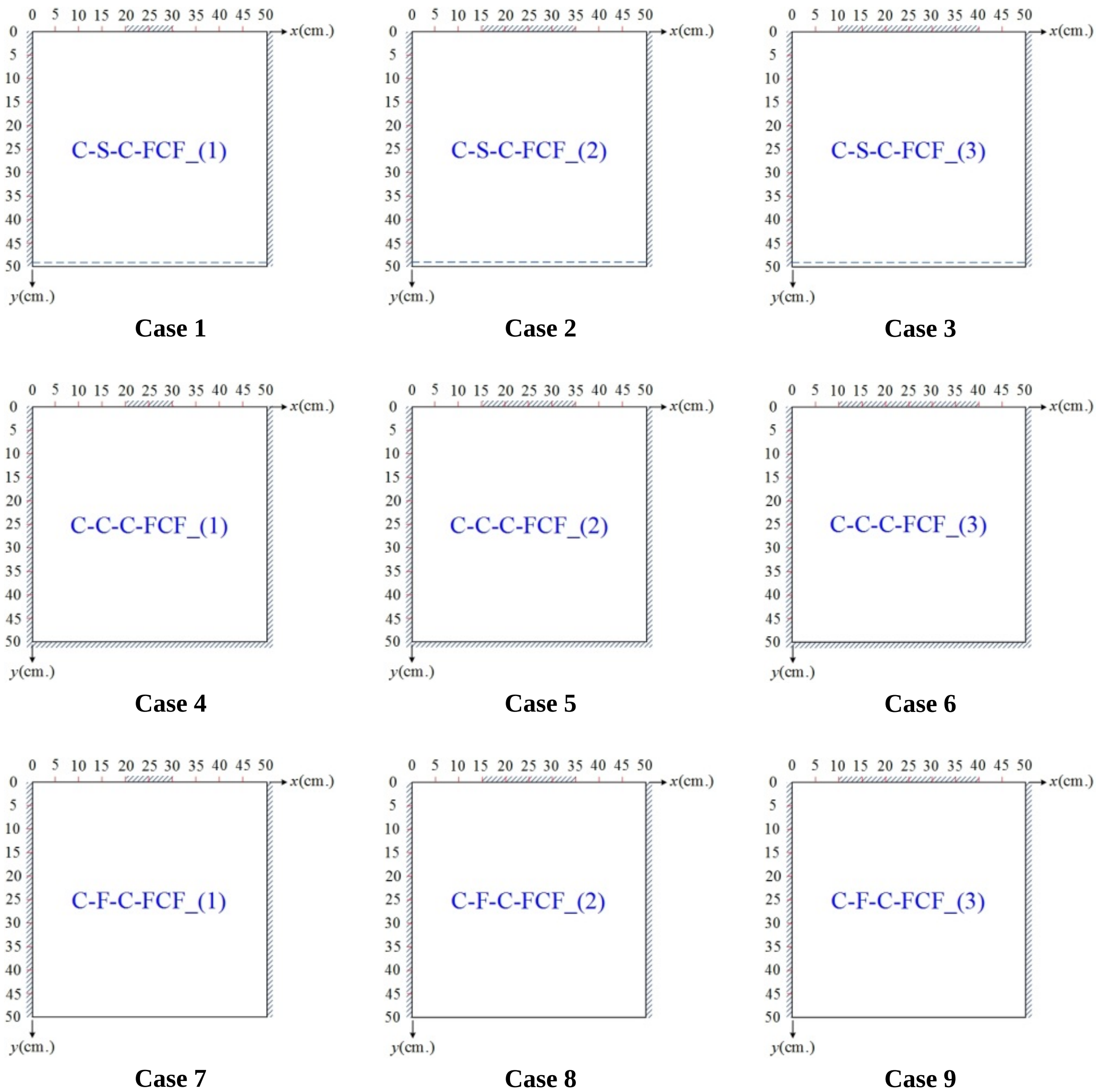

***Fig. 1** Square plates having partial clamped support at the center of one edge.*

2. FINITE ELEMENT DETERMINATION

Numerical results for the present work can be obtained by using the academic well-known ANSYS finite element program [7]. SHELL181 element type for quadrilateral shape of the ANSYS Library [10] that shown in Fig. 2 is used to model the plates. This element has four nodes with six degrees of freedom at each node (translations in the x, y, and z directions, and rotations about the x, y, and z-axis).

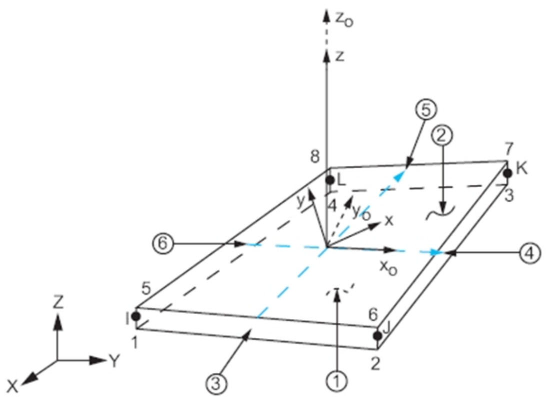

Fig. 2 *Quadrilateral ANSYS SHELL181 element [10].*

Utilizing the concept of finite element method [11], Fig. 3 demonstrates the discretization of single plate with a uniform mesh of 100 square elements. In this investigation, the plate has the plane dimensions of 50 cm. by 50 cm. and 3 mm. in thickness, and they are subjected to a uniformly distributed load with intensity of 100 kg/m^2. All calculations will be performed taking the Young's modulus (E) and Poisson's ratio (υ) equal to 55.43 GPa and 0.27, respectively. These material properties are of the aluminum plate that have been tested and used in Booncharoen et al. [9].

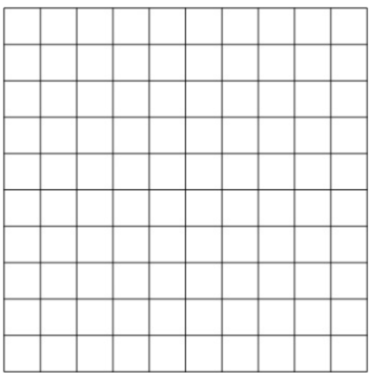

Fig. 3 *Mesh of plate discretization.*

Before processing the ANSYS program, boundary conditions of the plate are needed to apply in the finite element plate model. For this purpose, the criterion to be used in applying the boundary conditions at the transition points (discontinuous points) of mixed boundary supports (point having two different boundary support types) can be described that stronger boundary support conditions will be chosen to represent the boundary support conditions at that transition point. For example, consider the plate of case 1 that depicted in Fig. 1, the point of the plate at $x = 20$ cm. and $y = 0$ cm. is one of the transition points where the clamped support changes to the free edge. Thus, the clamped support condition will be chosen to represent the conditions at that point in the finite element support model.

3. NUMERICAL RESULTS

To efficiently demonstrate the obtained finite element results, SigmaPlot program [12] is used for this purpose. It can plot a smooth sharp variation in dependent discretized values within 2D and 3D data sets.

As seen in Figs. 4 to 12, the distributions for the deflections along two middle lines at $y = 25$ cm. and at $x = 25$ cm., along the diagonal line at $y = x$, and along the free edge at $y = 50$ cm. of the plates with different boundary support conditions are simultaneously presented.

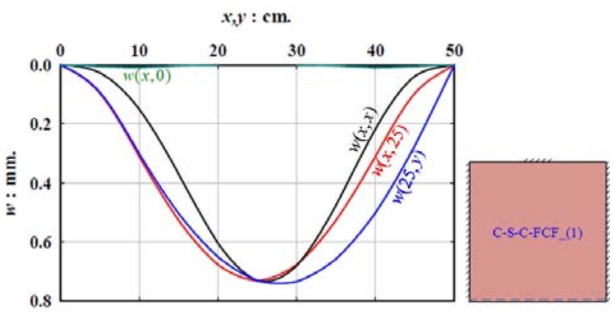

Fig. 4 *Deflection curves (w) for the plate of case 1.*

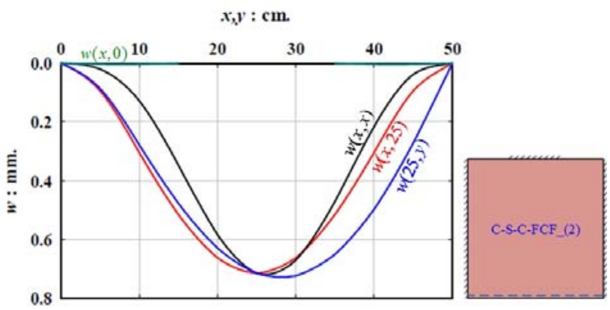

Fig. 5 *Deflection curves (w) for the plate of case 2.*

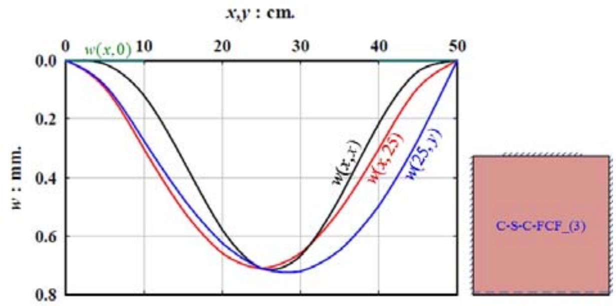

Fig. 6 *Deflection curves (w) for the plate of case 3.*

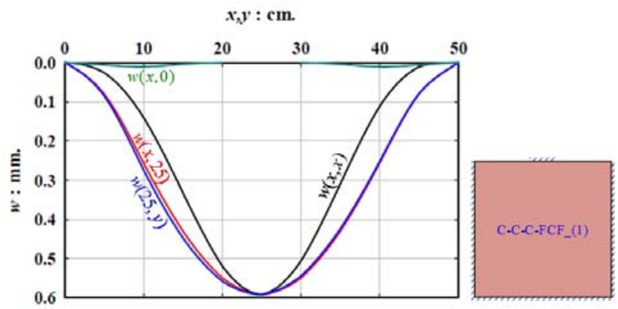

Fig. 7 *Deflection curves (w) for the plate of case 4.*

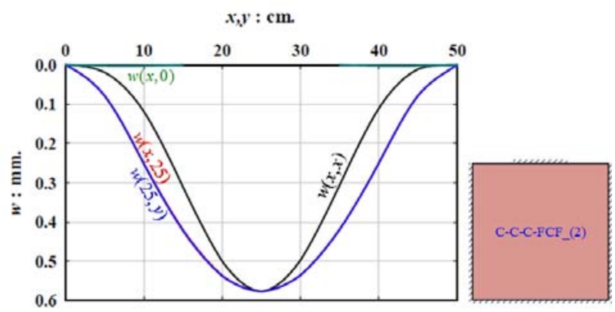

Fig. 8 *Deflection curves (w) for the plate of case 5.*

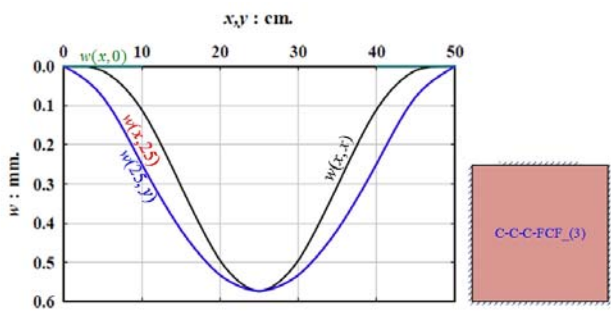

Fig. 9 *Deflection curves (w) for the plate of case 6.*

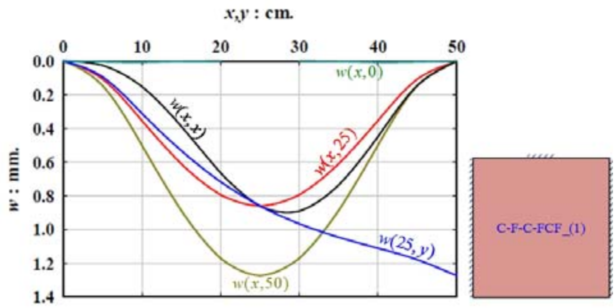

Fig. 10 *Deflection curves (w) for the plate of case 7.*

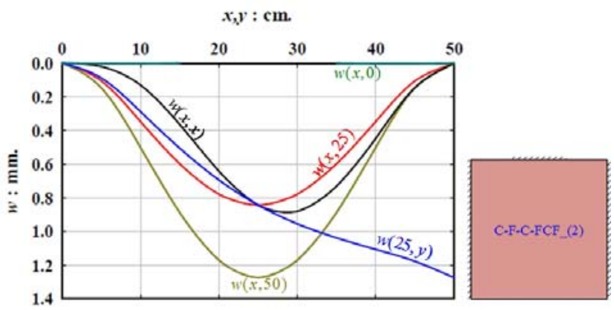

Fig. 11 *Deflection curves (w) for the plate of case 8.*

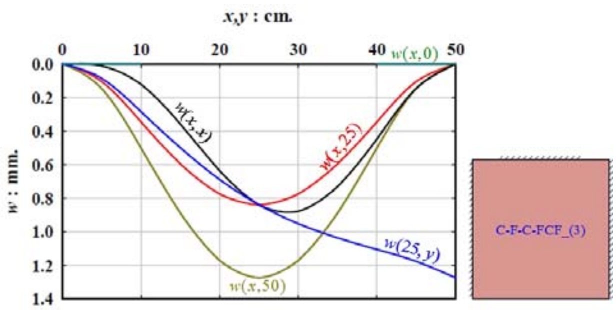

Fig. 12 *Deflection curves (w) for the plate of case 9.*

In addition, Tables 1 to 9 provide, respectively, the numerical values for the deflections corresponding to each case of the plates that shown in Figs. 4 to 12. It can be noted that deflection distributions along two middle lines for the plates of case 5 and case 6 are much closed together. The differences can, however, be seen in Tables 5 and 6.

Figs. 13 to 17 illustrate the comparative deflection distributions along the middle lines at $y = 25$ cm. and at $x = 25$ cm., along the free edges at $y = 0$ cm. and at $y = 50$ cm., and along the diagonal line at $y = x$, respectively. It can be observed in Figs. 13, 14, and 17 that when the length of clamped support at the upper edge of the plates is increased, the difference in two adjacent deflection curves is decreased within the same boundary support conditions. For the deflection curves along the free edge at $y = 0$ cm. corresponding to the plates of cases 3, 6, and 9 as shown in Fig. 15, they have very small quantities in comparison with other cases of the plate. Consider the deflection curves along the free edge at $y = 50$ cm. that shown in Fig. 16, they are all nearly the same in magnitude.

Finally, the overall deflected plate-bending behaviors can clearly be seen in Figs. 18 and 19 on the variation of deflection contours and deflection surfaces for nine cases of the plate.

Table 1 *Deflection values for the plate of case 1: C-S-C-FCF_(1).*

x,y **: cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**
0	0.00000	0.00000	0.00000	0.00000
5	0.09570	0.09190	0.02670	0.00502
10	0.30947	0.29970	0.14951	0.00826
15	0.52526	0.50198	0.36894	0.00296
20	0.67622	0.64990	0.60344	0.00000
25	0.73014	0.73014	0.73014	0.00000
30	0.67621	0.73317	0.67946	0.00000
35	0.52525	0.65656	0.47404	0.00296
40	0.30946	0.50004	0.21674	0.00826
45	0.09570	0.27257	0.03810	0.00502
50	0.00000	0.00000	0.00000	0.00000

Table 2 *Deflection values for the plate of case 2: C-S-C-FCF_(2).*

x,y **: cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**
0	0.00000	0.00000	0.00000	0.00000
5	0.09370	0.08600	0.02010	0.00325
10	0.30297	0.27710	0.12734	0.00337
15	0.51355	0.47534	0.34632	0.00000
20	0.66110	0.62807	0.58237	0.00000
25	0.71337	0.71337	0.71337	0.00000
30	0.66110	0.72187	0.66895	0.00000
35	0.51355	0.64920	0.46911	0.00000
40	0.30297	0.49585	0.21523	0.00337
45	0.09370	0.27064	0.03790	0.00325
50	0.00000	0.00000	0.00000	0.00000

Table 3 *Deflection values for the plate of case 3: C-S-C-FCF_(3).*

x,y **: cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**
0	0.00000	0.00000	0.00000	0.00000
5	0.09330	0.08430	0.01290	0.00010
10	0.30119	0.27345	0.11938	0.00000
15	0.51060	0.46931	0.33999	0.00000
20	0.65716	0.62252	0.57708	0.00000
25	0.70918	0.70918	0.70918	0.00000
30	0.65715	0.71895	0.66632	0.00000
35	0.51059	0.64738	0.46789	0.00000
40	0.30119	0.49478	0.21484	0.00000
45	0.09330	0.27017	0.03780	0.00010
50	0.00000	0.00000	0.00000	0.00000

***Table 4** Deflection values for the plate of case 4: C-C-C-FCF_(1).*

x,y **: cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**
0	0.00000	0.00000	0.00000	0.00000
5	0.08000	0.08480	0.02620	0.00622
10	0.25595	0.27321	0.13981	0.01100
15	0.42967	0.44702	0.33188	0.00449
20	0.54904	0.55725	0.51878	0.00000
25	0.59139	0.59139	0.59139	0.00000
30	0.54904	0.54399	0.50554	0.00000
35	0.42967	0.42346	0.30902	0.00449
40	0.25595	0.25168	0.11136	0.01100
45	0.08000	0.07870	0.01110	0.00622
50	0.00000	0.00000	0.00000	0.00000

***Table 5** Deflection values for the plate of case 5: C-C-C-FCF_(2).*

x,y **: cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**
0	0.00000	0.00000	0.00000	0.00000
5	0.07820	0.07930	0.01940	0.00253
10	0.25020	0.25217	0.11855	0.00264
15	0.41922	0.42219	0.31065	0.00000
20	0.53564	0.53731	0.49943	0.00000
25	0.57644	0.57644	0.57644	0.00000
30	0.53564	0.53454	0.49667	0.00000
35	0.41922	0.41788	0.30530	0.00000
40	0.25020	0.24924	0.11046	0.00264
45	0.07820	0.07800	0.01100	0.00253
50	0.00000	0.00000	0.00000	0.00000

***Table 6** Deflection values for the plate of case 6: C-C-C-FCF_(3).*

x,y **: cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**
0	0.00000	0.00000	0.00000	0.00000
5	0.07790	0.07780	0.01230	0.00009
10	0.24858	0.24876	0.11094	0.00000
15	0.41659	0.41661	0.30471	0.00000
20	0.53209	0.53219	0.49458	0.00000
25	0.57271	0.57271	0.57271	0.00000
30	0.53209	0.53204	0.49447	0.00000
35	0.41659	0.41653	0.30443	0.00000
40	0.24858	0.24856	0.11029	0.00000
45	0.07790	0.07780	0.01100	0.00009
50	0.00000	0.00000	0.00000	0.00000

Table 7 *Deflection values for the plate of case 7: C-F-C-FCF_(1).*

***x,y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**	***w*(*x*,50) : mm.**
0	0.00000	0.00000	0.00000	0.00000	0.00000
5	0.10759	0.09420	0.02640	0.00423	0.14631
10	0.35338	0.31030	0.15179	0.00647	0.49970
15	0.60964	0.53156	0.38691	0.00195	0.88683
20	0.79351	0.71607	0.66285	0.00000	1.17040
25	0.85998	0.85998	0.85998	0.00000	1.27370
30	0.79351	0.96598	0.89065	0.00000	1.17040
35	0.60964	1.04520	0.73574	0.00195	0.88683
40	0.35338	1.11070	0.44872	0.00647	0.49970
45	0.10759	1.17890	0.14500	0.00423	0.14631
50	0.00000	1.27370	0.00000	0.00000	0.00000

Table 8 *Deflection values for the plate of case 8: C-F-C-FCF_(2).*

***x,y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**	***w*(*x*,50) : mm.**
0	0.00000	0.00000	0.00000	0.00000	0.00000
5	0.10563	0.08820	0.02000	0.00353	0.14656
10	0.34683	0.28758	0.12977	0.00365	0.50011
15	0.59783	0.50480	0.36424	0.00000	0.88714
20	0.77825	0.69409	0.64165	0.00000	1.17050
25	0.84307	0.84307	0.84307	0.00000	1.27370
30	0.77825	0.95457	0.88003	0.00000	1.17050
35	0.59783	1.03770	0.73076	0.00000	0.88714
40	0.34683	1.10650	0.44721	0.00365	0.50011
45	0.10563	1.17700	0.14487	0.00353	0.14656
50	0.00000	1.27370	0.00000	0.00000	0.00000

Table 9 *Deflection values for the plate of case 9: C-F-C-FCF_(3).*

***x,y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**	***w*(*x*,50) : mm.**
0	0.00000	0.00000	0.00000	0.00000	0.00000
5	0.10519	0.08650	0.01280	0.00010	0.14653
10	0.34502	0.28388	0.12178	0.00000	0.50009
15	0.59484	0.49872	0.35786	0.00000	0.88708
20	0.77426	0.68848	0.63630	0.00000	1.17040
25	0.83882	0.83882	0.83882	0.00000	1.27360
30	0.77426	0.95159	0.87736	0.00000	1.17040
35	0.59484	1.03590	0.72953	0.00000	0.88708
40	0.34502	1.10530	0.44684	0.00000	0.50009
45	0.10519	1.17640	0.14482	0.00010	0.14653
50	0.00000	1.27360	0.00000	0.00000	0.00000

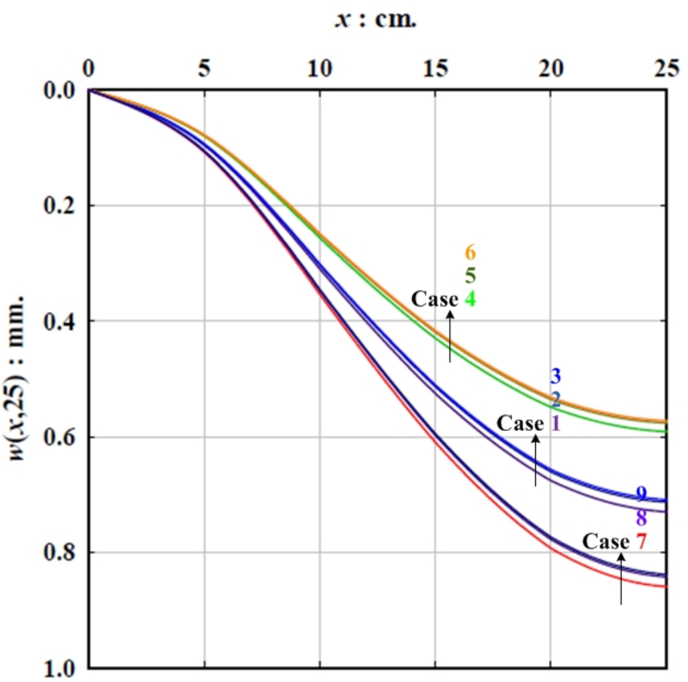

Fig. 13 *Comparative deflections along the middle line w(x,25) of the plates.*

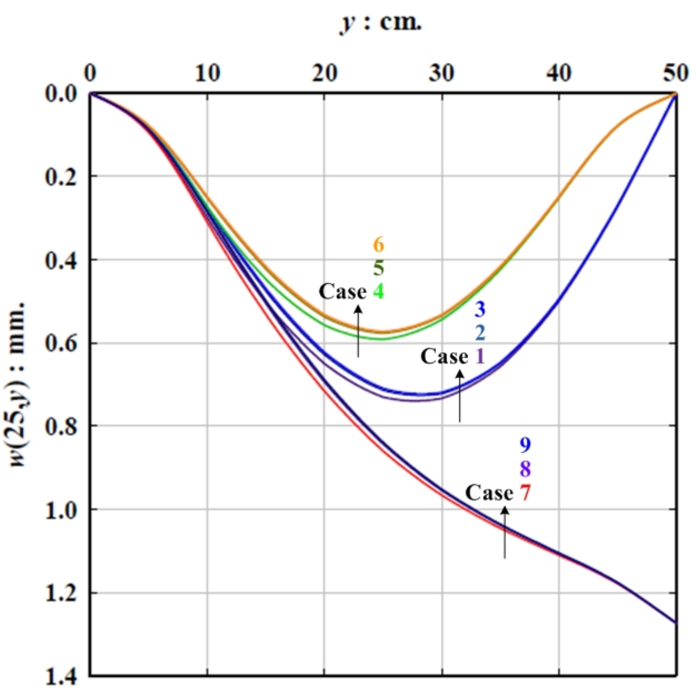

Fig. 14 *Comparative deflections along the middle line w(25,y) of the plates.*

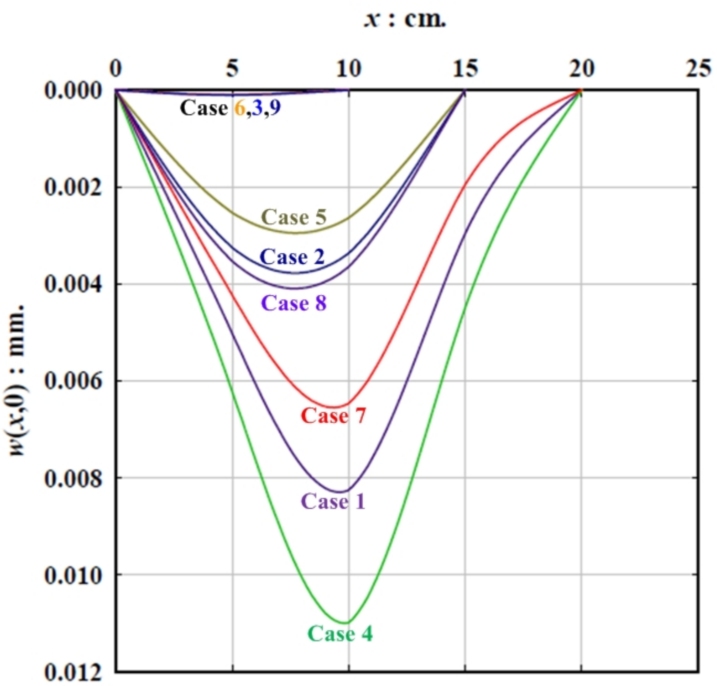

Fig. 15 *Comparative deflections along the free edge w(x,0) of the plates.*

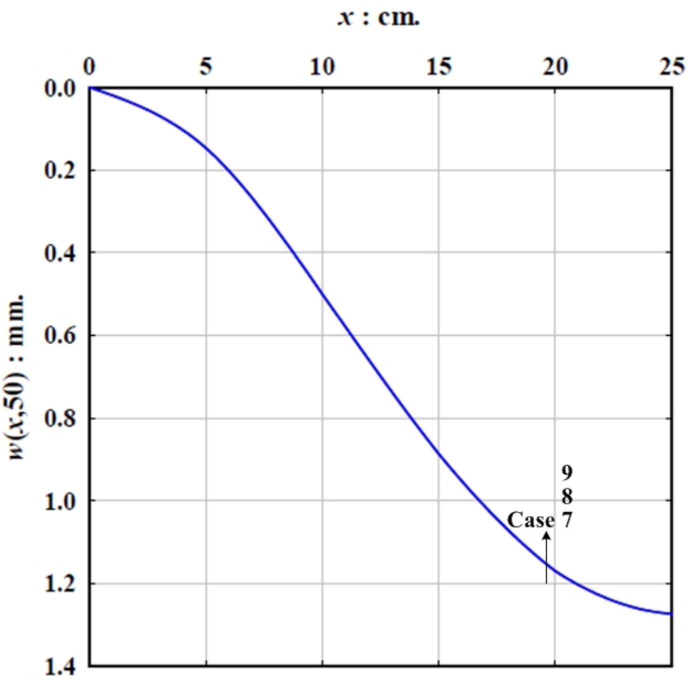

Fig. 16 *Comparative deflections along the free edge, w(x,50) of the plates.*

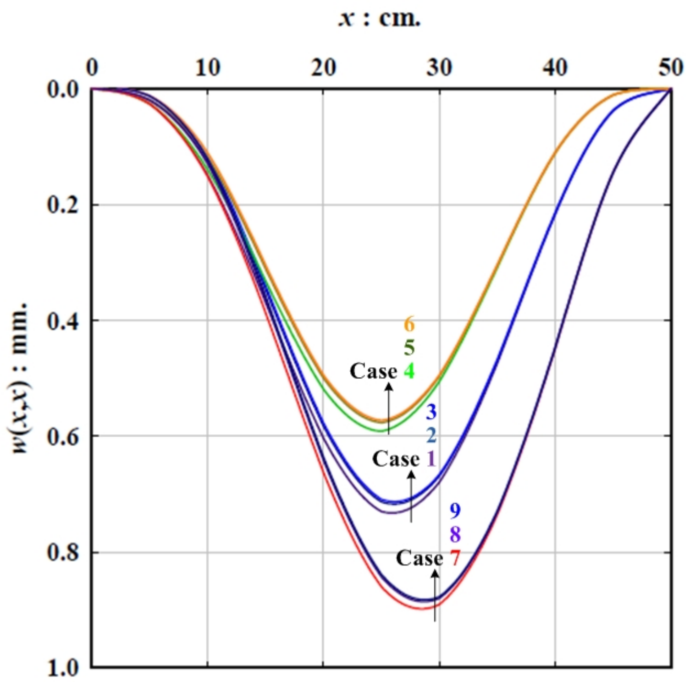

Fig. 17 *Comparative deflections along the diagonal line w(x,x) of the plates.*

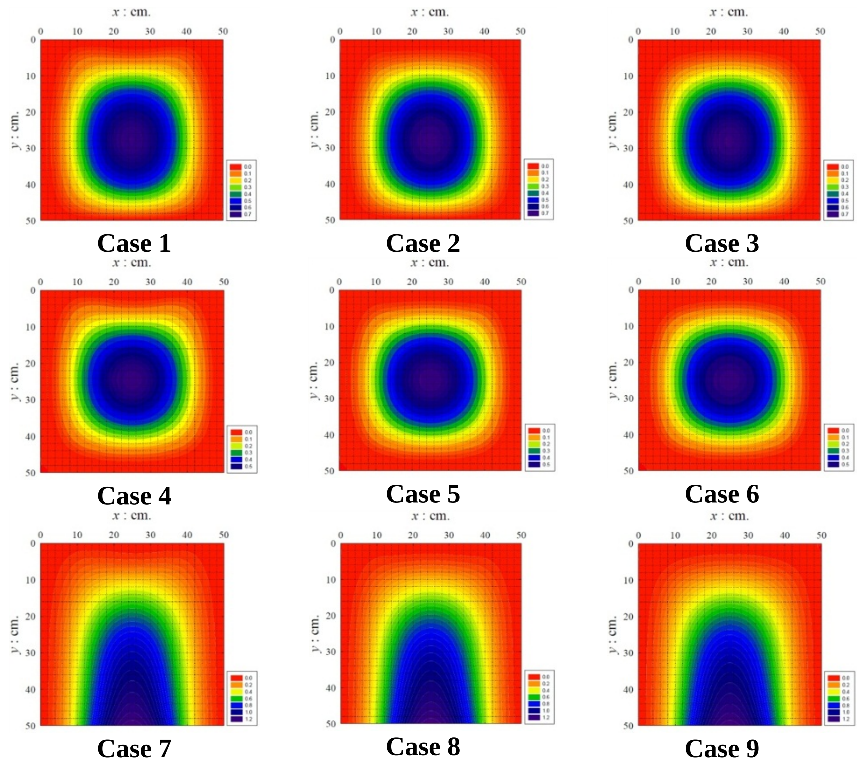

Fig. 18 *Deflection contours w(x,y) for different plate configurations.*

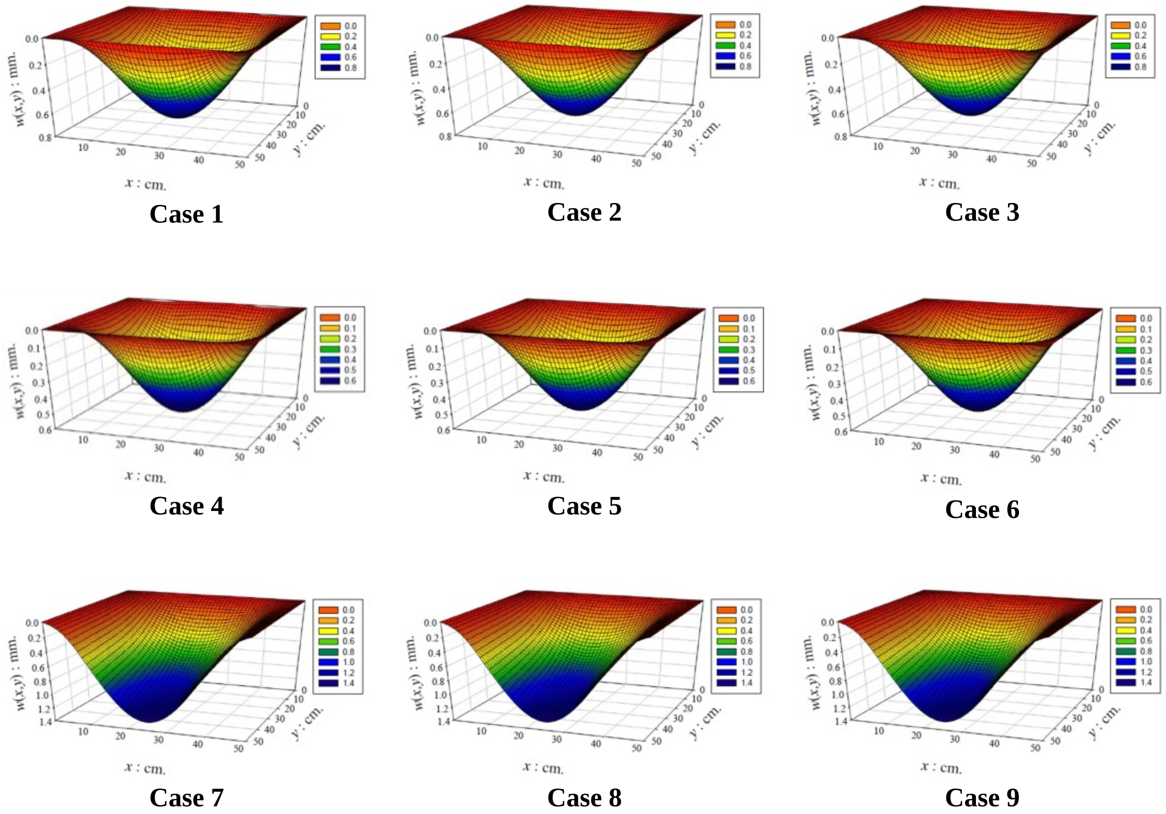

Fig. 19 *Deflection surfaces w(x,y) for different plate configurations.*

4. CONCLUSIONS

This paper numerically presents the deflection responses of square plates having one edge mixed between clamped and free supports, loaded by uniformly distributed load. Results can be carried out using the application of ANSYS computer program based on finite element concept. Nine different configurations of the plate are analyzed and their numerical results concerning the deflection distributions along the middle line, along the diagonal line, and along the free edge of the plate are given in the form of graph and table.

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