Bending of Square Plates with Mixed Conditions between Simply Supported and Free Edges

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ABSTRACT

Nine different configurations for uniformly loaded square plate with mixed boundary conditions between simply supported and free edges are proposed in the present investigation. Results for the plate deflection distribution along the middle line, along the diagonal line, and along the free edge can be determined numerically using the finite element software package named ANSYS. The obtained results are demonstrated graphically and also given numerically in tabular form for easy reference.

Keywords : *Square plate, Mixed support conditions, Finite element method*.

1. INTRODUCTION

Plates have widely been used as part of fundamental structural components throughout various engineering designs and applications [1]. Many solutions concerning the bending of plates with regular or common boundary conditions can be found in Timoshenko and Woinowsky-Krieger [2] or in several scientific literatures. However, it is observed that closed-form solutions can be obtained for the case of regular boundary conditions, but not for the case of mixed boundary conditions. In the latter case, numerical method is an alternative method to be used for approximating the solutions in this class of problem.

The finite element method is one of the powerful numerical methods, which has been used with considerable success to determine the solution of various problems. Salamon et al. [3] proposed the finite element formulation and the iterative algorithm to numerically study the response of a square plate under uniform pressure load that is unilaterally supported around the periphery by discrete elastic springs. Papanikolaou and Doudoumis [4] dealt with a numerical methodology for the elastic analysis of nine different single plates, which takes into account any unilateral support conditions (uplift potential) using the MSC /NASTRAN for Windows finite element software.

Another efficient numerical method was applied by Hu and Hartley [5], who investigated a thin plate supported on unilateral elastic edge supports. The direct boundary element method was developed as the analysis procedure employing iteration to decouple support forces. Examples of square, triangular and hexagonal plates on elastic simple edge supports that subjected to both uniform and concentrated loads were given for a range of elastic support stiffnesses.

In the present work, the bending problem of plate with mixed edge boundary conditions between simple and free supports is considered for finding the deflection responses. Nine different plate configurations for combinations of support types are listed in Fig. 1 showing the plate dimensions, coordinates, and boundary support conditions. They are all loaded by uniformly distributed load. However, it can be noted in Fig. 1 that the letter symbols are designated for representing three classical boundary support conditions. Different letters designating the simple, clamped, and free supports are S, C, F, respectively. Since the plate has more than one type of boundary support conditions along the edge (called mixed boundary conditions), that edge will be designated by a sequence of successive combinatorial letters similar to that supposed by Boonchareon et al. [6].

In the analysis, the ANSYS finite element computer program [7] is used in the same manner with Pramon et al. [8] and Toyaboot et al. [9] for the cases of plate having simply supported-clamped and clamped-free combinations of edge boundary conditions, respectively. Additionally, the material of plate used in this study is of the aluminum plate having the Young's modulus (*E*) and Poisson's ratio (υ) taken as 55.43 GPa and 0.27, respectively [6]. These material properties have been tested in accordance with the standard of JIS Z 2241-1998: Tensile Test through the universal testing machine model (UTM): AG-100KNI M2 from the Iron and Steel Institute of Thailand (ISIT).

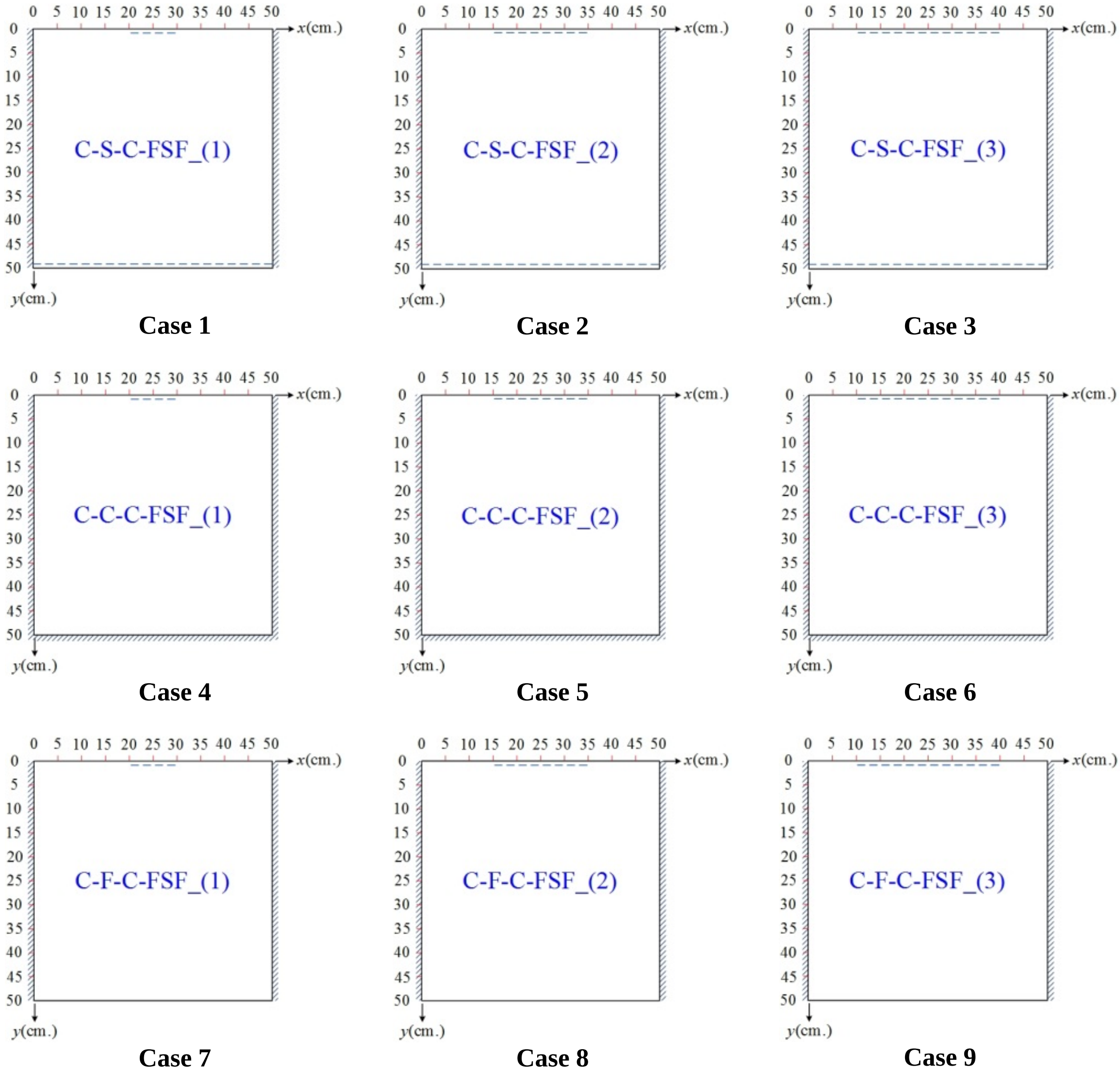

***Fig. 1** Square plates having partial simple support at the center of one edge.*

2. METHOD OF INVESTIGATION

SHELL181 element type of the ANSYS Library [10] which is a four-node element with six degrees of freedom at each node (translations in the *x*, *y*, and *z* directions, and rotations about the *x*, *y*, and *z*-axis) as depicted in Fig. 2 is used to model and analyze the plate-bending problem. This element allows one to analyze thin to moderately-thick shell of plate structures.

In the finite element concept [11], the square plate with plane dimensions of 50 cm. by 50 cm. and uniform thickness of 3 mm. is then discretized with a uniform mesh of 100 square elements as shown in Fig. 3 so that it has 726 degrees of freedom. The intensity of applied uniformly distributed load is equal to 100 kg/m^2. This distributed load will be transformed to an equivalent point load applied to the node of elements.

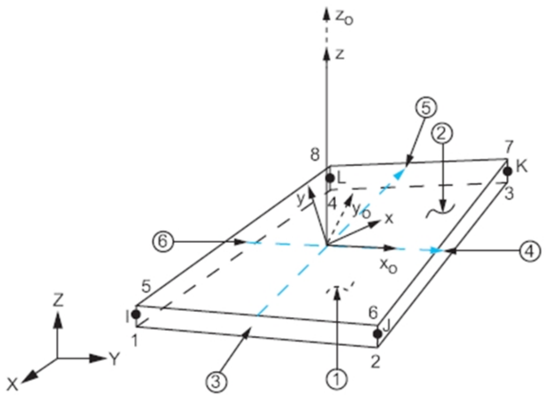

Fig. 2 *Quadrilateral ANSYS SHELL181 element [10].*

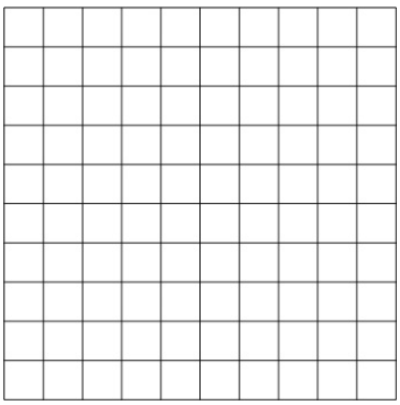

Fig. 3 *Mesh of plate discretization.*

3. NUMERICAL RESULTS

Numerical results for the deflection responses of the plate are all graphically presented using the SigmaPlot program [12]. This program can efficiently illustrate a smooth sharp variation in dependent discretized values within 2D and 3D data sets. Figs. 4 to 12 simultaneously present the distributions for the deflections along two middle lines at $y = 25$ cm. and at $x = 25$ cm., along the diagonal line at $y = x$, and along the free edges at $y = 0$ cm. and at $y = 50$ cm. of the plates with different boundary support conditions. Tables 1 to 9 are their numerical values that correspond with Figs. 4 to 12, respectively.

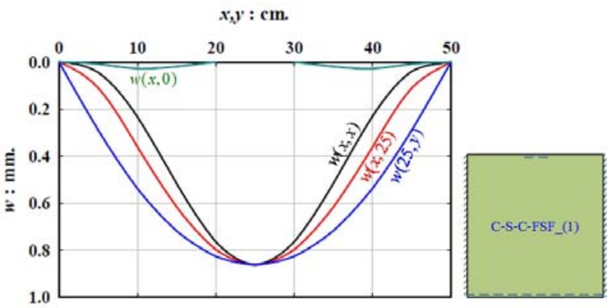

Fig. 4 *Deflection curves (w) for the plate of case 1.*

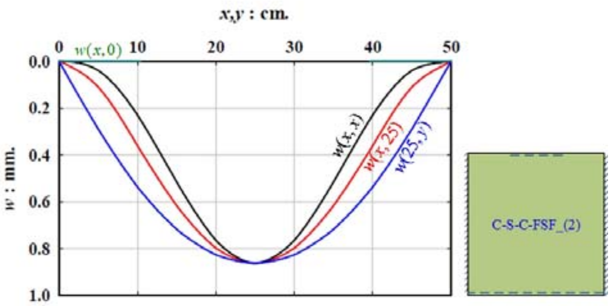

Fig. 5 *Deflection curves (w) for the plate of case 2.*

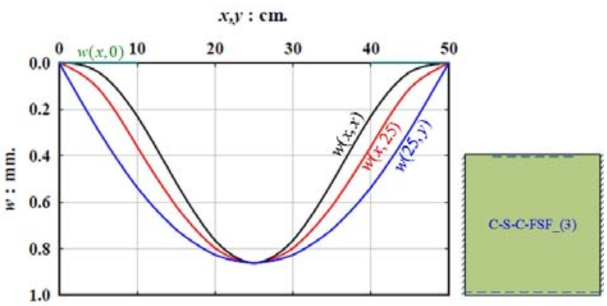

Fig. 6 *Deflection curves (w) for the plate of case 3.*

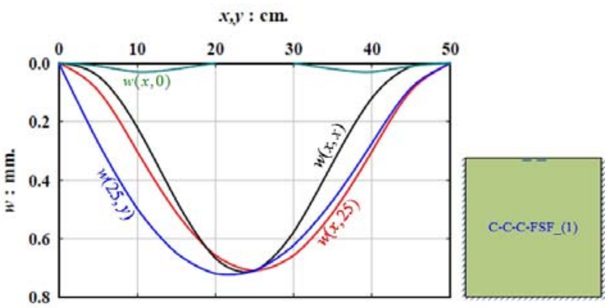

Fig. 7 *Deflection curves (w) for the plate of case 4.*

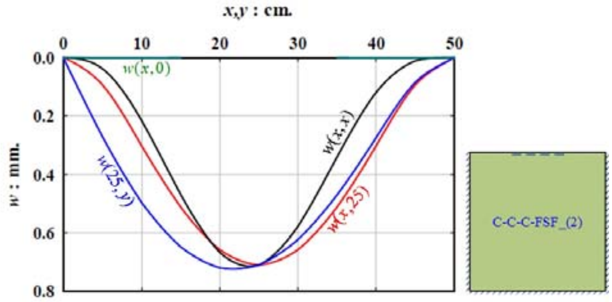

Fig. 8 *Deflection curves (w) for the plate of case 5.*

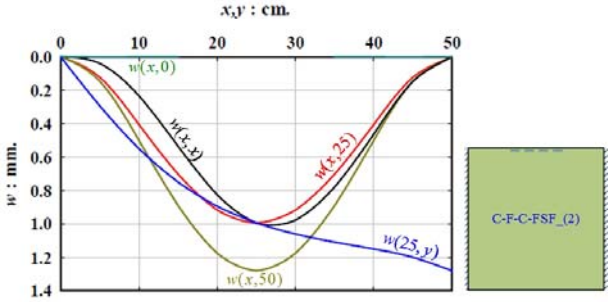

Fig. 11 *Deflection curves (w) for the plate of case 8.*

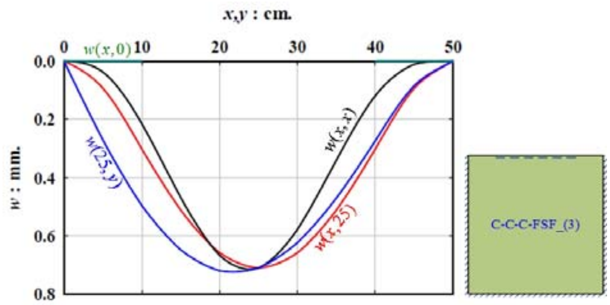

Fig. 9 *Deflection curves (w) for the plate of case 6.*

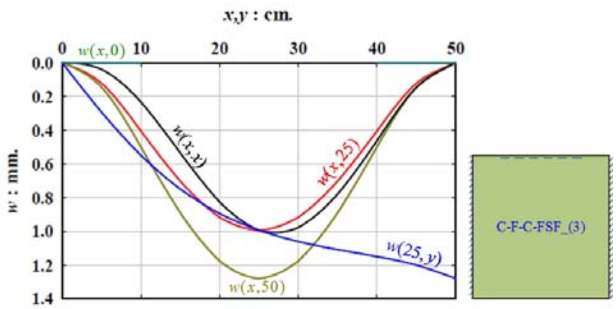

Fig. 12 *Deflection curves (w) for the plate of case 9.*

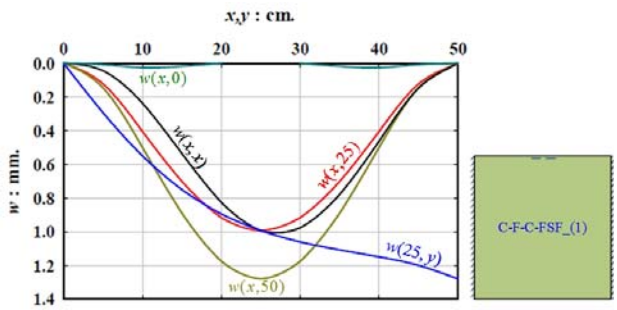

Fig. 10 *Deflection curves (w) for the plate of case 7.*

Figs. 13 to 16 intentionally demonstrate the comparative deflection curves for all cases of the plate. However, the difference cannot be observed from three groups of the plate, which are: (1) simply supported at $y = 50$ cm., (2) clamped at $y = 50$ cm., and (3) free edge at $y = 50$ cm. Figs. 17 and 18 illustrate the variations of deflection contour and deflection surface for all cases of the plate, respectively.

Table 1 *Deflection values for the plate of case 1: C-S-C-FSF_(1).*

***x,y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**
0	0.00000	0.00000	0.00000	0.00000
5	0.11041	0.28913	0.04490	0.00879
10	0.35979	0.53723	0.23366	0.02770
15	0.61535	0.71800	0.51631	0.01910
20	0.79661	0.82579	0.76405	0.00000
25	0.86141	0.86141	0.86141	0.00000
30	0.79661	0.82524	0.76346	0.00000
35	0.61535	0.71665	0.51465	0.01910
40	0.35979	0.53561	0.22967	0.02770
45	0.11041	0.28892	0.03970	0.00879
50	0.00000	0.00000	0.00000	0.00000

Table 2 *Deflection values for the plate of case 2: C-S-C-FSF_(2).*

x,y : cm.	*w*(*x*,25) : mm.	*w*(25,*y*) : mm.	*w*(*x*,*x*) : mm.	*w*(*x*,0) : mm.
0	0.00000	0.00000	0.00000	0.00000
5	0.11055	0.28919	0.03970	0.00022
10	0.36017	0.53614	0.22990	0.00001
15	0.61595	0.71740	0.51520	0.00000
20	0.79735	0.82612	0.76427	0.00000
25	0.86216	0.86216	0.86216	0.00000
30	0.79735	0.82612	0.76427	0.00000
35	0.61595	0.71740	0.51521	0.00000
40	0.36017	0.53614	0.22992	0.00001
45	0.11055	0.28919	0.03980	0.00022
50	0.00000	0.00000	0.00000	0.00000

Table 3 *Deflection values for the plate of case 3: C-S-C-FSF_(3).*

x,y : cm.	*w*(*x*,25) : mm.	*w*(25,*y*) : mm.	*w*(*x*,*x*) : mm.	*w*(*x*,0) : mm.
0	0.00000	0.00000	0.00000	0.00000
5	0.11055	0.28919	0.03980	0.00021
10	0.36017	0.53614	0.22990	0.00000
15	0.61594	0.71739	0.51520	0.00000
20	0.79735	0.82611	0.76427	0.00000
25	0.86216	0.86216	0.86216	0.00000
30	0.79734	0.82611	0.76426	0.00000
35	0.61594	0.71739	0.51519	0.00000
40	0.36017	0.53613	0.22988	0.00000
45	0.11055	0.28918	0.03970	0.00021
50	0.00000	0.00000	0.00000	0.00000

Table 4 *Deflection values for the plate of case 4: C-C-C-FSF_(1).*

x,y : cm.	*w*(*x*,25) : mm.	*w*(25,*y*) : mm.	*w*(*x*,*x*) : mm.	*w*(*x*,0) : mm.
0	0.00000	0.00000	0.00000	0.00000
5	0.09310	0.27007	0.04340	0.00992
10	0.30078	0.49585	0.21880	0.02970
15	0.50989	0.64794	0.46900	0.02000
20	0.65631	0.71852	0.66601	0.00000
25	0.70831	0.70831	0.70831	0.00000
30	0.65631	0.62150	0.57616	0.00000
35	0.50989	0.46858	0.33923	0.02000
40	0.30078	0.27290	0.11855	0.02970
45	0.09310	0.08430	0.01150	0.00992
50	0.00000	0.00000	0.00000	0.00000

Table 5 *Deflection values for the plate of case 5: C-C-C-FSF_(2).*

x,y : cm.	*w*(*x*,25) : mm.	*w*(25,*y*) : mm.	*w*(*x*,*x*) : mm.	*w*(*x*,0) : mm.
0	0.00000	0.00000	0.00000	0.00000
5	0.09320	0.27011	0.03790	0.00048
10	0.30112	0.49467	0.21485	0.00070
15	0.51041	0.64724	0.46780	0.00000
20	0.65695	0.71877	0.66615	0.00000
25	0.70895	0.70895	0.70895	0.00000
30	0.65695	0.62223	0.57683	0.00000
35	0.51041	0.46913	0.33963	0.00000
40	0.30112	0.27321	0.11868	0.00070
45	0.09320	0.08440	0.01150	0.00048
50	0.00000	0.00000	0.00000	0.00000

Table 6 *Deflection values for the plate of case 6: C-C-C-FSF_(3).*

***x*,*y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**
0	0.00000	0.00000	0.00000	0.00000
5	0.09320	0.27015	0.03780	0.00019
10	0.30115	0.49475	0.21483	0.00000
15	0.51046	0.64732	0.46786	0.00000
20	0.65701	0.71885	0.66623	0.00000
25	0.70902	0.70902	0.70902	0.00000
30	0.65701	0.62228	0.57687	0.00000
35	0.51046	0.46916	0.33965	0.00000
40	0.30115	0.27322	0.11868	0.00000
45	0.09320	0.08440	0.01150	0.00019
50	0.00000	0.00000	0.00000	0.00000

Table 7 *Deflection values for the plate of case 7: C-F-C-FSF_(1).*

***x*,*y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**	***w*(*x*,50) : mm.**
0	0.00000	0.00000	0.00000	0.00000	0.00000
5	0.12252	0.29319	0.04470	0.00801	0.14576
10	0.40435	0.54993	0.23666	0.02600	0.49935
15	0.70089	0.74960	0.53563	0.01820	0.88868
20	0.91546	0.89379	0.82512	0.00000	1.17470
25	0.99296	0.99296	0.99296	0.00000	1.27900
30	0.91546	1.05980	0.97623	0.00000	1.17470
35	0.70089	1.10730	0.77765	0.01820	0.88868
40	0.40435	1.14880	0.46236	0.02600	0.49935
45	0.12252	1.19890	0.14666	0.00801	0.14576
50	0.00000	1.27900	0.00000	0.00000	0.00000

Table 8 *Deflection values for the plate of case 8: C-F-C-FSF_(2).*

***x*,*y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**	***w*(*x*,50) : mm.**
0	0.00000	0.00000	0.00000	0.00000	0.00000
5	0.12266	0.29325	0.03980	0.00056	0.14580
10	0.40473	0.54890	0.23308	0.00036	0.49952
15	0.70149	0.74904	0.53459	0.00000	0.88900
20	0.91620	0.89413	0.82536	0.00000	1.17510
25	0.99372	0.99372	0.99372	0.00000	1.27950
30	0.91620	1.06070	0.97708	0.00000	1.17510
35	0.70149	1.10810	0.77826	0.00000	0.88900
40	0.40473	1.14960	0.46265	0.00036	0.49952
45	0.12266	1.19950	0.14672	0.00056	0.14580
50	0.00000	1.27950	0.00000	0.00000	0.00000

Table 9 *Deflection values for the plate of case 9: C-F-C-FSF_(3).*

***x*,*y* : cm.**	***w*(*x*,25) : mm.**	***w*(25,*y*) : mm.**	***w*(*x*,*x*) : mm.**	***w*(*x*,0) : mm.**	***w*(*x*,50) : mm.**
0	0.00000	0.00000	0.00000	0.00000	0.00000
5	0.12266	0.29322	0.03990	0.00022	0.14580
10	0.40471	0.54886	0.23309	0.00000	0.49952
15	0.70147	0.74900	0.53456	0.00000	0.88900
20	0.91617	0.89409	0.82532	0.00000	1.17510
25	0.99369	0.99369	0.99369	0.00000	1.27950
30	0.91617	1.06070	0.97705	0.00000	1.17510
35	0.70147	1.10810	0.77825	0.00000	0.88900
40	0.40471	1.14960	0.46264	0.00000	0.49952
45	0.12266	1.19950	0.14672	0.00022	0.14580
50	0.00000	1.27950	0.00000	0.00000	0.00000

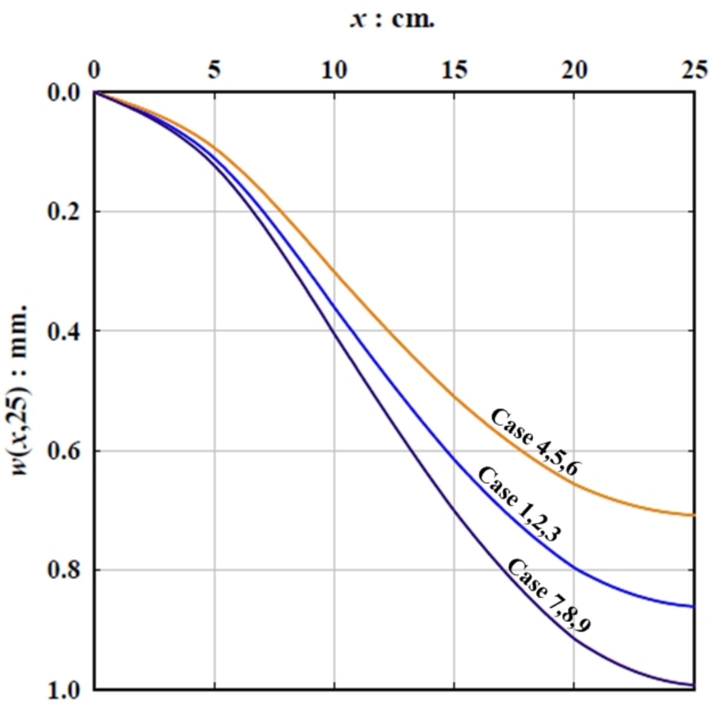

Fig. 13 *Comparative deflections along the middle line w(x,25) of the plates.*

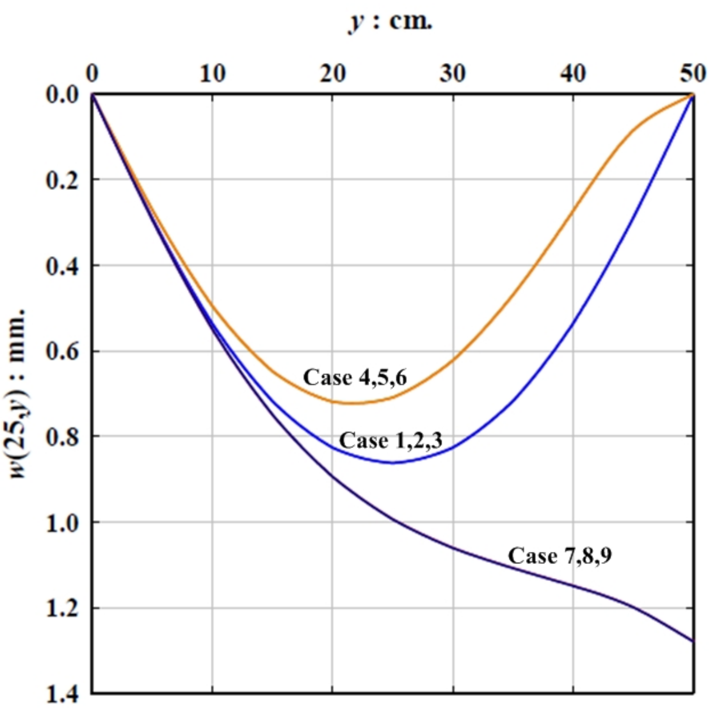

Fig. 14 *Comparative deflections along the middle line w(25,y) of the plates.*

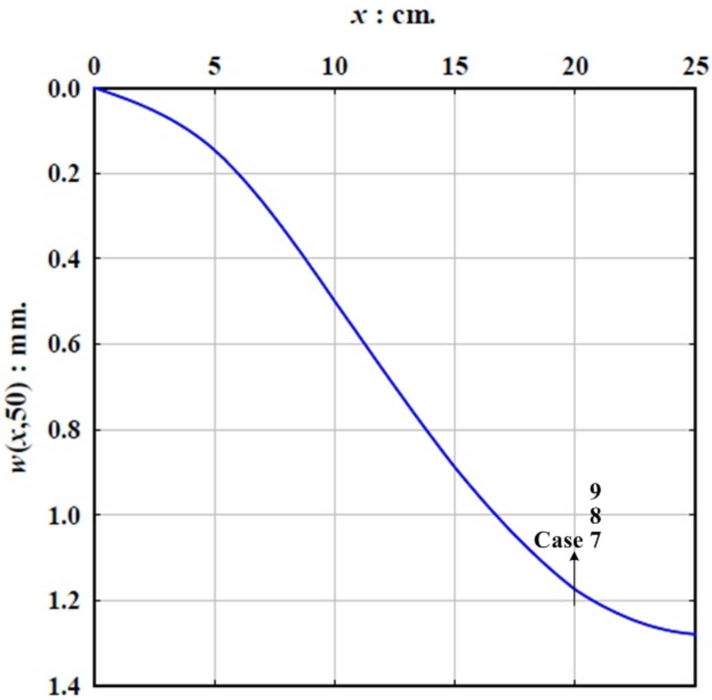

Fig. 15 *Comparative deflections along the free edge w(x,50) of the plates.*

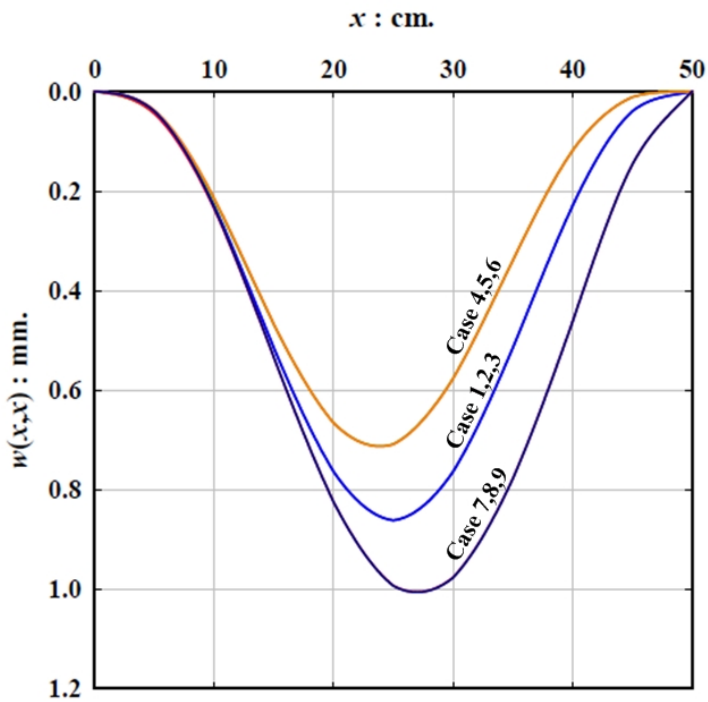

Fig. 16 *Comparative deflections along the diagonal line w(x,x) of the plates.*

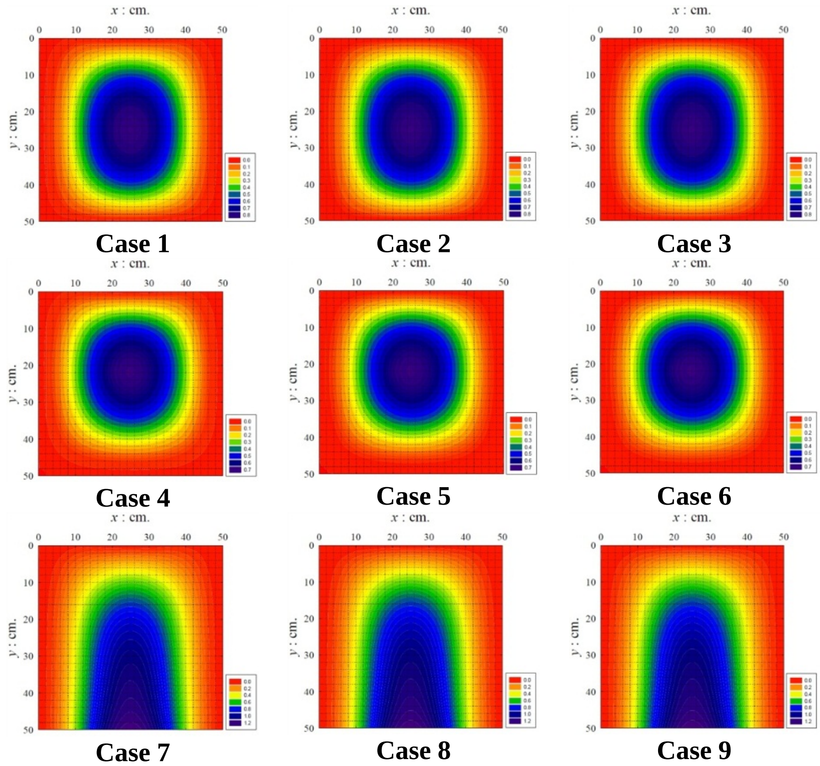

Fig. 17 *Deflection contours w(x,y) for square plates.*

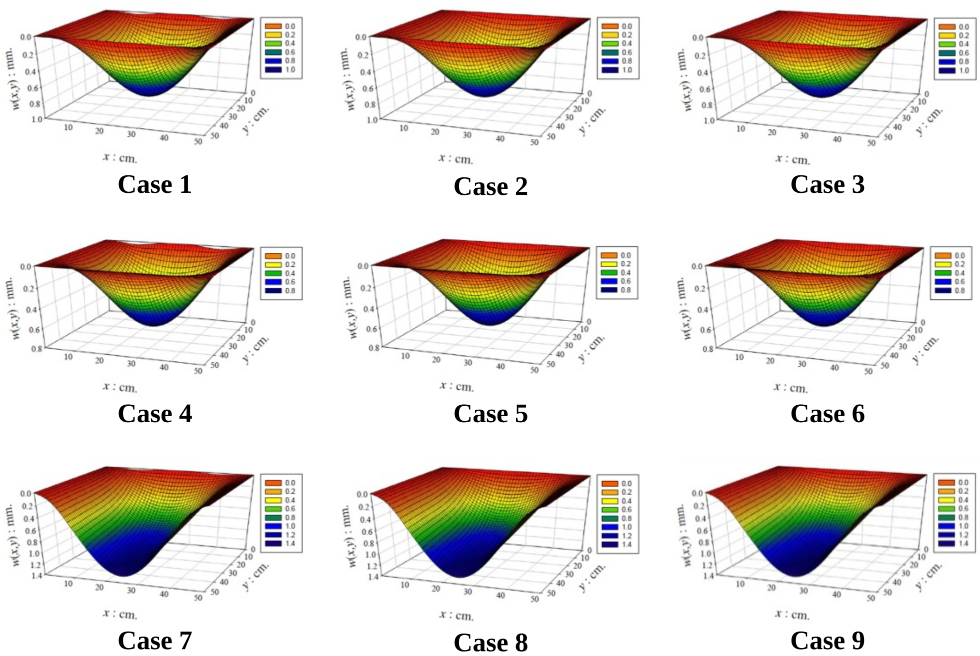

Fig. 18 *Deflection surfaces w(x,y) for square plates.*

4. CONCLUSIONS

With the implementation of ANSYS computer finite element software for windows, the deflection responses of nine different configurations for uniformly loaded square plate with mixed boundary support conditions between simply supported and free edges are determined numerically in the present investigation. The obtained results are prepared and given for the deflection distributions along the middle line, along the diagonal line, and along the free edge of the plates in the form of graph and table. The latter is for easy reference by other investigators.

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