Numerical Finite Element Determination on Free, Transverse Vibrations of Circular Plates

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ABSTRACT

This study presents an approximate numerical finite element solution for the higher natural frequencies of circular plates having three different classical boundary support conditions namely, clamped, simply supported, and free edges. The first eighty frequency coefficients are reported and tabulated in the form of table for easy reference, which can be used as benchmark for other alternative methods. In addition, the modal patterns are also graphically demonstrated for showing vibration behaviors.

Keywords : *Circular plate, Free vibration, Natural frequency, Finite element method*.

1. INTRODUCTION

The flat plate is a very common structural component in engineering applications, particularly in civil, mechanical and aerospace structures. Much attention was received to investigate the bending [1] and vibration [2], [3] of the plates. Although theoretical analysis is valuable for providing basic understanding, in general, it is not always easily attainable. This is due to the difficulty lied in the requirements to satisfy both, the governing partial differential equation and boundary conditions exactly.

In the exact vibration analysis of a uniform circular plate, polar coordinates are conveniently used to obtain separable solutions to the partial differential equation of motion. Significantly, the exact vibration mode shape is found to be expressed as a series of products of Bessel and modified Bessel functions [3], [4]. This yields an infinite number of solutions in frequency equation. Thus, numerical calculation is the principal drawback in this analysis. Nevertheless, Rajalingham et al.[5] studied the vibrations of clamped uniform elliptical plate in modified polar coordinates with the exact modes of an analogous circular plate as the shape functions in the Rayleigh-Ritz method.

For the engineering applications, an alternative numerical method is one of the most important approaches for obtaining full solutions for theoretical analysis and engineering design. Hence, a variety of numerical methods have been developed and used for the vibration analysis of circular plates. Olson and Lindberg [6] developed finite plate-bending elements in polar coordinates for solving several static and dynamic problems of plates. Numerical results indicated the good engineering accuracy to be expected with the presented method.

Laura et al.[7] presented two independent solution methods: the optimized Rayleigh-Ritz method and the standard finite element ALGOR code, for determining fundamental frequency of orthotropic annular plates having non-uniform thickness and a free inner edge. SAMCEF finite element code was applied to investigate the clamped and simply supported circular plates carrying a central, concentrated mass [8]. The mesh of 1513 elements with 7275 degrees of freedom was used to generate the model of the plates. Laura et al.[9] dealt with the determination of the fundamental frequency coefficients for vibrating annular plate with free edges and two intermediate concentric circular supports. The optimized Rayleigh-Ritz method and a finite element algorithmic procedure based on the ALGOR code were used. Singh [10] proposed a polynomial version of the finite element method for the free vibration analysis of circular and elliptical plates. Natural frequencies were calculated for the plates having clamped and simply supported boundary conditions.

2. FINITE ELEMENT DETERMINATION

For the problems at hand, the clamped, simply supported and free circular plates with uniform thickness *h* and radius *a* that depicted in Figs.1 to 3, respectively, are considered. An approximate solution for the frequency coefficients can be carried out numerically by making use of the standard finite element code ANSYS [11]. All calculations are performed for a Poisson's ratio taken as 0.3.

Numerical experiments are run using the net shown in Fig.4 employing quadrilateral shape of SHELL181 element type [12] with 12720 elements and 73482 degrees of freedom. The same discretization is used for three types of boundary conditions.

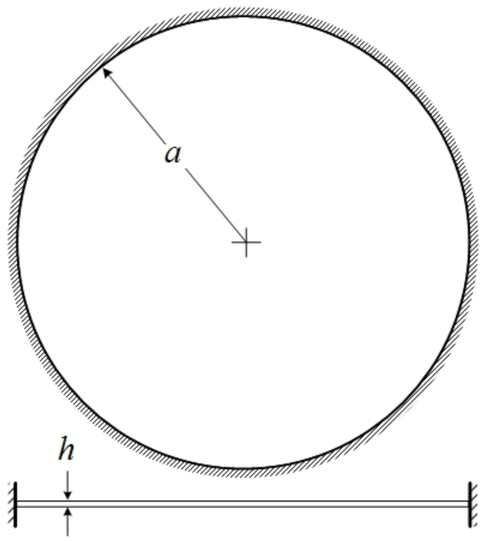

Fig. 1 *Configuration of clamped circular plate.*

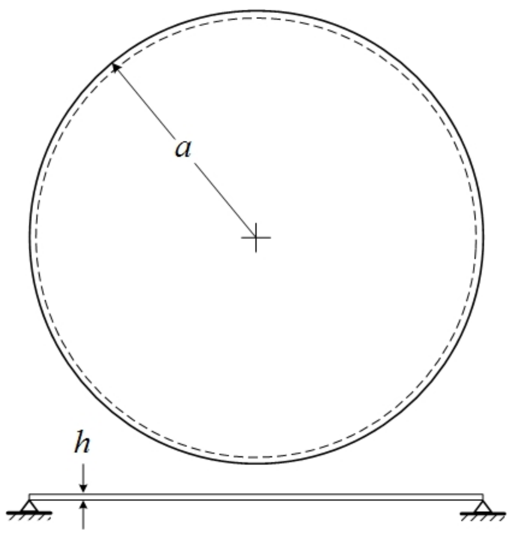

Fig. 2 *Configuration of simply supported circular plate.*

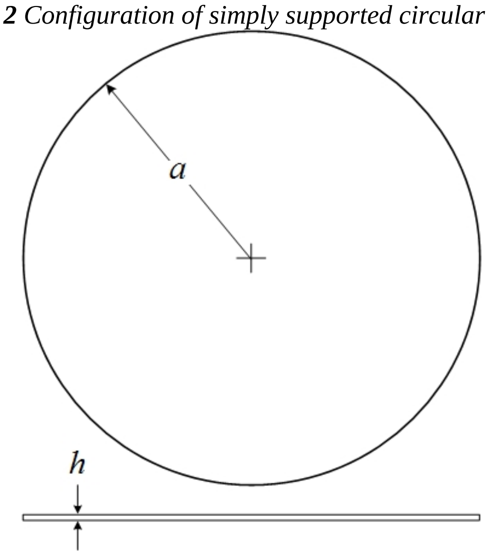

Fig. 3 *Configuration of completely free circular plate.*

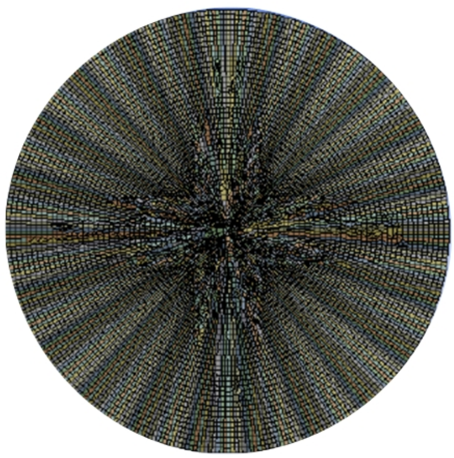

Fig. 4 *Mesh of finite element net used in circular plate.*

3. NUMERICAL RESULTS

The results obtained from the ANSYS computer program are given in terms of natural frequencies (*f*) with the unit of hertz.

However, it is convenient to express them in the form of frequency coefficients as $\lambda^2 = 2\pi fa^2\sqrt{\rho h / D}$ where ρ and D are mass density per unit area of the plate and plate's flexural rigidity, respectively.

Fig. 5 shows the first thirty frequency coefficients with respect to the mode number of vibrations. The upper, middle, and lower lines are for the plates having clamped, simply supported and free edges, respectively. It can be noted only for the case of completely free circular plate that the first six modes give the frequency coefficients equal to zero. These modes are corresponded with the rigid body motions of the plate.

For the higher mode of free vibrations, the additional frequency coefficients up to the 80th mode are prepared and given in Table 1. Figs. 6 to 8 present the first twenty vibration mode patterns corresponding to clamped, simply supported and free circular plates, respectively.

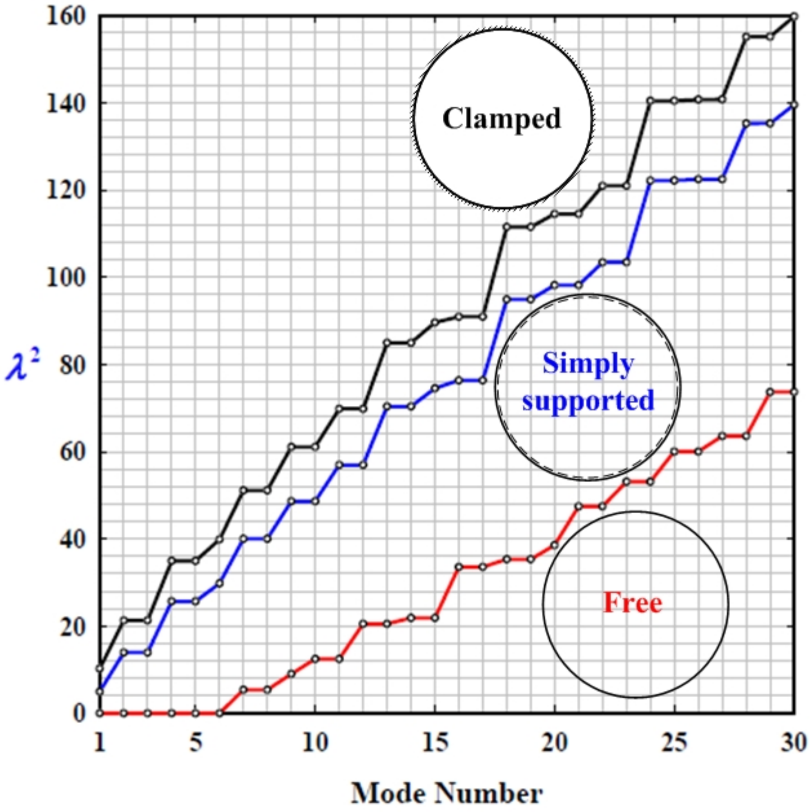

Fig. 5 *Frequency coefficients for clamped, simply supported and free circular plates.*

4. SUMMARY

With the implementation of a well-known ANSYS finite element software package, frequency coefficients for the free vibrations of circular plates having three different types of support can be determined numerically. A dense net of 12720 SHELL181 elements is used for obtaining accurate values. Numerical results for the first eighty frequency coefficients are provided in tabular form for easy use by other researchers and assessing other analytical and numerical methods.

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***Table 1** Frequency coefficients for the first 80 vibration modes of circular plates*

Mode	Clamped	Simply supported	Free	Mode	Clamped	Simply supported	Free
1	10.217	4.936	0.000	**41**	207.142	184.719	112.340
2	21.279	13.906	0.000	**42**	207.163	184.730	119.691
3	21.280	13.907	0.000	**43**	231.544	207.378	119.717
4	34.917	25.634	0.000	**44**	231.651	207.465	122.787
5	34.918	25.635	0.000	**45**	232.554	209.608	122.787
6	39.853	29.763	0.000	**46**	232.579	209.623	124.992
7	51.099	39.997	5.359	**47**	244.252	219.968	124.997
8	51.100	39.997	5.359	**48**	244.268	219.974	142.949
9	61.025	48.599	9.005	**49**	246.154	220.886	142.980
10	61.035	48.605	12.441	**50**	246.231	220.948	149.839
11	69.775	56.908	12.441	**51**	250.998	225.387	149.839
12	69.780	56.908	20.489	**52**	267.987	243.401	151.249
13	84.914	70.329	20.490	**53**	268.013	243.417	151.254
14	84.924	70.334	21.840	**54**	274.001	247.743	154.407
15	89.584	74.476	21.841	**55**	274.052	247.784	154.443
16	90.906	76.311	33.511	**56**	283.930	257.801	158.236
17	90.911	76.317	33.511	**57**	283.966	257.832	176.430
18	111.494	94.864	35.293	**58**	293.404	265.814	176.440
19	111.494	94.864	35.294	**59**	293.414	265.819	176.861
20	114.467	98.170	38.505	**60**	303.559	275.241	176.861
21	114.467	98.170	47.414	**61**	303.575	275.267	182.248
22	120.921	103.369	47.415	**62**	305.774	279.552	182.259
23	120.952	103.394	53.063	**63**	305.810	279.573	192.112
24	140.422	122.126	53.063	**64**	319.138	290.836	192.122
25	140.427	122.152	59.974	**65**	319.231	290.902	200.104
26	140.730	122.444	59.979	**66**	326.198	298.238	200.134
27	140.766	122.449	63.526	**67**	326.213	298.254	206.060
28	155.073	135.214	63.526	**68**	343.371	313.566	206.060
29	155.109	135.239	73.630	**69**	343.658	313.787	212.689
30	159.774	139.505	73.636	**70**	345.924	318.067	212.704
31	168.766	149.122	81.833	**71**	345.960	318.088	215.760
32	168.776	149.127	81.838	**72**	358.935	328.053	215.775
33	172.636	152.100	84.642	**73**	359.078	328.181	232.790
34	172.647	152.105	84.647	**74**	364.000	332.816	232.902
35	191.932	169.884	88.159	**75**	367.009	336.686	237.444
36	191.937	169.889	96.899	**76**	367.091	336.743	237.444
37	199.478	178.178	96.899	**77**	370.961	341.192	245.144
38	199.494	178.188	102.323	**78**	371.074	341.284	245.221
39	201.534	178.706	102.328	**79**	388.427	358.940	249.276
40	201.565	178.737	112.335	**80**	388.473	358.965	251.691

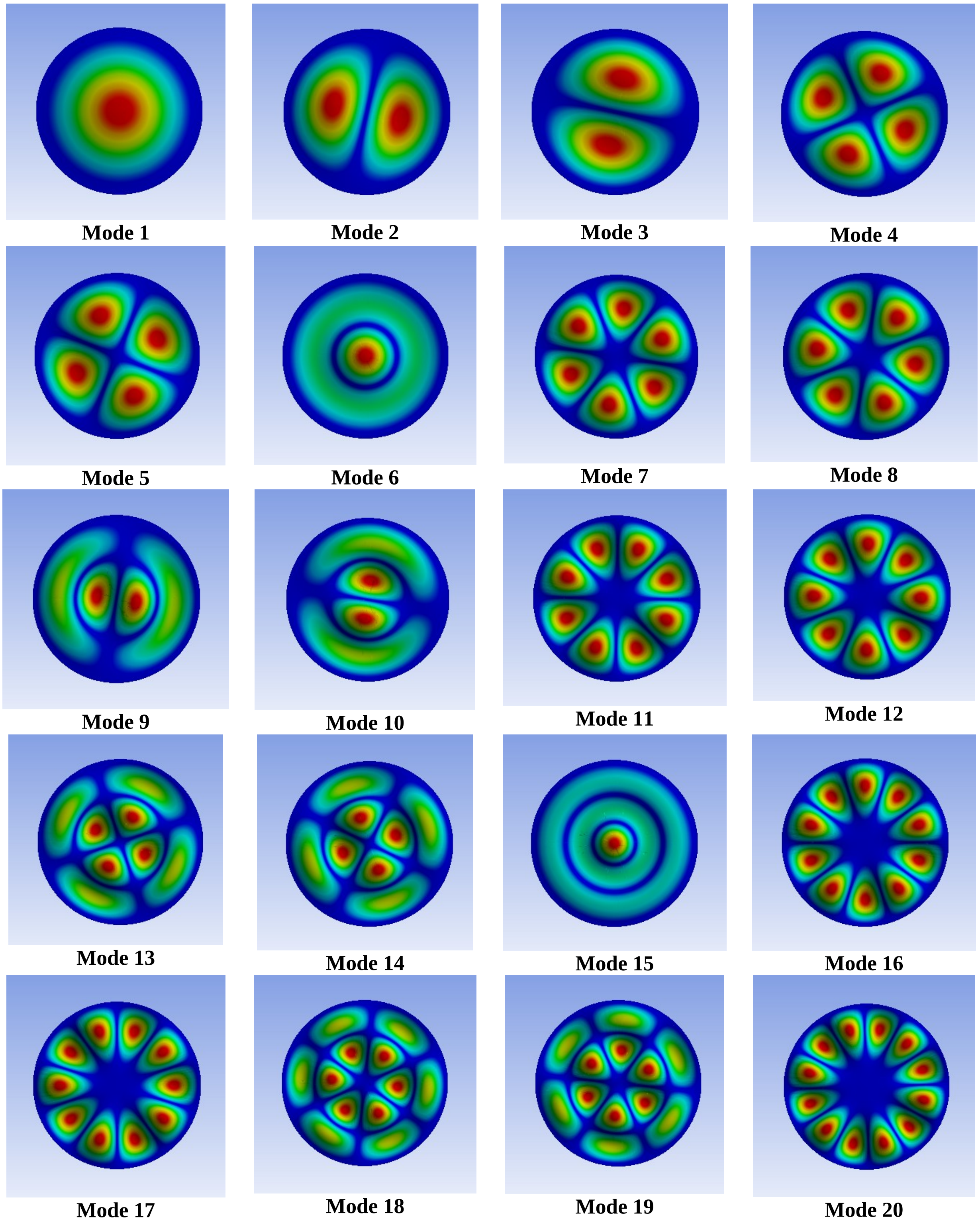

Fig. 6 *First 20 vibration mode contours for clamped circular plate.*

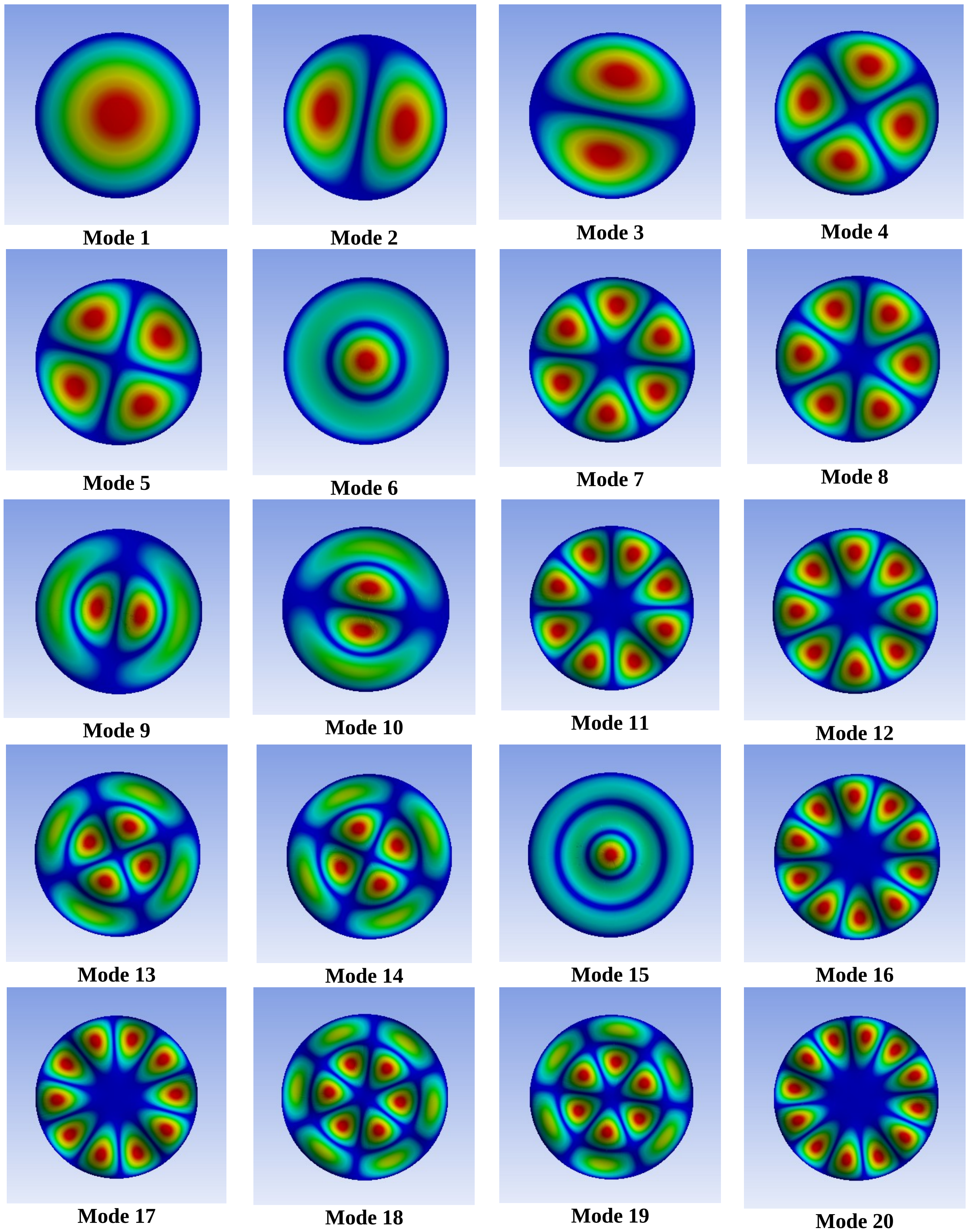

Fig. 7 *First 20 vibration mode contours for simply supported circular plate.*

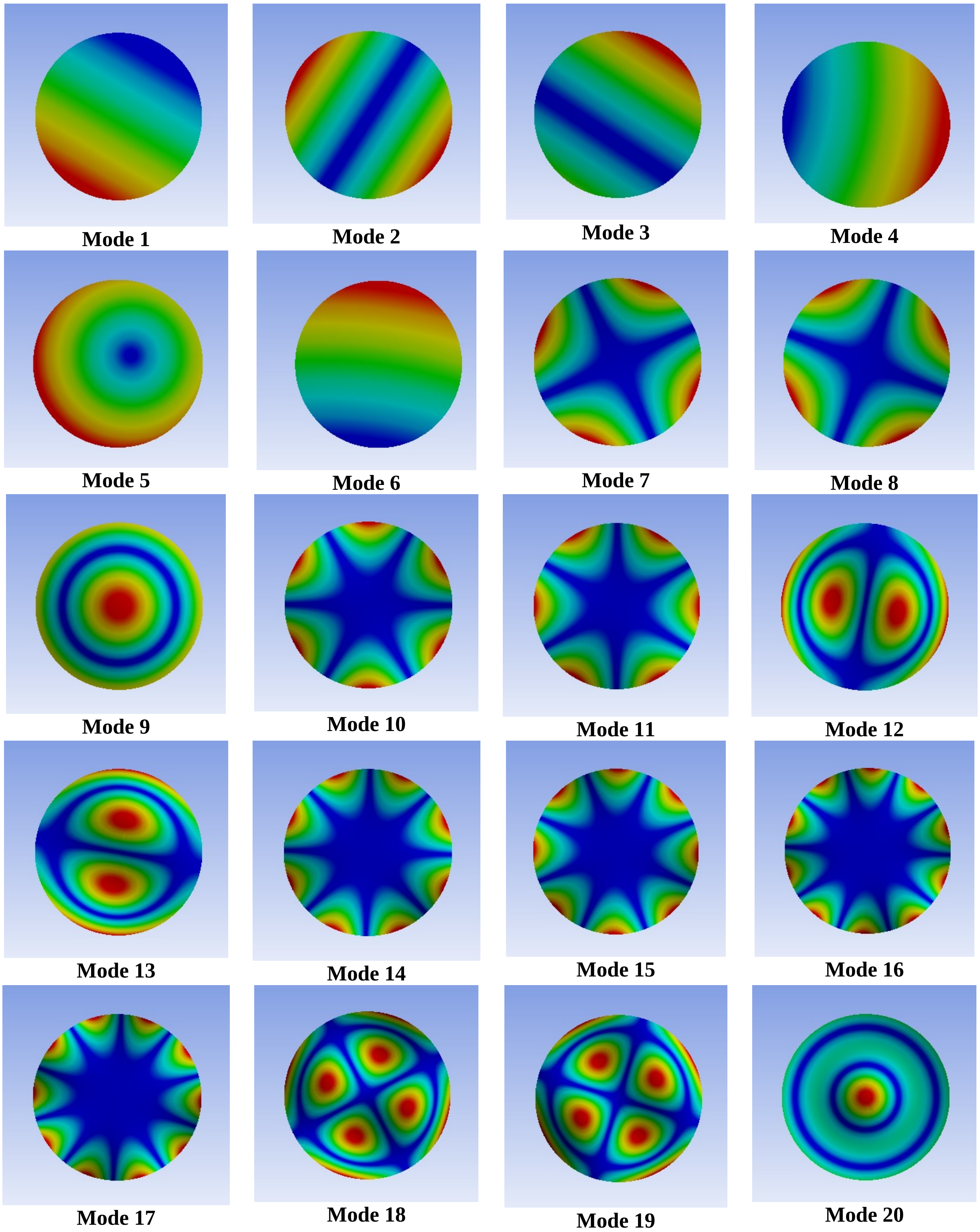

Fig. 8 *First 20 vibration mode contours for completely free circular plate.*

REFERENCES

[1] S.P. Timoshenko and S. Woinowsky-Krieger, Theory of Plates and Shells. 2nd ed., McGraw-Hill, Singapore, 1959.

[2] S.P. Timoshenko and D.H. Young, Vibration Problems in Engineering, 3rd ed., D. Van Nostrand Company, Inc., New Jersey, 1955.

[3] A.W. Leissa, Vibration of Plates, Reprinted ed., Acous-tical Society of America, Washington, D.C., 1993.

[4] L. Meirovitch, Analytical Methods in Vibrations, Macmillan Publishing Co., Inc., New York, 1967.

[5] C. Rajalingham, R.B. Bhat and G.D. Xistris, “A note on elliptical plate vibration modes as a bifurcation from cir-cular plate modes”, International Journal of Mechanical Sciences, vol. 37, pp. 61-75, 1995.

[6] M.D. Olson and G.M. Lindberg, “Annular and circular sector finite elements for plate bending”, International Journal of Mechanical Sciences, vol. 12, pp. 17-33, 1970.

[7] P.A.A. Laura, R.E. Rossi, D.A. Vega, S.A. Vera and M.D. Sanchez, “Vibrations of orthotropic, circular annular plates of non-uniform thickness and a free inner edge”, Journal of Sound and Vibrations, vol. 218, pp. 159-163, 1998.

[8] P.A.A. Laura, E. Romanelli, V. Sonzogni, and S. Idelsohn, “Numerical experiments on vibrating circular plates of rectangular orthotropy and carrying a central, concentrated mass”, Journal of Sound and Vibrations, vol. 221, pp. 737-749, 1999.

[9] P.A.A. Laura, R.H. Gutierrez and R.E. Rossi, “Transverse vibrations of a circular, annular plate with free edges and two, intermediate concentric circular supports”, Journal of Sound and Vibrations, vol. 226, pp. 1043-1047, 1999.

[10] A.V. Singh, “A numerical free vibration analysis of annular elliptic plates”, Multidiscipline Modeling in Materials and Structures, vol. 1, pp. 53-62, 2005.

[11] ANSYS, Inc., ANSYS Mechanical APDL Theory Refe-rence, Release 14.5, 2012.

[12] ANSYS, Inc., ANSYS Mechanical APDL Element Refe-rence, Release 14.5, 2012.

Ronnachai Malison, photograph and biography not available at the time of publication.

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