Effect of K^{+} Potential and Nuclear Equation of State on the Invariant Cross-Section of K^{+} Production in Nucleus-Nucleus Collisions

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Abstract

This work examines and concentrates on the effect of K^{+} potential and nuclear equation of state on the invariant cross-section of K^{+} production in nucleus-nucleus collisions. The invariant cross-section of K^{+} production as a function of kinetic energy in heavy-ion collisions at incident energy 0.8 and 1.0 A GeV (GeV/Nucleon) in ^{12}C + ^{12}C collisions with impact parameter between 3.70 and 5.73 fm by using the quantum molecular dynamics model (QMD) is studied. The calculations energized with and without the Brown-Rho parameters ($K^{+}N$ potential) as well as the soft and hard equation of state (soft and hard EOS). In addition, the invariant cross-section of K^{+} production as a function of kinetic energy is computed and compared with KaoS experiments. The results show that K^{+} production is measured by utilizing the QMD model with a soft EoS, which similar to a hard EoS. The theoretical calculations with soft and hard EoS while expanding the $K^{+}N$ potential tend to be consistent with the KaoS experiments, Consequently, this work refers that the invariant cross-section of K^{+} production in heavy-ion collision at intermediate is sensitive observable to probe the nuclear equation of state in dense nuclear matter.

Keywords: the quantum molecular dynamics model (QMD), K^{+} production, nucleus-nucleus collisions

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1. Introduction

The nuclear matter properties at high temperature and high densities are described by the relativity of heavy-ion collisions as well as the nuclear equation of state of nuclear matter plays an important role in physics phenomenon and astrophysical such as the dynamics of the modeling of neutron stars and a supernova explosion which the nuclear matter is compressed to high density and temperature conditions (Oyamatsu & Sumiyoshi, 1998).

According to former research of kaon production in heavy-ion reactions at intermediate energies (Fuchs, 2006), the theoretical predictions kaon-nucleon potentials in a nuclear matter are repulsive for K^+ but attractive for K^- mesons which were reviewed, in heavy-ion reaction at high sensitivity to the dense nuclear matter is shown under the threshold of kaon production. Subthreshold kaon production as a probe of the nuclear equation of state (Aichelin & Ko, 1985) for central collisions between heavy nuclei at incident energies around 700 MeV/nucleon, the numbers of produced kaons differs by a factor ~3, depending on the equation of state. Moreover, in the relativistic heavy-ion collisions in the center of mass system of subthreshold energy which a nuclear density around 2-3 ρ_0 (ρ_0 is a normal nuclear density) was studied (Hartnack et al., 1994; Hartnack, 2004). The research presented the strange mesons production below the production thresholds of these particles in the collisions of free nucleon-nucleon (NN), which is a sensitive investigation to analyze the nuclear matter behavior at high densities. The kaons rest mass is equal to 0.454 GeV. Also, in the NN collisions (in the laboratory frame) the threshold of K^+ production is determined by the combined production of , which is 1.58 GeV and for the K^- production through the pair formation is 2.5 A GeV. The density reached in the reaction region depends on the stiffness of nuclear matter, wherewith their particular production mechanism and their rather long path (5 fm at normal nuclear density). Consequently, K^+ mesons are ideal probes for test the high-density phase of a heavy-ion reaction and studying the stiffness of the nuclear equation of state (EoS) (Ko, 1984; Oeschler, 2006; Fuchs et al., 2001; Hartnack et al., 2006).

In this work, we studied the invariant cross-section of K^+ production as a function of kinetic energy at an incident energy of 0.8 and 1.0A GeV by applying the quantum molecular dynamics (QMD) model. The QMD model is an n-body theory that all of the information can find the solution of the n-body Lioville equation. The QMD model is more predominant than the concept of the solution equation by using the equation of Boltzmann-Unlenback-Uehling

(BUU) and Vlasov-Unlenback-Uehling (VUU) that can observe only one-body. All of the above model comparisons can be studied from this work (Bhalerao et al., 2003; Chaimongkon et al., 2019). The results of the calculation of the K^{+} mesons cross-section production in with and without the kaon potential by using the nuclear equation of state are compared to the KaoS experiments (Hartnack et al., 2012).

2. Theories

The QMD Model

By and large, an n-body theory generated in the heavy-ion collisions at intermediate energies on computational physics has been described by the Quantum Molecular Dynamics model (QMD model) (Aichelin et al., 1987; Aichelin & Stöcker, 1986; Maruyama et al., 1990; Aichelin, 1991). In the QMD model, the baryon dynamics are described which each nucleon is indicated by a coherent state of the form

$$\psi(\mathbf{r},\ \mathbf{p},\ t) = \frac{1}{(2\pi L)^{3/4}} \exp\left\{-\frac{(\mathbf{r}-\mathbf{r}_0)^2}{4L}\right\} \exp\{i\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}_0)\}, \tag{1}$$

here the $\mathbf{r}_0$ represents the time-dependent centre of the Gaussian wave packet. The parameter L relates to the extension of the wave packet in coordinate space which the with L = 1.08 fm^2 is stored constant. Accordingly, the density of the system with N nucleons in coordinate space is given below:

$$\rho(\mathbf{r},\ t) = \sum_{i=1}^{N} \frac{1}{(2\pi L)^{3/2}} \exp\left\{\frac{-(\mathbf{r}-\mathbf{r}_{i0})^2}{2L}\right\}. \tag{2}$$

The time evolution of the *N*-body distribution is imposed by the motion of the centroid of the Gaussian $\{\dot{\mathbf{r}}_{i0},\ \dot{\mathbf{p}}_{i0}\}$, which is broadcasted by the Poisson brackets,

$$\dot{\mathbf{r}}_{i0} = \{\mathbf{p}_{i0},\ H\}, \tag{3}$$

and

$$\dot{\mathbf{p}}_{i0} = \{\mathbf{r}_{i0},\ H\}, \tag{4}$$

which the nuclear Hamiltonian (H) can be read as below;

$$H = \sum_{i=1}^{N} \sqrt{\mathbf{p}_{i0}{}^2 + m_i^2} + \sum_{i<j}^{N} \left(U_{ij}^{Str} + U_{ij}^{Coul}\right). \tag{5}$$

In this circumstance, U_{ij}^{Str} and U_{ij}^{Coul} are a nuclear mean-field and a Coulomb interaction, serially.

The strength of the nuclear condensation is explained by the compressibility (K) (Hartnack et al., 1998), which the compressibility is determined by the constant value can be written in the form

$$K = 9\rho^2 \frac{\partial^2}{\partial \rho^2}\left(\frac{E}{A}\right), \tag{6}$$

the energy per nucleon (E/A) is commonly as a function of the density. The potential (U) is calculated selfconsistently and corresponded to the real part of the Brueckner G-matrix. The ground-state properties of nuclear matter, one has to readjust the parameters.

$$U = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^{\gamma}. \tag{7}$$

In this circumstance, the ρ is the nuclear density and the ρ_0 indicates nuclear density which is calibrated in a unit of the situation density ($\rho_0 \approx 0.16$ fm^{-3}). Moreover, the relation of the α, β, and γ is demonstrated in table 1. The Skyrme parameterizations are estimated by utilizing three parameters from table 1, these show the differential value of the nuclear equation of state, a soft EoS is exhibited by a value of K = 200 MeV, while a hard EoS is exhibited by a value of K = 380 MeV. The differential value of the K constant is displayed in figure 1.

Table 1. The parameters are used in the equations (6) and (7) for the soft/hard nuclear equation of state (EoS) (Hartnack et al., 1998).

K (MeV)	α (MeV)	β (MeV)	γ (MeV)	EoS
200	356	303	7/6	Soft
380	-124	70.5	2	Hard

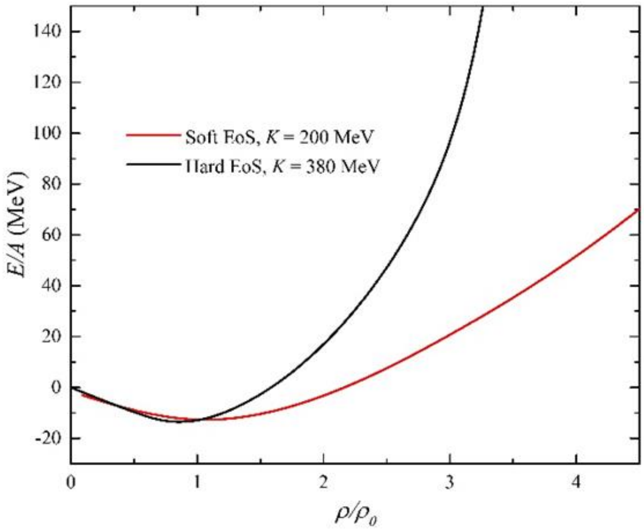

Figure 1. The equations of state in the Skyrme parameterizations are estimated by utilizing three parameters in table 1. (Aichelin, 1991; Hartnack et al., 1998).

Kaons in dense matter

The chiral perturbation theory (ChPT), is a common framework to investigate the interactions between pseudoscalar mesons and baryons at low energies by using the Chiral Lagrangian. The Euler-Lagrange equations are utilized for the derivation of the field equations for the $K^{\pm}$ mesons can be written in the form (Ko, 2001).

$$\left[\partial_\mu \partial^\mu \pm \frac{3i}{4f_\pi^{*2}} j_\mu \partial^\mu + \left(m_K^2 - \frac{\Sigma_{KN}}{f_\pi^{*2}}\rho_s\right)\right]\phi_{K^\pm}(x) = 0 . \tag{8}$$

In the equation (8) j_μ is the baryon four-vector current, ρ_s is the baryon scalar density, f_π^{*2} is the in-medium pion decay constant. Leading to the kaonic vector potential, which read as below

$$V_\mu = \frac{3}{8f_\pi^{*2}} j_\mu , \tag{9}$$

and the equation (8) is rewritten in the form (Fuchs et al., 1998).

$$\left[(\partial_\mu \pm iV_\mu)^2 + m_K^{*2}\right]\phi_{K^\pm}(x) = 0 . \tag{10}$$

Consequently, the vector field is guided by the minimal coupling into the Klein-Gordon equation. The effective mass m_K^* of the kaon is provided by

$$m_K^* = \sqrt{m_K^2 - \frac{\Sigma_{KN}}{f_\pi^{*2}}\rho_s + V_\mu V^\mu} , \tag{11}$$

in this instance, m_K^* = 0.496 GeV is the rest mass of K meson. The parameters in the equation (8) and (11) are clarified as follows: the kaon nucleon: the kaon–nucleon sigma term Σ_{KN} = 0.450 GeV, the in-medium pion decay constant at normal nuclear matter density (ρ_0), $f_\pi^{*2} = \sqrt{0.6} f_\pi^2$ and the vacuum pion decay constant f_π = 0.093 GeV (Brown et al., 1996)

The $K^{\pm}$ single-particle energies are illuminated as,

$$\omega_{K^{\pm}}\left(\mathbf{k}, \rho\right) = \sqrt{\mathbf{k}^{*2} + m_K^{*2}} \pm V_0 , \tag{12}$$

here $\mathbf{k}^* = \mathbf{k} \mp \mathbf{V}$ refer to the kaon effective momentum $V_\mu = (V_0, \mathbf{V})$, The kaon vector field is introduced by minimal coupling into the Klein-Gordon with adverse signal for K^+ and K^- . The kaon and antikaon potentials $U_{K^{\pm}}$ are defined as (13)

$$U_{K^{\pm}}\left(\mathbf{k}, \rho\right) = \omega_{K^{\pm}}(\mathbf{k}, \rho) - \sqrt{\mathbf{k}^2 + m_K^2} . \tag{13}$$

Following Ref. (Zheng et al., 2004), the Brown and Rho parameterization Σ_{KN} = 450 MeV is utilized, f_π^{*2} = $0.6 f_\pi^2$ for the vector field, and f_π^{*2} = f_π^2 for the scalar part is given by $-\sum_{KN} / f_\pi^{*2} \rho_s$. From this parameterization, the Brown and Rho potential (K^+N) at saturation density is 30 MeV ($U_K \approx 30$). In another group estimates by utilizing parameterization Σ_{KN} = 450 MeV as well as f_π^{*2} = f_π^2. This parameterization is called Ko and Li parameterization which weaker potential ($U_K\left(\rho\right) \approx$ 30) (Li & Ko, 1995; Ko, 2001). In this work, we use the Brown and Rho potential calculation of kaons potential.

Invariant cross-section

An invariant cross-section is usually used in high energy physics for describing the production of particles in the heavy-ion reaction, the invariant cross-section defined as

$$\sigma_{inv} = E \frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3} , \tag{14}$$

when p is represented as a kinematic variable. The differential cross-section can be read as $\mathrm{d}^3\sigma / \mathrm{d}p_x \, \mathrm{d}p_y \mathrm{d}p_z$. The quantity is frequently denoted as $\mathrm{d}^3\sigma / \mathrm{d}p^3$, with an understanding that $\mathrm{d}p^3$ is the elementary volume in the p space as revealed in figure 2. It appears that the product $\mathrm{d}^3\sigma / \mathrm{d}p^3$ and *E* is also an invariant quantity.

The volume element in the cylindrical coordinate, the $\mathrm{d}p^3$ can be express as

$$\mathrm{d}p^3 = p_t \mathrm{d}p_t \mathrm{d}p_z \mathrm{d}\phi , \tag{15}$$

where

$$p_t = \left(p_x^2 + p_y^2\right)^{1/2} , \tag{16}$$

p_t and $\mathrm{d}\phi$ are the Lorentz invariant concerning longitudinal transformations. Since a particle is moving by the momentum in a plane perpendicular to the z-axis. Thus, the invariant cross-section in cylindrical coordinate can be written by,

$$E\frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3} = \frac{\mathrm{d}^3\sigma}{p_t \mathrm{d}p_t \mathrm{d}\phi \mathrm{d}p_z / E}, \quad (17)$$

where, we have $\mathrm{d}p_z / E = \mathrm{d}y$ and y is the rapidity, (Chaimongkon et al., 2019) so that

$$E\frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3} = \frac{\mathrm{d}^3\sigma}{p_t \mathrm{d}p_t \mathrm{d}\phi \mathrm{d}y} \quad (18)$$

To calculate the invariant cross-section (σ) on after heavy-ion collision. the σ is defined as (Chaimongkon et al., 2019)

$$\sigma = P_{prob} \int_0^{b_{max}} 2\pi b \mathrm{d}b \quad (19)$$

where b is an impact parameter variable and P_{prob} is the probability of the particle production after the heavy-ion collision and b_{max} is the maximum impact parameter in a heavy-ion collision (Chaimongkon et al., 2019).

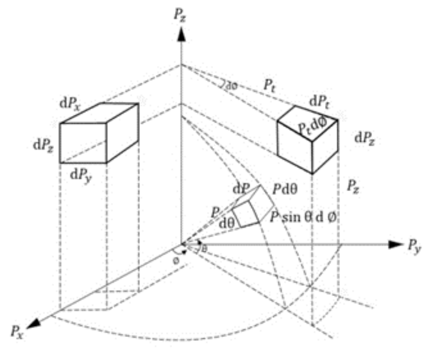

Figure 2. The volume element of the p space in the cylindrical coordinate

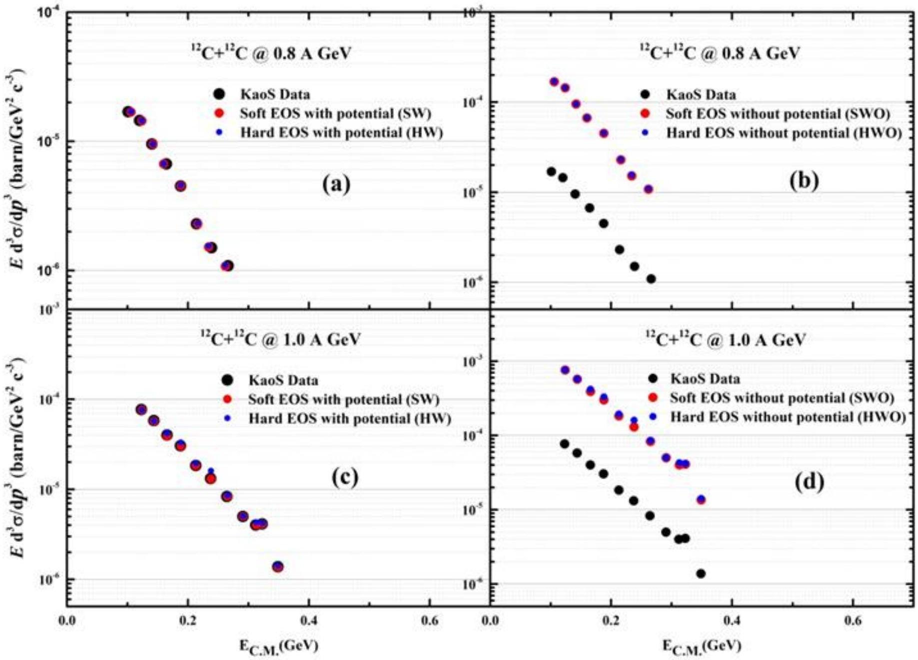

Figure 3. Shows the invariant cross-section of K^+ production as a function of kinetic energy in ^{12}C + ^{12}C collisions at 0.8 and 1.0 A GeV with b from 3.70 to 5.73 fm. The theoretical calculations compare with the experimental data form KaoS data.

3. Result and Discussion

Figure 3 displays the invariant cross-section of K+ production as a function of kinetic energy in ^{12}C + ^{12}C collisions at an incident energy of 0.8 and 1.0 A GeV with the impact parameter from 3.70 to 5.73 fm. (a) the black circle symbols refer to the KaoS data, the red circle symbols refer to the results calculated by applying the soft EOS with the K^+N potential at incident energy 0.8 A GeV and the blue circle symbols refer to the results calculated by applying the hard EOS with the K^+N potential at incident energy 0.8 A GeV. (b) the red circle symbols are the results calculated by applying the soft EOS without the K^+N potential at incident energy 0.8 A GeV and the blue circle symbols are the calculated results by applying the hard EOS without the K^+N potential at incident energy 0.8 A GeV. (c) the red circle symbols are the calculated results by applying the soft EOS with the K^+N potential at incident energy 1.0 A GeV and the blue circle symbols refer to the results which calculated by using the hard EOS with the K^+N potential at incident energy 1.0 A GeV. (d) the red circle symbols refer to the calculated results by applying the soft EOS without the K^+N potential at incident energy 1.0 A GeV and the blue circle symbols refer to the results which calculated by using the hard EOS without the K^+N potential at incident energy 1.0 A GeV. From the results of theoretical calculation with soft EOS is similar to the hard EOS. The invariant cross-section will decrease as the kinetic energy of the particle K^+ increases because the potential of K^+ is repulsive (Srisawad et al., 2018). Therefore, after the collision of ^{12}C + ^{12}C, K^+ has high kinetic energy, and it will be pushed away from the center of the collision at high velocity. As a result, the high kinetic energy K^+ particles are difficult to measure, as shown in Figure 3. Moreover, the calculations are performed without the K^+N potential and using a soft EOS and hard EOS are decrease with high kinetic energy and increases with low kinetic energy. After taking into the K^+N potential, which is repulsive potential, the K^+ mesons are repelled from nucleons and bring about the right experimental data. Thereby, the calculated results with soft and hard EOS presented that inclusive K^+N potential has a tendency compatible with the KaoS data. This intimate that this calculation gives a good result and reasonable in describing the experimental data. In addition, the similar research's results are found in reference (Chaimongkon et al., 2019).

4. Conclusion

The quantum molecular dynamics model is used to analyze the K^+ production by the simulation at incident energy of 0.8 and 1.0 A GeV and impact parameter from 3.70 to 5.73 fm. The production of K^+ mesons cross-section as a function of the laboratory kinetic energy from ^{12}C + ^{12}C collisions in polar angles 90°±10° is calculated by QMD model as well. The results are compared with the KaoS data. We found that the calculation of K^+ production with soft and hard EOS and K^+N potential. The results are consistent with the KaoS data. This mean that the information of the K^+N potential at high densities can be extracted from the sensitive investigations of the invariant cross-section of K^+ production as a function of kinetic energy.

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