The Diophantine Equations $8^x + p^y = z^3$ and $8^x - p^y = z^3$

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Abstract

Let p be a prime. In this paper, we show that all non-negative integer solutions of the Diophantine equation $8^x + p^y = z^3$ are of the form $(p,x,y,z) = (3\cdot 2^{2n} + 3\cdot 2^n + 1,\ n, 1, 2^n + 1)$, where n is a non-negative integer. All non-negative integer solutions of the Diophantine equation $8^x - p^y = z^3$ are of the form (p,x,y,z), where $(p,x,y,z) \in \{(2,n,3n,0),(p,0,0,0),(13,3,2,7),(3\cdot 2^{2n} - 3\cdot 2^n + 1, n, 1, 2^n - 1) \mid n \in \mathbb{N} \cup \{0\}\}$

Keywords: Diophantine equation, non-negative integer solution

1. Introduction

A Diophantine equation is a famous topic in number theory with a vital importance in the field of Geometry, Computer Science, Chemistry, Business and many more. Recently, there have been an increasing interest in finding non-negative integer solutions to Diophantine equations of the form $a^x + b^y = z^c$ where a, b and c are positive integers. For instance, Suvarnamani (2011) studied non-negative integer solutions of the Diophantine equation $2^x + p^y = z^2$ where p is a prime number. Chotchaisthit (2012) showed that the Diophantine equation $4^x + p^y = z^2$, where p is a prime, has infinitely many non-negative integer solutions. Bacani & Rabago (2015) showed that the Diophantine equation $p^x + q^y = z^2$, where p, q are twin primes, has infinitely many positive integer solutions. In the same year, Qi & Li (2015) published a paper on finding positive integer solutions of the Diophantine equation $8^x + p^y = z^2$ where p is an odd prime. Meanwhile, Mina & Bacani (2019) presented results that guarantee the non-existence of positive integer solutions of the Diophantine equations of the form $p^x + q^y = z^{2n}$ where the values p and q are fixed primes and n is a positive integer.

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Burshtein (2017) proved that the Diophantine equation $p^3+q^2=z^3$ has exactly four positive integer solutions. In the following year, Burshtein (2018) found that the Diophantine equation $p^3+q=z^3$ has infinitely many positive integer solutions. Burshtein (2020) also studied positive integer solutions for the Diophantine equation of type $p^x+q^y=z^3$ where p,q are primes and $x,y\in\{1,2\}$. Recently, Burshtein (2021) found that the Diophantine equation $p^3+q^y=z^3$, when p and q are distinct odd primes and $y>3$, has no solution in positive integers.

In this paper, we consider two Diophantine equations in the particular forms of $8^x+p^y=z^3$ and $8^x-p^y=z^3$ where p is a prime and x,y,z are non-negative integers.

2. Results

We first consider the non-negative integer solutions of the Diophantine equation $8^x+p^y=z^3$. Later, we study the non-negative integer solutions of the Diophantine equation $8^x-p^y=z^3$.

Lemma 1 The Diophantine equation

$$8^x+2^y=z^3 \qquad (1)$$

has no non-negative integer solution.

Proof. From (1), we get $(z-2^x)(z^2+z2^x+2^{2x})=2^y$ which implies that $z-2^x=2^u$ and $z^2+z2^x+2^{2x}=2^{y-u}$, for some a non-negative integer u. We have $2^{2u}+3\cdot2^{x+u}+3\cdot2^{2x}=2^{y-u}$. If $x=u$, then $7\cdot2^{2x}=2^{y-x}$. It is impossible. If $x>u$, then $2^{2u}(1+3\cdot2^{x-u}+3\cdot2^{2x-2u})=2^{y-u}$. Thus, there exists a non-negative integer t such that $1+3\cdot2^{x-u}+3\cdot2^{2x-2u}=2^t$. Since $x>u$, we have $1+3\cdot2^{x-u}+3\cdot2^{2x-2u}$ is odd. It follows that 2^t is also odd. Then $t=0$. Thus, $1+3\cdot2^{x-u}+3\cdot2^{2x-2u}=1$. It is impossible.

Similarly, if $x<u$, then $2^{2x}(2^{2u-2x}+3\cdot2^{u-x}+3)=2^{y-u}$. Thus, $2^{2u-2x}+3\cdot2^{u-x}+3=1$. It is impossible. Hence, the equation (1) has no non-negative integer solution.

Lemma 2 The Diophantine equation

$$8^x+3^y=z^3 \qquad (2)$$

has no non-negative integer solution.

Proof. From (2), we get $(z-2^x)(z^2+z2^x+2^{2x})=3^y$ which implies that $z-2^x=3^v$ and $z^2+z2^x+2^{2x}=3^{y-v}$, for some a non-negative integer v. It follows that $y>2v$ and $3^{2v}+3^{v+1}\cdot 2^x+3\cdot 2^{2x}=3^{y-v}$. If $v=0$, then $1+3\cdot 2^x+3\cdot 2^{2x}=3^y$. Since $y>0$, we have $3\mid 1$, a contradiction. Then $v>0$ and $3^{2v-1}+3^v\cdot 2^x+2^{2x}=3^{y-v-1}$. Since $2v-1>0$ and $y-v-1>0$, we get $3\mid 2^{2x}$. It is impossible. Hence, the equation (2) has no non-negative integer solution.

Lemma 3 Let p be an odd prime and y be an even integer. Then, the Diophantine equation

$$p^y-3(2^x+1)2^x=1 \tag{3}$$

has no non-negative integer solution.

Proof. By (3) and our supposition, we have $y=2k$, for some positive integer k and

$$(p^k-1)(p^k+1)=3(2^x+1)2^x. \tag{4}$$

Since p is an odd prime and (4), we get $x\geq 2$. Since two consecutive integers are relatively prime, it implies that $\gcd\left(\frac{p^k-1}{2},\frac{p^k+1}{2}\right)=1$.

Thus,
$$\left(\frac{p^k-1}{2}\right)\left(\frac{p^k+1}{2}\right)=3(2^x+1)2^{x-2}. \tag{5}$$

From (5), we consider four cases:

Case 1: $3\left|\left(\frac{p^k-1}{2}\right)\right.$ and $2^{x-2}\left|\left(\frac{p^k-1}{2}\right)\right.$. Since $\gcd(3,2^{x-2})=1$, we have $3\cdot 2^{x-2}\left|\left(\frac{p^k-1}{2}\right)\right.$. Then there is a positive integer a such that $p^k-1=3\cdot 2^{x-1}a$, and so $p^k+1=3\cdot 2^{x-1}a+2$. From (4), we have $(3\cdot 2^{x-1}a)(3\cdot 2^{x-1}a+2)=3(2^x+1)2^x$. Then $2^{x-1}(4-3\cdot a^2)=2a-2$. If $a=1$, then $2^{x-1}=0$, a contradiction. If $a>1$, then $2^{x-1}(4-3\cdot a^2)<2a-2$. This is impossible.

Case 2: $3\left|\left(\frac{p^k-1}{2}\right)\right.$ and $2^{x-2}\left|\left(\frac{p^k+1}{2}\right)\right.$. Then there are positive integers a,b such that $p^k-1=6a$ and $p^k+1=2^{x-1}b$. Thus, $6a+2=2^{x-1}b$. From (4), we have $(6a)(2^{x-1}b)=3(2^x+1)2^x$. Then $ab=2^x+1$. It follows that $6(2^x+1)+2b=2^{x-1}b^2$. Thus, $2^{x-2}(b^2-12)=b+3$ which contradicts to the fact that $x\geq 2$ and b are positive integers.

Case 3: $3\left|\left(\frac{p^k+1}{2}\right)\right.$ and $2^{x-2}\left|\left(\frac{p^k-1}{2}\right)\right.$. Then there are positive integers a,b such that $p^k-1=2^{x-1}a$ and $p^k+1=6b$. Thus, $2^{x-1}a+2=6b$. From (4), we have $(2^{x-1}a)(6b)=3(2^x+1)2^x$. Then $ab=2^x+1$. It follows that $2^{x-1}a^2+2a=6(2^x+1)$.
Thus, $2^{x-2}(12-a^2)=a-3$ which contradicts to the fact that $x\geq 2$ and a are positive integers.

Case 4: $3\left|\left(\frac{p^k+1}{2}\right)\right.$ and $2^{x-2}\left|\left(\frac{p^k+1}{2}\right)\right.$. Then there is a positive integer a such that $p^k+1=3\cdot 2^{x-1}a$. Thus, $p^k-1=3\cdot 2^{x-1}a-2$. From (4), we get $\left(3\cdot 2^{x-1}a-2\right)\left(3\cdot 2^{x-1}a\right)=3\left(2^x+1\right)2^x$. Then $2^{x-1}\left(3\cdot a^2-4\right)=2a+2$ which contradicts to the fact that $x\geq 2$ and a are positive integers.

Thus, the lemma is proved.

Lemma 4 Let p be a prime. Then, all non-negative integer solutions of the Diophantine equation

$$8^x+p=z^3 \tag{6}$$

are of the form $\left(p,x,z\right)=\left(3\cdot 2^{2n}+3\cdot 2^n+1, n, 2^n+1\right)$, where n is a non-negative integer.

Proof. From (6) it follows that $\left(z-2^x\right)\left(z^2+z2^x+2^{2x}\right)=p$. Thus, $z-2^x=1$ and $z^2+z2^x+2^{2x}=p$. It implies that $z=2^x+1$ and $p=3\cdot 2^{2x}+3\cdot 2^x+1$. Hence, all non-negative integer solutions of (6) are of the following form

$$\left(p,x,z\right)=\left(3\cdot 2^{2n}+3\cdot 2^n+1, n, 2^n+1\right),$$

where n is a non-negative integer and $3\cdot 2^{2n}+3\cdot 2^n+1$ is a prime.

Theorem 5 Let p be a prime. Then, all non-negative integer solutions of the Diophantine equation

$$8^x+p^y=z^3 \tag{7}$$

are of the form $\left(p,x,y,z\right)=\left(3\cdot 2^{2n}+3\cdot 2^n+1, n, 1, 2^n+1\right)$, where n is a non-negative integer.

Proof. In cases $p=2,3$, it follows that (7) has no non-negative integer solution, by Lemma 1 and 2. We only consider case $p>3$. From (7), we have

$$\left(z-2^x\right)\left(z^2+z2^x+2^{2x}\right)=p^y. \tag{8}$$

Then there is a non-negative integer u such that $z-2^x=p^u$ and $z^2+z2^x+2^{2x}=p^{y-u}$.

Therefore,

$$p^{2u}+3\cdot 2^x\cdot p^u+3\cdot 2^{2x}=p^{y-u}. \tag{9}$$

Assume that $u\geq 1$. If $y-u=0$, then it immediately follows that $p^{2u}+3\cdot 2^x\cdot p^u+3\cdot 2^{2x}=1$ which is impossible. Meanwhile, if $y-u\geq 1$, then $p\left|3\cdot 2^{2x}\right.$. Thus, $p=2$ or 3 which is impossible. Hence, $u=0$. It implies that $z=2^x+1$ and

$$p^y=3\cdot 2^{2x}+3\cdot 2^x+1. \tag{10}$$

If y is even, then (10) has no non-negative integer solution, by Lemma 3. If $y\neq 1$ is odd, then $y=2h+1$, for some positive integer h. From (10) we have $p^y\equiv 1\left(\bmod 3\right)$, and so

$p \equiv -1, 1 \pmod 3$. Assume that $p \equiv -1 \pmod 3$. Since y is odd, we have $p^y \equiv -1 \pmod 3$, a contradiction. Thus $p \equiv 1 \pmod 3$. From (10), we get $(p-1)(p^{2h} + p^{2h-1} + \cdots + 1) = 3(2^x + 1)2^x$. Since p is odd, then $p^{2h} + p^{2h-1} + \cdots + 1$ is odd, and so $\gcd(2^x, p^{2h} + p^{2h-1} + \cdots + 1) = 1$. It implies that $2^x \mid (p-1)$. Therefore $p - 1 = 3 \cdot 2^x m$, for m is a positive integer. Hence, $m(p^{2h} + p^{2h-1} + \cdots + 1) = 2^x + 1$. We have $p = 3 \cdot 2^x m + 1 > 2^x + 1 \geq p^{2h} + p^{2h-1} + \cdots + 1$ which is impossible. Thus, $y = 1$. By Lemma 4, all non-negative integer solutions of (7) are of the following form $(p, x, y, z) = (3 \cdot 2^{2n} + 3 \cdot 2^n + 1, n, 1, 2^n + 1)$, where n is a non-negative integer and $3 \cdot 2^{2n} + 3 \cdot 2^n + 1$ is a prime.

Lemma 6 All non-negative integer solutions of the Diophantine equation

$$8^x - 2^y = z^3 \tag{11}$$

are of the following form $(x, y, z) = (n, 3n, 0)$ where n is a non-negative integer.

Proof. From (11) it follows that $(2^x - z)(2^{2x} + 2^x z + z^2) = 2^y$. Then $2^x - z = 2^u$ and $2^{2x} + 2^x z + z^2 = 2^{y-u}$, for some a non-negative integer u. It follows that $x \geq u$ and $2^{2u}(3 \cdot 2^{2x-2u} - 3 \cdot 2^{x-u} + 1) = 2^{y-u}$. This implies that $3 \cdot 2^{2x-2u} - 3 \cdot 2^{x-u} + 1 = 1$. Thus, $x = u$, $z = 0$ and $y = 3u$. Here we conclude that all non-negative integer solutions of (11) are of the following form $(x, y, z) = (n, 3n, 0)$, where n is a non-negative integer.

Lemma 7 The Diophantine equation

$$8^x - 3^y = z^3 \tag{12}$$

has only one non-negative integer solution, namely $(x, y, z) = (0, 0, 0)$.

Proof. If (12) has a solution, we obtain $(2^x - z)(2^{2x} + 2^x z + z^2) = 3^y$. Then there is a non-negative integer v such that $2^x - z = 3^v$ and $2^{2x} + 2^x z + z^2 = 3^{y-v}$. It follows that $3 \cdot 2^{2x} - 3^{v+1} \cdot 2^x + 3^{2v} = 3^{y-v}$. Assume that $v \geq 1$, we have that $3(2^{2x} - 3^v \cdot 2^x + 3^{2v-1}) = 3^{y-v}$. It follows that $y - v = 1$. Thus, $x = 0$ and $z = 1$. Then we get $0 = 3^v$. It is impossible. Hence, $v = 0$. This implies that $3 \cdot 2^{2x} - 3 \cdot 2^x + 1 = 3^y$. We have $x = y = z = 0$.

Lemma 8 Let p be an odd prime and y be an even integer. Then, all non-negative integer solutions of the Diophantine equation

$$p^y - 3(2^x - 1)2^x = 1 \tag{13}$$

are of the form $(p, x, y) \in \{(p, 0, 0), (13, 3, 2)\}$.

Proof. Since y is even, there is a non-negative integer k such that $y=2k$. Form (13), we have

$$\left(p^k-1\right)\left(p^k+1\right)=3\left(2^x-1\right)2^x. \tag{14}$$

If $k=0$, we have $x=y=0$. Then $(p,x,y)=(p,0,0)$ is a solution of (13). Now, we suppose $k\geq 1$.

Since p is an odd prime and (14), it follows that $x\geq 2$ and $\gcd\left(\frac{p^k-1}{2},\frac{p^k+1}{2}\right)=1$. Thus,

$$\left(\frac{p^k-1}{2}\right)\left(\frac{p^k+1}{2}\right)=3\left(2^x-1\right)2^{x-2} \tag{15}$$

From (15), we consider four cases:

Case 1: $3\left|\left(\frac{p^k-1}{2}\right)\right.$ and $2^{x-2}\left|\left(\frac{p^k-1}{2}\right)\right.$. Then there is a positive integer a such that $p^k-1=3\cdot 2^{x-1}a$. Thus, $p^k+1=3\cdot 2^{x-1}a+2$. From (14), we have $\left(3\cdot 2^{x-1}a\right)\left(3\cdot 2^{x-1}a+2\right)=3\left(2^x-1\right)2^x$. Then $2^{x-1}\left(4-3\cdot a^2\right)=2a+2$. It follows that only one possible case is $a=1$ and $x=3$. From (13), we obtain $y=2$ and $p=13$. In this case, $(p,x,y)=(13,3,2)$ is the only one solution.

Case 2: $3\left|\left(\frac{p^k-1}{2}\right)\right.$ and $2^{x-2}\left|\left(\frac{p^k+1}{2}\right)\right.$. Then there are two positive integers a,b such that $p^k-1=6a$ and $p^k+1=2^{x-1}b$. Thus, $6a+2=2^{x-1}b$. From (14), we have $(6a)\left(2^{x-1}b\right)=3\left(2^x-1\right)2^x$. Then $ab=2^x-1$. It follows that $6\left(2^x-1\right)+2b=2^{x-1}b^2$. Thus, $2^{x-2}\left(b^2-12\right)=b-3$ which contradicts to the fact that $x\geq 2$ and b are positive integers.

Case 3: $3\left|\left(\frac{p^k+1}{2}\right)\right.$ and $2^{x-2}\left|\left(\frac{p^k-1}{2}\right)\right.$. Then there are two positive integers a,b such that $p^k-1=2^{x-1}a$ and $p^k+1=6b$. Thus, $2^{x-1}a+2=6b$. From (14), we have $\left(2^{x-1}a\right)(6b)=3\left(2^x-1\right)2^x$. Then $ab=2^x-1$. It follows that $2^{x-1}a^2+2a=6\left(2^x-1\right)$. Thus, $2^{x-2}\left(12-a^2\right)=a+3$ which contradicts to the fact that $x\geq 2$ and a are positive integers.

Case 4: $3\left|\left(\frac{p^k+1}{2}\right)\right.$ and $2^{x-2}\left|\left(\frac{p^k+1}{2}\right)\right.$. Then there is a positive integer a such that $p^k+1=3\cdot 2^{x-1}a$. Thus, $p^k-1=3\cdot 2^{x-1}a-2$. From (14), we have $\left(3\cdot 2^{x-1}a-2\right)\left(3\cdot 2^{x-1}a\right)=3\left(2^x-1\right)2^x$. Then $2^{x-1}\left(3\cdot a^2-4\right)=2a-2$ which contradicts to the fact that $x\geq 2$ and a are positive integers.

Therefore, $\{(p,0,0),(13,3,2)\}$ is a set of all non-negative integer solutions of (13).

Lemma 9 Let p be a prime. Then, all non-negative integer solutions of the Diophantine equation

$$8^x - p = z^3 \tag{16}$$

are of the form $(p,x,z)=(3\cdot 2^{2n}-3\cdot 2^n+1,\, n,\, 2^n-1)$ where n is a non-negative integer.

Proof. From (16) it follows that $(2^x-z)(2^{2x}+2^x z+z^2)=p$. Thus, $2^x-z=1$ and $2^{2x}+2^x z+z^2=p$. It implies that $z=2^x-1$ and $p=3\cdot 2^{2x}-3\cdot 2^x+1$. Hence, all non-negative integer solutions of (16) are of the following form $(p,x,z)=(3\cdot 2^{2n}-3\cdot 2^n+1,\, n,\, 2^n-1)$ where n is a non-negative integer and $3\cdot 2^{2n}-3\cdot 2^n+1$ is a prime.

Theorem 10 Let p be a prime. Then, all non-negative integer solutions of the Diophantine equation

$$8^x - p^y = z^3 \tag{17}$$

are of the form (p,x,y,z), where

$$(p,x,y,z)\in\left\{(2,n,3n,0),(p,0,0,0),(13,3,2,7),(3\cdot 2^{2n}-3\cdot 2^n+1,\, n,1,\, 2^n-1)\,\middle|\, n\in\mathbb{N}\cup\{0\}\right\}.$$

Proof. In cases $p=2,3$, it follows from Lemma 6 and 7 that

$$(p,x,y,z)\in\{(2,n,3n,0),(3,0,0,0)\}$$

where n is a non-negative integer. We only consider case $p>3$. From (17), we have

$$(2^x-z)(2^{2x}+2^x z+z^2)=p^y. \tag{18}$$

Then there is non-negative integer u such that $2^x-z=p^u$ and $2^{2x}+2^x z+z^2=p^{y-u}$.

Therefore,
$$3\cdot 2^{2x}-3\cdot 2^x\cdot p^u+p^{2u}=p^{y-u}. \tag{19}$$

Assume that $u\ge 1$. If $y-u=0$, then it immediately follows that $2^{2x}+2^x z+z^2=1$. Thus, $x=0$ and $z=0$. This implies $u=0$. This is a contradiction. Meanwhile, if $y-u\ge 1$, then it follows from (19) that $p\,|\,3\cdot 2^{2x}$. Hence, either $p=2$ or 3 which is impossible. Thus, $u=0$. We have $z=2^x-1$ and

$$p^y=3\cdot 2^{2x}-3\cdot 2^x+1 \tag{20}$$

If y is even, then it follows form Lemma 8 that all non-negative integer solutions (p,x,y) of (20) are of the forms $(p,0,0)$ and $(13,3,2)$. It follows that $z=0$ and $z=7$, respectively. Then $(p,x,y,z)\in\{(p,0,0,0),(13,3,2,7)\}$. If $y\ne 1$ is odd, then $y=2h+1$, for some positive integer h. From (20) we have $p\equiv 1 \pmod 3$ and $(p-1)(p^{2h}+p^{2h-1}+\cdots+1)=3(2^x-1)2^x$. Since p is odd, then obviously $p^{2h}+p^{2h-1}+\cdots+1$ is odd. Then, $2^x\,|\,(p-1)$. Thus, $p-1=3\cdot 2^x m$, for m is positive integer. Hence, $m(p^{2h}+p^{2h-1}+\cdots+1)=2^x-1$. We have

$p = 3\cdot 2^x m + 1 > 2^x - 1 \geq p^{2h} + p^{2h-1} + \cdots + 1$ which is impossible. Thus, $y = 1$. By Lemma 9, we have $(p, x, y, z) = (3\cdot 2^{2n} - 3\cdot 2^n + 1, n, 1, 2^n - 1)$ where n is a non-negative integer and $3\cdot 2^{2n} - 3\cdot 2^n + 1$ is a prime.

3. Conclusions

In the paper, we have shown that all non-negative integer solutions of the Diophantine equation $8^x + p^y = z^3$ are of the following form $(p, x, y, z) = (3\cdot 2^{2n} + 3\cdot 2^n + 1, n, 1, 2^n + 1)$, where p is a prime and n is a non-negative integer. All non-negative integer solutions of the Diophantine equation $8^x - p^y = z^3$ are of the following form

$$(p, x, y, z) \in \{(2, n, 3n, 0), (p, 0, 0, 0), (13, 3, 2, 7), (3\cdot 2^{2n} - 3\cdot 2^n + 1, n, 1, 2^n - 1)\},$$

where p is prime and n is a non-negative integer.

4. Discussion

In Theorem 5, it is to be noted that a prime p must be in the form $3\cdot 2^{2n} + 3\cdot 2^n + 1$ where n is a non-negative integer. For example, $p = 7, 19$ and 61. Otherwise, the Diophantine equation $8^x + p^y = z^3$ has no non-negative integer solution. Similarly, in Theorem 10, if $y = 1$, then a prime p must be in the form $3\cdot 2^{2n} - 3\cdot 2^n + 1$ where n is a non-negative integer. For example, $p = 7, 37$ and $12,097$. Moreover, it is interesting to find the non-negative integer solutions of the Diophantine equations with a more general form, that is $q^x + p^y = z^3$ where p and q are primes.

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