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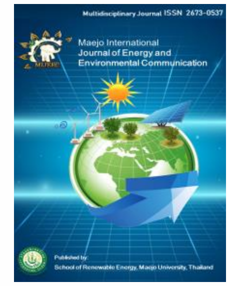

ARTICLE

The application of the KdV type equation in engineering simulation

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ABSTRACT

Bores propagating in shallow water transform into undular bores and, finally, into trains of solitons. The observed number and height of these undulations and later discrete solitons are strongly dependent on the propagation length of the bore. Empirical results show that the final height of the leading soliton in the far-field is twice the initial mean bore height. The complete disintegration of the initial bore into a train of solitons requires very long propagation, but unfortunately, these required distances are usually not available in experimental tests of nature. Therefore, the analysis of the bore decomposition for experimental data into solitons is complicated and requires different approaches. Previous studies have shown that by applying the nonlinear Fourier transform based on the Ko- rteweg–de Vries equation (KdV-NFT) to bores and long-period waves propagating in constant depth, the number and height of all solitons can be reliably predicted already based on the initial bore-shaped free surface. Against this background, this study presents the systematic analysis of the leading-soliton amplitudes for non-breaking and breaking bores with different strengths in different water depths to validate the KdV-NFT results for non-breaking bores to show the limitations of wave breaking on the spectral results. The analytical results are compared with data from experimental tests, numerical simulations and other approaches from the literature.

1. Introduction

A nonlinear problem is critical in all areas of mathematics and physics. The complexity of physical processes hides the majority of the exciting consequences. These occurrences can only be investigated using nonlinear problem-solving techniques (Demiray & Antar, 1997). Kortweg and De-Vries developed the Kortweg De-Vries (KdV) equation in 1895 to model ripples on belowground bodies of water. it's a dispersive non - linear set of equations with correct and precise remedies. Nonlinear fractional derivative formulas, namely the fractional equation of KdVBurgers (Wang, 1996), the fractional equations of Schrödinger-Kortewegde Varies (Golmankhaneh et al., 2011) and the fractional equations of Burgers (Gaitonde & Visbal, 1998), have recently been introduced to classify many essential circumstances and evolving physics methods. A few theories of the KdVB equation have received much interest, like the travelling-wave remedy. In recent years, several precise surface waves remedies to KdVB formulas have been discovered. A similar stability procedure was used to acquire impose solutions to a compound KdVB equation. Hassan (Hassan, 2004) Surface wave remedies were created both for the mixed KdVB formula and for the application of the specific abbreviated optimization technique to the general two-dimensional KdVB formula. The procedure Expfunction is used in (Soliman, 2009) for

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generic isolation and recurrent remedies to the KdVB equation. Over the last few generations, a few more authors have investigated mathematical solutions to overcome the KdVB techniques. A novel technique for finding explicit and numerical solutions to the KdVB equations was presented, with no transitions, equations, or presumptions of weak non - linearity (Kaya, 2004). The analytical simulation of KdV-Burgers was found using the precise and specific restrictive Taylor estimation (RTA) (Ismail, 2014). To replicate the nonlinear scattering wavefront issues mentioned by the KdVB equations in (Li & Visbal, 2006), more excellent compact variational strategies were combined with a higher top filter and the traditional fourth order Runge-Kutta arrangement. The component-free Galerkin (EFG) technique for numerical integration of the chemical KdVB equation was discussed (Rong-Jun & Yu-Min, 2011). A greater portable variation mechanism for various derivatives based on implicit interpolations was developed in 1992 (Lele, 1992). Such ambiguous strategies were precise inhomogeneous areas and had frequency pixel density properties when using the worldwide matrix. In (Koto, 2004) Li and Visbal used portable strategies in conjunction and a high filter to solve KdV-Burger’s equations. In recent years, it has become popular to solve hyperbolic formulas using the less diffusional and much less oscillatory CIP scheme established by (Takewaki et al., 1985). The traditional CIP strategies, mainly specified as semi-Lagrangian formulations, were stable and had low dispersion. The system would fix foolish formulas with third-order exactness in the area (Utsumi et al., 1997). On the other hand, the first CIP technique (Li & Visbal, 2006) uses a supplementary boundary layer for locational contour knowledge. Typically, to obtain the optimum interpretation values on the device, the equation with spatial variable must be differentiated. When the velocity is constant, the procedure is simple, although it is hard for complex algorithms. This article presents a numerical procedure to solve the KdV-Burger’s equations called compact-type CIP strategies, based on a set of CIP and more incredible portable strategies. The existing system is primarily predicated on the notion of the distinctive procedure; as a novel component, the more extensive moveable system is employed to amass derivatives instead of differentiating the equation with information, permitting the construction of a CIP theme and acquiring element density properties. Unlike the traditional portable scheme for fixing KdV-Burger’s equations, this scheme does not use a filter to conquer non-physical oscillations. The merits of our current method for the KdVB equation are demonstrated in this paper, and contrast of numerical methods with the optimal solution is performed to demonstrate the method's capability for nonlinear dispersive equations. This equation is commonly referred to as the Rosenau-KdV equation. The researcher also presented the two constants for the simplistic Rosenau-KdV formula (Takewakiet al., 1985; Li & Visbal, 2006). In specific, in addition to the ansatz method for obtaining the singular 1-solution solution, perturbation theory was used to acquire the adiabatic variable evolution of the water waves. The ansatz procedure (Lele 1992) used This paper uses linear functions to discuss real-world applications, including essential characteristics and how to graph them from various forms.

2. The common linear relationships in the field of science

There are three linear relationships in science: speed, Hooke's law, and ohm's law.

1. A motion detector was used to demonstrate the linear relationships between constant speed, distance, and time in this lesson. Distance and time, for example, are inextricably linked because speed remains constant.
2. Hooke's law shows the relationship between a spring's length and the pressure acting on it. There is a spring constant, k, for each spring that acts as a multiplying factor. Hooke's law convey as $F = KX$, where F is that the force on the spring (in newtons), X is the modification in spring length (in metres), and k is the spring constant (in N/m).
3. Ohm's law also demonstrates a direct relationship between voltage and resistance (potential). In equation form, Ohm's law is $V = iR$, wherever V is voltage (in volts), R is resistance (in ohms), and I is current (in amperes).

3. Numerical applications

KdV equation is a widely used wave propagation prototype in hydrodynamics, plasma quantum mechanics, elastomer media, and nonlinear optics. To study magneto-acoustic waves in plasma, the Homotopy perturbation remodels methodology (HPTM) is employed to repair the unifying framework of non - linear fifth order KdV equations. KdV equation shows the cumulative influence of the smallest, non-linearity and the easiest lengthy dispersion, which explains its broad range of applicability. The multi-symplectic finite distinction theme is often obtained by group action multi-symplectic equations equivalent to equation (5) over the domain associated, choosing a useful approximation to the integral terms in each the t and x directions. The centred box scheme is employed to satisfy the concatenated centre rule. This scheme is inherent, and it's conjointly referred to as the Preissman box scheme, which is widely employed in hydraulics.

In this section, we look at the nonlinear fractional forced KdV equation to determine the applicability and efficacy of the suggested algorithm. The homotopy analysis method solves the forced Korteweg-de Vries (fKdV) equations (HAM). HAM is an exact and approximate technique that offers a novel approach to obtaining series solutions to nonlinear problems. The MOL is an effective method for solving partial differential equations (PDEs). It entails approximating the spatial derivatives and lowering the issue to an ODE system (Schiesser, 1994). A period integrator can also be used to correct this ODE system. Until an improvised statistical procedure for solving ODEs is used, The maximum principal advantage of the MOL technique is that it combines the benefit of clean and unique methods with the dominance (balance benefit) of implied methods. Greater estimations in speculative temporal partitioning are possible without significantly increasing computation cost. This method applies to a variety of physicochemical processes modelled of the PDE, such as the differential delay equations (Koto, 2004), the two-dimensional Gordon sinusoidal equations (Bratsos, 2007), the Boussinesq equation extended to one dimension of Nwogu (Hamdi et al., 2005), the Boussinesq equation of the fourth order, the Kaup-Kupershmidt equation of the fifth-order and a fifth-order equation (Saucez et al., 2004).

3.1. Numerical calculation and simulations

A collocation methodology with the septic B-spline interpolation performed is planned for simulations of the Rosenau-KDV equation. The precision of the procured schemes is demonstrated by determining the mistake standard L2 and L∞. The MOL is an effective method for solving temporary deferential (10) formulas (PDEs). It entails estimating the identify and reducing the problem to an ODE system (Schiesser, 1994). Furthermore, this ODE system can be solved using a temporal integrator. A higher-order estimate in the partitioning of order derivatives is feasible without significantly increasing computation cost. This method applies to various physicochemical processes modelled by PDEs, such as the slow system of formulas, two-dimensional sine-Gordon (11) formula, fourth Boussinesq formulas, fifth Kaup-Kupershmidt formulas, and the elongated (KDV5 fifth Korteweg-de Vries formula.

So, applicability and efficiency of the proposed algorithm:

$$D_t^{\alpha} M(x,t) + c\left(\left((Fr-1) - \frac{3}{2}\frac{u(x,t)}{h}\right)\frac{\partial u(x,t)}{\partial t} - \frac{1}{6}h^2\frac{\partial^3 u(x,t)}{\partial t^3} + \frac{1}{2}\frac{\partial}{\partial t}\left(0.1e^{-\tau}+1\right)\right) = 0,\ 0<\alpha<1 \quad (1)$$

Initial condition:

$$u(x,0) = -\frac{2e^x}{(1+e^x)^2} \quad (2)$$

$$N^{+}[D_t^{\alpha}u(x,t)] - cN^{+}\left[\left((F_r-1) - \frac{3}{2}\frac{u}{h}\right)\frac{\partial u}{\partial t} - \frac{1}{6}h^2\frac{\partial^3 m}{\partial t^3} + \frac{1}{2}\frac{\partial}{\partial t}(0.1e^{-\tau}+1)\right] \quad (3)$$

The operator is defined as:

$$\frac{s^{\alpha}}{\omega^{\alpha}}N^{+}[u(x,t)] - \sum_{k=0}^{n-1}\frac{s^{\alpha-(k+1)}}{\omega^{\alpha-k}}[D^k u]_{t=0} = -cN^{+}[\left((Fr-1) - \frac{3}{2}\frac{u(x,t)}{h}\right)\frac{\partial u(x,t)}{\partial t} - \frac{1}{6}h^2\frac{\partial^3 u(x,t)}{\partial t^3} + \frac{1}{2}\frac{\partial}{\partial t}\left(0.1e^{-\tau}+1\right)] \quad (4)$$

From the above equation:

$$N^{+}[u(x,t)] = \frac{1}{s}\left[-\frac{2e^x}{(1+e^x)^2}\right] - \frac{c\omega^{\alpha}}{s^{\alpha}}N^{+}[\left(F_r-1) - \frac{3}{2}\frac{u(x,t)}{h}\right)\frac{\partial u(x,t)}{\partial t} - \frac{1}{6}h^2\frac{\partial^3 u(x,t)}{\partial t^3} + \frac{1}{2}\frac{\partial}{\partial t}(0.1e^{-\tau}+1)]. \quad (5)$$

Using the inverse of the previous equation:

$$u(x,t) = -\frac{2e^x}{(1+e^x)^2} - cN^{-1}[\frac{\omega^{\alpha}}{s^{\alpha}}N^{+}[\left(F_r-1) - \frac{3}{2}\frac{u(x,t)}{h}\right)\frac{\partial u(x,t)}{\partial t} - \frac{1}{6}h^2\frac{\partial^3 u(x,t)}{\partial t^3} + \frac{1}{2}\frac{\partial}{\partial t}(0.1e^{-\tau}+1)]] \quad (6)$$

Let us consider that the series solution for $\mu\ (x, t)$ is;

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \quad (7)$$

$$\sum_{n=0}^{\infty} u_n(x,t) = -\frac{2e^x}{(1+e^x)^2} - cN^{-1}[\frac{\omega^{\alpha}}{s^{\alpha}}N^{+}[\left((F_r-1) - \frac{3}{2}\frac{u(x,t)}{h}\right)\frac{\partial u(x,t)}{\partial t} - \frac{1}{6}h^2\frac{\partial^3 u(x,t)}{\partial t^3} + \frac{1}{2}\frac{\partial}{\partial t}(0.1e^{-\tau}+1)]] \quad (8)$$

The stability analysis of the method is shown to be unconditionally stable. The obtained schemes are tested through a single solitary wave in which the analytic solution is known, then extended to study the interaction of solitary waves and the evolution of solitons where no analytic solution is known. The obtained numerical results and simulations have shown that the applied method is an efficient method to analyze behaviours of the dispersive shallow-water waves.

We consider the KDVB equation;

$$u_t + (\alpha + \beta u)uu_x + \gamma u_{xx} - \delta u_{xxx} = 0 \quad (9)$$

With the initial solution for $\gamma = 0$, $\delta = -1$.

$$u(x,0) = -\frac{\alpha}{2\beta}\left(1 + tanh\left(\frac{\alpha}{2\sqrt{(6\beta)}}(x)\right)\right) \quad (10)$$

We show the numerical solutions for different values of α and β in Figure 1. If we let $\beta = 0$, $\alpha = 2$, $\gamma = -5$, $\delta = -3$.

The exact solution for this case is (Takewaki et al., 1985).

$$u(x,t) = \frac{1}{3}(sech^2(\theta/2) + 2tanh\theta/2) + 2) \quad (11)$$

Where $\theta = \frac{1}{3}x + \frac{2}{3}t$…The numerical and analytical solutions are shown in Figure 2. The numerical solutions are identical to exact solutions.

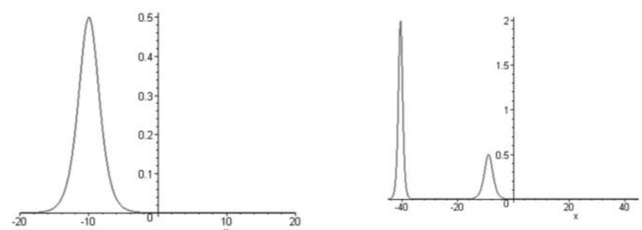

Figure 1 Graph of KdvB equation.

4. Conclusion

This paper presented a technique to calculate fifth order KDV equations, fifth order nonhomogeneous KDV equations, and Kawahara's formula combining Laplace transform with homogeneous perturbation. This combination creates a powerful method known as the Homotopy Disturbance Transformation (HPTM) procedure. This procedure was used to simulate the nonlinear characteristics of magnetoacoustic ripples in plasma numerically. The Homotopy Disturbance Transformation (HPTM) procedure was used to find the precise solutions of the fifth-order nonlinear KdV formulas with a primary situation, demonstrating the importance of the technique. As a result, it can resolve complex numeric values and variational equations of higher order.

The limited computation work required by this new method is a significant advantage. The homotopy perturbation transform technique is a powerful tool for finding approximate analytical solutions to a wide range of nonlinear problems. The procedure is able to reduce the quantity of work involving calculation while retaining solid numerical accuracy. Unlike the decomposition method, the HPTM resolve nonlinear equations without using a domain's polynomials. The proposed algorithm has been expected to be broadly used in science and engineering to solve other nonlinear equations. The Kudryashov, usable variable, and representational data processing systems have all been used effectively to investigate the strain wave equation in microstructure solids. A few analytical solutions to this model have been officially extracted, and they agree with previous literature reports. Furthermore, the procedure, particularly the Kudryashov procedure, can be utilized to an extensive range of nonlinear wave formulas, including higher-order nonlinear partial differential formulas

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

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