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การประยุกต์ใช้วอลเปเปอร์กรุปประเภท P6m สร้างลวดลายผ้ามัดหมี่ทอมือ

Application of Wallpaper Group P6m to Creation of Handwoven Mudmee Silk Patterns

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บทคัดย่อ

บทความนี้เสนอการสร้างลวดลายผ้ามัดหมี่ทอมือให้มีความสมมาตรประเภท P6m โดยพัฒนาบนวอลเปเปอร์กรุป แม่ลายที่ใช้พัฒนามาจากส่วนหนึ่งของลายดั้งเดิมที่ใช้ทอผ้ามัดหมี่ในพื้นที่จังหวัดบุรีรัมย์ ประเทศไทย ซึ่งใช้วิธีการเลือกลายแบบเจาะจงทั้งหมด 2 ลาย ได้แก่ ลายโคมประทีป และลายขอบินไทย ผลการศึกษาพบว่า ทั้งสองแม่ลายสามารถพัฒนาสู่ลายมัดหมี่ที่มีความสมมาตร

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ประเภท P6m ได้โดยใช้เทคนิคการค้นหมี่แบบหมี่ลวดร่วมกับการใช้เทคนิคการมัดหมี่แบบสะท้อนลาย

คำสำคัญ: วอลเปเปอร์กรุป ลวดลาย มัดหมี่ ผ้าทอ

ABSTRACT

This article presents the creation of wallpaper which are classified as Mudmee group P6m. Motifs used for pattern development are the original patterns used in Mudmee weaving in Buriram Province, Thailand. Two patterns, Khom Prateep and Khor Bin Thai, are specifically chosen. Results of the study reveal that both primary patterns can be developed into Mudmee fabric pattern having the symmetry of group P6m, using point group P6m in combination with tie-dyeing (IKAT) technique and “Mee Luad” technique.

Keywords: Wallpaper group, Patterns, Mudmee, Woven textile

1. Introduction

Patterns are an art form which has been practiced by many people groups for millennium. Patterns are found everywhere in nature, artwork and handicrafts, architecture, and mathematics. Some mathematicians are interested in patterns employing repetitions and have tried classifying patterns using geometric principles of rotation, reflection, translation, and glide-reflection which is consistent with group theory concepts.

Fedorov [3] and Polya [10] proved that there are only 17 equivalence classes of wallpaper groups. Mathematicians have been interested in and study knowledge related to ancient patterns of handwoven textiles, which also includes architecture, to determine similarities within the 17 groups of wallpaper patterns [1 4 6 7 9 12]. Different wallpaper groups are found in the patterns of textiles and architectures studied; patterns with three and six fold rotations occur with a lesser frequency.

Wallpaper pattern P6m is a symmetrical six-fold rotation on hexagonal lattice and rarely found in ancient architecture or woven textiles. This includes a pattern known as Mudmee, which employs resistant dyeing on yarn prior to the dyeing and weaving process for fabrics. The symmetrical types often found in Mudmee textiles are Cm, Cmm and Pm. Moreover, Thai Mudmee fabric is a handicraft that uses a very complicated technique in creating patterns on the selected cloth. Thus, there is a question whether it is possible to create Mudmee fabric with wallpaper group P6m.

One purpose of this article seeks to determine how to create the Mudmee pattern with wallpaper group P6m using primary patterns derived from the original ones to preserve the uniqueness of local art. The patterns would be created on the basis of wallpaper groups. Results of the study would assist understanding related to the possibility of wallpaper group P6m being woven into Mudmee fabric; and if so, how to create those patterns. The resulting patterns will contribute to a diversity of patterns in the market and may also affect the diversity of Mudmee products.

2. Wallpaper Groups

A group $(G,*)$ is a nonempty set G together with a binary operation $*: G\times G \to G$ that satisfies three properties: associative, identity, and inverse properties. If B is a nonempty subset of G and $(B,*)$ is a group; then, B is called a subgroup of G. In particular, let X be the set of all bijections of the plane $\mathbb{R}^2$ onto itself. If $\circ$ is the composition; then, $(X,\circ)$ is a group [8]. We now define and investigate the class of functions that we will use to model symmetries.

Definition 2.1 [2] An **isometry** $f:\mathbb{R}^2 \to \mathbb{R}^2$ is a distance-preserving function from the plane to itself. This means that $\|f(a)-f(b)\| = \|a-b\|$ for all $a,b \in \mathbb{R}^2$. The set of all isometries is denoted by $\mathrm{Isom}(\mathbb{R}^2)$.

It was shown in [2] that if a function $f:\mathbb{R}^2 \to \mathbb{R}^2$ is an isometry; then, f is a bijection from the plane onto itself. Thus, $\mathrm{Isom}(\mathbb{R}^2)$ is a subgroup of X. Moreover,

there are only three basic types of isometries: translations (τ), rotations (r) and reflections (m). A glide-reflections (g) is an isometry that consists of a reflection over a line and then translation along that line.

Definition 2.2 [8] Let P be any nonempty subset of points in the plane. Define a subset $\mathrm{Sym}(P)$ of $\mathrm{Isom}(\mathbb{R}^2)$ by $\mathrm{Sym}(P)=\left\{\alpha\in \mathrm{Isom}(\mathbb{R}^2) \;\middle|\; \alpha(P)=P\right\}$. That is $\alpha(p)\in P$ for all $p\in P$.

Example 2.1 Let $P=\left\{m\left(\cos\frac{n\pi}{3},\sin\frac{n\pi}{3}\right)\;\middle|\; n\in\{1,2,\ldots,6\} \text{ and } m\in\{0,0.2,0.4,\ldots,1\}\right\}$.

Define the function $r:\mathbb{R}^2\to\mathbb{R}^2$ by

$$r(x,y)=\begin{pmatrix}\cos\frac{\pi}{3} & -\sin\frac{\pi}{3}\\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3}\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} \text{ for all } (x,y)\in\mathbb{R}^2.$$

Let $(x_1,y_1),(x_2,y_2)\in\mathbb{R}^2$. Consider,

$$\begin{aligned}\|r(x_1,y_1)-r(x_2,y_2)\| &= \left\|\left(\left(x_1-x_2\right)\cos\frac{\pi}{3}+\left(y_2-y_1\right)\sin\frac{\pi}{3},\left(x_1-x_2\right)\sin\frac{\pi}{3}+\left(y_1-y_2\right)\cos\frac{\pi}{3}\right)\right\|\\ &= \sqrt{\left(x_1-x_2\right)^2\left(\cos^2\frac{\pi}{3}+\sin^2\frac{\pi}{3}\right)+\left(y_1-y_2\right)^2\left(\sin^2\frac{\pi}{3}+\cos^2\frac{\pi}{3}\right)}\\ &= \sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}\\ &= \left\|\left(x_1,y_1\right)-\left(x_2,y_2\right)\right\|.\end{aligned}$$

Hence, $r\in\mathrm{Isom}(\mathbb{R}^2)$.

Moreover, $r(x,y)\in P$ for all $(x,y)\in P$. Therefore, $r\in\mathrm{Sym}(P)$.

The finding demonstrated in [2] that $\mathrm{Sym}(P)$ is a subgroup of $\mathrm{Isom}(\mathbb{R}^2)$

Definition 2.3 Let O_2 be the set of all matrices of the form

$$R_\theta=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} \quad \text{or} \quad M_{\frac{\theta}{2}}=\begin{pmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{pmatrix}$$

where R_θ is the function of rotation θ counter-clockwise around the origin and

$M_{\frac{\theta}{2}}$ is the function of reflection on a straight line through the origin such that the line is at $\frac{\theta}{2}$-degree angle on x-axis.

Noting that the function r in Example 2.1 is $R_{\frac{\pi}{3}} \in O_2$ in Definition 2.3.

Proposition 2.1 [2] Let $A \in O_2$, $z, b \in \mathbb{R}^2$ and let the function $\pi : \mathrm{Isom}(\mathbb{R}^2) \to O_2$ be given by $\pi(f) = A$ if $f(z) = Az + b$. Then, π is a surjective mapping from $\mathrm{Isom}(\mathbb{R}^2)$ onto O_2 and π is a homomorphism.

The finding demonstrated in [2] that if H is a subgroup of $\mathrm{Isom}(\mathbb{R}^2)$, let $T_H = T \cap H$ where T is the set of all translations in $\mathrm{Isom}(\mathbb{R}^2)$. The function $\pi : \mathrm{Isom}(\mathbb{R}^2) \to O_2$ restricts to a function with the smaller domain H. The kernel of this restricted function is the subgroup T_H. Thus, from the First Isomorphism Theorem, $\pi(H) \cong H/T_H$ and $\pi(H)$ is called the point group of H.

Definition 2.4 [2] A subgroup H of $\mathrm{Isom}(\mathbb{R}^2)$ is a **wallpaper group** if the translation subgroup T_H of H is generated by translations by two linear vectors: $T_H = \langle \tau_a, \tau_b \rangle$ where a and b are linearly independent vectors in the plane and the point group $\pi(H)$ is finite.

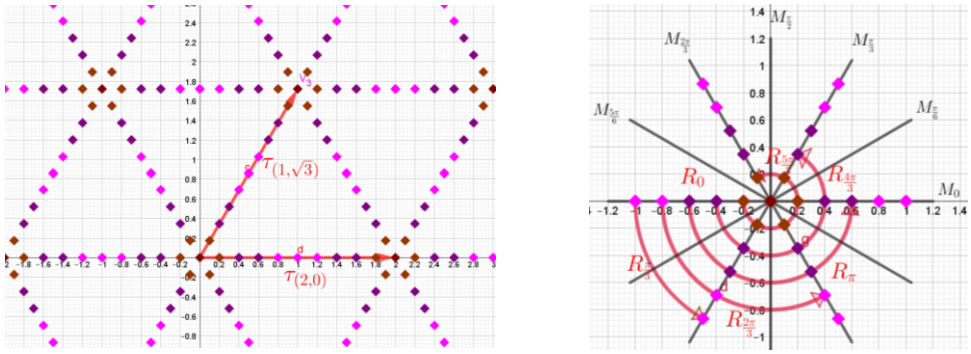
Example 2.2 Consider the set P in Example 2.1.

Let $P' = \left\{ (x + 2h + k, y + \sqrt{3}k) \,\middle|\, (x, y) \in P \text{ and } h, k \in \mathbb{Z} \right\}$ as shown in Figure 2.1 (a).

Then P' is a wallpaper group where

$$\pi(P') = \left\{ R_0, R_{\frac{\pi}{3}}, R_{\frac{2\pi}{3}}, R_{\pi}, R_{\frac{4\pi}{3}}, R_{\frac{5\pi}{3}}, M_0, M_{\frac{\pi}{6}}, M_{\frac{\pi}{3}}, M_{\frac{\pi}{2}}, M_{\frac{2\pi}{3}}, M_{\frac{5\pi}{6}} \right\}$$

as shown in Figure 2.1 (b) and $T_{P'} = \left\langle \tau_{(2,0)}, \tau_{(1,\sqrt{3})} \right\rangle$.



(a) Some points in P' (b) All elements in $\pi(P')$

Figure 2.1 Example of wallpaper group

Theorem 2.1 ([3] and [10]) There are seventeen equivalence classes of wallpaper groups.

In this paper, we use their IUCr (International Union of Crystallography) symbols and classify wallpaper groups as shown in Table 2.1.

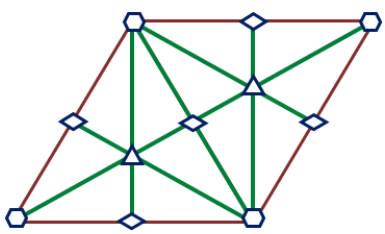
Table 2.1 Seventeen wallpaper groups and their symmetries (adopted from [1])

IUCr Notation	Lattice	Highest Order of Rotation	Reflection	Non-Trivial Glide Reflection
P1	parallelogram	1	no	no
P2	parallelogram	2	no	no
Pm	rectangular	1	yes	no
Pg	rectangular	1	no	yes
Cm	rhombic	1	yes	yes
Pmm	rectangular	2	yes	no
Pmg	rectangular	2	yes	yes
Pgg	rectangular	2	no	yes
Cmm	rhombic	2	yes	yes
P4	square	4	no	no

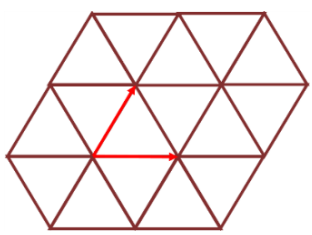
IUCr Notation	Lattice	Highest Order of Rotation	Reflection	Non-Trivial Glide Reflection
P4m	square	4	yes	yes
P4g	square	4	yes	yes
P3	hexagonal	3	no	no
P3m1	hexagonal	3	yes	yes
P31m	hexagonal	3	yes	yes
P6	hexagonal	6	no	no
P6m	hexagonal	6	yes	yes

3. Development of Original Patterns into Wallpaper Group P6m

Creation of the wallpaper group P6m patterns can be done using Doris Schattschneider's method [13] that consists of two steps. The first step generates a primitive cell as shown in Figure 3.1 (a). Then, the primitive cell generated in the first step is glided along vector direction on the hexagonal lattice as shown in Figure 3.1 (b).



(a) Generation of primitive cell



(b) Primitive cell's glide on hexagonal lattice

Reflection axis

Center of 2-fold rotation

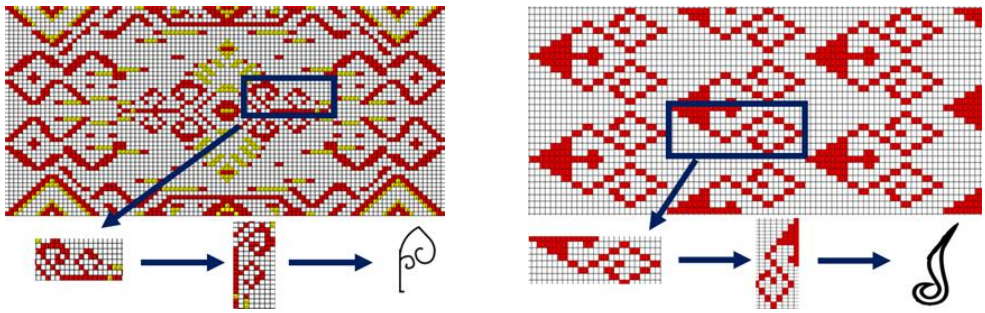
Center of 3-fold rotation

Center of 6-fold rotation

Direction vector of primary pattern's glide

Figure 3.1 Method for creating patterns of wallpaper group P6m

The original motifs used to form wallpaper patterns include “Khom Prateep” and “Khor Bin Thai”, which are local Mudmee patterns woven in Buriram Province. The chosen patterns and drafted motifs are shown in Figure 3.2.



(a) Copying of motif from a part of “Khom Prateep” pattern

(b) Copying of motif from a part of “Khor Bin Thai” pattern

Figure 3.2 A step of copying motifs from a part of “Khom Prateep” and “Khor Bin Thai” patterns

The motif obtained from a part of “Khom Prateep” pattern is used to produce a primitive cell in accordance with the method for creating wallpaper group P6m as shown in Figure 3.3.

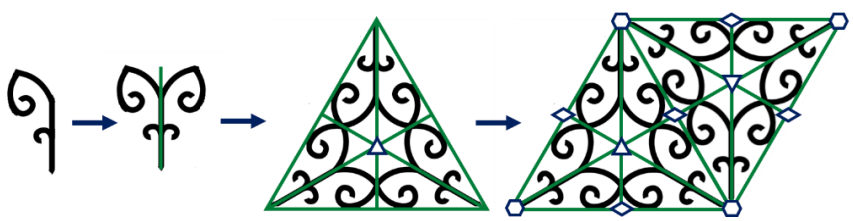
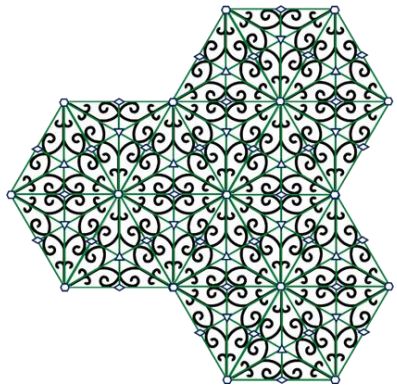
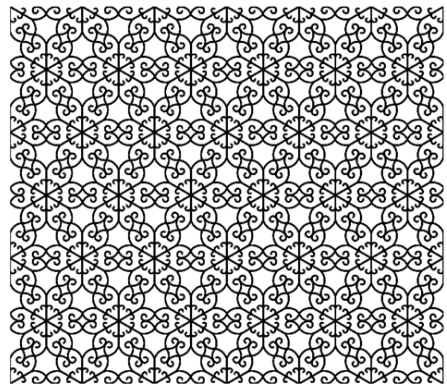


Figure 3.3 Process of creating a primitive cell of wallpaper group P6m from the motif obtained from “Khom Prateep” pattern

After obtaining the primitive cell, glide it along the hexagonal lattice as shown in Figure 3.4 (a) and continue to glide the primitive cell along the hexagonal lattice until the pattern of wallpaper group P6m is created as shown in Figure 3.4 (b).



(a) Gliding the primitive cell along the hexagonal lattice



(b) Pattern of wallpaper group P6m

Figure 3.4 Process of gliding the primitive cell along the hexagonal lattice until a pattern of wallpaper group P6m is obtained

Repeat the same process for the pattern created from a part of “Khor Bin Thai” pattern, starting with creation of the primitive cell as shown in Figure 3.5.

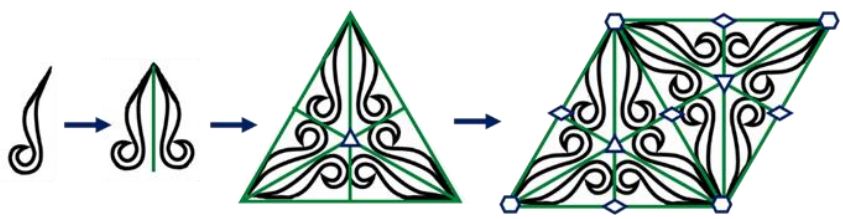
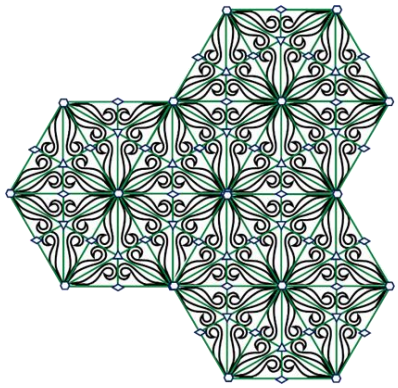
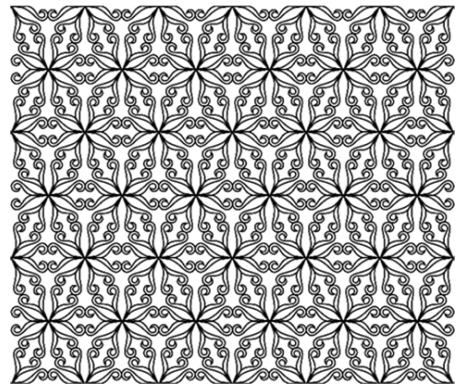


Figure 3.5 Process of creating a primitive cell of wallpaper group P6m from the motif obtained from “Khor Bin Thai” pattern

Keep gliding the primitive cell along the hexagonal lattice as shown in Figure 3.6 (a) to obtain a pattern of wallpaper group P6m as shown in Figure 3.6 (b).



(a) Gliding the primitive cell along the hexagonal lattice

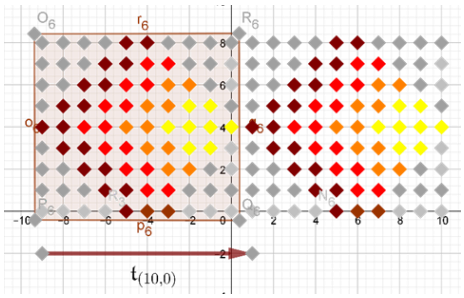


(b) Pattern of wallpaper group P6m

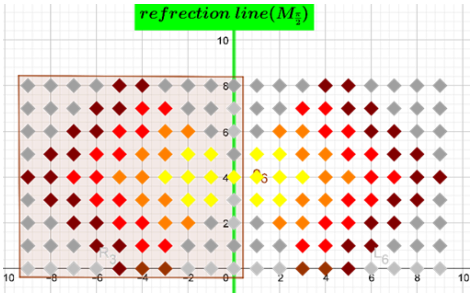
Figure 3.6 Gliding the primitive cell along the hexagonal lattice until a pattern of wallpaper group P6m is obtained

4. Applying Wallpaper Group P6m to Weaving Processes

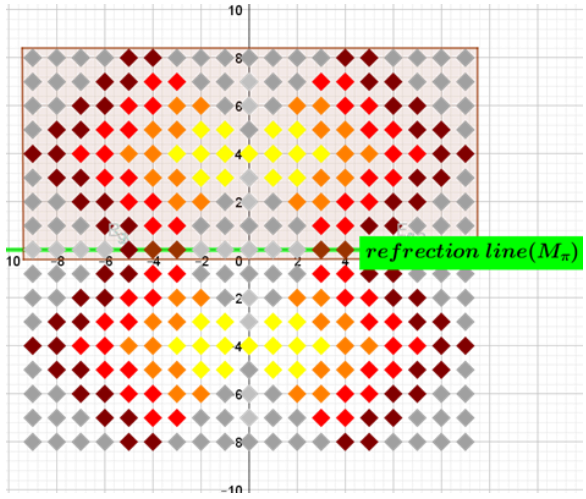
Hobanthad, Prajonsant and Vongpramate [5] found that the skills for creating Mudmee patterns is developed from tie-dyeing process to make differences on the weft. By representing the colored positions with numeric codes on the matrix, the Mudmee formation process is presented; then, the formation of the pattern is explained by using the matrix transformation which consists of two steps. The two steps are use of matrix transformation for reflection of motif generated by the tie-dyeing technique and the matrix transformation for reflection of the motif generated by the "Mee Luad" technique. In this paper, the Mudmee formation process will be represented again by elements of $\mathbf{Isom}(\mathbb{R}^2)$. Let P be a nonempty subset of points in the plane. If P is a motif, then tie-dyeing translation technique, tie-dyeing reflection technique and Mee-Luad technique are τ_a, $M_{\frac{\pi}{2}}$ and M_{π}, respectively (see Figure 4.1).



(a) Tie-dyeing by translation technique



(b) Tie-dyeing by reflection technique



(c) Tie-dyeing by reflection technique in combination with Mee-Luad technique

Figure 4.1 Mudmee formation process

When weavers have been given tables determining the positions for dyeing that have been thoroughly analyzed relative to the pattern formation process and how to dye the selected yarn to create the desired symmetrical patterns, it will give the weavers significant guidelines in the dyeing process. After reviewing P6m patterns from the two motifs, patterns will be entered into the table created by the GeoGebra program by specifying the positions of colors in Mudmee patterns tied with wefts. This is accomplished in accordance with coloring positions in the program and taking into account the order of double dyeing. Let $\boldsymbol{P}$ be a nonempty subset of points in the plane such that $\boldsymbol{P}$ is the motif in right triangle shape (Figure 4.2) from a part of hexagonal lattice. This starts from creation of the set $\boldsymbol{P'}$ using point group P6m as follows:

$P' = \bigcup_{i=1}^{6} P_i$ where

$$P_1 = \left\{ (a,b) \;\middle|\; \begin{pmatrix} \cos\frac{2\pi}{3} & \sin\frac{2\pi}{3} \\ \sin\frac{2\pi}{3} & -\cos\frac{2\pi}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \text{ and } (x,y) \in P \right\},$$

$$P_2 = \left\{ (a,b) \;\middle|\; \begin{pmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \text{ and } (x,y) \in P \cup P_1 \right\},$$

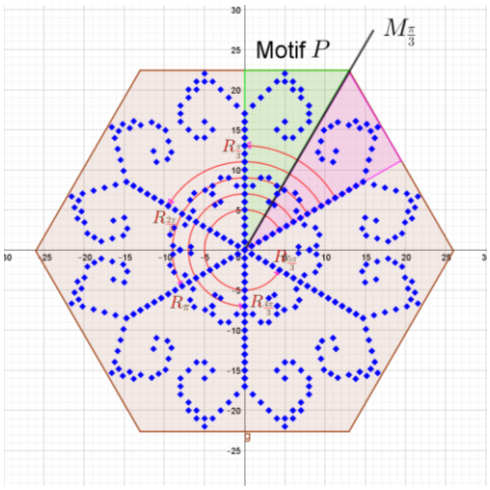
$$P_3 = \left\{ (a,b) \;\middle|\; \begin{pmatrix} \cos\frac{2\pi}{3} & -\sin\frac{2\pi}{3} \\ \sin\frac{2\pi}{3} & \cos\frac{2\pi}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \text{ and } (x,y) \in P \cup P_1 \right\},$$

$$P_4 = \left\{ (a,b) \;\middle|\; \begin{pmatrix} \cos\pi & -\sin\pi \\ \sin\pi & \cos\pi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \text{ and } (x,y) \in P \cup P_1 \right\},$$

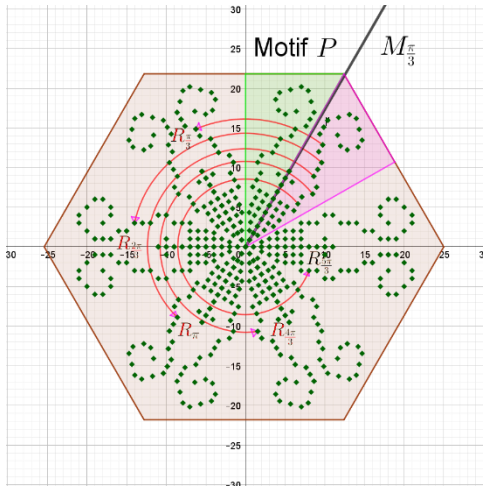
$$P_5 = \left\{ (a,b) \;\middle|\; \begin{pmatrix} \cos\frac{4\pi}{3} & -\sin\frac{4\pi}{3} \\ \sin\frac{4\pi}{3} & \cos\frac{4\pi}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \text{ and } (x,y) \in P \cup P_1 \right\},$$

and $$P_6 = \left\{ (a,b) \;\middle|\; \begin{pmatrix} \cos\frac{5\pi}{3} & -\sin\frac{5\pi}{3} \\ \sin\frac{5\pi}{3} & \cos\frac{5\pi}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \text{ and } (x,y) \in P \cup P_1 \right\}$$

as shown in Figure 4.2.



(a) Creation of P' from motif obtained from "Khom Prateep" pattern



(b) Creation of P' from motif obtained from "Khor Bin Thai" pattern

Figure 4.2 Creation of P' from motif using point group P6m

If τ_a and τ_b are translation vectors where a and b are linearly independent vectors in the plane, then translation of all elements in P' by $\langle \tau_a, \tau_b \rangle$ on hexagonal lattice is shown in Figure 4.3.

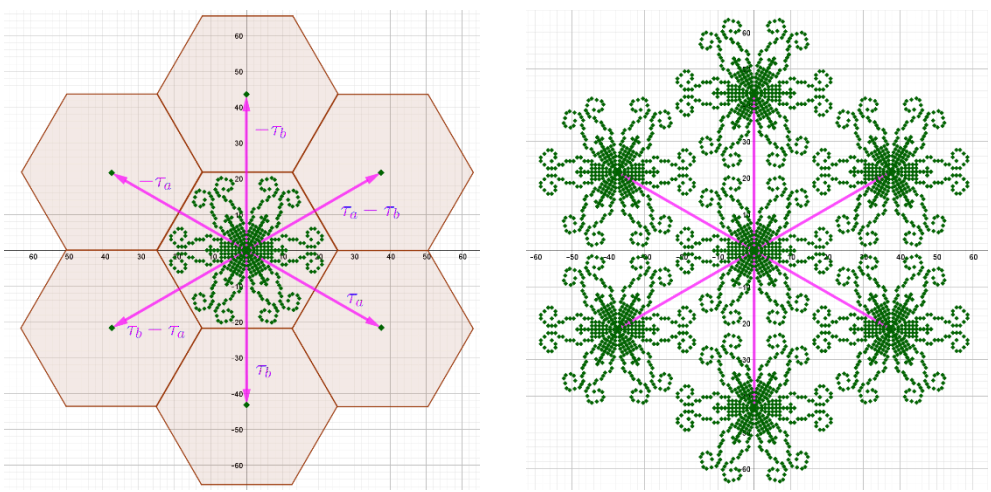
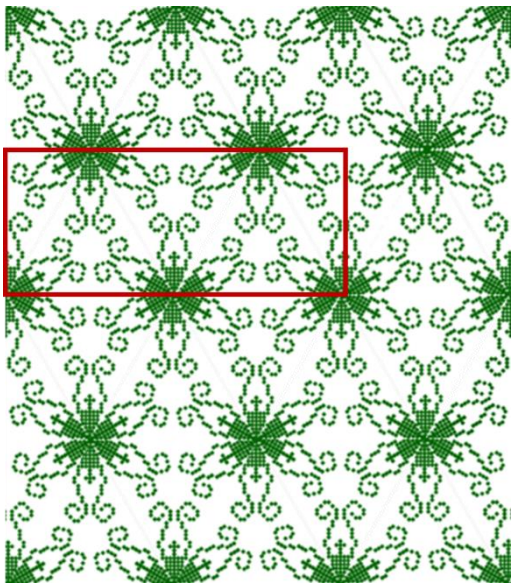


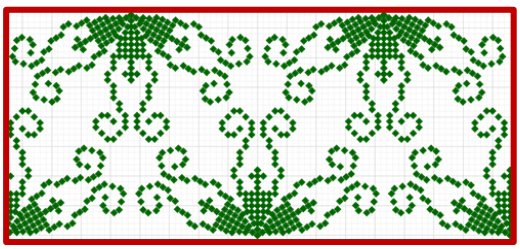
Figure 4.3 Translation P' by $\langle \tau_a, \tau_b \rangle$ on hexagonal lattice

If P'' is the result of translation P' by $\langle \tau_a, \tau_b \rangle$ on hexagonal lattice, then P'' is a wallpaper group P6m pattern as shown in Figure 4.4 (a) and the rectangle is cropped

from a part of P'' such that rectangle is on a reflection line of wallpaper group P6m pattern (Figure 4.4 (b)) for tie-dyeing process as shown in Figure 4.4 (c).



(a) Wallpaper group P6m of P''

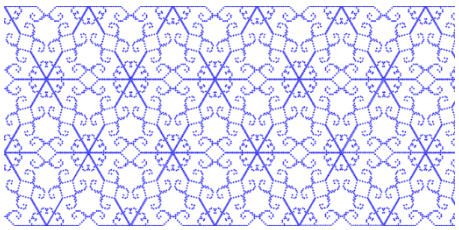


(b) Crop rectangle from a part of P''

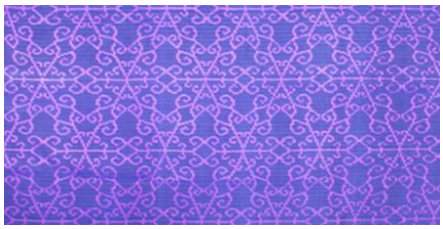
(c) tie-dyeing process

Figure 4.4 Wallpaper group P6m pattern to tie-dyeing process

From recording the pattern developed from a part of "Khom Prateep" pattern in the table with the pattern laid down in a manner that corresponds to the width of the cloth, which is about 100 centimeters, to the process of simulation of the Mudmee pattern with the pattern of the reflection technique and the Mee Luad technique, a simulated P6m pattern using the GeoGebra program is obtained as shown in Figure 4.5 (a). When weavers from Ban Non Samran in Muang District of Buriram Province perform the tie-dyeing and weaving process, the pattern matching the simulated image is obtained as shown in Figure 4.5 (b).



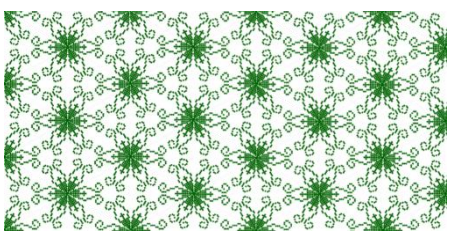
(a) Simulation of Mudmee pattern formed by Mee Luad technique and tie-dyeing technique of pattern reflection



(b) Result of actual weaving using the design technique to obtain the pattern of wallpaper group P6m

Figure 4.5 A comparison of simulated pattern and the actual pattern of handwoven Mudmee developed from a part of “Khom Prateep” pattern

Likewise, the pattern developed from a part of “Khor Bin Thai” pattern is recorded in the table created by the GeoGebra program before undergoing the process of simulation of Mudmee pattern using the pattern reflection technique and the Mee Luad technique to obtain a simulated pattern of wallpaper group P6m as shown in Figure 4.6 (a). When the actual tie-dyeing and weaving process is performed by weavers from Ban Nong Tad Noy in Muang District from Buriram Province, the pattern that matches the simulated image is obtained as shown in Figure 4.6 (b).



(a) Simulation of Mudmee pattern formed by Mee Luad technique and tie-dyeing technique of pattern reflection

(b) Result of actual weaving using the design technique to obtain the pattern of wallpaper group P6m

Figure 4.6 A comparison of simulated pattern and the actual pattern of handwoven Mudmee developed from a part of “Khor Bin Thai” pattern

Consider, the results of actual weaving have maximum order of six rotations and have an axis reflection as shown in Figure 4.7.

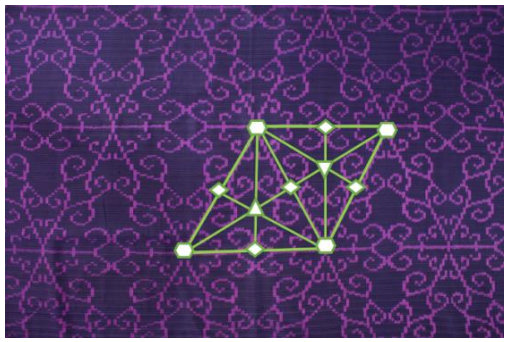


Figure 4.7 Result of actual weaving obtain the pattern of wallpaper group P6m

From Table 2.1 and Figure 3.1, the result of actually weaving obtains the P6m pattern. Besides, pattern simulation yields the same result with a research study that develops a program to simulate Mudmee pattern using Mee Luad technique [14]. However, it cannot be said that the two patterns created by actually weaving are truly symmetrical P6m patterns, because the resulting patterns do not actually resemble the models in every side and corner due to the number of yarns used as wefts are not in proportion to the warps [11] and that tie-dyeing technique requires a high level of expertise to determine the distance of each position of yarn tying. Therefore, it is difficult to actually obtain symmetrical P6m pattern. However, symmetry of the developed patterns is very close with that of wallpaper group P6m and absolutely different from that of the original patterns. Thus, new patterns are created as per the desired outcome as shown in Figure 4.8.

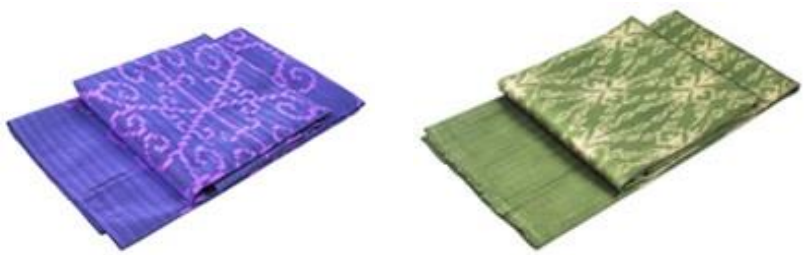


Figure 4.8 Handwoven fabrics developed from original patterns to obtain wallpaper group P6m

5. Conclusion

This article presented the procedures for creating handwoven Mudmee P6m wallpaper pattern from original fabric patterns. Results of the study revealed that parts of the original patterns can be used to create the patterns of wallpaper group P6m. When the simulated patterns are used in the weaving process, it was found that the Mee Luad technique and the tie-dyeing technique of pattern reflection must be used. Regarding the process for generating the primitive cell, the positions of the pattern in the table must be determined for the purpose of tying and dyeing before weaving by the weavers. The weaving results matched the simulated patterns in GeoGebra program. However, the patterns obtained from the actual weaving process are not consistent in every side and corner due to disproportion between the numbers of wefts and warps and the weavers' technique to determine the positions to tie the yarns. However, the patterns obtained from this research are considered original, because there have been no similar patterns of this symmetrical group which have been identified to date. Thus, this is a way to improve the diversity of patterns and may further lead to a diversity of products.

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