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Pancyclicity and Panconnectedness of Line Graphs of Graphs having at Most One Cycle

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Abstract. Let G be a simple graph. The *line graph* of G, denoted by $L(G)$, is the graph obtained by taking the edges of G as vertices and joining two of these vertices whenever the corresponding edges in G have a vertex in common. A graph is called *pancyclic* if it contains cycles of length three up to its Hamiltonian cycle and it is called *panconnected* if, between any pair of distinct vertices u and v, it contains paths of length $d(u, v)$ up to its Hamiltonian path. In this paper, we consider a tree and a unicyclic graph, and give necessary and sufficient conditions for their line graphs to be pancyclic and panconnected, respectively.

Keywords: pancyclic; panconnected, Hamiltonian cycle, line graph.

1 Introduction

All graphs considered in this paper are simple. That is, they have neither loops nor multiple edges. Let G be a graph. The *line graph* of G, denoted by $L(G)$, is the graph obtained by taking the edges of G as vertices and joining two of these vertices whenever the corresponding edges of G have a vertex in common. A *dominating circuit* of G is a circuit of G with the property that every edge of G either belongs to the circuit or is adjacent to an edge of the circuit. Harary and Nash-Williams (Harary and Nash-Williams, 1971) gave a classic result that for a graph G without isolated vertices, $L(G)$ is Hamiltonian if and only if G is the graph $K_{1,n}$, for some $n \geq 3$, or G contains a dominating circuit. A graph is *pancyclic* if it contains a cycle of each length. Thus, a pancyclic graph is necessary Hamiltonian. A graph is *panconnected* if, between any pair of distinct vertics, it contains a path of each length at least distance between the two vertices. Note that (i) a complete graph K_n is pancyclic and also panconnected, and (ii) a cycle $C_n, n \geq 4$ is not pancyclic and is not panconnected. In Figure 1, G_1 is panconnedted and G_2 is pancyclic.

In this paper, we consider a connected graph with no cycles, called a *tree*, and a connected graph with only one cycle, called a *unicyclic graph*. The discussion of a panconnected property of a unicyclic graph was shown in (Chia et al., 2017).

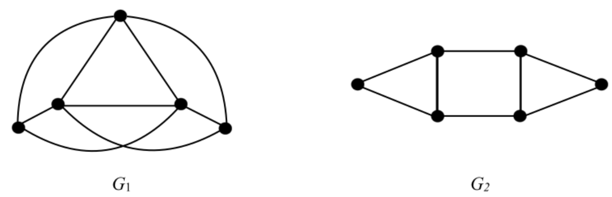


Figure 1: G_1 is panconnedted and G_2 is pancyclic.

If a tree contains one edge, then its line graph is a complete graph K_1. In the case that a tree contains two edges, the tree is a path P_3 and its line graph is a path P_2. In both cases, their line graph has no cycle. Then, we consider a tree with at least three edges. For a unicylic graph G, assume that the only one cycle of G also contains at least three edges. We investigate trees and unicycle graphs and find the necessary and sufficient conditions for their line graphs to be pancyclic and panconnected, in sections 2 and 3, respectively.

2 Pancyclicity

Recall that an edge xy is a *cut-edge* of a graph G if the number of components of $G - xy$ is more than the number of components of G, and a vertex v is a *cut-vertex* of a graph G if the number of components of $G - v$ is more than the number of components of G.

Since all edges of a tree is a cut-edge, we focus the cut-edge xy such that both x and y have degree at least two, called such edge an *internal cut-edge*. We can see that if e is an internal cut-edge in G, then e is a cut-vertex in $L(G)$. Before obtaining the necessary condition for $L(G)$ to be pancyclic, we shall recall a well-know fact that if a graph has a Hamiltonian cycle, then the deletion of any s vertices from it will result in a graph at most s components.

Lemma 1. *Let G be a connected graph. If $L(G)$ is pancyclic, then G has no internal cut-edges.*

Proof. Assume that $L(G)$ is pancyclic and suppose that G has an internal cut-edge, say $e = uv$. Then, e is a cut-vertex of $L(G)$. Let $S = \{e\}$. Then $L(G) - S$ contains at least two components. Therefore, $L(G)$ contains no Hamiltonian cycle which is a contradiction. Hence, G has no internal cut-edges. □

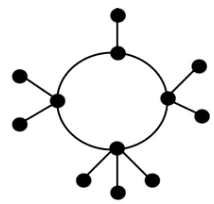
The next theorem obtains a sufficient and necessary condition for a tree such that its line graph is pancyclic.

Theorem 1. *Let G be a tree with $q(\geq 3)$ edges. Then, $L(G)$ is pancyclic if and only if G is a star $K_{1,q}$.*

Proof. Let G be a tree with $q(\geq 3)$ edges such that $L(G)$ is pancyclic. By Lemma 1, G has no internal cut-edge. Then, G must be a star $K_{1,q}$.

Next, assume that G is a star $K_{1,q}$ for some integer $q(\geq 3)$. Then, $L(G)$ is the complete graph K_q. Since $q \geq 3$, $L(G)$ contains cycles of length ℓ for all $\ell \in \{3, 4, \ldots, q\}$. That is, $L(G)$ is pancyclic. □

The *sun flower graph*, denoted by $SF(m; n_1, n_2, \ldots, n_m)$, is a unicyclic graph obtained from a cycle $v_1 v_2 \cdots v_m v_1$ and m sets of independent vertices, $A_1, A_2, \ldots, A_m$, which each set contains $n_1, n_2, \ldots, n_m$ vertices, respectively, by joining each vertex of A_i to v_i for $i \in \{1, 2, \ldots, m\}$. Here, let $e_i = v_i v_{i+1}$ for $i \in \{1, 2, \ldots, m-1\}$, $e_m = v_m v_1$ and edges joining v_i to each vertices of A_i are $e_i^1, e_i^2, \ldots e_i^{n_i}$ for $i \in \{1, 2, \ldots, m\}$. Then, $SF(m; n_1, n_2, \ldots, n_m)$ is a unicyclic graph with no internal cut-edge. In Figure 2, G_3 is $SF(4; 1, 2, 2, 3)$.



G_3

Figure 2: G_3 is $SF(4; 1, 2, 2, 3)$.

Lemma 2. *The line graph of* $SF(3; n_1, n_2, n_3)$ *is pancyclic.*

Proof. Let G denote the graph $SF(3; n_1, n_2, n_3)$. Since $|V(L(G))| = |E(G)| = n_1 + n_2 + n_3 + 3$, we shall find cycles of length ℓ in $L(G)$ where $\ell \in \{3, 4, \ldots, n_1 + n_2 + n_3 + 3\}$. Since the subgraph of $L(G)$ induced by the set $\{e_1, e_3, e_1^1, e_1^2, \ldots, e_1^{n_1}\}$ is the complete graph K_{n_1+2}, there is a cycle of length ℓ in $L(G)$ for all $\ell \in \{3, 4, \ldots, n_1 + 2\}$. The cycle of length $n_1 + 3$ is of the form $e_1 e_2 e_3 e_1^1 e_1^2 \cdots e_1^{n_1} e_1$.

Next, we shall construct the cycle of length ℓ in $L(G)$ where $\ell \in \{n_1 + 4, n_1 + 5, \ldots, n_1 + n_2 + 3\}$. For $i \in \{1, 2, \ldots, n_2\}$, let P_i be the path $e_2^1 e_2^2 \cdots e_2^i$ in $L(G)$. Then, the cycle of length $n_1 + 3 + i$ in $L(G)$ for $i \in \{1, 2, \ldots, n_2\}$ is of the form $e_1 P_i e_2 e_3 e_1^1 e_1^2 \cdots e_1^{n_1} e_1$. Hence, the cycles of length ℓ in $L(G)$ where $\ell \in \{n_1 + 4, n_1 + 5, \ldots, n_1 + n_2 + 3\}$ are obtained.

Finally, we shall construct the cycle of length ℓ in $L(G)$ for all $\ell \in \{n_1+n_2+4, n_1+n_2+5, \ldots, n_1 + n_2 + n_3 + 3\}$ by extending the cycle $e_1 P_{n_2} e_2 e_3 e_1^1 e_1^2 \cdots e_1^{n_1} e_1$ of length $n_1 + n_2 + 3$ in $L(G)$. For $i \in \{1, 2, \ldots, n_3\}$, let Q_i be the path $e_3^1 e_3^2 \cdots e_3^i$ in $L(G)$. Then, the cycle of length n_1+n_2+3+i in $L(G)$ for $i \in \{1, 2, \ldots, n_3\}$ is of the form $e_1 P_{n_2} e_2 Q_i e_3 e_1^1 e_1^2 \cdots e_1^{n_1} e_1$. Hence, the cycles of length ℓ where $\ell \in \{n_1 + n_2 + 4, n_1 + n_2 + 5, \ldots, n_1 + n_2 + n_3 + 3\}$ are obtained.

This completes the proof. □

The next lemma is an important property of $L(G)$ where G is $SF(m; n_1, n_2, \ldots, n_m)$.

Lemma 3. *Let* $m > 3$, G *be the graph* $SF(m; n_1, n_2, \ldots, n_m)$ *and* $n = \max\{n_1, n_2, \ldots, n_m\}$. *If* $m - n > 3$, *then* $L(G)$ *contains no cycle of length* $n + 3$.

Proof. Without loss of generality, let $n = n_1$. Assume that $m - n > 3$. If $L(G)$ contains a cycle C of length $n + 3$, then there exists $n + 3$ vertices $a_1, a_2, \ldots, a_{n+3}$ of $L(G)$ with the property that a_1 is adjacent to a_{n+3} and a_2, a_i is adjacent to vertices a_{i-1} and a_{i+1} for $i \in \{2, 3, \ldots, n + 2\}$ and a_{n+3} is adjacent to a_{n+2} and a_1. Thus, C is the cycle $a_1 a_2 \cdots a_{n+3} a_1$. We consider in two cases.

Case (i) : $n + 3$ vertices of C must come from vertices in $\{e_1, e_2, \ldots, e_m\}$ of $L(G)$. Since $m > n+3$ and the subgraph of $L(G)$ induced by $\{e_1, e_2, \ldots, e_m\}$ is a cycle C_m, C is not a cycle.

Case (ii) : $n + 3$ vertices of C must come from vertices $e_i^1, e_i^2, \ldots, e_i^{n_i}, e_{i-1}, e_i$ and other $n - n_i + 1$ vertices of $L(G)$ for some $i \in \{1, 2, \ldots, m\}$. The vertices $e_i^1, e_i^2, \ldots, e_i^{n_i}, e_{i-1}$, e_i induce the complete graph K_{n_i+2} in $L(G)$ and $L(G)$ contains the cycle $C' : e_i^1 e_i^2 \cdots e_i^{n_i} e_{i-1} e_i e_i^1$. In order to complete the $(n + 3)$-cycle C, $n - n_i + 1$ vertices $w_1, w_2, \ldots, w_{n-n_i+1}$ of $L(G)$ added to C must come from the vertices in $A = \{e_{i-1}^1, e_{i-1}^2, \ldots, e_{i-1}^{n_{i-1}}\}$ or $B = \{e_{i+1}^1, e_{i+1}^2, \ldots, e_{i+1}^{n_{i+1}}\}$. Thus, we replace the edge $e_{i-1}e_i$ of C' by $e_{i-1} w_1 w_2 \cdots w_{n-n_i+1} e_i$.

If $w_1, w_2, \ldots, w_{n-n_i+1} \in A$, then w_{n-n_i+1} is not adjacent to e_i in $L(G)$, which is a contradiction.

If $w_1, w_2, \ldots, w_{n-n_i+1} \in B$, then w_1 is not adjacent to e_{i-1} in $L(G)$ which is a contradiction.

If some vertices of $n - n_i + 1$ are in A and the remaining vertices are in B, then without loss of generality, there is an integer t such that $1 \leq t \leq n - n_i$, $w_1, w_2, \ldots, w_t \in A$ and $w_{t+1}, w_{t+2}, \ldots, w_{n-n_i+1} \in B$. Then, w_t is not adjacent to w_{t+1} in $L(G)$, which is a contradiction.

Therefore, there is no cycle of length $n+3$ in $L(G)$. □

From Lemma 2 and 3, we can have sufficient and necessary conditions for $L(G)$ to be pancyclic where G is a unicyclic graph.

Theorem 2. *Let G be a unicyclic graph. Then, $L(G)$ is pancyclic if and only if*

(i) G is $SF(3; n_1, n_2, n_3)$, or

(ii) if $m > 3$, then G is $SF(m; n_1, n_2, \ldots, n_m)$ such that $m - n \leq 3$ where $n = \max\{n_1, n_2, \ldots, n_m\}$.

Proof. For the necessity part, let G be a unicycle graph such that $L(G)$ is pancyclic. By Lemma 1, G has no internal cut-edges. Then, G is $SF(m; n_1, n_2, \ldots, n_m)$ for some integers $m (\geq 3)$, $n_1, n_2, \ldots, n_m$. Let $n = \max\{n_1, n_2, \ldots, n_m\}$. If $m > 3$ and $m - n > 3$, then, by Lemma 3, $L(G)$ contains no cycle of length $n+3$, which is a contradiction. Therefore, $m - n \leq 3$.

For the sufficient part, we can see by Lemma 2 that if G is $SF(3; n_1, n_2, n_3)$, then $L(G)$ is pancyclic. If $m > 3$, then we consider in three cases that (i) $m - n = 3$; (ii) $m - n = 2$; or (iii) $m - n \leq 1$. Without loss of generality, assume that $n = n_1$.

Case (i) : $m - n = 3$. Then, $m = n + 3 = n_1 + 3$. We shall find a cycle of length ℓ for all $\ell \in \{3, 4, \ldots, 2n_1 + n_2 + \ldots + n_m + 3\}$. Since the subgraph of $L(G)$ induced by the set $\{e_1, e_m, e_1^1, e_1^2, \ldots, e_1^{n_1}\}$ is the complete graph K_{n_1+2}, there is a cycle of length ℓ for all $\ell \in \{3, 4, \ldots, n_1 + 2\}$. The cycle of length $n_1 + 3$ is of the form $e_1 e_2 e_3 \cdots e_{m-1} e_m e_1$.

For $i \in \{1, 2, \ldots, m\}$, let P_i^j be the path $e_i^1 e_i^2 e_i^3 \cdots e_i^j$ for $j \in \{1, 2, \ldots, n_i\}$.

Next, we shall construct cycles in $L(G)$ of length ℓ where $\ell \in \{n_1+4, n_1+5, \ldots, 2n_1+3\}$ by using the cycle $e_1 e_2 e_3 \cdots e_{m-1} e_m e_1$ of length $m = n_1 + 3$. Then, for $j \in \{1, 2, \ldots, n_1\}$, the cycle of length ℓ where $\ell = n_1 + 3 + j$ is of the form $e_1 e_2 e_3 \cdots e_{m-1} e_m P_1^j e_1$.

For the cycle in $L(G)$ of length ℓ where $\ell \in \{2n_1+4, 2n_1+5, \ldots, 2n_1+n_2\}$, we modify the cycle $e_1 e_2 e_3 \cdots e_{m-1} e_m P_1^{n_1} e_1$ of length $2n_1 + 3$ by adding path P_i^j. Then, for $j \in \{1, 2, \ldots, n_2\}$, the cycle of length ℓ where $\ell = 2n_1 + 3 + j$ is of the form $e_1 P_2^j e_2 e_3 \cdots e_{m-1} e_m P_1^{n_1} e_1$.

By the recursive process, the cycle of length $2n_1 + n_2 + \ldots + n_k + j$ for $j \in \{1, 2, \ldots, n_k\}$ is of the form

$$e_1 P_2^{n_2} e_2 P_3^{n_3} e_3 \cdots e_{k-1} P_k^j e_k e_{k+1} \cdots e_m P_1^{n_1} e_1.$$

Case (ii) : $m - n = 2$. Then $m = n + 2 = n_1 + 2$. We shall find a cycle of length ℓ for all $\ell \in \{3, 4, \ldots, 2n_1 + n_2 + \ldots + n_m + 2\}$. Since the subgraph of $L(G)$ induced by the set $\{e_1, e_m, e_1^1, e_1^2, \ldots, e_1^{n_1}\}$ is the complete graph K_{n_1+2}, there is a cycle of length ℓ for all $\ell \in \{3, 4, \ldots n_1 + 2\}$.

For $i \in \{1, 2, \ldots, m\}$, let P_i^j be the path $e_i^1 e_i^2 e_i^3 \cdots e_i^j$ for $j \in \{1, 2, \ldots, n_i\}$.

Next, we shall construct cycles in $L(G)$ of length ℓ where $\ell \in \{n_1+3, n_1+4, \ldots, 2n_1+2\}$ by using the cycle $e_1 e_2 e_3 \cdots e_{m-1} e_m e_1$ of length $m = n_1 + 2$. Then, for $j \in \{1, 2, \ldots, n_1\}$, the cycle of length ℓ where $\ell = n_1 + 2 + j$ is of the form $e_1 e_2 e_3 \cdots e_{m-1} e_m P_1^j e_1$.

For the cycle in $L(G)$ of length ℓ where $\ell \in \{2n_1 + 3, 2n_1 + 4, \ldots, 2n_1 + n_2 + 2\}$, we modify the cycle $e_1 e_2 e_3 \cdots e_{m-1} e_m P_1^{n_1} e_1$ of length $2n_1 + 2$ by adding path P_i^j. Then, for $j \in \{1, 2, \ldots, n_2\}$, the cycle of length ℓ where $\ell = 2n_1 + 2 + j$ is of the form $e_1 P_2^j e_2 e_3 \cdots e_{m-1} e_m P_1^{n_1} e_1$.

By the recursive process, the construction of the cycle of length $2n_1 + n_2 + \ldots + n_k + j$ for $j \in \{1, 2, \ldots, n_k\}$ is in the form

$$e_1 P_2^{n_2} e_2 P_3^{n_3} e_3 \cdots e_{k-1} P_k^j e_k e_{k+1} \cdots e_m P_1^{n_1} e_1.$$

Case (iii) : $m - n \leq 1$. In this case, first, we construct the cycle of length ℓ where $\ell \in \{m, m+1, m+2, \ldots, m+n_1+n_2+\cdots+n_m\}$. Next, the constructions of the cycle of length ℓ where $\ell \in \{3, 4, 5, \ldots, n+2\}$ are obtained. Finally, we show that $n+2 \geq m-1$. Thus, G contains cycles of length ℓ where $\ell \in \{3, 4, 5, \ldots, m+n_1+n_2+\ldots+n_m\}$.

Since $v_1v_2\cdots v_mv_1$ is a cycle in G, vertices $e_1, e_2, \ldots e_m$ in $L(G)$ induce the cycle $e_1e_2e_3\cdots e_{m-1}e_me_1$ of length m in $L(G)$. Then, $L(G)$ contains a cycle of length m.

For $i \in \{1, 2, \ldots, m\}$, let P_i^j be the path $e_i^1e_i^2e_i^3\cdots e_i^j$ for $j \in \{1, 2, \ldots, n_i\}$.

For the cycle in $L(G)$ of length ℓ where $\ell \in \{m+1, m+2, \ldots, m+n_1\}$, we modify the cycle $e_1e_2e_3\cdots e_{m-1}e_me_1$ of length m by adding path P_i^j. Then, for $j \in \{1, 2, \ldots, n_1\}$, the cycle of length ℓ where $\ell = m+j$ is in the form $e_1e_2e_3\cdots e_{m-1}e_mP_1^je_1$. For $j \in \{1, 2, \ldots, n_2\}$, the cycle of length ℓ where $\ell = m+n_1+j$ is of the form $e_1P_2^je_2e_3\cdots e_{m-1}e_mP_1^{n_1}e_1$.

By the recursive process, the construction of the cycle of length $m+n_1+n_2+\ldots+n_k+j$ for $j \in \{1, 2, \ldots, n_k\}$ is in the form $e_1P_2^{n_2}e_2P_3^{n_3}e_3\cdots e_{k-1}P_k^je_ke_{k+1}\cdots e_mP_1^{n_1}e_1$.

Thus, the cycles of length $m, m+1, m+2, \ldots, m+n_1+n_2+\cdots+n_m$ are obtained.

Since the subgraph of $L(G)$ induced by the set $\{e_1, e_m, e_1^1, e_1^2, \ldots, e_1^{n_1}\}$ is the complete graph K_{n_1+2}, there is a cycle of length ℓ for all $\ell \in \{3, 4, \ldots n+2\}$.

We shall show that $n+2 \geq m-1$. Suppose to the contrary that $n+2 < m-1$. Then, $m-n > 1$, which is a contradiction. Thus $n+2 \geq m-1$.

Hence, G contains cycles of length ℓ for all $\ell \in \{3, 4, \ldots, m+n_1+n_2+\ldots+n_m\}$.

This completes the proof. □

3 Panconnectedness

We first recall the well-known fact that if v is a cut-vertex in a connected graph G and u and w are vertices in distinct components of $G - v$, then v lies on every $u - w$ path in G.

Lemma 4. *Let G be a graph such that $L(G)$ is panconnected. Then, G has no internal cut-edge.*

Proof. Let G be a graph. Suppose that G has an internal cut-edge e which x and y are the end-vertices. Then, e is a cut-vertex of $L(G)$. Since deg $x \geq 2$ and deg $y \geq 2$, let e_1 and e_2 be edges incident with x and y, respectively, where $e_1 \neq e$ and $e_2 \neq e$.

We shall show that there is no Hamiltonian path from e_1 to e in $L(G)$. Suppose to the contrary that there is a Hamiltonian path P from e_1 to e in $L(G)$. Then, P and the edge e_1e form a cycle C in $L(G)$. Thus, the path from e_1 to e_2 obtained from C by deleting e_1e and ee_2 has no e in it, this contradicts to the property of e. Hence, $L(G)$ has no Hamiltonian path and this implies that $L(G)$ is not panconnected. □

Since the star has no intenal cut-edge and the line graph of the star is a complete graph, the following result obtains directly from Lemma 4.

Theorem 3. *Let G be a tree. Then, $L(G)$ is panconnected if and only if G is a star.*

The following lemma is an important property of a minimal vertex-cut of a connected graph.

Lemma 5. *Let G be a 2-connected graph with at least 4 vertices and let $U = \{x, y\}$ be a vertex-cut. Then, there is no Hamiltonian path from x to y in G.*

Proof. Suppose to the contrary that there is a Hamiltonian path P from x to y in G. Let x' and y' be vertices that adjacent to x and y in P, respectively. Then, $G - U$ contains a Hamiltonian path $P - U$ from x' to y'. That is, $G - U$ is a connected graph which is a contradiction. □

Next, we obtain a sufficient and necessary condition for a unicyclic graph G that $L(G)$ is panconnected.

Theorem 4. *Let G be a unicyclic graph. Then, $L(G)$ is panconnected if and only if G is C_3.*

Proof. Assume that G is a unicyclic such that G is not a cycle C_3 and $L(G)$ is panconnected. By Lemma 4, G has no internal cut-edges. Then, G must be $SF(m; n_1, n_2, \ldots, n_m)$ for some non-negative integers $m, n_1, n_2, \ldots, n_m$.

Case (i) : $m > 3$. Since $L(G)$ is a 2-connected graph and $U = \{e_1, e_3\}$ is a vertex-cut of $L(G)$, by Lemma 5, there is no Hamiltonian path from e_1 to e_3 in $L(G)$, which is a contradiction.

Case (ii) : $m = 3$ and $n_i \neq 0$ for some $i \in \{1, 2, 3\}$. Without of loss of generality, let $n_1 \neq 0$. Since $L(G)$ is a 2-connected graph and $U = \{e_1, e_3\}$ is a vertex-cut of $L(G)$, $L(G) - U$ contains two components H_1 and H_2 such that $V(H_1) = \{e_1^1, e_1^2, e_1^3, \ldots, e_1^{n_1}\}$ and $V(H_2) = E(G) - (V(H_1) \cup \{e_1, e_3\})$. By Lemma 5, there is no Hamiltonian path from e_1 to e_3 in $L(G)$, which is a contradiction.

Therefore $L(G)$ must be C_3.

Conversely, if G is C_3, then $L(G)$ is C_3. Thus, for two distinct vertices, there is a path of length 1 and 2. Therefore, G is panconnected. □

4 Conclusions

The *cyclomatic number* of a graph G, denoted by $c(G)$, is $|E(G)| - |V(G)| + 1$. Then, $c(G) = 0$ if and only if G is a tree and $c(G) = 1$ if and only if G is a unicyclic graph. This article obtains necessary and sufficient conditions for a line graph of a graph at most one cycle to be pancyclic and panconnected, respectively. That is, such conditions give for graphs with cyclomatic number 0 and 1. In the future research, we shall find necessary and sufficient conditions for a line graph of a graph with cyclomatic number more that one to be pancyclic or panconnected.

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