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Confidence Intervals for the Common Inverse Mean of Several Normal Populations with Unknown Coefficients of Variation

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Abstract. This paper proposes confidence intervals for the common inverse mean of the normal distributions with unknown coefficients of variation (CVs). The generalized confidence interval (GCI), large sample, adjusted method of variance estimates recovery (adjusted MOVER) approaches were proposed to construct the confidence intervals. The confidence intervals were compared with existing confidence interval for the common inverse mean of the normal distributions based on the GCI proposed by Thangjai et al. (2017a). The coverage probability and average length of the proposed confidence intervals were considered for performance criterion. The results indicate that the GCI and the adjusted MOVER confidence interval perform satisfactorily in terms of the coverage probability and average length for large sample sizes. The GCI and the adjusted MOVER approaches are better than the other approaches for constructing the confidence intervals for the common inverse mean of the normal distributions with unknown CVs. Finally, two real data in finance and medical science are given to illustrate the proposed confidence intervals.

Keywords: GCI approach, Large sample approach, Adjusted MOVER approach, Monte Carlo simulation

1. Introduction

A normal distribution is the most important and most widely used in statistics. This distribution is often used in natural sciences and social sciences. The mean and the standard deviation of distribution are defined as μ an σ, respectively. The ratio of the standard deviation to the mean is the coefficient of variation (CV).

An inverse mean is defined as the ratio of one to the mean. It is widely used in many fields. For instance, in experimental nuclear physics, the inverse of the track curvature of a particle $(1/\mu)$ is a charged particle momentum (Lamanna et al. (1981) and Treadwell (1982)). In economics, the inverse mean is used to estimate the marginal propensity to consume in a simple Keynesian model (Braulke (1982)). The estimation of the inverse of the CV is a related problem with the inverse of the mean. Estimating the inverse of the CV is more difficult to remove when the expected value of the CV of normal distribution is infinite (Johnson and Kotz (1970) and Chaturvedi and Rani (1996)). The inverse of the CV is used to estimate the signal-to-noise ratio (μ/σ) in electrical and electronic engineering (Brown et al. (2001)). Furthermore, the inverse of the CV is used a reliability index in structural design and construction (Zubeck and Kvinson (1996) and Duerr (2008)).

The point estimation for the inverse powers of a normal mean is presented by Withers and Nadarajah (2013). The confidence interval estimation for the inverse mean of normal distribution has been studied by several researchers, e.g. Wongkhao et al. (2013), Niwitpong and Wongkhao (2015) and Niwitpong and Wongkhao (2016). Several measuring instruments are utilized to measure the products produced by the same production process to estimate the average quality and different laboratories are employed to measure the amount of toxic waste in a river. If the samples collected by independent studies are from normal populations with a common inverse mean, then the problem of interest may be to construct a confidence interval for the common inverse mean of these populations. Therefore, Thangjai et al. (2016) and Thangjai et al. (2017a) constructed the confidence intervals for the common inverse mean of normal distributions.

In practice, if the CVs of the populations are unknown, then the CVs need to be estimated. Therefore, several researchers have been studied the mean of normal distribution with unknown CV, see the research papers of Srivastava (1980), Sahai (2004), Sahai and Acharya (2016), Sodanin et al. (2016), and Thangjai et al. (2017b). Recently, the confidence intervals for the inverse mean and difference of inverse means of normal distributions with unknown CVs are constructed by Thangjai et al. (2019). Consequently, estimating the common inverse mean of several normal populations with unknown CVs is most interesting problems in statistical inference.

Let $X=(X_1,X_2,\ldots,X_n)$ be a random variable from all possible distributions. Let $L(X)$ and $U(X)$ be the lower and upper limits for the common mean with unknown CVs with nominal confidence level $1-\alpha$. By definition, $1/U(X)$ and $1/L(X)$ are the lower and upper limits for the common inverse mean with unknown CVs with nominal confidence level $1-\alpha$.

This paper extends the paper works of Thangjai et al. (2016), Thangjai et al. (2017a), and Thangjai et al. (2019) to construct confidence intervals for the common inverse mean of the normal distributions with unknown CVs. The confidence intervals were constructed based on the generalized confidence interval (GCI), large sample, and adjusted method of variance estimates recovery (Adjusted MOVER) approaches. Firstly, The GCI approach has been widely used to construct confidence interval; for example, see Tian (2005), Tian and Wu (2007), Thangjai et al. (2016), Thangjai et al. (2017a), Thangjai et al. (2017b), and Thangjai et al. (2019). The large sample approach is used to estimate confidence interval for common parameter in the research works of Tian and Wu (2007), Thangjai et al. (2016), and Thangjai et al. (2017a). Finally, the adjusted MOVER approach is motivated based on the MOVER approach of Zou and Donner (2008) and Zou et al. (2009). The adjusted MOVER approach is introduced by Thangjai and Niwitpong (2017) and Thangjai et al. (2017a). Three approaches are compared with the GCI approach for the common inverse mean of normal distributions of Thangjai et al. (2017a).

This paper is organized as follows. In Section 2, the proposed approaches are described. In Section 3, simulation results are presented to evaluate the performances of the proposed approaches and the existing approach on coverage probabilities and average lengths. Section 4, illustrates the proposed approaches and the existing approach with two real examples. And finally, Section 5 summarizes this paper.

2. The Confidence Intervals for the Common Inverse Mean of Several Normal Populations with Unknown CVs

Let $X=(X_1,X_2,\ldots,X_n)$ be a random variable from a normal distribution with mean μ and variance σ^2. Let $\bar{X}$ and S^2 be sample mean and sample variance for X, respectively.

Following Srivastava (1980), the estimator of the mean of normal population with unknown CV has the following form

$$\hat{\eta}=\frac{\bar{X}}{1+(S^2/n\bar{X}^2)}=\frac{n\bar{X}}{n+(S^2/\bar{X}^2)}. \quad (1)$$

The inverse mean of normal population with unknown CV is

$$\theta=\frac{1}{\eta}=\frac{1+(\sigma^2/n\mu^2)}{\mu}=\frac{n+(\sigma^2/\mu^2)}{n\mu}. \quad (2)$$

The estimator of θ is

$$\hat{\theta}=\frac{1}{\hat{\eta}}=\frac{1+(S^2/n\bar{X}^2)}{\bar{X}}=\frac{n+(S^2/\bar{X}^2)}{n\bar{X}}. \quad (3)$$

According to Thangjai et al. (2019), the mean and variance of $\hat{\theta}$ are

$$E(\hat{\theta})=\left(1+\frac{\sigma^2}{n\mu^2+\sigma^2}+\frac{2\sigma^6+4n\mu^2\sigma^4}{(n\mu^2+\sigma^2)^3}\right)\theta \quad (4)$$

and

$$Var(\hat{\theta}) = \left(\frac{1}{\mu} + \frac{1}{\mu} \left(\frac{\sigma^2}{n\mu^2 + \sigma^2} \right) \left(1 + \frac{2\sigma^4 + 4n\mu^2\sigma^2}{(n\mu^2 + \sigma^2)^2} \right) \right)^2$$

$$\text{x} \left(\frac{\left(\frac{n\sigma^2}{n\mu^2 + \sigma^2} \right)^2 \left(\frac{2}{n} + \frac{2\sigma^4 + 4n\mu^2\sigma^2}{(n\mu^2 + \sigma^2)^2} \right)}{\left(n + \left(\frac{n\sigma^2}{n\mu^2 + \sigma^2} \right) \left(1 + \frac{2\sigma^4 + 4n\mu^2\sigma^2}{(n\mu^2 + \sigma^2)^2} \right) \right)^2} + \frac{\sigma^2}{n\mu^2} \right) \quad (5)$$

.

For $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, n_i$, let $X_i = (X_{i1}, X_{i2}, \ldots, X_{in_i})$ be a random variable from the i-th normal distribution with the common inverse mean $1/\mu$ and possibly unequal variances σ_i^2. Let $\bar{X}_i$ and S_i^2 be sample mean and sample variance for X_i, respectively. Also, let $\bar{x}_i$ and s_i^2 be the observed sample of $\bar{X}_i$ and S_i^2, respectively.

The estimator of the inverse mean of normal population with unknown CV based on i-th sample is

$$\hat{\theta}_i = \frac{1}{\hat{\eta}_i} = \frac{1 + (S_i^2 / n_i \bar{X}_i^2)}{\bar{X}_i} = \frac{n_i + (S_i^2 / \bar{X}_i^2)}{n_i \bar{X}_i}. \quad (6)$$

The variance of $\hat{\theta}_i$ is obtained by

$$Var(\hat{\theta}_i) = \left(\frac{1}{\mu_i} + \frac{1}{\mu_i} \left(\frac{\sigma_i^2}{n_i\mu_i^2 + \sigma_i^2} \right) \left(1 + \frac{2\sigma_i^4 + 4n_i\mu_i^2\sigma_i^2}{(n_i\mu_i^2 + \sigma_i^2)^2} \right) \right)^2$$

$$\text{x} \left(\frac{\left(\frac{n_i\sigma_i^2}{n_i\mu_i^2 + \sigma_i^2} \right)^2 \left(\frac{2}{n_i} + \frac{2\sigma_i^4 + 4n_i\mu_i^2\sigma_i^2}{(n_i\mu_i^2 + \sigma_i^2)^2} \right)}{\left(n_i + \left(\frac{n_i\sigma_i^2}{n_i\mu_i^2 + \sigma_i^2} \right) \left(1 + \frac{2\sigma_i^4 + 4n_i\mu_i^2\sigma_i^2}{(n_i\mu_i^2 + \sigma_i^2)^2} \right) \right)^2} + \frac{\sigma_i^2}{n_i\mu_i^2} \right). \quad (7)$$

2.1 The GCI

Definition 1: Let $X = (X_1, X_2, \ldots, X_n)$ be a random variable from a distribution $F(x \mid \delta)$, where $x = (x_1, x_2, \ldots, x_n)$ is an observed value of X, $\delta = (\theta, v)$ is a vector of unknown parameters, θ is a parameter of interest, and v is a vector of nuisance parameters. Let $R(X; x, \delta)$ be a function of X, x, and δ. The random quantity $R(X; x, \delta)$ is called a generalized pivotal quantity if it has the following two properties; see Weerahandi (1993):

(i) The distribution of $R(X; x, \delta)$ is free of all unknown parameters.

(ii) The observed value of $R(X; x, \delta)$, $X = x$, does not depend on the vector of nuisance parameters.

The GCI is computed using the percentiles of the generalized pivotal quantity. Let $R(\alpha)$ be the $100(\alpha)$-th percentile of $R(X; x, \delta)$, then $R(\alpha)$ is a $100(1-\alpha)\%$ lower bound of one-sided GCI for θ and $[R(\alpha/2), R(1-\alpha/2)]$ is the $100(1-\alpha)\%$ two-sided GCI for θ.

Consider k independent normal populations with a common inverse mean with unknown CVs. Recall that

$$\frac{(n_i - 1)S_i^2}{\sigma_i^2} = \chi^2_{n_i - 1}, \quad (8)$$

where $\chi^2_{n_i-1}$ denotes a chi-squared distribution with $n_i - 1$ degrees of freedom. The generalized pivotal quantity for σ_i^2 is obtained by

$$R_{\sigma_i^2} = \frac{(n_i - 1)s_i^2}{\chi^2_{n_i - 1}}. \quad (9)$$

The mean can be written as

$$\mu_i \approx \bar{X}_i - \frac{Z_i}{\sqrt{U_i}} \sqrt{\frac{(n_i - 1)S_i^2}{n_i}}, \quad (10)$$

where Z_i and U_i denote a standard normal distribution and a chi-squared distribution with $n_i - 1$ degrees of freedom, respectively. The generalized pivotal quantity for μ_i is given by

$$R_{\mu_i} = \bar{x}_i - \frac{Z_i}{\sqrt{U_i}}\sqrt{\frac{(n_i-1)s_i^2}{n_i}}\,. \qquad (11)$$

The generalized pivotal quantity for θ_i is

$$R_{\theta_i} = \frac{n_i + (R_{\sigma_i^2}/R_{\mu_i}^2)}{n_i R_{\mu_i}}\,, \qquad (12)$$

where $R_{\sigma_i^2}$ and R_{μ_i} are defined in Equation (9) and Equation (11), respectively.

From Equation (5), the generalized pivotal quantity for variance of $\hat{\theta}_i$ is

$$R_{Var(\hat{\theta}_i)} = \left(\frac{1}{R_{\mu_i}} + \frac{1}{R_{\mu_i}}\left(\frac{R_{\sigma_i^2}}{n_i R_{\mu_i}^2 + R_{\sigma_i^2}}\right)\left(1 + \frac{2R_{\sigma_i^2}^2 + 4n_i R_{\mu_i}^2 R_{\sigma_i^2}}{(n_i R_{\mu_i}^2 + R_{\sigma_i^2})^2}\right)\right)^2$$

$$\text{x}\left(\frac{\left(\frac{n_i R_{\sigma_i^2}}{n_i R_{\mu_i}^2 + R_{\sigma_i^2}}\right)^2\left(\frac{2}{n_i} + \frac{2R_{\sigma_i^2}^2 + 4n_i R_{\mu_i}^2 R_{\sigma_i^2}}{(n_i R_{\mu_i}^2 + R_{\sigma_i^2})^2}\right)}{\left(n_i + \left(\frac{n_i R_{\sigma_i^2}}{nR_{\mu_i}^2 + R_{\sigma_i^2}}\right)\left(1 + \frac{2R_{\sigma_i^2}^2 + 4n_i R_{\mu_i}^2 R_{\sigma_i^2}}{(n_i R_{\mu_i}^2 + R_{\sigma_i^2})^2}\right)\right)^2} + \frac{R_{\sigma_i^2}}{n_i R_{\mu_i}^2}\right). \qquad (13)$$

Following Ye et al. (2010), the generalized pivotal quantity for the common inverse mean with unknown CVs is a weighted average of the generalized pivotal quantity R_{θ_i} based on k individual samples given by

$$R_\theta = \sum_{i=1}^{k}\frac{R_{\theta_i}}{R_{Var(\hat{\theta}_i)}}\Bigg/\sum_{i=1}^{k}\frac{1}{R_{Var(\hat{\theta}_i)}}\,, \qquad (14)$$

where R_{θ_i} and $R_{Var(\hat{\theta}_i)}$ are defined in Equation (12) and Equation (13), respectively.

To construct confidence interval based on R_θ, we need to confirm that R_θ in Equation (14) satisfies the two conditions in Definition 1. The value of R_{θ_i} in Equation (12) at $(\bar{X}_i, S_i^2) = (\bar{x}_i, s_i^2)$ is $1/\mu_i$, where $i = 1,2,\ldots,k$. Therefore, $R_\theta = 1/\mu$ at $(\bar{X}, S^2) = (\bar{x}, s^2)$. It is also clear from Equation (12), that for given $(\bar{x}, s^2)$, the distribution of R_θ is independent of any unknown parameters. Therefore, R_θ is generalized pivotal quantity, and its percentiles can be used to construct confidence interval for the common inverse mean with unknown CVs.

Therefore, the $100(1-\alpha)\%$ two-sided confidence interval for the common inverse mean with unknown CVs based on the GCI approach is

$$CI_{GCI} = [L_{GCI}, U_{GCI}] = [R_\theta(\alpha/2), R_\theta(1-\alpha/2)]\,, \qquad (15)$$

where $R_\theta(\alpha/2)$ and $R_\theta(1-\alpha/2)$ denote the $100(\alpha/2)$-th and $100(1-\alpha/2)$-th percentiles of R_θ, respectively.

The following algorithm was used to construct the GCI:

Algorithm 1

For g = 1 to m

Generate $\chi^2_{n_i-1}$ and compute $R_{\sigma_i^2}$ from Equation (9)

Generate Z_i and U_i, and then compute R_{μ_i} from Equation (11)

Compute R_{θ_i} and $R_{Var(\hat{\theta}_i)}$, and then compute R_θ from Equation (14)

End g loop

Compute the $100(\alpha/2)$-th and the $100(1-\alpha/2)$-th percentiles of R_θ.

2.2 The Large Sample Confidence Interval

According to Graybill and Deal (1959), the large sample estimate of inverse mean with unknown CVs is a pooled estimate of the inverse mean with unknown CV is

$$\hat{\theta} = \sum_{i=1}^{k}\frac{\hat{\theta}_i}{Var(\hat{\theta}_i)}\Bigg/\sum_{i=1}^{k}\frac{1}{Var(\hat{\theta}_i)}\,, \qquad (16)$$

where $\hat{\theta}_i$ is defined in Equation (6) and $Var(\hat{\theta}_i)$ is an estimate of $Var(\hat{\theta}_i)$ in Equation (7) with μ_i and σ_i^2 replaced by $\bar{x}_i$ and s_i^2, respectively.

The distribution of $\hat{\theta}$ is approximately normal distribution when the sample size is large. The confidence interval for the common inverse mean with unknown CVs is constructed using the quantile of the normal distribution. Therefore, the $100(1-\alpha)\%$ two-sided confidence interval for the common inverse mean with unknown CVs based on the large sample approach is

$$CI_{LS}=[L_{LS},U_{LS}]=\left[\hat{\theta}-z_{1-\alpha/2}\sqrt{1\Big/\sum_{i=1}^{k}\frac{1}{Var(\hat{\theta}_i)}},\hat{\theta}+z_{1-\alpha/2}\sqrt{1\Big/\sum_{i=1}^{k}\frac{1}{Var(\hat{\theta}_i)}}\right]$$
, (17)

where $z_{1-\alpha/2}$ denotes the $(1-\alpha/2)$-th quantile of the standard normal distribution.

2.3 The Adjusted MOVER Confidence Interval

Let θ_1 and θ_2 be the parameters of interest. Zou and Donner (2008) and Zou et al. (2009) proposed the MOVER approach to construct $100(1-\alpha)\%$ two-sided confidence interval $[L,U]$ for the sum of two parameters ($\theta_1+\theta_2$), where L and U denote the lower and upper limits of the confidence interval. By the central limit theorem and under the assumption of independence between the point estimates $\hat{\theta}_1$ and $\hat{\theta}_2$, the lower limit L is

$$L=\hat{\theta}_1+\hat{\theta}_2-z_{\alpha/2}\sqrt{Var(\hat{\theta}_1)+Var(\hat{\theta}_2)}, \qquad (18)$$

where $z_{\alpha/2}$ denotes the $(\alpha/2)$-th quantile of the standard normal distribution.

Let $[l_i,u_i]$ be a $100(1-\alpha)\%$ two-sided confidence interval for θ_i, where $i=1,2$. It is well known that the lower limit L must be closer to l_1+l_2 than to $\hat{\theta}_1+\hat{\theta}_2$. According to the central limit theorem, the variance estimate for $\hat{\theta}_i$ at $\theta_i=l_i$ is

$$Var(\hat{\theta}_i)=\frac{(\hat{\theta}_i-l_i)^2}{z_{\alpha/2}^2}. \qquad (19)$$

Substituting back into Equation (18) yields

$$L=\hat{\theta}_1+\hat{\theta}_2-\sqrt{(\hat{\theta}_1-l_1)^2+(\hat{\theta}_2-l_2)^2} \qquad (20)$$

and similarly

$$U=\hat{\theta}_1+\hat{\theta}_2+\sqrt{(u_1-\hat{\theta}_1)^2+(u_2-\hat{\theta}_2)^2} \qquad (21).$$

Let $\theta_1,\theta_2,\ldots,\theta_k$ be the parameters of interest. The MOVER approach is motivated to construct $100(1-\alpha)\%$ two-sided confidence interval $[L,U]$ for the sum of k parameters ($\theta_1+\theta_2+\ldots+\theta_k$). Let $[l_1,u_1],[l_2,u_2],\ldots,[l_k,u_k]$ be the $100(1-\alpha)\%$ two-sided confidence intervals for $\theta_1,\theta_2,\ldots,\theta_k$, respectively. The lower limit L must be closer to $l_1+l_2+\ldots+l_k$ than to $\hat{\theta}_1+\hat{\theta}_2+\ldots+\hat{\theta}_k$. Using the central limit theorem and the assumption of independence between the point estimates $\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_k$, the lower limit L is

$$L=\hat{\theta}_1+\ldots+\hat{\theta}_k-z_{\alpha/2}\sqrt{Var(\hat{\theta}_1)+\ldots+Var(\hat{\theta}_k)}$$

$$=\hat{\theta}_1+\ldots+\hat{\theta}_k-\sqrt{(\hat{\theta}_1-l_1)^2+\ldots+(\hat{\theta}_k-l_k)^2}. \qquad (22)$$

Similarly, the upper limit U must be closer to $u_1+u_2+\ldots+u_k$ than to $\hat{\theta}_1+\hat{\theta}_2+\ldots+\hat{\theta}_k$. Using the central limit theorem and the assumption of independence between the point estimates $\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_k$, the upper limit U is

$$U=\hat{\theta}_1+\ldots+\hat{\theta}_k+z_{\alpha/2}\sqrt{Var(\hat{\theta}_1)+\ldots+Var(\hat{\theta}_k)}$$

$$=\hat{\theta}_1+\ldots+\hat{\theta}_k+\sqrt{(u_1-\hat{\theta}_1)^2+\ldots+(u_k-\hat{\theta}_k)^2}. \qquad (23)$$

The concepts of the large sample and MOVER approaches given in Equations (16) - (23) are used to construct the confidence interval for the common inverse mean with unknown CVs. This approach is called the adjusted MOVER approach. The common inverse mean with unknown CVs is weighted average of the inverse mean with unknown CV based on k individual samples is

$$\hat{\theta}=\sum_{i=1}^{k}\frac{\hat{\theta}_i}{Var(\hat{\theta}_i)}\Bigg/\sum_{i=1}^{k}\frac{1}{Var(\hat{\theta}_i)}, \qquad (24)$$

where the variance estimate for $\hat{\theta}_i$ at $\theta_i = l_i$ and $\theta_i = u_i$ is

$$Var(\hat{\theta}_i) = \frac{1}{2}\left(\frac{(\hat{\theta}_i - l_i)^2}{z_{\alpha/2}^2} + \frac{(u_i - \hat{\theta}_i)^2}{z_{\alpha/2}^2} \right), \quad (25)$$

where $z_{\alpha/2}$ denotes the $(\alpha/2)$-th quantile of the standard normal distribution.

Therefore, the lower and upper limits for the common inverse mean with unknown CVs are

$$L_{AM} = \hat{\theta} - z_{1-\alpha/2}\sqrt{1 \Big/ \sum_{i=1}^{k} \frac{z_{\alpha/2}^2}{(\hat{\theta}_i - l_i)^2}} \quad (26)$$

and

$$U_{AM} = \hat{\theta} + z_{1-\alpha/2}\sqrt{1 \Big/ \sum_{i=1}^{k} \frac{z_{\alpha/2}^2}{(u_i - \hat{\theta}_i)^2}}, \quad (27)$$

where $z_{\alpha/2}$ and $z_{1-\alpha/2}$ denote the $(\alpha/2)$-th and $(1-\alpha/2)$-th quantiles of the standard normal distribution.

Therefore, the $100(1-\alpha)\%$ two-sided confidence interval for the common inverse mean with unknown CVs based on the adjusted MOVER approach is

$$CI_{AM} = [L_{AM}, U_{AM}] = \left[\hat{\theta} - z_{1-\alpha/2}\sqrt{1 \Big/ \sum_{i=1}^{k} \frac{z_{\alpha/2}^2}{(\hat{\theta}_i - l_i)^2}}, \hat{\theta} + z_{1-\alpha/2}\sqrt{1 \Big/ \sum_{i=1}^{k} \frac{z_{\alpha/2}^2}{(u_i - \hat{\theta}_i)^2}} \right]$$

. (28)

According to Niwitpong and Wongkhao (2016), the $100(1-\alpha)\%$ two-sided confidence interval for the inverse mean based on i-th sample is

$$[l_i, u_i] = \left[\frac{\sqrt{n_i}}{d_i S_i + \sqrt{n_i}\bar{X}_i}, \frac{\sqrt{n_i}}{-d_i S_i + \sqrt{n_i}\bar{X}_i} \right], \quad (29)$$

where d_i denotes the $(1-\alpha/2)$-th quantile of the Student's t-distribution with $n_i - 1$ degrees of freedom.

Substituting l_i and u_i are defined in Equation (29) back into Equation (28), the confidence interval for the common inverse mean with unknown CVs based on the adjusted MOVER approach is obtained.

Next, the GCI approach of Thangjai et al. (2017a) is briefly reviewed for constructing the confidence interval for the common inverse mean of the normal distributions. The generalized pivotal quantity for θ_i is

$$R_{\theta_i} = \frac{1}{R_{\mu_i}},$$

(30)

where $R_{\mu_i} = \bar{x}_i - (Z_i s_i / \sqrt{U_i})$, Z_i is the standard normal distribution, and U_i is chi-squared distribution with $n_i - 1$ degrees of freedom

The generalized pivotal quantity for the common inverse mean of the normal distributions is

$$R_{\theta.TH} = \sum_{i=1}^{k} \frac{R_{\theta_i}}{R_{\mathrm{Var}(\hat{\theta}_i)}} \Bigg/ \sum_{i=1}^{k} \frac{1}{R_{\mathrm{Var}(\hat{\theta}_i)}}, \quad (31)$$

where $R_{\mathrm{Var}(\hat{\theta}_i)} = R_{\sigma_i^2} / (n_i (R_{\mu_i})^4)$,

$R_{\sigma_i^2} = (n_i - 1)s_i^2 / \chi^2_{n_i - 1}$, and $\chi^2_{n_i-1}$ denotes a chi-squared distribution with $n_i - 1$ degrees of freedom.

Therefore, the $100(1-\alpha)\%$ two-sided confidence interval for the common inverse mean of the normal distributions based on the GCI approach of Thangjai et al. (2017a) is

$$CI_{TH} = [L_{TH}, U_{TH}] = [R_{\theta.TH}(\alpha/2), R_{\theta.TH}(1-\alpha/2)]$$

(32)

,

where $R_{\theta.TH}(\alpha/2)$ and $R_{\theta.TH}(1-\alpha/2)$ denote the $100(\alpha/2)$-th and $100(1-\alpha/2)$-th percentiles of $R_{\theta.TH}$, respectively.

3. Simulation Studies

Simulation studies were carried out to evaluate the performance of the GCI, large sample, and adjusted MOVER approaches for the common inverse mean with unknown CVs, comparison studies were also conducted using the GCI approach for the common inverse mean of Thangjai et al. (2017a). The GCI was defined as CI_{GCI}, the large sample confidence interval was defined as CI_{LS}, the adjusted MOVER confidence intervals was defined as CI_{AM}, and the GCI of Thangjai et al. (2017a) was defined as CI_{TH}. The performances of these four approaches were evaluated through the coverage probabilities and the average lengths. In particular, the confidence interval was chosen when the simulated coverage probability was greater than or close to the nominal confidence level $(1-\alpha)$ and the simulated average length was the shortest average length.

The following algorithm was used to estimate the coverage probability and average length:

Algorithm 2

For M, m, k, $(n_1, n_2, \ldots, n_k)$, μ, $(\sigma_1, \sigma_2, \ldots, \sigma_k)$, θ

For h = 1 to M

Generate x_{ij} from $N(\mu, \sigma_i^2)$, $i = 1,2,\ldots,k$, $j = 1,2,\ldots,n_i$

Compute $\bar{x}_i$ and s_i^2

Construct $[L_{GCI(h)}, U_{GCI(h)}]$, $[L_{LS(h)}, U_{LS(h)}]$, and $[L_{AM(h)}, U_{AM(h)}]$

If $L_{(h)} \le \theta \le U_{(h)}$, set $p_{(h)}$ = 1; else set $p_{(h)}$ = 0

Compute $U_{(h)} - L_{(h)}$

End h loop

Compute means of probability $p_{(h)}$ and length $U_{(h)} - L_{(h)}$.

In the simulation, each confidence interval was computed at the nominal confidence level of 0.95. The sample cases were used k = 2, 4, and 6, with the sample sizes $n_1 = n_2 = \ldots = n_k = n =$ 20, 30, 50, 100, and 200. Following Thangjai et al. (2017a), the common inverse mean of normal data within each population was $1/\mu =$ 1.00, and the population standard deviations were $\sigma_1 = \sigma_2 = \ldots = \sigma_{k/2} =$ 0.10 and $\sigma_{(k/2)+1} = \sigma_{(k/2)+2} = \ldots = \sigma_k =$ 0.01, 0.03, 0.05, 0.07, 0.09, 0.10, 0.30, 0.50, and 0.70. For each parameter setting, 5000 random samples were generated and thus 2500 R_θ's were obtained for each of the random samples.

Tables 1-3 presents the coverage probabilities and average lengths of 95% two-sided confidence intervals for the common inverse mean with unknown CVs for k = 2, 4, and 6 sample cases, respectively. For k = 2, the coverage probabilities of CI_{GCI} were closer to the nominal confidence level of 0.95 when the sample size was large. Moreover, CI_{LS} and CI_{AM} provided low coverage probability especially for small sample size. The coverage probabilities of CI_{TH} were close to the nominal confidence level regardless of the sample sizes. For k = 4 and k = 6, the coverage probabilities of CI_{GCI} and CI_{TH} were stable. For small sample size, the coverage probabilities of CI_{LS} and CI_{AM} tended to increase when the value of σ was increase. For large sample size, CI_{GCI} and CI_{AM} performed as well as CI_{TH} in terms of the coverage probability and average length. Overall, the coverage probabilities of CI_{TH} were close to nominal confidence level than CI_{GCI} and CI_{AM} when the sample size was small ($n \le$ 30), whereas CI_{GCI} and CI_{AM} performed as well as CI_{TH} in term of the coverage probability and average length when the sample size was large ($n >$ 30). Therefore, CI_{AM} can be an alternative to estimate the confidence interval for the common inverse mean with unknown CVs when the sample size was small and value of σ was large. Moreover, CI_{GCI} and CI_{AM} were recommended to construct the confidence intervals when the sample size was large.

4. An Empirical Application

Two examples were exhibited to illustrate the proposed approaches given in Section 2 and the GCI approach of Thangjai et al. (2017a).

Example 1: Walpole et al. (2012) and Thangjai et al. (2017a) considered rates of return on equity for 24 randomly selected firms. The data were categorized into four groups depending on the level of financial leverages: control, low, medium, and high. Thangjai et al. (2017a) analyzed that the four data sets come from normal populations. From the data, the following information regarding the estimates of sample size, mean, variance, and inverse mean are $n_1 = 6$, $n_2 = 6$, $n_3 = 6$, $n_4 = 6$, $\bar{x}_1 = 4.3833$, $\bar{x}_2 = 5.1000$, $\bar{x}_3 = 8.4167$, $\bar{x}_4 = 8.3333$, $s_1^2 = 4.8257$, $s_2^2 = 3.8840$, $s_3^2 = 5.9937$, $s_4^2 = 5.4707$, $1/\bar{x}_1 = 0.2281$, $1/\bar{x}_2 = 0.1961$, $1/\bar{x}_3 = 0.1188$, and $1/\bar{x}_4 = 0.1200$. The 95% two-sided confidence intervals for the common inverse mean with unknown CVs are evaluated. The GCI is CI_{GCI} = [0.1258, 0.2473] with a length of interval of 0.1215. The large sample confidence interval is CI_{LS} = [-0.2897, 0.6384] with a length of interval of 0.9281. The adjusted MOVER confidence interval is CI_{AM} = [0.1104, 0.1630] with a length of interval of 0.0526. Finally, the GCI of Thangjai et al. (2017a) is CI_{TH} = [0.0980, 0.1644] with a length of interval of 0.0664. It seems that CI_{AM} performs better than the other confidence intervals in term of length when the sample size is small and the variance is large.

Example 2: Tian (2005) and Fung and Tsang (1998) considered the data about measurements of Hb, RBC, MCV, Hct, WBC and Platelet from 1995 and 1996 surveys. Fung and Tsang (1998) presented that the data come from normal distributions. For 1995 survey, the sample size, sample mean, sample variance, coefficient of variation, and inverse mean are extracted: $n_1 = 63$, $\bar{x}_1 = 84.13$, $s_1^2 = 3.390$, $\tau_1 = 0.0406$, and $1/\bar{x}_1 = 0.0119$. For 1996 survey, the summary statistics are as follows: $n_2 = 72$, $\bar{x}_2 = 85.68$, $s_2^2 = 2.946$, $\tau_2 = 0.0346$, and $1/\bar{x}_2 = 0.0117$. The confidence intervals for the common inverse mean with unknown CVs are computed. The GCI is CI_{GCI} = [0.0117, 0.0118] with a length of interval of 0.0001. The large sample confidence interval is CI_{LS} = [0.0085, 0.0150] with a length of interval of 0.0065. The adjusted MOVER confidence interval is CI_{AM} = [0.0117, 0.0118] with a length of interval of 0.0001. The GCI of Thangjai et al. (2017a) is CI_{TH} = [0.0117, 0.0118] with a length of interval of 0.0001. It is clear from the above results that CI_{GCI}, CI_{AM}, and CI_{TH} perform well in term of length when the sample size is large. These results support the simulation results in the previous section.

5. Discussion and Conclusions

The aim of this paper was to propose novel approaches for constructing confidence interval for the common inverse mean of normal distributions with unknown CVs. The proposed confidence intervals were constructed based on the GCI, large sample, and adjusted MOVER approaches, compared with the existing approach, using the GCI approach of Thangjai et al. (2017a) to estimate the confidence interval for the common inverse mean. The GCI approach provided more stable coverage probability and it was recommended to construct confidence interval for the common inverse mean with unknown CVs when the sample size was large. The large sample approach was not recommended to estimate the confidence interval. The adjusted MOVER approach performed satisfactorily in terms of the coverage probability and average length of the confidence interval when the sample size was large. However, the adjusted MOVER approach was easy to use more than the GCI approach. This is because the adjusted MOVER approach was computed by the simple formula, whereas the GCI approach was based on a computational approach. Therefore, the adjusted MOVER approach can be an alternative to construct confidence interval for the common inverse mean with unknown CVs when the sample size was large.

Table 1 The coverage probabilities (CP) and average lengths (AL) of 95% two-sided confidence intervals for the common inverse mean of normal distributions with unknown CVs: 2 sample cases

n	σ_1	σ_2	CI_{GCI}		CI_{LS}		CI_{AM}		CI_{TH}	
			CP	**AL**	**CP**	**AL**	**CP**	**AL**	**CP**	**AL**
20	0.10	0.01	0.9454	0.0092	0.9296	0.0086	0.9456	0.0092	**0.9518**	0.0094
		0.03	**0.9584**	0.0269	0.9374	0.0247	**0.9540**	0.0264	**0.9622**	0.0276
		0.05	**0.9520**	0.0423	0.9288	0.0383	0.9440	0.0409	**0.9572**	0.0433
		0.07	0.9496	0.0543	0.9260	0.0488	0.9410	0.0521	**0.9556**	0.0556
		0.09	**0.9518**	0.0633	0.9240	0.0568	0.9414	0.0607	**0.9584**	0.0648
		0.10	**0.9530**	0.0671	0.9242	0.0602	0.9406	0.0643	**0.9570**	0.0687
		0.30	**0.9502**	0.0902	0.9296	0.0815	0.9460	0.0872	**0.9538**	0.0922
		0.50	**0.9518**	0.0932	0.9320	0.0846	0.9472	0.0905	**0.9544**	0.0955
		0.70	**0.9552**	0.0944	0.9356	0.0856	**0.9524**	0.0916	**0.9588**	0.0970
30	0.10	0.01	0.9436	0.0074	0.9320	0.0070	0.9450	0.0074	**0.9510**	0.0075
		0.03	0.9488	0.0215	0.9358	0.0204	0.9458	0.0212	**0.9514**	0.0219
		0.05	**0.9526**	0.0336	0.9368	0.0315	0.9476	0.0328	**0.9576**	0.0341
		0.07	0.9456	0.0431	0.9320	0.0402	0.9410	0.0419	**0.9508**	0.0438
		0.09	0.9498	0.0504	0.9346	0.0469	0.9444	0.0490	**0.9540**	0.0512
		0.10	**0.9536**	0.0532	0.9336	0.0495	0.9472	0.0517	**0.9570**	0.0541
		0.30	0.9460	0.0713	0.9326	0.0669	0.9430	0.0699	0.9498	0.0724
		0.50	**0.9510**	0.0739	0.9396	0.0695	0.9496	0.0726	**0.9556**	0.0751
		0.70	**0.9538**	0.0746	0.9402	0.0704	**0.9502**	0.0735	**0.9566**	0.0759
50	0.10	0.01	0.9430	0.0056	0.9372	0.0055	0.9432	0.0056	**0.9502**	0.0057
		0.03	0.9452	0.0163	0.9392	0.0158	0.9460	0.0162	**0.9502**	0.0165
		0.05	**0.9538**	0.0255	0.9438	0.0245	**0.9502**	0.0252	**0.9554**	0.0258
		0.07	**0.9512**	0.0328	0.9422	0.0314	0.9470	0.0322	**0.9512**	0.0330
		0.09	**0.9542**	0.0384	0.9464	0.0367	**0.9520**	0.0377	**0.9566**	0.0387
		0.10	**0.9526**	0.0405	0.9424	0.0387	0.9480	0.0397	**0.9552**	0.0408
		0.30	**0.9510**	0.0542	0.9446	0.0522	0.9484	0.0536	**0.9528**	0.0547
		0.50	0.9472	0.0559	0.9406	0.0541	0.9460	0.0555	0.9494	0.0565
		0.70	0.9496	0.0564	0.9406	0.0545	0.9484	0.0559	0.9498	0.0569
100	0.10	0.01	**0.9504**	0.0039	0.9484	0.0039	**0.9512**	0.0039	**0.9560**	0.0040
		0.03	0.9430	0.0114	0.9404	0.0112	0.9418	0.0114	0.9446	0.0115
		0.05	**0.9526**	0.0178	0.9482	0.0174	**0.9510**	0.0177	**0.9542**	0.0179
		0.07	**0.9540**	0.0228	0.9464	0.0223	**0.9510**	0.0226	**0.9544**	0.0229
		0.09	**0.9528**	0.0266	0.9480	0.0260	**0.9524**	0.0264	**0.9550**	0.0268
		0.10	**0.9518**	0.0282	0.9468	0.0275	**0.9516**	0.0279	**0.9556**	0.0283
		0.30	0.9496	0.0378	0.9456	0.0371	0.9476	0.0376	**0.9506**	0.0380
		0.50	**0.9500**	0.0390	0.9468	0.0383	**0.9502**	0.0388	**0.9506**	0.0392
		0.70	0.9452	0.0394	0.9434	0.0388	0.9458	0.0393	0.9462	0.0396
200	0.10	0.01	**0.9500**	0.0028	0.9486	0.0028	0.9494	0.0028	**0.9514**	0.0028
		0.03	0.9476	0.0080	0.9456	0.0080	0.9468	0.0080	0.9490	0.0080
		0.05	0.9444	0.0125	0.9422	0.0124	0.9442	0.0124	0.9452	0.0125
		0.07	0.9498	0.0160	0.9480	0.0158	0.9496	0.0159	0.9498	0.0160
		0.09	0.9464	0.0187	0.9438	0.0185	0.9456	0.0186	0.9498	0.0187
		0.10	**0.9520**	0.0198	**0.9528**	0.0195	**0.9536**	0.0197	**0.9540**	0.0198
		0.30	0.9484	0.0265	0.9480	0.0262	0.9484	0.0264	**0.9500**	0.0266
		0.50	0.9494	0.0274	0.9468	0.0271	0.9492	0.0273	**0.9510**	0.0274
		0.70	**0.9550**	0.0276	**0.9554**	0.0274	**0.9566**	0.0276	**0.9572**	0.0277

Table 2 The CP and AL of 95% two-sided confidence intervals for the common inverse mean of normal distributions with unknown CVs: 4 sample cases

n	σ_1	σ_3	CI_{GCI}		CI_{LS}		CI_{AM}		CI_{TH}	
			CP	**AL**	**CP**	**AL**	**CP**	**AL**	**CP**	**AL**
20	0.10	0.01	0.9412	0.0066	0.9080	0.0060	0.9282	0.0064	**0.9538**	0.0068
		0.03	0.9450	0.0193	0.9180	0.0171	0.9348	0.0183	**0.9522**	0.0197
		0.05	**0.9584**	0.0302	0.9250	0.0266	0.9460	0.0284	**0.9636**	0.0309
		0.07	**0.9536**	0.0389	0.9258	0.0342	0.9416	0.0365	**0.9602**	0.0398
		0.09	**0.9554**	0.0453	0.9212	0.0396	0.9436	0.0424	**0.9600**	0.0463
		0.10	**0.9580**	0.0480	0.9258	0.0420	0.9442	0.0449	**0.9616**	0.0491
		0.30	0.9488	0.0644	0.9232	0.0566	0.9408	0.0606	**0.9522**	0.0659
		0.50	0.9490	0.0667	0.9216	0.0586	0.9392	0.0627	**0.9508**	0.0682
		0.70	0.9494	0.0678	0.9254	0.0596	0.9428	0.0637	**0.9530**	0.0695
30	0.10	0.01	0.9434	0.0053	0.9262	0.0049	0.9384	0.0051	**0.9554**	0.0054
		0.03	0.9498	0.0153	0.9314	0.0142	0.9442	0.0148	**0.9552**	0.0156
		0.05	**0.9528**	0.0240	0.9340	0.0220	0.9452	0.0230	**0.9568**	0.0243
		0.07	**0.9566**	0.0308	0.9330	0.0283	0.9464	0.0295	**0.9608**	0.0313
		0.09	**0.9524**	0.0360	0.9328	0.0329	0.9436	0.0344	**0.9546**	0.0365
		0.10	**0.9578**	0.0380	0.9390	0.0347	**0.9504**	0.0363	**0.9638**	0.0386
		0.30	**0.9558**	0.0511	0.9374	0.0469	0.9468	0.0490	**0.9580**	0.0518
		0.50	**0.9500**	0.0527	0.9338	0.0485	0.9462	0.0506	**0.9520**	0.0535
		0.70	0.9482	0.0533	0.9308	0.0490	0.9410	0.0512	**0.9508**	0.0541
50	0.10	0.01	0.9472	0.0040	0.9360	0.0038	0.9428	0.0039	**0.9530**	0.0041
		0.03	**0.9534**	0.0116	0.9440	0.0111	0.9488	0.0114	**0.9576**	0.0117
		0.05	**0.9572**	0.0181	0.9474	0.0173	**0.9540**	0.0177	**0.9586**	0.0183
		0.07	0.9460	0.0233	0.9376	0.0221	0.9436	0.0227	**0.9502**	0.0235
		0.09	**0.9536**	0.0272	0.9398	0.0258	0.9486	0.0265	**0.9552**	0.0275
		0.10	**0.9532**	0.0287	0.9438	0.0272	**0.9510**	0.0279	**0.9550**	0.0290
		0.30	**0.9526**	0.0386	0.9444	0.0367	0.9498	0.0376	**0.9560**	0.0390
		0.50	0.9488	0.0399	0.9388	0.0379	0.9454	0.0389	0.9494	0.0402
		0.70	**0.9512**	0.0402	0.9422	0.0383	0.9488	0.0392	**0.9534**	0.0406
100	0.10	0.01	0.9458	0.0028	0.9406	0.0027	0.9448	0.0028	**0.9504**	0.0028
		0.03	**0.9514**	0.0081	0.9470	0.0079	0.9498	0.0080	**0.9522**	0.0081
		0.05	**0.9546**	0.0126	0.9476	0.0123	**0.9520**	0.0125	**0.9558**	0.0127
		0.07	**0.9592**	0.0162	**0.9540**	0.0158	**0.9568**	0.0160	**0.9606**	0.0163
		0.09	**0.9522**	0.0189	0.9470	0.0184	**0.9506**	0.0186	**0.9550**	0.0190
		0.10	**0.9528**	0.0200	0.9468	0.0194	**0.9504**	0.0197	**0.9532**	0.0201
		0.30	0.9494	0.0268	0.9460	0.0261	0.9498	0.0264	**0.9508**	0.0269
		0.50	0.9476	0.0277	0.9434	0.0270	0.9466	0.0273	**0.9502**	0.0278
		0.70	0.9452	0.0279	0.9412	0.0273	0.9436	0.0276	0.9480	0.0280
200	0.10	0.01	0.9460	0.0020	0.9432	0.0019	0.9450	0.0020	0.9466	0.0020
		0.03	0.9448	0.0057	0.9414	0.0056	0.9440	0.0056	0.9460	0.0057
		0.05	**0.9508**	0.0088	0.9468	0.0087	0.9484	0.0088	**0.9510**	0.0089
		0.07	**0.9518**	0.0114	0.9488	0.0112	**0.9508**	0.0113	**0.9542**	0.0114
		0.09	**0.9522**	0.0132	0.9498	0.0131	**0.9514**	0.0131	**0.9514**	0.0133
		0.10	**0.9500**	0.0140	0.9476	0.0138	**0.9500**	0.0139	**0.9506**	0.0140
		0.30	**0.9532**	0.0188	0.9498	0.0185	**0.9514**	0.0186	**0.9550**	0.0188
		0.50	**0.9524**	0.0194	**0.9518**	0.0192	**0.9532**	0.0193	**0.9532**	0.0194
		0.70	**0.9504**	0.0196	0.9482	0.0193	**0.9508**	0.0195	**0.9518**	0.0196

Table 3 The CP and AL of 95% two-sided confidence intervals for the common inverse mean of normal distributions with unknown CVs: 6 sample cases

n	σ_1	σ_4	CI_{GCI}		CI_{LS}		CI_{AM}		CI_{TH}	
			CP	**AL**	**CP**	**AL**	**CP**	**AL**	**CP**	**AL**
20	0.10	0.01	0.9388	0.0055	0.9048	0.0048	0.9242	0.0052	**0.9596**	0.0056
		0.03	**0.9522**	0.0158	0.9214	0.0139	0.9404	0.0148	**0.9584**	0.0162
		0.05	**0.9570**	0.0248	0.9236	0.0216	0.9432	0.0231	**0.9622**	0.0254
		0.07	**0.9530**	0.0318	0.9224	0.0276	0.9388	0.0295	**0.9600**	0.0325
		0.09	**0.9536**	0.0373	0.9226	0.0323	0.9448	0.0346	**0.9576**	0.0381
		0.10	**0.9564**	0.0394	0.9240	0.0341	0.9434	0.0365	**0.9606**	0.0402
		0.30	0.9490	0.0531	0.9158	0.0460	0.9392	0.0492	**0.9524**	0.0542
		0.50	0.9464	0.0547	0.9174	0.0475	0.9364	0.0508	0.9484	0.0560
		0.70	0.9498	0.0554	0.9188	0.0480	0.9416	0.0513	**0.9514**	0.0567
30	0.10	0.01	0.9424	0.0043	0.9234	0.0040	0.9348	0.0042	**0.9598**	0.0044
		0.03	**0.9558**	0.0126	0.9326	0.0116	0.9458	0.0121	**0.9634**	0.0128
		0.05	**0.9516**	0.0196	0.9306	0.0179	0.9424	0.0187	**0.9542**	0.0199
		0.07	**0.9504**	0.0252	0.9312	0.0230	0.9420	0.0240	**0.9558**	0.0256
		0.09	**0.9510**	0.0295	0.9254	0.0268	0.9406	0.0280	**0.9544**	0.0299
		0.10	**0.9570**	0.0312	0.9372	0.0284	**0.9500**	0.0296	**0.9588**	0.0317
		0.30	0.9480	0.0419	0.9322	0.0381	0.9434	0.0399	**0.9502**	0.0425
		0.50	0.9440	0.0432	0.9290	0.0394	0.9402	0.0412	0.9464	0.0439
		0.70	0.9454	0.0437	0.9292	0.0398	0.9408	0.0416	0.9472	0.0444
50	0.10	0.01	0.9492	0.0033	0.9376	0.0031	0.9434	0.0032	**0.9598**	0.0033
		0.03	**0.9544**	0.0095	0.9452	0.0091	**0.9516**	0.0093	**0.9588**	0.0096
		0.05	**0.9504**	0.0149	0.9398	0.0141	0.9450	0.0144	**0.9538**	0.0150
		0.07	0.9466	0.0190	0.9352	0.0180	0.9416	0.0185	**0.9502**	0.0192
		0.09	0.9492	0.0222	0.9366	0.0210	0.9446	0.0215	**0.9516**	0.0224
		0.10	**0.9534**	0.0235	0.9408	0.0222	0.9474	0.0228	**0.9532**	0.0237
		0.30	**0.9504**	0.0316	0.9414	0.0299	0.9498	0.0307	**0.9516**	0.0319
		0.50	0.9494	0.0326	0.9424	0.0309	0.9492	0.0317	**0.9520**	0.0329
		0.70	**0.9504**	0.0329	0.9412	0.0312	0.9474	0.0320	**0.9520**	0.0332
100	0.10	0.01	0.9462	0.0023	0.9416	0.0022	0.9446	0.0023	**0.9508**	0.0023
		0.03	**0.9522**	0.0066	0.9464	0.0065	0.9486	0.0065	**0.9520**	0.0067
		0.05	**0.9508**	0.0103	0.9450	0.0100	0.9486	0.0102	**0.9524**	0.0104
		0.07	0.9498	0.0132	0.9450	0.0129	0.9476	0.0130	**0.9520**	0.0133
		0.09	**0.9518**	0.0154	0.9468	0.0150	0.9494	0.0152	**0.9522**	0.0155
		0.10	**0.9504**	0.0163	0.9450	0.0159	0.9480	0.0161	0.9496	0.0164
		0.30	**0.9512**	0.0219	0.9428	0.0213	0.9470	0.0216	**0.9510**	0.0220
		0.50	**0.9532**	0.0226	0.9480	0.0220	**0.9506**	0.0223	**0.9528**	0.0227
		0.70	**0.9506**	0.0228	0.9462	0.0222	0.9486	0.0225	**0.9516**	0.0229
200	0.10	0.01	**0.9508**	0.0016	0.9472	0.0016	0.9494	0.0016	**0.9500**	0.0016
		0.03	**0.9512**	0.0046	0.9498	0.0046	**0.9508**	0.0046	**0.9524**	0.0047
		0.05	**0.9514**	0.0072	0.9476	0.0071	0.9498	0.0072	**0.9506**	0.0072
		0.07	**0.9514**	0.0093	0.9494	0.0091	**0.9506**	0.0092	**0.9534**	0.0093
		0.09	**0.9510**	0.0108	0.9488	0.0107	0.9498	0.0107	**0.9510**	0.0108
		0.10	0.9466	0.0114	0.9430	0.0113	0.9448	0.0113	0.9448	0.0115
		0.30	**0.9508**	0.0153	0.9484	0.0151	**0.9502**	0.0152	**0.9504**	0.0154
		0.50	0.9486	0.0158	0.9472	0.0156	0.9486	0.0157	**0.9502**	0.0159
		0.70	**0.9518**	0.0160	**0.9516**	0.0158	**0.9524**	0.0159	**0.9530**	0.0160

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