SHORT COMMUNICATION

Some Examples of a Hypergroup $(\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$

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Abstract. Let G be a group and N a normal subgroup of G. If the hyperoperation $\circ_N$ is defined by $x \circ_N y = (xy)N$ for all $x, y \in G$, then $(G, \circ_N)$ is a hypergroup. Since $m\mathbb{Z}_n$ is normal subgroup of $\mathbb{Z}_n$, $(\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$ is hypergroup. In this paper, we let $G|_N H = \{g \circ_N H | g \in G\}$ and give some example that $G|_{m\mathbb{Z}_n} k\mathbb{Z}_n$ equals to $\mathbb{Z}_n / k\mathbb{Z}_n$. We take a hyperoperation $\circ_N$ to construct cosets of any subgroup H of G instead of coset multiplication by the binary operation of G and studies some examples of this new structure of cosets.

Keywords: Hypergroup, Coset, Normal Subgroup

1. Introduction

A hypergroup is a branch of algebraic structure that extends the concept of a group into a more general form. In this paper, the hypergroup we are interested in studying will be defined from the group G and the normal subgroup N of G, that is, if G is a group and N is a normal subgroup of G, then $(G, \circ_N)$ is a hypergroup where the hyperoperation $\circ_N$ is defined by $x \circ_N y = (xy)N$ for all $x, y \in G$.

For any group G and N is a subgroup of G for each $x \in G$, $xN = \{xn \mid n \in N\}$ is called a *left coset* of N at x in G, and $Nx = \{nx \mid n \in N\}$ is called a *right coset* of N at x in G .Then N is a normal subgroup if $xN = Nx$ for all $x \in G$. A *hyperoperation* on a nonempty set H is a function $\circ : H \times H \to P(H) \setminus \{\varnothing\}$ where $P(H)$ is the power set of H. The value of $(x, y) \in H \times H$ under the function $\circ$ denoted by $x \circ y$, is called the *hyperproduct* of x and y. A system $(H, \circ)$ is called a *hypergroupoid*, that is, $(H, \circ)$ has the *closure property*. For $A, B \subseteq H$ and $x \in H$ we define

$$A \circ B = \bigcup_{\substack{a \in A \\ b \in B}} (a \circ b), A \circ x = \bigcup_{a \in A} a \circ x \quad \text{and} \quad x \circ A = \bigcup_{a \in A} x \circ a.$$

For a semihypergroup $(H, \circ)$ is called a *hypergroup* if and only if $x \circ H = H \circ x = H$ for all $x \in H$, that is, the system $(H, \circ)$ has the reproductive law.

Coset of a hypergroup $(\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$

Let G be a group and N is a normal subgroup of G, we define $\circ_N$ by

$$x \circ_N y = (xy)N \text{ for all } x, y \in G.$$

Then we have $(G, \circ_N)$ is a hypergroup. Moreover, if H is a subgroup of G such that $N \subseteq H$ and $g \in G$, then

$$g \circ_N H = \bigcup_{h \in H} g \circ_N h$$

is called a left coset of H in $(G, \circ_N)$. We will define the set of all left cosets of H in $(G, \circ_N)$ by $G|_N H$, that is,

$$G|_N H = \{g \circ_N H \mid g \in G\}.$$

In this section, we will consider some examples of the sets of all left cosets $k\mathbb{Z}_n$ in $(\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$,where $G=\mathbb{Z}_n$, $N=m\mathbb{Z}_n$ and $H=k\mathbb{Z}_n$.
Case: $n=p$ for some prime number p.

Example 1. $G=\mathbb{Z}_2$, $N=0\mathbb{Z}_2$, $H=0\mathbb{Z}_2$. We have

$$
\begin{aligned}
G|_N H &= \{\bar{0}\circ_N H, \bar{1}\circ_N H\} \\
&= \left\{ \bigcup_{h\in 0\mathbb{Z}_2} \bar{0}\circ_N h, \bigcup_{h\in 0\mathbb{Z}_2} \bar{1}\circ_N h \right\} \\
&= \{\bar{0}\circ_{0\mathbb{Z}_2} \bar{0}, \bar{1}\circ_{0\mathbb{Z}_2} \bar{0}\} \\
&= \{\bar{0}+\bar{0}+0\mathbb{Z}_2, \bar{1}+\bar{0}+0\mathbb{Z}_2\} \\
&= \{\bar{0}+0\mathbb{Z}_2, \bar{1}+0\mathbb{Z}_2\} \\
&= \{\{\bar{0}\},\{\bar{1}\}\} \\
&= \mathbb{Z}_2/0\mathbb{Z}_2 \\
&= G/N \\
&= G/H
\end{aligned}
$$

Example 2. $G=\mathbb{Z}_2, N=0\mathbb{Z}_2, H=1\mathbb{Z}_2$. We have

$$
\begin{aligned}
G|_N H &= \{\bar{0}\circ_N H, \bar{1}\circ_N H\} \\
&= \left\{ \bigcup_{h\in 1\mathbb{Z}_2} \bar{0}\circ_N h, \bigcup_{h\in 1\mathbb{Z}_2} \bar{1}\circ_N h \right\} \\
&= \{(\bar{0}\circ_{0\mathbb{Z}_2} \bar{0})\cup(\bar{0}\circ_{0\mathbb{Z}_2} \bar{1}), \\
&\quad (\bar{1}\circ_{0\mathbb{Z}_2} \bar{0})\cup(\bar{1}\circ_{0\mathbb{Z}_2} \bar{1})\} \\
&= \{(\bar{0}+\bar{0}+0\mathbb{Z}_2)\cup(\bar{0}+\bar{1}+0\mathbb{Z}_2), \\
&\quad (\bar{1}+\bar{0}+0\mathbb{Z}_2)\cup(\bar{1}+\bar{1}+0\mathbb{Z}_2)\} \\
&= \{(\bar{0}+0\mathbb{Z}_2)\cup(\bar{1}+0\mathbb{Z}_2), \\
&\quad (\bar{1}+0\mathbb{Z}_2)\cup(\bar{0}+0\mathbb{Z}_2)\} \\
&= \{\{\bar{0}\}\cup\{\bar{1}\}, \{\bar{1}\}\cup\{\bar{0}\}\} \\
&= \{\mathbb{Z}_2\} \\
&= \{\bar{0}+1\mathbb{Z}_2\} \\
&= \mathbb{Z}_2/1\mathbb{Z}_2 \\
&= G/H.
\end{aligned}
$$

Example 3. $G=\mathbb{Z}_2$, $N=1\mathbb{Z}_2$, $H=1\mathbb{Z}_2$. We have

$$
\begin{aligned}
G|_N H &= \{\bar{0}\circ_N H, \bar{1}\circ_N H\} \\
&= \left\{ \bigcup_{h\in 1\mathbb{Z}_2} \bar{0}\circ_N h, \bigcup_{h\in 1\mathbb{Z}_2} \bar{1}\circ_N h \right\} \\
&= \{(\bar{0}+1\mathbb{Z}_2)\cup(\bar{1}+1\mathbb{Z}_2), \\
&\quad (\bar{1}+1\mathbb{Z}_2)\cup(\bar{0}+1\mathbb{Z}_2)\} \\
&= \{\mathbb{Z}_2, \mathbb{Z}_2\} \\
&= \{\mathbb{Z}_2\} \\
&= \{\bar{0}+1\mathbb{Z}_2\} \\
&= \mathbb{Z}_2/1\mathbb{Z}_2 \\
&= G/N \\
&= G/H
\end{aligned}
$$

Example 4. $G=\mathbb{Z}_3$, $N=0\mathbb{Z}_3$, $H=0\mathbb{Z}_3$. We have

$$
\begin{aligned}
G|_N H &= \{\bar{0}\circ_N H, \bar{1}\circ_N H, \bar{2}\circ_N H\} \\
&= \left\{ \bigcup_{h\in 0\mathbb{Z}_3} \bar{0}\circ_N h, \bigcup_{h\in 0\mathbb{Z}_3} \bar{1}\circ_N h, \bigcup_{h\in 0\mathbb{Z}_3} \bar{2}\circ_N h \right\} \\
&= \{\bar{0}+0\mathbb{Z}_3, \bar{1}+0\mathbb{Z}_3, \bar{2}+0\mathbb{Z}_3\} \\
&= \{\{\bar{0}\},\{\bar{1}\},\{\bar{2}\}\} \\
&= \mathbb{Z}_3/0\mathbb{Z}_3 \\
&= G/N \\
&= G/H
\end{aligned}
$$

Example 5. $G=\mathbb{Z}_3, N=0\mathbb{Z}_3, H=2\mathbb{Z}_3$. We have

$$
\begin{aligned}
\mathrm{w}G|_N H &= \{\bar{0}\circ_N H, \bar{1}\circ_N H, \bar{2}\circ_N H\} \\
&= \{(\bar{0}+0\mathbb{Z}_3)\cup(\bar{1}+0\mathbb{Z}_3)\cup(\bar{2}+0\mathbb{Z}_3), \\
&\quad (\bar{1}+0\mathbb{Z}_3)\cup(\bar{2}+0\mathbb{Z}_3)\cup(\bar{0}+0\mathbb{Z}_3), \\
&\quad (\bar{2}+0\mathbb{Z}_3)\cup(\bar{0}+0\mathbb{Z}_3)\cup(\bar{1}+0\mathbb{Z}_3)\} \\
&= \{\{\bar{0}\}\cup\{\bar{1}\}\cup\{\bar{2}\}, \{\bar{1}\}\cup\{\bar{2}\}\cup\{\bar{0}\}, \{\bar{2}\}\cup\{\bar{0}\}\cup\{\bar{1}\}\} \\
&= \{\mathbb{Z}_3\} \\
&= \{\bar{0}+2\mathbb{Z}_3\} \\
&= \mathbb{Z}_3/2\mathbb{Z}_3 \\
&= G/H
\end{aligned}
$$

Example 6. $G=\mathbb{Z}_3, N=1\mathbb{Z}_3, H=2\mathbb{Z}_3$. We have

$$\begin{aligned} G|_N\ H &= \{\bar{0}\circ_N H, \bar{1}\circ_N H, \bar{2}\circ_N H\} \\ &= \Big\{\bigcup_{h\in 2\mathbb{Z}_3} \bar{0}\circ_N h\Big\} \cup \Big\{\bigcup_{h\in 2\mathbb{Z}_3} \bar{1}\circ_N h\Big\} \\ &\quad \cup \Big\{\bigcup_{h\in 2\mathbb{Z}_3} \bar{2}\circ_N h\Big\} \\ &= \{(\bar{0}+1\mathbb{Z}_3)\cup(\bar{1}+1\mathbb{Z}_3)\cup(\bar{2}+1\mathbb{Z}_3), \\ &\quad (\bar{1}+1\mathbb{Z}_3)\cup(\bar{2}+1\mathbb{Z}_3)\cup(\bar{0}+1\mathbb{Z}_3), \\ &\quad (\bar{2}+1\mathbb{Z}_3)\cup(\bar{0}+1\mathbb{Z}_3)\cup(\bar{1}+1\mathbb{Z}_3)\} \\ &= \{\mathbb{Z}_3\} \\ &= \{\bar{0}+1\mathbb{Z}_3\} \\ &= \mathbb{Z}_3/1\mathbb{Z}_3 \\ &= G/N \\ &= G/H \end{aligned}$$

Case: $n=pq$ where p and q are distinct primes.

Example 7. $G=\mathbb{Z}_6, N=0\mathbb{Z}_6, H=0\mathbb{Z}_6$. We have

$$\begin{aligned} G|_N\ H &= \{\bar{0}\circ_N H, \bar{1}\circ_N H, \ldots, \bar{5}\circ_N H\} \\ &= \Big\{\bigcup_{h\in 0\mathbb{Z}_6} \bar{0}\circ_N h, \bigcup_{h\in 0\mathbb{Z}_6} \bar{1}\circ_N h \\ &\quad ,\ldots, \bigcup_{h\in 0\mathbb{Z}_6} \bar{5}\circ_N h\Big\} \\ &= \{\bar{0}+0\mathbb{Z}_6, \bar{1}+0\mathbb{Z}_6, \ldots, \bar{5}+0\mathbb{Z}_6\} \\ &= \{\{\bar{0}\},\{\bar{1}\},\ldots,\{\bar{5}\}\} \\ &= \mathbb{Z}_6/0\mathbb{Z}_6 \\ &= G/N \\ &= G/H \end{aligned}$$

Example 8.
$G=\mathbb{Z}_6, N=0\mathbb{Z}_6, H=3\mathbb{Z}_6=\{\bar{0},\bar{3}\}$. We have

$$\begin{aligned} G|_N\ H &= \{\bar{0}\circ_N H, \bar{1}\circ_N H, \ldots, \bar{5}\circ_N H\} \\ &= \Big\{\bigcup_{h\in 3\mathbb{Z}_6} \bar{0}\circ_N h, \bigcup_{h\in 3\mathbb{Z}_6} \bar{1}\circ_N h \\ &\quad ,\ldots, \bigcup_{h\in 3\mathbb{Z}_6} \bar{5}\circ_N h\Big\} \\ &= \Big\{(\bar{0}+0\mathbb{Z}_6)\cup(\bar{3}+0\mathbb{Z}_6), \\ &\quad (\bar{1}+0\mathbb{Z}_6)\cup(\bar{4}+0\mathbb{Z}_6), \\ &\quad \vdots \\ &\quad (\bar{5}+0\mathbb{Z}_6)\cup(\bar{2}+0\mathbb{Z}_6)\Big\} \\ &= \{\{\bar{0},\bar{3}\},\{\bar{1},\bar{4}\},\{\bar{5},\bar{2}\}\} \\ &= \{\bar{0}+3\mathbb{Z}_6, \bar{1}+3\mathbb{Z}_6, \bar{2}+3\mathbb{Z}_6\} \\ &= \mathbb{Z}_6/3\mathbb{Z}_6 \\ &= G/H \end{aligned}$$

Example 9. $G=\mathbb{Z}_6, N=0\mathbb{Z}_6, H=5\mathbb{Z}_6(=\mathbb{Z}_6)$. We have

$$\begin{aligned} G|_N\ H &= \{\bar{0}\circ_N H, \bar{1}\circ_N H, \ldots, \bar{5}\circ_N H\} \\ &= \Big\{\bigcup_{h\in 5\mathbb{Z}_6} \bar{0}\circ_N h, \bigcup_{h\in 5\mathbb{Z}_6} \bar{1}\circ_N h \\ &\quad ,\ldots, \bigcup_{h\in 5\mathbb{Z}_6} \bar{5}\circ_N h\Big\} \\ &= \Big\{(\bar{0}+0\mathbb{Z}_6)\cup(\bar{1}+0\mathbb{Z}_6) \\ &\quad \cup\ldots\cup(\bar{5}+0\mathbb{Z}_6), \\ &\quad (\bar{1}+0\mathbb{Z}_6)\cup(\bar{2}+0\mathbb{Z}_6) \\ &\quad \cup\ldots\cup(\bar{0}+0\mathbb{Z}_6), \\ &\quad \vdots \\ &\quad (\bar{5}+0\mathbb{Z}_6)\cup(\bar{0}+0\mathbb{Z}_6) \\ &\quad \cup\ldots\cup(\bar{4}+0\mathbb{Z}_6)\Big\} \\ &= \{\{\bar{0}\}\cup\{\bar{1}\}\cup\{\bar{2}\}\ldots\cup\{\bar{5}\}\} \\ &= \{\mathbb{Z}_6\} \\ &= \{\bar{0}+5\mathbb{Z}_6\} \\ &= \mathbb{Z}_6/5\mathbb{Z}_6 \\ &= G/H \end{aligned}$$

Example 10.
$G=\mathbb{Z}_6, N=2\mathbb{Z}_6, H=2\mathbb{Z}_6=\{\bar{0},\bar{2},\bar{4}\}$. We have

$$\begin{aligned} G|_N\ H &= \{\bar{0}\circ_N H, \bar{1}\circ_N H, \ldots, \bar{5}\circ_N H\} \\ &= \Big\{\bigcup_{h\in 2\mathbb{Z}_6} \bar{0}\circ_N h, \bigcup_{h\in 2\mathbb{Z}_6} \bar{1}\circ_N h \\ &\quad ,\ldots, \bigcup_{h\in 2\mathbb{Z}_6} \bar{5}\circ_N h\Big\} \\ &= \Big\{(\bar{0}+2\mathbb{Z}_6)\cup(\bar{2}+2\mathbb{Z}_6)\cup(\bar{4}+2\mathbb{Z}_6), \\ &\quad (\bar{1}+2\mathbb{Z}_6)\cup(\bar{3}+2\mathbb{Z}_6)\cup(\bar{5}+2\mathbb{Z}_6), \\ &\quad \vdots \end{aligned}$$

$$
\begin{aligned}
&(\overline{5}+2\mathbb{Z}_6)\cup(\overline{1}+2\mathbb{Z}_6)\\
&\cup(\overline{3}+2\mathbb{Z}_6)\big\}\\
&=\{\{\overline{0},\overline{2},\overline{4}\},\{\overline{1},\overline{3},\overline{5}\}\}\\
&=\{\overline{0}+2\mathbb{Z}_6,\overline{1}+2\mathbb{Z}_6\}\\
&=\mathbb{Z}_6/2\mathbb{Z}_6\\
&=G/N\\
&=G/H
\end{aligned}
$$

Example 11. $G=\mathbb{Z}_6, N=2\mathbb{Z}_6, H=5\mathbb{Z}_6(=\mathbb{Z}_6)$. We have

$$
\begin{aligned}
G|_N\ H &= \{\overline{0}\circ_N H,\overline{1}\circ_N H,\ldots,\overline{5}\circ_N H\}\\
&=\Big\{\bigcup_{h\in 5\mathbb{Z}_6}\overline{0}\circ_N h,\ \bigcup_{h\in 5\mathbb{Z}_6}\overline{1}\circ_N h\\
&\quad,\ldots,\bigcup_{h\in 5\mathbb{Z}_6}\overline{5}\circ_N h\Big\}\\
&=\big\{(\overline{0}+2\mathbb{Z}_6)\cup(\overline{1}+2\mathbb{Z}_6)\\
&\quad\cup\ldots\cup(\overline{5}+2\mathbb{Z}_6),\\
&\quad(\overline{1}+2\mathbb{Z}_6)\cup(\overline{2}+2\mathbb{Z}_6)\\
&\quad\cup\ldots\cup(\overline{0}+2\mathbb{Z}_6),\\
&\quad\vdots\\
&\quad(\overline{5}+2\mathbb{Z}_6)\cup(\overline{0}+2\mathbb{Z}_6)\\
&\quad\cup\ldots\cup(\overline{4}+2\mathbb{Z}_6)\big\}\\
&=\{(\overline{0}+2\mathbb{Z}_6)\cup(\overline{1}+2\mathbb{Z}_6)\}\\
&=\{\{\overline{0},\overline{2},\overline{4}\}\cup\{\overline{1},\overline{3},\overline{5}\}\}\\
&=\{\mathbb{Z}_6\}\\
&=\{\overline{0}+5\mathbb{Z}_6\}\\
&=\mathbb{Z}_6/5\mathbb{Z}_6\\
&=G/H
\end{aligned}
$$

$Case: n=p^k$ for some prime number p and $k\geq 2$

Example 12. $G=\mathbb{Z}_8, N=4\mathbb{Z}_8, H=2\mathbb{Z}_8$. We have

$$
\begin{aligned}
G|_N\ H &= \{\overline{0}\circ_N H,\overline{1}\circ_N H,\ldots,\overline{7}\circ_N H\}\\
&=\{\bigcup_{h\in 2\mathbb{Z}_8}\overline{0}\circ_N h,\ \bigcup_{h\in 2\mathbb{Z}_8}\overline{1}\circ_N h,\\
&\quad\cdots,\bigcup_{h\in 2\mathbb{Z}_8}\overline{7}\circ_N h\}\\
&=\{(\overline{0}+4\mathbb{Z}_8)\cup(\overline{2}+4\mathbb{Z}_8)\\
&\quad\cup(\overline{4}+4\mathbb{Z}_8)\cup(\overline{6}+4\mathbb{Z}_8),\\
&\quad(\overline{1}+4\mathbb{Z}_8)\cup(\overline{3}+4\mathbb{Z}_8)\\
&\quad\cup(\overline{5}+4\mathbb{Z}_8)\cup(\overline{7}+4\mathbb{Z}_8),\\
&\quad\vdots\\
&\quad(\overline{7}+4\mathbb{Z}_8)\cup(\overline{1}+4\mathbb{Z}_8)\\
&\quad\cup(\overline{3}+4\mathbb{Z}_8)\cup(\overline{5}+4\mathbb{Z}_8)\}\\
&=\{(\overline{0}+4\mathbb{Z}_8)\cup(\overline{2}+4\mathbb{Z}_8),\\
&\quad(\overline{1}+4\mathbb{Z}_8)\cup(\overline{3}+4\mathbb{Z}_8)\}\\
&=\{\{\overline{0},\overline{4}\}\cup\{\overline{2},\overline{6}\},\\
&\quad\{\overline{1},\overline{5}\}\cup\{\overline{3},\overline{7}\}\}\\
&=\{\{\overline{0},\overline{2},\overline{4},\overline{6}\},\{\overline{1},\overline{3},\overline{5},\overline{7}\}\}\\
&=\{\overline{0}+2\mathbb{Z}_8,\overline{1}+2\mathbb{Z}_8\}\\
&=\mathbb{Z}_8/2\mathbb{Z}_8\\
&=G/H
\end{aligned}
$$

\

Example 13. $G=\mathbb{Z}_{16}, N=8\mathbb{Z}_{16}, H=2\mathbb{Z}_{16}$. We have

$$
\begin{aligned}
G|_N\ H &= \{\overline{0}\circ_N H,\overline{1}\circ_N H,\ldots,\overline{15}\circ_N H\}\\
&=\{\bigcup_{h\in 2\mathbb{Z}_{16}}\overline{0}\circ_N h,\ \bigcup_{h\in 2\mathbb{Z}_{16}}\overline{1}\circ_N h\\
&\quad,\cdots,\bigcup_{h\in 2\mathbb{Z}_{16}}\overline{15}\circ_N h\}\\
&=\{(\overline{0}+8\mathbb{Z}_{16})\cup(\overline{2}+8\mathbb{Z}_{16})\\
&\quad\cup\cdots\cup(\overline{14}+8\mathbb{Z}_{16}),\\
&\quad(\overline{1}+8\mathbb{Z}_{16})\cup(\overline{3}+8\mathbb{Z}_{16})\\
&\quad\cup\cdots\cup(\overline{15}+8\mathbb{Z}_{16}),\\
&\quad\vdots\\
&\quad(\overline{15}+8\mathbb{Z}_{16})\cup(\overline{1}+8\mathbb{Z}_{16})\\
&\quad\cup\cdots\cup(\overline{13}+8\mathbb{Z}_{16})\}\\
&=\{(\overline{0}+8\mathbb{Z}_{16})\cup(\overline{2}+8\mathbb{Z}_{16})\\
&\quad\cup(\overline{4}+8\mathbb{Z}_{16})\cup(\overline{6}+8\mathbb{Z}_{16}),\\
&\quad(\overline{1}+8\mathbb{Z}_{16})\cup(\overline{3}+8\mathbb{Z}_{16})\\
&\quad\cup(\overline{5}+8\mathbb{Z}_{16})\cup(\overline{7}+8\mathbb{Z}_{16})\}\\
&=\{\{\overline{0},\overline{8}\}\cup\{\overline{2},\overline{10}\}\cup\{\overline{4},\overline{12}\}\cup\{\overline{6},\overline{14}\},\\
&\quad\{\overline{1},\overline{9}\}\cup\{\overline{3},\overline{11}\}\cup\{\overline{5},\overline{13}\}\\
&\quad\cup\{\overline{7},\overline{15}\}\}\\
&=\{\{\overline{0},\overline{2},\overline{4},\overline{6},\overline{8},\overline{10},\overline{12},\overline{14}\},\\
&\quad\{\overline{1},\overline{3},\overline{5},\overline{7},\overline{9},\overline{11},\overline{13},\overline{15}\}\}
\end{aligned}
$$

$$= \{\overline{0} + 2\mathbb{Z}_{16}, \overline{1} + 2\mathbb{Z}_{16}\}$$
$$= \mathbb{Z}_{16} / 2\mathbb{Z}_{16}$$
$$= G / H$$

From the analysis of examples of the set of all left cosets of H in G under the hyperoperation $\circ_N$ where $G = \mathbb{Z}_n$, $N = m\mathbb{Z}_n$, $H = k\mathbb{Z}_n$ and $N \subseteq H$. It can be seen that the three main cases be analyzed are the cases where $n = p$, $n = pq$ and $n = p^k$ when p and q are distinct primes, we have the conclusion that every left coset $ao_N H$ under the hyperoperation $\circ_N$ is the same as the left coset aH under the binary operation in G. But what is different is the structure of each coset $ao_N H$, that is, it is a collection of different cosets of the cosets of N , that is, where each coset $(ah)N$ is a subset of the coset aH and for each $(ah)N$ is combined, it becomes the coset aH , that is,

$$ao_N H = \bigcup_{h \in H} (ah)N = aH \quad \text{for all} \quad a \in G.$$

The inclusion of the example hypergroup $(\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$ is a commendable strength of this work. This concrete example significantly enhances the paper's accessibility by providing a clear, illustrative context for the defined hyperoperation $\circ_N$. It will be highly beneficial for readers attempting to visualize and understand the general construction of the hypergroup $(G, \circ_N)$ and will aid future research in this area.

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