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Bayesian Modelling of Inflation in Nigeria with Threshold Autoregressive Model

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Abstract

In this work, we analyze Nigerian inflation rate with Bayesian threshold autoregressive from 1960 to 2019. The threshold models that are special cases where another variable will be the threshold trigger apart from first lag of dependent was considered. This condition may take longer than a period to induce the regime switch. Prior sensitivity analysis was also carried out to know how sensitive the posterior is to changes in prior information while delay parameter that allocates most of the probability through the posterior distribution will be determined. All results from the analysis are robust to changes to in the prior.

Keywords: Inflation, Nigerian, Posterior distribution, prior information, threshold

1. Introduction

Dynamics in many macroeconomics variables have necessitated evolution of different autoregressive delineation used in different regimes. Threshold autoregressive (TAR) models are typical example and popular of such Regime Switching Model (RSM). The major advantage of TAR model is its determination of one or more threshold values that allow the estimation of different linear models for different regimes Aydin and Esen [1].

So many notable works have been carried out on inflation rate with the aid of TAR model; these are Emmanuel, et al. [2], Munir, et al. [3], Aydin and Esen [1], Dammak and Helali [4]. Emmanuel, et al. [2] examined inflation rate of Ghana under threshold models. The threshold models were compared with standard linear autoregressive models in terms of their fitness and forecasting performance. It was found out that both the self-exiting threshold autoregressive and logistic smooth threshold autoregressive models fit the data used better.

The role of inflation threshold effects in the relationship between economic growth and unemployment was investigated using Turkey data by Aydin and Esen [1]. Their aim was to whether inflation plays a role on the interaction between unemployment and economic growth

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with two-regime TAR model. Their results indicate that there is linear and inverse relationship between the unemployment and growth.

Existence of threshold effects in the relationship between inflation rate and growth rate of GDP in Malaysia with new endogenous TAR models proposed by Hansen [5] was utilized by Munir et al. [3]. Their finding clearly shows that one inflation threshold value exists which means that there is a non-linear relationship between both the growth and inflation.

Bayesian inference has a way of updating prior beliefs into posterior beliefs conditional on observation data. It also offers a rationalist theory of personalistic beliefs in contexts of uncertainty Bernado and Smith [6]. In Classical methods, the existence of unit roots and cointegration has been problems which make both the estimators and test Statistics to be complex. However, Bayesian used observed data with prior beliefs to overcome these problems. Early work on Bayesian threshold autoregressive model was recorded in Geweke and Terui [7]. Geweke and Terui [7] provided a Bayesian method to a Statistical inference in the threshold autoregressive model for time series. They derived the exact posterior density function for both the delay and threshold parameters while their derived methods were applied to different data sets. Other notable works on Bayesian threshold autoregressive model are: Chen and Lee [8], Chen [9], Calderon and Nietto [10], Pan, et al. [11] among others.

Bayesian threshold autoregressive models have been applied to many dynamics of macroeconomics variables (see McCulloch and Tsay [12], Koop and Potter [13]). However, the impact of inflation on economy recovery cannot be underemphasized in any economy especially with use of Bayesian method in order to obtain the inflation threshold effects. This aim of the study is to examine inflation in Nigeria by employing a Bayesian threshold autoregressive through an objective choice that will determine threshold values. In this case, we assume that another variable other than first lag of dependent can trigger the regime switch which can take longer than a period to induce the regime switch. We will also carry out a prior sensitivity analysis to know how sensitive the posterior is to changes in prior information. To demonstrate this, we will find out which delay parameter that allocates most of the probability through the posterior distribution and also determine the time of the inflation rate.

The remainder of the paper is organized as follows. Section 2 gives an overview of threshold autoregressive model. Section 3 describes Bayesian method for threshold autoregressive model. Section 4 presents the data as well as the results from the analysis and Section 5 renders the concluding remarks.

2. Overview of threshold autoregressive model

The threshold autoregressive model with different regimes, process of p^{th} order autoregression, and k-equations is given as:

$$y_{t=}\begin{cases} \emptyset_0^{(1)} + \sum_{i=1}^{P_1} \emptyset_i^{(1)} y_{t-i} + \varepsilon_t^{(1)} & \text{if } y_{t-d} < \tau \\ \emptyset_0^{(2)} + \sum_{i=1}^{P_2} \emptyset_i^{(2)} y_{t-i} + \varepsilon_t^{(2)} & \text{if } y_{t-d} < \tau \\ \quad \ldots\ldots\ldots. & \\ \emptyset_0^{(k)} + \sum_{i=1}^{P_k} \emptyset_i^{(k)} y_{t-i} + \varepsilon_t^{(k)} & \text{if } y_{t-d} \geq \tau \end{cases} \tag{1}$$

where d is the delay parameter and y_{t-d} is either a function of lags of the dependent variable or exogenous variable, τ is the threshold parameter for which the system switches from one regime to other.

3. Bayesian technique

In this study, we consider a two regime TAR model with a variable y_t where $t = p+1, \ldots, T$.

$$y_t = \beta_{10} + \beta_{11} y_{t-1} + \ldots + \beta_{1p} y_{t-p} + \varepsilon_t \quad \text{if } y_{t-1} \le \tau \tag{2}$$

$$y_t = \beta_{20} + \beta_{21} y_{t-1} + \ldots + \beta_{2p} y_{t-p} + \varepsilon_t \quad \text{if } y_{t-1} > \tau \tag{3}$$

If we assume that the model (2) and (3) has a variable, Z, known as the threshold trigger and this variable will take longer than the period to induce the regime switch, we then write (2) and (3) as:

$$y_t = \beta_{10} + \beta_{11} y_{t-1} + \ldots + \beta_{1p} y_{t-p} + \varepsilon_t \quad \text{if } Z_{t-d} \le \tau \tag{4}$$

$$y_t = \beta_{20} + \beta_{21} y_{t-1} + \ldots + \beta_{2p} y_{t-p} + \varepsilon_t \quad \text{if } Z_{t-d} > \tau \tag{5}$$

where d is the delay parameter, Z_{t-d} is either exogenous or functions of the lags.

The order of autoregressive for (4) and (5) is $P = 4$.

Thus, threshold trigger can be simply be defined as:

$$Z_{t-d} = \frac{\sum_{d=1}^{P} y_{t-d}}{d} \tag{6}$$

We also assume that parameter; d is unknown and has a non-informative prior over $1,\ldots,k$ which means that:

$$P(d = i) = \frac{1}{k} \tag{7}$$

And Normal-Gamma (NG) priors with

$$\beta, h \sim NG(\beta^o, M^o, (S^{-2})^o, v^o) \tag{8}$$

Since Z_{t-d} is either functions of lags of dependent or exogenous variable, thus we can condition the same way as we conditioned X. Therefore, we can write the TAR model as:

$$y = X\beta + \varepsilon \tag{9}$$

And X is $(T-k) \times 2(k+1)$ matrix with $t-th$ row which is given as:

$$[\, D_t, D_t\, y_{t-1}, \ldots, D_t\, y_{t-k}, (1-D_t), (1-D_t), y_{t-1}, \ldots, (1-D_t), y_{t-k}] \tag{10}$$

where D_t is the dummy and is defined as:

$$D_t = \begin{cases} 1, & if\ Z_{t-d} \le \tau \\ 0, & if\ Z_{t-d} > \tau \end{cases}$$

$$y = (y_{k+1}, \ldots, y_T)'$$

$$\varepsilon = \begin{pmatrix} \varepsilon_{k+1} \\ \vdots \\ \varepsilon_N \end{pmatrix} \text{ and } \beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix}$$

while

$$\varepsilon \sim \mathrm{N}(0, h^{-1})$$

Hence, we can simply apply Koop, et al. [14] to obtain the posterior density function for parameters β and h conditioned on both d and τ and simply written as:

$$\beta, h | \tau, d \sim NG(\beta^*, M^*, (S^*)^{-2}, v^*) \quad (11)$$

where

$$M^* = ((M^0)^{-1} \ \beta^0 + X'X)^{-1}$$

$$\beta^* = M^*((M^0)^{-1} \ \beta^0 + X'X \ \hat{\beta})$$

$$v^*(S^*)^2 = v^o(S^o)^2 + SSE + (\hat{\beta} - \beta^o)' \ X'X \ M^*(M^0)^{-1} \ (\hat{\beta} - \beta^o)$$

But $\hat{\beta}$ and SSE are OLS estimator and Sum of Squares of Error respectively.

N.B: The parameters with o and * are prior and posterior distributions.

The prior specification for hyperparameters is given as:

$$v^o = 5, \beta^o = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad (S^O)^2 = 1, \quad M^0 = 0.25 \ I_{10}, P = 4,$$

4. Presentation and discussion of results

The data considered in this study are quarterly Nigeria inflation rate. These data span from January, 1960 to December, 2019 were sourced from World Bank indicators website. In this study, we set τ to the mean of Z_{t-d} and the two regimes can be simply interpreted as:

$$Regime = \begin{cases} Below \ the \ inflation \ rate \ regime \\ Above \ the \ inflation \ rate \ regime \end{cases} \quad (12)$$

In Table 1, we present the posterior means and standard deviation of inflation rates for parameters of different prior. The posterior mean and standard deviation for c = 0.25 and 1 are virtually the same thing for all the parameters except for c = 100 which is extremely not informative. As the prior gets larger (noinformative), the posterior means and standard deviation also increases for parameters β_{10} - β_{14} while as the prior gets larger (noinformative), the posterior means and standard deviation also decreases for parameters β_{20} - β_{24}.

Table 2 shows different values of delay parameters for posterior distribution. For c = 0.25 and 1, d allocates most probability, d= 1, which means that, it is the average inflation rate over the last first quarters that triggers the change in regime. However, for c = 100, d allocates most probability d= 4, indicating that inflation rate averaged over the past four quarters. Inflation rate four quarters ago plays a great role in regime shifting in Nigerian inflation rate.

Figures 1, 2, and 3 show the plot of posterior density of the delay parameter for different prior specification. These figures show some interesting pattern.

Table 1: Posterior results of inflation rate for TAR model with unknown threshold and delay.

Parameter	Prior specification	Posterior mean	Posterior SD
β_{10}	$c = 0.25$	0.0812	0.0724
	$c = 1$	0.0800	0.0876
	$c = 100$	0.0183	0.1622
β_{11}	$c = 0.25$	0.0225	0.1635
	$c = 1$	0.0674	0.3142
	$c = 100$	0.5229	1.4962
β_{12}	$c = 0.25$	0.0090	0.1615
	$c = 1$	0.0109	0.3044
	$c = 100$	0.0431	1.5204
β_{13}	$c = 0.25$	0.0176	0.1593
	$c = 1$	0.0279	0.1593
	$c = 100$	0.0625	1.3798
β_{14}	$c = 0.25$	0.0348	0.1571
	$c = 1$	0.0798	0.2797
	$c = 100$	0.3066	0.9859
β_{20}	$c = 0.25$	0.1442	0.0861
	$c = 1$	0.1436	0.1289
	$c = 100$	0.0811	0.1635
β_{21}	$c = 0.25$	0.1386	0.1481
	$c = 1$	0.3020	0.2503
	$c = 100$	0.8479	0.4339
β_{22}	$c = 0.25$	0.0365	0.1489
	$c = 1$	-0.0046	0.2550
	$c = 100$	-0.4530	0.5309
β_{23}	$c = 0.25$	0.0240	0.1498
	$c = 1$	0.0127	0.2575
	$c = 100$	0.2924	0.5530
β_{24}	$c = 0.25$	0.0209	0.1504
	$c = 1$	-0.0030	0.2553
	$c = 100$	-0.1535	0.4731
σ^2	$c = 0.25$	0.1084	0.0205
	$c = 1$	0.1043	0.0197
	$c = 100$	0.0983	0.0186

Table 2: Posterior distribution values of delay parameters.

d	$c = 0.25$	$c = 1$	$c = 100$
1	0.2888	0.2864	0.2033
2	0.2469	0.2494	0.2066
3	0.2390	0.2383	0.2401
4	0.2253	0.2259	0.3299

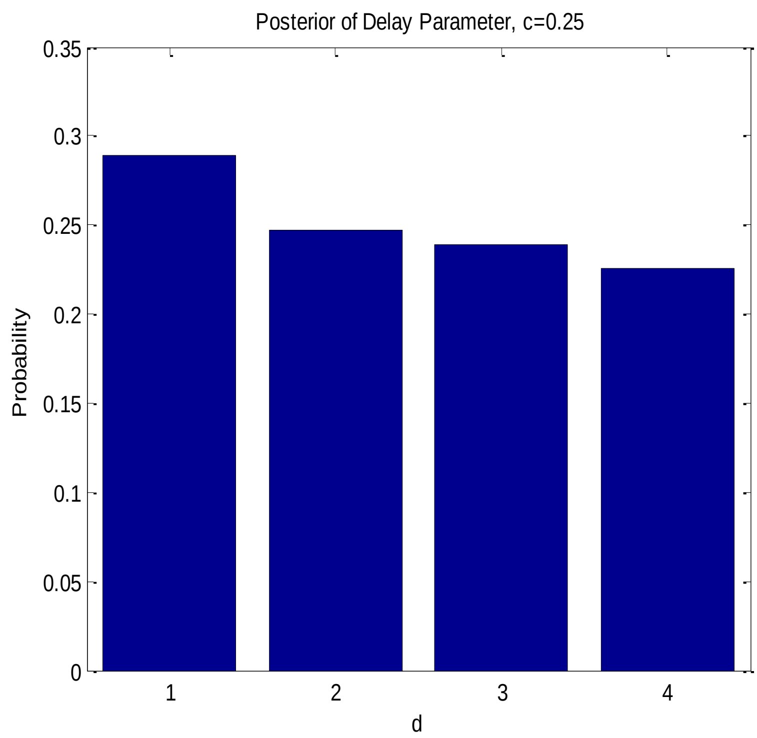

Figure 1: Plot of posterior density of the delay parameter when $C = 0.25$

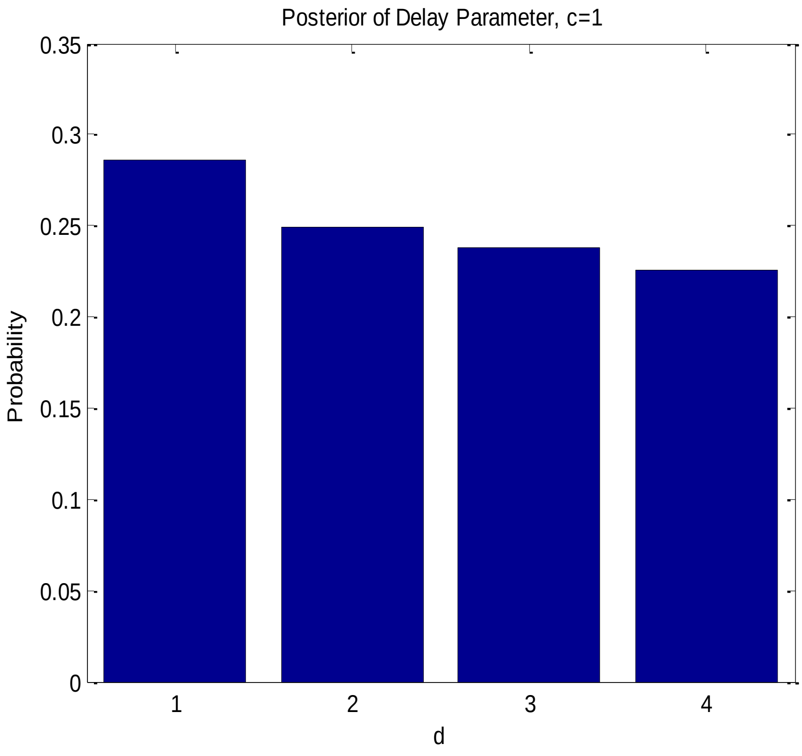

Figure 2: Plot of posterior density of the delay parameter when $C = 1$

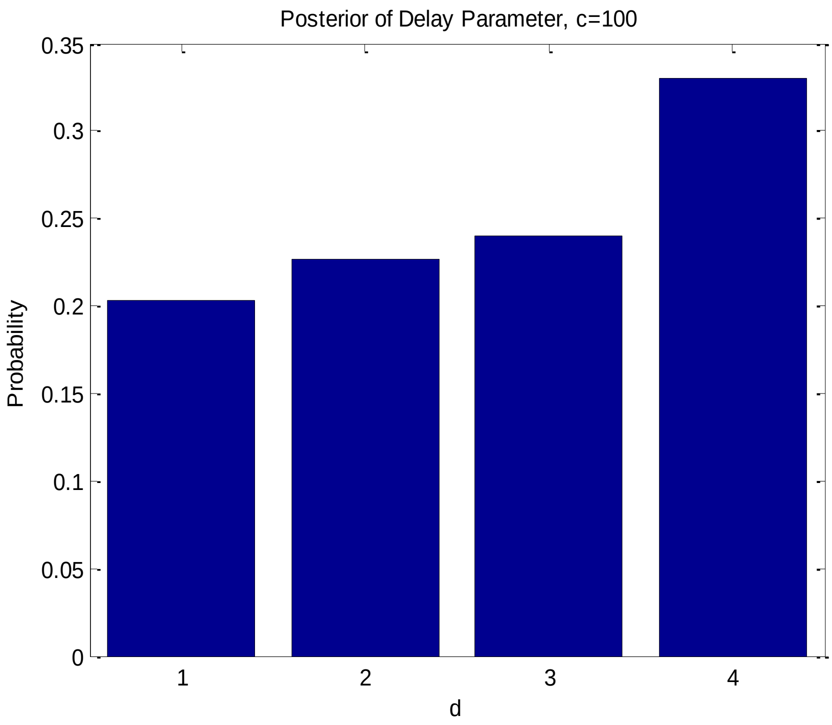

Figure 3: Plot of posterior density of the delay parameter when $C = 100$

5. Conclusion

Inflation plays a dominant role in any economy. It has plunged so many countries into long periods of instability. This study examined inflation in Nigeria by employing a Bayesian threshold autoregressive through an objective choice that will determine threshold values in threshold autoregressive model that trigger a regime shift in inflation. Prior elicitation is more important to a researcher in Bayesian modelling of dynamics of macroeconomic variables for different autoregressive delineation of different regimes. Therefore, prior sensitivity analysis was carried out to know how sensitive the posterior is to changes in prior information. It is apparent that as the prior ($c \rightarrow \infty$), the posterior also give infinite estimates for all the parameters of threshold autoregressive models. However, setting c to a small value will ensure more accurate estimation. The delay parameter, d allocates most probability d= 1, for prior, c = 0.05 and 1, that means the threshold trigger is average inflation rate over the last first quarters triggered the change in regime. But, for prior, c = 100, d allocates most probability d= 4, indicating that inflation rate averaged over the past four quarters triggered the regime shift. Prior information with c = 0.05 and 1 performed better than c = 100 for the posterior distribution. Hence, Inflation rate over last first quarters ago plays a great role in regime shifting in Nigerian inflation rate for the datasets. And also the inflation does not have reducing impact on economy. These findings will inform the decision makers that inflation has to be kept below the threshold level.

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