การทดสอบค่าสัมประสิทธิ์การแปรผันของปริมาณฝุ่นละอองขนาดเล็ก (PM2.5) ของอำเภอหาดใหญ่ จังหวัดสงขลา

TESTING FOR THE COEFFICIENT OF VARIATION IN FINE PARTICULATE MATTER (PM2.5) OF HAT YAI, SONGKHLA, THAILAND

วราฤทธิ์ พานิชกิจโกศลกุล[1*]
Wararit Panichkitkosolkul[1*]

บทคัดย่อ

ผลกระทบที่รุนแรงมากที่สุดของมลพิษทางอากาศต่อสุขภาพของประชาชนเป็นที่เข้าใจกันว่ามาจากการได้รับ PM2.5 ในระยะยาว ซึ่งจะเพิ่มความเสี่ยงต่อการเสียชีวิต โดยเฉพาะในบางช่วงอายุและมีอาการของโรคหัวใจและหลอดเลือด ซึ่งโดยปกติระดับของ PM2.5 มีการแจกแจงแกมมา อย่างไรก็ตาม บางสถานการณ์อาจสนใจการทดสอบค่าสัมประสิทธิ์การแปรผันของ PM2.5 บทความนี้เสนอการทดสอบค่าสัมประสิทธิ์การแปรผันของการแจกแจงแกมมา 2 วิธี คือ การทดสอบที่ใช้วิธีของสคอร์ และวิธีของวาล์ว การเปรียบเทียบประสิทธิภาพใช้วิธีการจำลองภายใต้การแจกแจงแกมมา ซึ่งมีพารามิเตอร์รูปร่างแตกต่างกัน โดยพิจารณาจากขนาดของการทดสอบเชิงประจักษ์และกำลังของการทดสอบเชิงประจักษ์ การเปรียบเทียบประสิทธิภาพยังได้ประยุกต์ใช้กับข้อมูล PM2.5 รายชั่วโมงของอำเภอหาดใหญ่ จังหวัดสงขลา ผลการจำลองพบว่า การทดสอบที่ใช้วิธีของวาล์วมีประสิทธิภาพมากกว่าการทดสอบที่ใช้วิธีของสคอร์ เมื่อพิจารณาจากขนาดของการทดสอบเชิงประจักษ์และกำลังของการทดสอบเชิงประจักษ์ โดยการทดสอบที่ใช้วิธีของวาล์วเป็นวิธีที่แนะนำให้กับผู้ใช้ในสถานการณ์ที่คล้ายคลึงกัน

คำสำคัญ: การทดสอบสมมติฐาน, ค่าวัดการกระจาย, การแจกแจงแกมมา, มลพิษทางอากาศ

[1] รองศาสตราจารย์ สาขาวิชาคณิตศาสตร์และสถิติ คณะวิทยาศาสตร์และเทคโนโลยี มหาวิทยาลัยธรรมศาสตร์ ศูนย์รังสิต

[*] Corresponding author e-mail: wararit@mathstat.sci.tu.ac.th

Submission: 20 January 2021, **Received:** 3 March 2021, **Accepted:** 29 March 2021

Abstract

The biggest impact of particulate air pollution on public health is understood to be from long-term exposure to fine particulate matter (PM2.5), which increases the age-specific mortality risk, particularly from cardiovascular causes. Usually, the hourly PM2.5 level fits a gamma distribution, and this has led us to consider testing this via the coefficient of variation (CV) of these data. Herein, we present two statistical methods for testing the CV in a gamma population based on the Score and Wald methods. To compare their performances, a simulation study was conducted under several shape parameter values for a gamma distribution. The performances of the test statistics were compared based on their empirical type I error rates and powers of the test. Their performances were then illustrated by applying them to the hourly PM2.5 level in Hat Yai, Songkhla, Thailand. The simulation results show that the test statistic based on the Wald method performed better than the one based on the Score method in terms of the attained nominal significance level and is thus recommended for analysis in similar scenarios.

Keyword: Hypothesis testing, Measure of dispersion, Gamma distribution, Air pollution

Introduction

The population coefficient of variation (CV) is a unit-free measure of variability relative to the population mean (Albatineh et al., 2017). It is defined as the ratio of the population standard deviation σ to the population mean μ, namely $\theta = \sigma / \mu$, where $\mu \neq 0$. Furthermore, it has been more widely used than the standard deviation for comparing the variations of several variables obtained by different units.

The estimator of CV has been widely applied in many fields of science, including the medical sciences, engineering, economics and others (see Nairy and Rao, 2003). For example, the applicability of the CV method for analyzing synaptic plasticity was studied by Faber and Korn (1991). Calif and Soubdhan (2016) used the CV to measure the spatial and temporal correlation of global solar radiation. Reed et al. (2002) used the CV in assessing the variability of quantitative assays. Bedeian and Mossholder (2000) used the CV for comparing diversity in work groups. Kang et al. (2007) applied the CV for monitoring variability in statistical process control. Castagliola et al. (2011) proposed a new method to monitor the CV by means of two one-sided EWMA charts of the CV squared. The effect of the CV of operation times on the optimal allocation of storage space in production line systems was studied by Frederick and Kut (1991). Döring and Reckling (2018) proposed a method to adjust the standard CV to account for the systematic dependence of population variance from the population mean. The CV for the injection pressure recorded during the operation of the engine were evaluated by Bakowskia et al. (2017).

The literature on testing the CV in a gamma distribution is limited. However, there are many methods available for estimating the confidence interval for a population CV. For example, the confidence interval for the CV based on the chi-squared distribution was proposed by McKay (1932). Vangel (1996) proposed a modified McKay's approximate confidence interval for the CV. Panichkitkosolkul (2009) improved the modified McKay's approximate confidence interval by replacing the typical sample estimate of the CV with the maximum likelihood estimate in the case of a normal distribution. In addition, the confidence intervals for the CV based on a ranked set sampling was proposed by Albatineh et al. (2014). Recently, Sangnawakij and Niwitpong (2017) presented two confidence intervals for the CV in a gamma distribution using the Score and Wald methods. These confidence intervals for the CV can be used to test the hypothesis for the CV.

Air pollution has become a major problem for people living in industrial cities. It harms human health, and it also has various indirect implications for society and the economy by, for example, undermining a country's economic growth potential through a reduction of people's work hours and a loss of agricultural productivity (World Bank, 2016). Air pollutants include gaseous pollutants and particle matters (PM). Fine particulate matter with a diameter of up to 2.5 micrometers (PM2.5) are so small and light that they tend to stay longer in the air than heavier particles. PM2.5 can be emitted from combustion sources, such as diesel engines and biomass burning, or formed in the atmosphere from chemical reactions of sulfur dioxide (SO_2), oxides of nitrogen (NO_x) and volatile organic compounds (VOCs) (Narita et al., 2019). PM2.5 has a potential risk for human health, especially premature mortality under long-term exposure.

Moreover, PM2.5 can penetrate deeply into the lung, irritate and corrode the alveolar wall, and consequently impair lung function (Xing et al., 2016).

Some circumstances have led us to consider testing levels of PM2.5 in terms of the CV. The information could be advantageous for the Thailand's Ministry of Natural Resources and Environment and other related organizations to realize and plan for solving or reducing the problem of levels of PM2.5. The results from a gamma quantile-quantile (Q-Q) plot (Figure 1(a)), a histogram (Figure 1(b)), and applying the Anderson-Darling goodness-of-fit test show that the hourly PM2.5 levels from Hat Yai station from January 11-16, 2021 fit well to a gamma distribution.

The objective of this study is to propose some methods for testing the population CV and identify the appropriate methods for practitioners. Two confidence intervals proposed by Sangnawakij and Niwitpong (2017) for testing the population CV are considered. Because a theoretical comparison is not possible, we conducted a simulation study to compare the performance of these methods and used these results to suggest a test statistic with high power to attain a nominal significance level for practitioners.

The structure of this paper is as follows. The second section reviews the point estimation of parameters in a gamma distribution. In the third section, we present the proposed statistical methods for testing the CV in a gamma distribution. The simulation technique and results are discussed in the fourth section. A numerical example is illustrated in the fifth section. Finally, discussion and conclusions are presented in the two last sections.

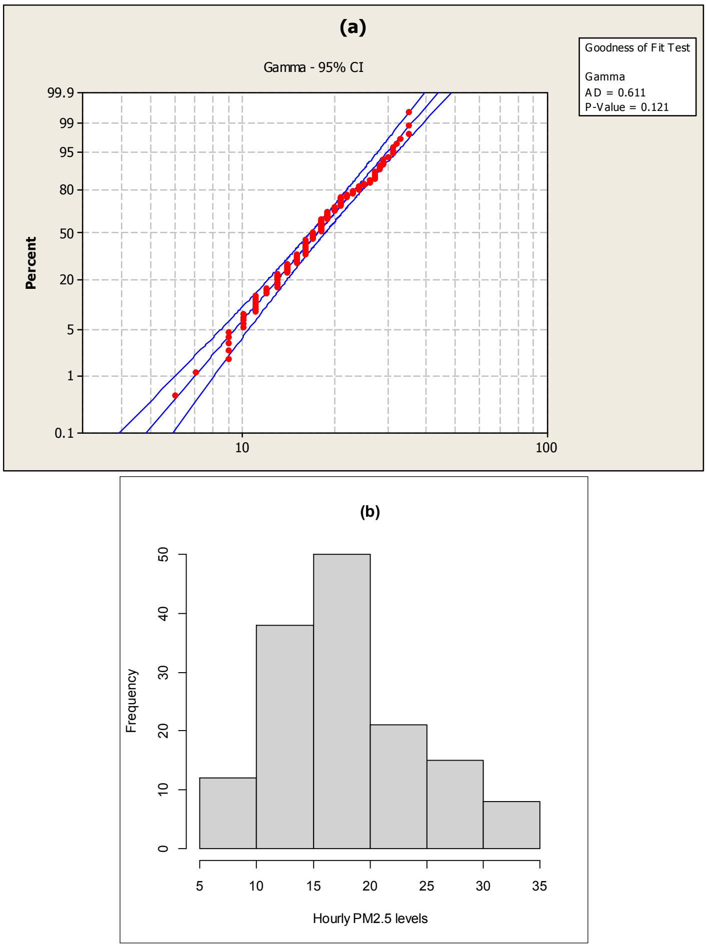

Figure 1. (a) gamma QQ plot (b) histogram of the hourly PM2.5 levels from Hat Yai station.

Point Estimation of Parameters in a Gamma Distribution

In this section, we explain the point estimation of parameters in a gamma distribution. Let $X=(X_1,\ldots,X_n)$ be a random sample from the gamma distribution with the shape parameter α and scale parameter β, denoted as $\mathrm{Gamma}(\alpha,\beta)$. The probability density function of X is given by

$$f(x;\alpha,\beta) \quad = \quad \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1}e^{-x/\beta}, \qquad x>0,\ \alpha>0,\ \beta>0$$

(1)

The mean and variance of X are $E(X)=\alpha\beta$ and $Var(X)=\alpha\beta^2$, respectively. Therefore, the CV of X is given by $\theta=1/\sqrt{\alpha}$. Since α is unknown parameter, it is required to be estimated.

We consider the maximum likelihood estimators for α and β. From the probability density function shown in (1), the log-likelihood function of α and β is given by

$$\ln L(\alpha,\beta) = -\frac{\sum_{i=1}^{n} X_i}{\beta} + (\alpha-1)\sum_{i=1}^{n}\ln(X_i) - n\ln\left(\Gamma(\alpha)\right) - n\alpha\ln(\beta).$$

Taking partial derivatives of the above equation with respect to α and β, respectively, the score function is derived as

$$U(\alpha,\beta) = \begin{bmatrix} \sum_{i=1}^{n}\ln(X_i) - n\ln(\alpha) + \frac{n}{2\alpha} - n\ln(\beta) \\ \frac{\sum_{i=1}^{n} X_i}{\beta^2} - \frac{n\alpha}{\beta} \end{bmatrix}.$$

Then, the maximum likelihood estimators can be conducted for α and β, respectively,

$$\hat{\alpha} = \frac{1}{2\left(\ln(\bar{X}) - \frac{\sum_{i=1}^{n}\ln(X_i)}{n}\right)}, \quad \hat{\beta} = \frac{\bar{X}}{\hat{\alpha}},$$

where $\bar{X} = n^{-1}\sum_{i=1}^{n} X_i$ is the sample mean of X. Also, the estimator of CV is given by $\hat{\theta} = 1/\sqrt{\hat{\alpha}}$.

Proposed Test Statistics

Let $X_1,\dots,X_n$ be an independent and identically distributed (i.i.d.) random sample of size n from a population with finite mean, μ and finite variance, σ^2. Let $\bar{X}$ be the sample mean and S be the sample standard deviation. Then $\hat{\theta} = S/\bar{X}$ is the estimated value of the population CV, $\theta = \sigma/\mu$.

We want to test for the population CV. The null and alternative hypotheses are defined as follows:

$$H_0 : \theta = \theta_0$$
$$H_1 : \theta \neq \theta_0.$$

In this section, we discuss two test statistics for the CV based on the Score and Wald methods.

1. Score method

Let α and β be the parameter of interest and the nuisance parameters, respectively. The score statistic is denoted as

$$W_1 = U^T(\alpha_0, \hat{\beta}_0) I^{-1}(\alpha_0, \hat{\beta}_0) U(\alpha_0, \hat{\beta}_0),$$

where $\hat{\beta}_0$ is the maximum likelihood estimator for β, under the null hypothesis $H_0' : \alpha = \alpha_0$, $U(\alpha_0, \hat{\beta}_0)$ is the vector of the Score function and $I(\alpha_0, \hat{\beta}_0)$ is the matrix of the Fisher information. Here, it is easy to derive that the Score function under H_0' is

$$U(\alpha_0, \hat{\beta}_0) = \begin{bmatrix} \sum_{i=1}^{n} \ln(X_i) + \frac{n}{2\alpha_0} - n\ln(\bar{X}) \\ 0 \end{bmatrix}.$$

The inverse of the matrix of the Fisher information can be found as follows

$$I^{-1}(\alpha_0, \hat{\beta}_0) = \begin{bmatrix} \frac{2\alpha_0^2}{n} & -\frac{2\bar{X}}{n} \\ -\frac{2\bar{X}}{n} & \frac{\bar{X}^2(2\alpha_0 + 1)}{n\alpha_0^3} \end{bmatrix}.$$

Using the property of the Score function, we can see that the pivotal

$$Z_{score} = \sqrt{\frac{2\alpha_0^2}{n}} \left(\sum_{i=1}^{n} \ln(X_i) + \frac{n}{2\alpha_0} - n\ln(\bar{X}) \right) \quad (2)$$

converges in distribution to the standard normal distribution. Since the variance of $\hat{\alpha}$ is $\frac{2\alpha_0^2}{n}$, it is approximated by substituting $\hat{\alpha}$ in its variance. Under H_0', the statistic in (2) is given as

$$Z_{score} \cong \sqrt{\frac{2\hat{\alpha}^2}{n}} \left(\sum_{i=1}^{n} \ln(X_i) + \frac{n}{2\alpha} - n\ln(\bar{X}) \right).$$

From the probability statement, $1-\gamma = P(-Z_{1-\gamma/2} \le Z_{score} \le Z_{1-\gamma/2})$, it can be simply written as $1-\gamma = P(l_s \le \theta \le u_s)$. Therefore, the $(1-\gamma)100\%$ confidence interval for θ based on the score method, CI_s, is given by

$$CI_s = [l_s, u_s] = \left[\sqrt{\frac{2}{n}\left(M - Z_{1-\gamma/2}\sqrt{\frac{n}{2\hat{\alpha}^2}} \right)}, \sqrt{\frac{2}{n}\left(M + Z_{1-\gamma/2}\sqrt{\frac{n}{2\hat{\alpha}^2}} \right)} \right],$$

where $M = n\ln(\bar{X}) - \sum_{i=1}^{n} \ln(X_i)$ and $Z_{1-\gamma/2}$ is the $(1-\gamma/2)^{\text{th}}$ quantile of the standard normal distribution. Thus, we will reject the null hypothesis, $H_0 : \theta = \theta_0$, if

$$\theta_0 < \sqrt{\frac{2}{n}\left(M - Z_{1-\gamma/2}\sqrt{\frac{n}{2\hat{\alpha}^2}} \right)}$$

or

$$\theta_0 > \sqrt{\frac{2}{n}\left(M + Z_{1-\gamma/2}\sqrt{\frac{n}{2\hat{\alpha}^2}}\right)}.$$

2. Wald method

The Wald statistic is an asymptotic statistic derived from the property of the maximum likelihood estimator. The general form of the Wald statistic under the null hypothesis $H_0' : \alpha = \alpha_0$ is defined as

$$W_2 = (\hat{\alpha} - \alpha_0)^T \left[I^{\alpha\alpha}(\hat{\alpha}, \hat{\beta}) \right]^{-1} (\hat{\alpha} - \alpha_0),$$

where $I^{\alpha\alpha}(\hat{\alpha}, \hat{\beta})$ is the estimated variance of $\hat{\alpha}$ obtained from the first row and the first column of $I^{-1}(\hat{\alpha}, \hat{\beta})$. Using the information of partial derivatives from the previous subsection, the inverse matrix is given by

$$I^{-1}(\hat{\alpha}, \hat{\beta}) = \begin{bmatrix} \frac{2\hat{\alpha}^2}{n} & -\frac{2\bar{X}}{n} \\ -\frac{2\bar{X}}{n} & \frac{\bar{X}^2(2\hat{\alpha}+1)}{n\hat{\alpha}^3} \end{bmatrix},$$

where $I^{\alpha\alpha}(\hat{\alpha}, \hat{\beta}) = \frac{2\hat{\alpha}^2}{n}$. Therefore, under H_0', we obtain the Wald statistic

$$Z_{wald} \cong \sqrt{\frac{n}{2\hat{\alpha}^2}}(\hat{\alpha} - \alpha), \quad (3)$$

which has the limiting distribution of a standard normal distribution. Therefore, the $(1-\gamma)100\%$ confidence interval for θ based on the Wald method, CI_w, is given by

$$CI_w = [l_w, u_w] = \left[1 \Big/ \sqrt{\hat{\alpha} + Z_{1-\gamma/2}\sqrt{\frac{2\hat{\alpha}^2}{n}}},\ 1 \Big/ \sqrt{\hat{\alpha} - Z_{1-\gamma/2}\sqrt{\frac{2\hat{\alpha}^2}{n}}} \right],$$

where $Z_{1-\gamma/2}$ is the $(1-\gamma/2)^{\text{th}}$ quantile of the standard normal distribution. Thus, we will reject the null hypothesis, $H_0 : \theta = \theta_0$, if

$$\theta_0 < 1 \Big/ \sqrt{\hat{\alpha} + Z_{1-\gamma/2}\sqrt{\frac{2\hat{\alpha}^2}{n}}}$$

or

$$\theta_0 > 1 \Big/ \sqrt{\hat{\alpha} - Z_{1-\gamma/2}\sqrt{\frac{2\hat{\alpha}^2}{n}}}.$$

Simulation Study

In this study, two methods for testing the population CV in a gamma distribution are considered. Since a theoretical comparison is not possible, a Monte Carlo simulation was conducted using the R version 4.0.3 statistical software (Ihaka and Gentleman, 1996) to compare the performance of the test statistics. The methods were compared in terms of their attainment of empirical type I error rates and the powers of their performance. The simulation results are presented only for the significant level γ = 0.05, since 1) γ = 0.05 is widely used to compare the power of the test and 2) similar conclusions were obtained for other values of γ.

To observe the behavior of small, moderate and large sample sizes, we used n = 10, 30, 50, 100, and 200. The number of simulations was fixed at 10,000. The data were generated from a gamma distribution with $\beta = 2$ and α was adjusted to obtain the required coefficient of variation θ. We set $\theta =$ 0.05, 0.10, 0.20, 0.28, 0.30 and 0.33.

As can be seen in the simulation results displayed in all the tables, the empirical type I error rates of the Wald method were close to the nominal significance level of 0.05 for all sample sizes while those of the Score method were close to the nominal significance level of 0.05 for larger sample sizes. The Score method performed well in terms of the power of the test for $\theta > \theta_0$. On the other hand, the Wald method performed better for $\theta < \theta_0$. We observed a general pattern; when the sample size increases, the power of the test also increases and the empirical type I error rate approaches 0.05. Also the power increases as the value of the CV departs from the hypothesized value of the CV. It was observed that for large sample sizes, the performance of the test statistics did not differ greatly in the sense of power and the attainment of the nominal significance level of the test. However, a significant difference was observed for small sample sizes.

Table 1. Empirical type I error rates and powers of tests for Gamma(400,2), θ = 0.05.

n	Method	θ_0								
		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
10	Score	0.955	0.165	0.009	0.054	**0.195**	0.432	0.688	0.865	0.956
	Wald	1.000	0.965	0.669	0.213	**0.028**	0.008	0.017	0.042	0.085
30	Score	0.831	0.426	0.106	0.039	**0.111**	0.277	0.514	0.746	0.896
	Wald	0.970	0.809	0.474	0.164	**0.045**	0.057	0.153	0.322	0.527
50	Score	1.000	1.000	0.987	0.338	**0.083**	0.636	0.977	1.000	1.000
	Wald	1.000	1.000	0.999	0.657	**0.045**	0.317	0.856	0.995	1.000
100	Score	1.000	1.000	1.000	0.769	**0.068**	0.837	1.000	1.000	1.000
	Wald	1.000	1.000	1.000	0.893	**0.050**	0.658	0.997	1.000	1.000
200	Score	1.000	1.000	1.000	0.983	**0.060**	0.977	1.000	1.000	1.000
	Wald	1.000	1.000	1.000	0.994	**0.050**	0.944	1.000	1.000	1.000

Bold numeric is the empirical type I error rates.

Table 2. Empirical type I error rates and powers of tests for Gamma(100,2), θ = 0.10.

n	Method	θ_0								
		0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14
10	Score	0.007	0.024	0.053	0.113	**0.198**	0.302	0.435	0.561	0.683
	Wald	0.668	0.417	0.208	0.087	**0.027**	0.009	0.008	0.009	0.017
30	Score	0.820	0.436	0.113	0.038	**0.104**	0.270	0.511	0.750	0.899
	Wald	0.972	0.823	0.473	0.162	**0.041**	0.056	0.148	0.325	0.535
50	Score	0.987	0.804	0.338	0.059	**0.082**	0.298	0.626	0.874	0.977
	Wald	0.998	0.947	0.654	0.217	**0.048**	0.099	0.313	0.615	0.852
100	Score	1.000	0.993	0.764	0.177	**0.065**	0.396	0.836	0.985	1.000
	Wald	1.000	0.998	0.891	0.369	**0.048**	0.210	0.653	0.939	0.997
200	Score	1.000	1.000	0.982	0.451	**0.055**	0.573	0.976	1.000	1.000
	Wald	1.000	1.000	0.994	0.599	**0.046**	0.415	0.943	1.000	1.000

Bold numeric is the empirical type I error rates.

Table 3. Empirical type I error rates and powers of tests for Gamma(25,2), $\theta = 0.20$.

n	Method	θ_0								
		0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23	0.24
10	Score	0.051	0.082	0.113	0.152	**0.196**	0.246	0.304	0.358	0.424
	Wald	0.220	0.142	0.088	0.051	**0.029**	0.016	0.010	0.007	0.006
30	Score	0.109	0.054	0.037	0.055	**0.102**	0.175	0.270	0.386	0.507
	Wald	0.480	0.306	0.160	0.081	**0.039**	0.040	0.056	0.096	0.151
50	Score	0.362	0.159	0.065	0.044	**0.073**	0.163	0.296	0.457	0.616
	Wald	0.670	0.435	0.237	0.105	**0.046**	0.046	0.094	0.180	0.300
100	Score	0.781	0.468	0.190	0.058	**0.063**	0.175	0.382	0.627	0.821
	Wald	0.906	0.686	0.379	0.141	**0.050**	0.079	0.201	0.404	0.637
200	Score	0.986	0.845	0.473	0.120	**0.055**	0.213	0.563	0.846	0.974
	Wald	0.994	0.919	0.630	0.221	**0.050**	0.123	0.408	0.732	0.937

Bold numeric is the empirical type I error rates.

Table 4. Empirical type I error rates and powers of tests for Gamma(12.76,2), $\theta =$ 0.28.

n	Method	θ_0								
		0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
10	Score	0.080	0.107	0.129	0.154	**0.188**	0.220	0.257	0.301	0.348
	Wald	0.142	0.097	0.068	0.049	**0.032**	0.019	0.014	0.009	0.009
30	Score	0.052	0.038	0.044	0.060	**0.101**	0.145	0.201	0.273	0.352
	Wald	0.294	0.186	0.115	0.069	**0.045**	0.039	0.041	0.055	0.081
50	Score	0.143	0.079	0.042	0.047	**0.077**	0.127	0.202	0.301	0.412
	Wald	0.420	0.264	0.148	0.086	**0.052**	0.045	0.060	0.096	0.159
100	Score	0.445	0.228	0.098	0.049	**0.063**	0.126	0.237	0.394	0.570
	Wald	0.667	0.432	0.231	0.109	**0.052**	0.054	0.107	0.206	0.349
200	Score	0.827	0.553	0.250	0.084	**0.051**	0.136	0.328	0.576	0.797
	Wald	0.908	0.700	0.394	0.155	**0.052**	0.075	0.205	0.427	0.676

Bold numeric is the empirical type I error rates.

Numerical Example

To illustrate the application of the two statistical methods for testing the CV introduced in the previous section, we used data on the hourly PM2.5 levels $(\mu g/m^3)$ obtained from Thailand's Pollution Control Department (http://air4thai.pcd.go.th). The hourly PM2.5 levels were measured from the Hat Yai station from January 11-16, 2021. The descriptive statistics are as follows: sample size = 144, mean = 18.29, standard deviation (SD) = 6.33, CV = 0.34, coefficient of skewness = 0.67, and kurtosis = –0.08. The distribution of the hourly PM2.5 levels is slightly right-skewed and it has slightly thin-tailed data distribution.

A gamma quantile-quantile (QQ) plot and histogram of the data are displayed in Figure 1, while density and Box and Whisker plots are shown in Figure 2. According to the Anderson-Darling (AD) goodness-of-fit test, the hourly PM2.5 levels follow a gamma distribution. Using a Minitab program (McKenzie, 2004), the hourly PM2.5 levels from the Hat Yai station were from a gamma distribution with a shape parameter, $\hat{\alpha} =$ 8.5972 and a scale parameter, $\hat{\beta} =$ 2.12763, while the estimator of the CV is $\hat{\theta} = 0.3411$.

Table 5. Empirical type I error rates and powers of tests for Gamma(11.11,2), $\theta =$ 0.30.

n	Method	θ_0								
		0.26	0.27	0.28	0.29	0.30	0.31	0.32	0.33	0.34
10	Score	0.080	0.107	0.129	0.154	**0.188**	0.220	0.257	0.301	0.348
	Wald	0.142	0.097	0.068	0.049	**0.032**	0.019	0.014	0.009	0.009
30	Score	0.052	0.038	0.044	0.060	**0.101**	0.145	0.201	0.273	0.352
	Wald	0.294	0.186	0.115	0.069	**0.045**	0.039	0.041	0.055	0.081
50	Score	0.143	0.079	0.042	0.047	**0.077**	0.127	0.202	0.301	0.412
	Wald	0.420	0.264	0.148	0.086	**0.052**	0.045	0.060	0.096	0.159
100	Score	0.445	0.228	0.098	0.049	**0.063**	0.126	0.237	0.394	0.570
	Wald	0.667	0.432	0.231	0.109	**0.052**	0.054	0.107	0.206	0.349
200	Score	0.761	0.505	0.232	0.080	**0.051**	0.121	0.286	0.526	0.751
	Wald	0.861	0.655	0.372	0.152	**0.053**	0.068	0.171	0.367	0.608

Bold numeric is the empirical type I error rates.

Table 6. Empirical type I error rates and powers of tests for Gamma(9.18,2), θ = 0.33.

n	Method	θ_0								
		0.29	0.30	0.31	0.32	0.33	0.34	0.35	0.36	0.37
10	Score	0.087	0.102	0.131	0.156	**0.187**	0.216	0.254	0.303	0.345
	Wald	0.127	0.098	0.065	0.048	**0.035**	0.021	0.014	0.010	0.009
30	Score	0.042	0.038	0.047	0.067	**0.097**	0.131	0.183	0.237	0.301
	Wald	0.231	0.157	0.108	0.066	**0.043**	0.034	0.037	0.050	0.062
50	Score	0.099	0.056	0.042	0.051	**0.076**	0.113	0.174	0.248	0.343
	Wald	0.327	0.214	0.133	0.077	**0.055**	0.040	0.053	0.077	0.121
100	Score	0.326	0.179	0.083	0.047	**0.060**	0.106	0.187	0.309	0.464
	Wald	0.538	0.366	0.198	0.099	**0.052**	0.052	0.083	0.156	0.264
200	Score	0.696	0.439	0.203	0.073	**0.051**	0.106	0.238	0.447	0.658
	Wald	0.810	0.589	0.331	0.140	**0.055**	0.059	0.142	0.296	0.504

Bold numeric is the empirical type I error rates.

Our interest was in testing the population CV of the hourly PM2.5 levels. Suppose the researcher wanted to test the claim that a population CV equals 0.35. The null and alternative hypotheses are respectively given as follows:

$$H_0 : \theta = 0.35$$
$$H_1 : \theta \neq 0.35.$$

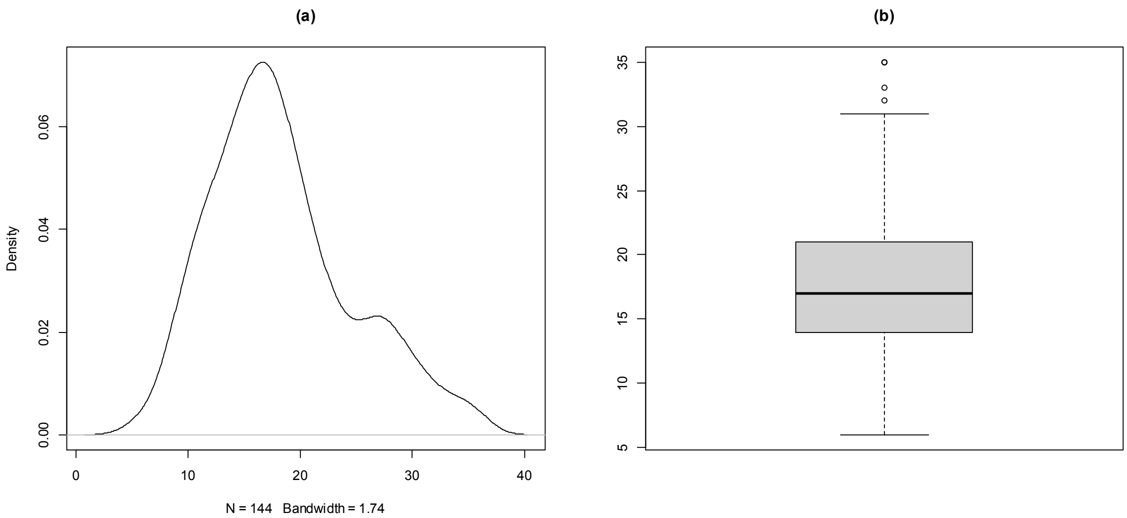

Figure 2. (a) density plot (b) Box and Whisker plot of the hourly PM2.5 levels from the Hat Yai station.

The lower and upper critical values of both test statistics were shown in Table 7. The null hypothesis H_0 was not rejected since $0.3020 \le \theta_0 \le 0.3820$ and $0.3104 \le \theta_0 \le 0.3927$ using test statistics based on the Score and Wald methods, respectively. We conclude that the population CV of the hourly PM2.5 levels does not differ from 0.35 at the 0.05 significance level.

Table 7. Critical values of test statistic based on the Score and Wald methods for the significance level of 0.05

Method	Critical values	
	Lower	Upper
Score	0.3020	0.3820
Wald	0.3104	0.3927

Discussion

The aim of this study is to identify potential methods that can be recommended to practitioners for testing the CV in a gamma distribution. From the simulation results presented in Tables 1-6, it is evident that the Wald method performed better than the Score method in terms of the empirical type I error rate. A general pattern was observed (as expected); as the sample size increased, the power of the test also increased and the empirical type I error rates approached 0.05. Moreover, the power increased as the value of CV departed from the hypothesized value of the CV. It can be observed that for large sample sizes, the performance of these methods did not differ greatly in terms of the power and attaining the nominal size of the test. Nevertheless, a significant difference was observed for small sample sizes.

Conclusions

In this study, two statistical methods for testing the population CV in a gamma distribution were derived. Since a theoretical comparison is not possible, a simulation study was conducted to compare the performance of these methods. Based on the simulation results, the Wald method performed better than the Score method in terms of the empirical type I error rate. The Score method performed well in the sense of the power of the test when the population CV was greater than the hypothesized value of the CV. On the other hand, the Wald method performed better when the population CV was smaller than the hypothesized value of the CV. In summary, we would recommend the Wald method for testing since its empirical type I error rate is close to the nominal significance level.

Acknowledgements

The author is grateful to the anonymous referees for their constructive comments and suggestions, which have significantly enhanced the quality and presentation of this paper. The author would also like to acknowledge the support provided by the Pollution Control Department, Ministry of Natural Resources and Environment, Thailand, in supplying the hourly PM2.5 levels.

References

Albatineh, A.N., Boubakari, I., & Kibra, B.M.G. (2017). New confidence interval estimator of the signal-to-noise ratio based on asymptotic sampling distribution, *Communication in Statistics-Theory and Methods*. 46(2): 574-590.

Albatineh, A.N., Kibria, B.M.G., Wilcox, M.L., & Zogheib, B. (2014). Confidence interval estimation for the population coefficient of variation using ranked set sampling, *Journal of Applied Statistics*. 41(4): 733-751.

Bakowskia, A., Radziszewskia, L., & Žmindak, M. (2017). Analysis of the coefficient of variation for injection pressure in a compression ignition engine, *Procedia Engineering*. 177: 297-302.

Bedeian, A.G., & Mossholder, K.W. (2000). On the use of the coefficient of variation as a measure of diversity, *Organizational Research Methods*. 3(3): 285-297.

Calif, R., & Soubdhan, T. (2016). On the use of the coefficient of variation to measure spatial and temporal correlation of global solar radiation, *Renewable Energy*. 88: 192-199.

Castagliola, P., Celano, G., & Psarakis, S. (2011). Monitoring the coefficient of variation using EWMA charts, *Journal of Quality Technology*. 43(3): 249-265.

Döring, T.F., & Reckling, M. (2018). Detecting global trends of cereal yield stability by adjusting the coefficient of variation, *European Journal of Agronomy*. 99: 30-36.

Faber, D.S., & Korn, H. (1991). Applicability of the coefficient of variation method for analyzing synaptic plasticity, *Biophysical Journal*. 60(5): 1288-1294.

Ihaka, R., & Gentleman, R. (1996). R: A Language for Data Analysis and Graphics. *Journal of Computational and Graphical Statistics*. 5(3): 299-314.

Kang, C.W., Lee, M.S., Seong, Y.J., & Hawkins, D.M. (2007). A control chart for the coefficient of variation, *Journal of Quality Technology*. 39(2): 151-158.

McKay, A.T. (1932). Distribution of the coefficient of variation and the extended t distribution, *Journal of the Royal Statistics Society*. 95(4): 695-698.

McKenzie, J.D. (2004). *Minitab Student Release 14: Statistical Software for Education*. Boston: Pearson Addison-Wesley.

Nairy, K.S., & Rao, K.A. (2003). Tests of coefficients of variation of normal population, *Communication in Statistics-Simulation and Computation*. 32(3): 641-646.

Narita, D., Oanh, N.T.K., Sato, K., Huo, M., Permadi, D.A., Chi, N.N.H., Ratanajaratroj, T., & Pawarmart, I. (2019). Pollution characteristics and policy actions on fine particulate matter in a growing Asian economy: the case of Bangkok metropolitan region, *Atmosphere*. 10(5): 227.

Panichkitkosolkul, W. (2009). Improved confidence intervals for a coefficient of variation of a normal distribution, *Thailand Statistician*. 7(2): 193-199.

Reed, G.F., Lynn, F., & Meade, B.D. (2002). Use of coefficient of variation in assessing variability of quantitative assays, *Clinical and Diagnostic Laboratory Immunology*. 9(6): 1235-1239.

Sangnawakij, P., & Niwitpong, S. (2017). Confidence intervals for functions of coefficients of variation with bounded parameter spaces in two gamma distributions, *Songklanakarin Journal of Science and Technology*. 39(1): 27-39.

Vangel, M. G. (1996). Confidence intervals for a normal coefficient of variation, *The American Statistician*. 50(1): 21-26.

World Bank. (2016). *The cost of air pollution*. Washington: World Bank.

Xing, Y.F., Xu, Y.H., Shi, M.H., & Lian, Y.X. (2016). The impact of PM2.5 on the human respiratory system, *Journal of Thoracic Disease*. 18(1): E69-74.