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***Original research article***

# Optimal and Near-Optimal Inventory Policies with Shipping Cost and Planned Backorders

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**ABSTRACT**

This article extends the Economic Order Quantity with backorders (EOQB) model with a mode of oversea shipping cost, namely full container load (FCL), by introducing a constraint of containers' capacity limitation to the model. Exact algorithm is proposed for straightforwardly solving a single product EOQB with shipping cost. The illustrative examples show that the proposed method yields the optimal solutions without using any complicated mathematical calculations. The model of multi-product EOQB with shipping cost is also presented. It is more complicated than the first one. We make use of the method of Lagrange multipliers to solve it systematically. The acceptable near-optimal solutions can be found; the differences ranges from 0.08-4.96% when compared to those obtained from the Excel solver. However, the second proposed method needs a skilled analyst to solve it on a spreadsheet.

**Keywords**: Economic order quantity; Freight cost; Full container load; Multi-product

## 1. Introduction

his research project was motivated by an automotive parts dealer who imports products from overseas and then sells to retailers in its territory. The company (dealer) has selected full container load (FCL) mode instead of less than container load (LCL) because of the lower freight rate. Additionally, as the products are unique and there are contracts between the dealer and retailers, shortages are allowed and can be delivered later. Accordingly, the problem at hand incorporates planned backorders and FCL costs. In conclusion, the question is, how many items should be ordered from oversea when the cost of backorders and FCL are in the consideration?

Furthermore, as the company is a small business, the method of calculation must be simple, straightforward, and able to be calculated by a medium skill worker. Accordingly, this study proposes the method

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of EOQB with shipping cost calculation that is acceptable to the company.

The rest of this article is organized as follows. In section 2, previous works related to this research are summarized. In section 3, the proposed EOQ model for a single product is presented. The example problems are illustrated in this section as well. In section 4, the proposed EOQ model for multiple products is described. Numerical examples are also presented. Finally, the conclusion and the direction of future research are drawn in section 5.

## 2. Related Works

The economic order quantity (EOQ) model was proposed by Harris in 1913 [1]. It is the most fundamental of all subsequent inventory models. The assumptions of the basic EOQ model are as follows [2]:

1. The demand rate is known and is a constant $\delta$ units per year.
2. Shortages are not permitted.
3. There is no order lead time.
4. The cost includes: a) setup cost $K$ per order, b) holding cost at $h$ per unit held per year.

The cost function of $q$ (order quantity) is expressed as Eq. (2.1). The first term of the right hand side is the ordering cost and the second term is the holding cost. The curves of functions are expressed in Fig. 1.

$$TC(q) = \frac{K\delta}{q} + \frac{hq}{2} \tag{2.1}$$

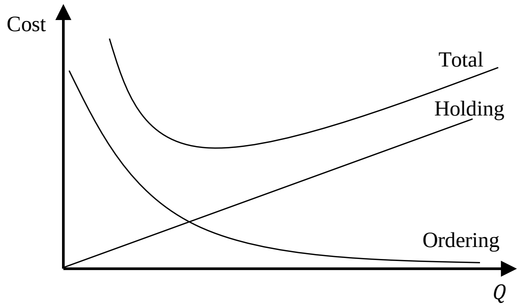


**Fig. 1.** Basic EOQ model cost functions.

In order to minimize the total cost, Eq. (2.2) and Eq. (2.3) are received by a classical calculus method.

$$q^* = \sqrt{\frac{2K\delta}{h}} \tag{2.2}$$

$$TC(q^*) = \sqrt{2K\delta h} \tag{2.3}$$

where $q^*$ is known as the economic order quantity ("economic" is just another word for "optimal" [3]).

The basic EOQ model is the most fundamental of all subsequent inventory models. The extension of the basic EOQ model relaxes some assumptions in order to comply with the realistic situations.

Among many extensions are the cases when shortage is permitted. If a customers is willing to wait for the late delivery at some penalty cost, it is the so-called backorder case. Besides, a customer may be not prepared to wait when shortage occurs. In this case, it is the so-called lost sales case [4].

Rezaei [5] considered the case of backorders which is the result of quality problem. The relaxation of infinite production rate is also in consideration. This makes filling up not instant. Consequently, the holding cost is affected by the model, see [2] and [3] for details.

As the topic of supply chain management is in the attention of the operation management research community, a number of extensions which incorporate transportation costs are published. Russell and Krajewski [6] proposed the case of less-than-truckload (LTL) cost included to the model. Burwell et al. [7] incorporated both all-unit discounts and freight discounts into the basic EOQ model. Swenseth and Godfrey [8] claimed that almost 50% of logistics cost is the transportation cost. Accordingly, the transportation cost should be accounted to the purchase plan of a company. The researchers added the freight rate function to the cost function and then solved it straightforwardly. However, the model did not explain the limit of transportation carriers.

Zhao et al. [9] accounted the restriction of vehicles' capacity into the model. They also considered the traveling time and its daily operating time in the model. Abad and Aggarwal [10] showed the flexibility of selling price and options of paying the freight. The reseller in this study was able to choose between less-than-full truckload and full-truckload shipments. Mendoza and Ventura [11] also proposed the model of EOQ with options of less-than-full truckload and full-truckload options. Furthermore, they also accounted all-unit discounts into the model. Rieksts and Ventura [12] considered two transportation modes, which were less-than-full truckload and full truckload choices. The authors also analyzed the option of the combination of two modes, which yielded more cost saving than only one mode consideration. The most relevant study to our research is the study of [9]. However, the key difference is that our model also accounts for backorder cost in the model. Furthermore, the approaching method is much easier than the existing one.

## 3. The Proposed EOQ Model for Single Product

### 3.1 Assumptions and Notation

The assumptions of this research are essentially the same as those of the basic EOQ model except for the backordered cost and FCL cost. To avoid any possible confusion, the assumptions and notation are described as follows [3]:

*Assumptions*

1. The demand is deterministic and is a constant rate. Furthermore, it is known in advance.
2. The lead time is zero—orders are received instantaneously.
3. The backorders are allowed—named *planned backorders*. However, the backorders are incurred with a penalty cost.
4. The ordering cost is fixed and does not depend on the order quantity.
5. The FCL cost is fixed per unit of container; in addition, its capacity is limited.

It is worth noting that the second assumption is not correct in a real situation. However, we would like to keep this assumption in this study. Practically, a practitioner may calculate a reorder point that can absorb the lead-time demand which he is facing.

Notation

$\delta$ = demand rate (units / year)
K= fixed ordering cost ($ / order)
q = order quantity (units / order)
y = fraction of demand that is backordered ($0 \le y < 1$)
f = fixed FCL cost ($ / container)
h = holding cost ($ / unit / year)
p = backorder penalty cost ($ / unit / year)
m = number of containers used per order (containers)
c = capacity of the container ( units / container)

### 3.2 The Model for Single Product

The total cost of the proposed EOQ model with planned backorders and FCL costs is expressed as Eq. (3.1) – Eq. (3.4) which we will call P1.

$$TC(q, y, m) = \frac{K\delta}{q} + \frac{hq(1-y)^2}{2} + \frac{pqy^2}{2} + \frac{mf\delta}{q} \quad (3.1)$$

Subject to

$$\frac{q}{c} \le m \quad (3.2)$$

$$m \in I^+ \quad (3.3)$$

$$0 \le y < 1, q \ge 0 \quad (3.4)$$

The total cost is the function of order quantity ( $q$ ) , fraction of demand that is backordered ($y$), and the number of deployed containers ($m$). As shown in Eq. (3.1), the first term of the right hand side is the ordering cost which depends on the number of orders per year, the second term expresses the holding cost per year, the third term is the penalty cost per year, and the last term is FCL cost per year. Furthermore, the model must be restricted by the size of container. Thus, the number of containers must not be less than the order quantity divided by the capacity of the container and it should be an integer, Eq. (3.2) and Eq. (3.3). Finally, the decision variable $y$ must be a fraction number and $q$ must be a nonnegative number, Eq. (3.4).

we would like to show the idea of inventory level under the planned backorder strategy, as shown in Fig. 2. The maximum inventory level is $q(1-y)$ units. The ordering cycle is $q/\delta$, as in the basic EOQ model. Accordingly, the area of a triangle above zero inventory level is given by $\frac{1}{2}\times q(1-y)\times q(1-y)/\delta$. Therefore, the average on- hand inventory is given by $\frac{\frac{1}{2}q^2(1-y)^2}{\delta}\times\frac{\delta}{q}=\frac{q(1-y)^2}{2}$, as shown in Eq. (3.1) . Similarly, each triangle under zero inventory level is given by $\frac{1}{2}\times qy\times\frac{qy}{\delta}=\frac{q^2y^2}{2\delta}$. Therefore, the average backorder is given by $\frac{q^2y^2}{2\delta}\times\frac{\delta}{q}=\frac{qy^2}{2}$, as shown in Eq. (3.1). See [3] for details.

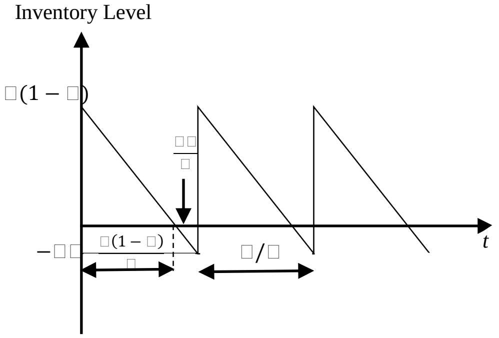


**Fig. 2.** Inventory level for EOQB.

In order to obtain the optimal decision variables, we take the partial derivatives with respect to the decision variables and set them equal to zero.

$$\frac{\partial TC(q,y,m)}{\partial q}=-\frac{K\delta}{q^2}+\frac{h(1-y)^2}{2}+\frac{py^2}{2}-\frac{mf\delta}{q^2} \quad (3.5)$$

$$\frac{\partial TC(q,y,m)}{\partial y}=-hq(1-y)+pqy \quad (3.6)$$

$$\frac{\partial TC(q,y,m)}{\partial m}=\frac{f\delta}{q} \quad (3.7)$$

We first look at **(3.6)**

$$-hq(1-y)+pqy=0 \quad (3.8)$$

$$h(1-y)=py \quad (3.9)$$

$$y^*=\frac{h}{h+p} \quad (3.10)$$

It looks like the analysis of the EOQB model; the fraction of backordered demand **(**$y$**)** does not depend on $q$ and $m$**.** As a result, even if we choose suboptimal $q$ and $m$, the optimal $y$ to choose is still the same as Eq**. (3.10).** Next, we substitute $y^*$ of Eq**. (3.10)** into Eq**. (3.1).**

$$TC(q,y^*,m)=\frac{K\delta}{q}+\frac{hq}{2}\left(\frac{p}{h+p}\right)^2+\frac{pq}{2}\left(\frac{h}{h+p}\right)^2+\frac{mf\delta}{q} \quad (3.11)$$

$$\Rightarrow=\frac{K\delta}{q}+\frac{q}{2}\left(\frac{p^2h+h^2p}{(h+p)^2}\right)+\frac{mf\delta}{q}$$

$$TC(q,y^*,m)=\frac{hp}{h+p}\frac{q}{2}+\frac{[K+mf]\delta}{q} \quad (3.12)$$

We have received problem P2 instead of P1**.** The problem P2 is shown as Eq**. (3.13)** – Eq**. (3.16).**

$$\text{P2}: TC(q,y^*,m)=\frac{hp}{h+p}\frac{q}{2}+\frac{[K+mf]\delta}{q} \quad (3.13)$$

Subject to

$$\frac{q}{c} \leq m \tag{3.14}$$

$$m \in I^{+} \tag{3.15}$$

$$q \geq 0 \tag{3.16}$$

Problem P2 looks like the EOQB model's total cost function, see [3] for details. The difference is $m$ in problem P2 is a decision variable. Nevertheless, suppose $m$ is a given constant integer number, problem P2 is exactly the same form as the EOQB cost function. It is a phase 1 of the proposed algorithm; Eq. (3.14) is ignored for a while. The optimal $q$ can be obtained by taking the derivative with respect to $q$ and setting it equal to zero, as shown in Eq. (3.17).

$$\frac{\partial TC(q,y^*)_m}{\partial q} = \frac{hp}{h+p}\left(\frac{1}{2}\right) - \frac{[K+mf]\delta}{q^2} \tag{3.17}$$

Set Eq.(3.17) equals zero, then, find the function of $q$.

$$\frac{hp}{h+p}\left(\frac{1}{2}\right) - \frac{[K+mf]\delta}{q^2} = 0 \tag{3.18}$$

$$\frac{[K+mf]\delta}{q^2} = \frac{hp}{h+p}\left(\frac{1}{2}\right) \tag{3.19}$$

$$q^*_{EOQM} = \sqrt{\frac{2[K+mf]\delta(h+p)}{hp}} \tag{3.20}$$

$q^*_{EOQM}$ is the optimal order quantity of single-product EOQ model with shipping cost and planned backorders Next, we substitute $q^*_{EOQM}$ of Eq. (3.20) into Eq. (3.13). We receive the minimum cost at a given $m$ as Eq. (3.21).

$$TC(q^*,y^*)_m = \sqrt{\frac{2[K+mf]\delta hp}{h+p}} \tag{3.21}$$

The fixed ordering, holding, FCL, and total cost at a given $m$ as a function of $q$ can be shown as Fig. 3.

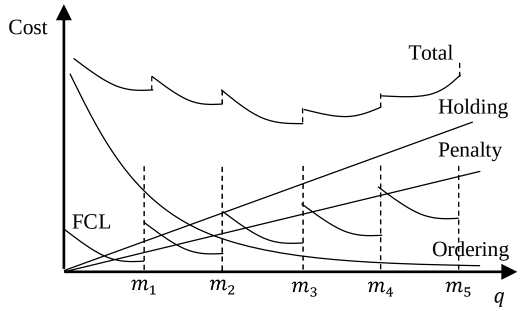


**Fig. 3.** Ordering, holding, FCL, and total cost functions.

The problem at hand right now is, how many containers should we use? Actually, P2 can be solved by any optimization solver which can deals with mixed-integer piecewise nonlinear optimization problem. However, we would like to develop a method that can solve this problem simply.

### 3.3 The Approaching Method

A trick of setting $m$ at a given constant integer number is a good choice. However, we need to confirm that at a given $m$, $TC(q,y^*)_m$ is a convex function. By taking the second derivative of Eq. (3.17). We receive a non-negative function as Eq. (3.22).

$$\frac{\partial^2 TC(q,y^*)_m}{\partial q^2} = \frac{[K+mf]\delta}{q^3} > 0 \tag{3.22}$$

As a result, $TC(q,y^*)_m$ is a convex function. Based on the above analysis, the following conclusions can be derived.

*Conclusion 1*. $TC(q,y^*)_m \geq TC(q,y^*)$. In other words, since FCL cost is added to the function, at any given $m$, the total cost is not less than the total cost of EOQB.

*Conclusion 2*. From the conclusion 1. $TC(q,y^*)_m = TC(q,y^*)$ when $f = 0$. We can observe from Eq.(3.20) and Eq. (3.21) at $f = 0$, the functions are the same as EOQB.

*Conclusion 3*. Lastly, it makes sense to order more than $q^*_{EOQB}$ of EOQB when we deal with FCL cost.

Based on the conclusions and clues we have, it does not make sense to set order quantity much more than $q^*_{EOQB}$ except changing $m$ by adding one more number; because the total cost incurs the FCL cost one more time, unreasonably.

**Theorem 3.1** For $TC(q, y^*, m)$ the optimal $q^*_{EOQM}$ lies between $q^*_{EOQB}$ and $mc$ where $m$ equals $\left\lceil \frac{q^*_{EOQB}}{c} \right\rceil$.

**Proof.** As shown in Eq. (3.20) and Eq. (3.21) , suppose $f = 0$, the minimum cost occurs at $q^*_{EOQB}$. Thus, when $f > 0$, it is reasonable to order more than $q^*_{EOQB}$. This means that $q^*_{EOQM}$ of P2 will not occur at any point $a$ where $a < q^*_{EOQB}$. Furthermore, at a given $m$, Eq. (3.20) and Eq. (3.21) never decrease when $m \leftarrow m + 1$. Keep in mind that $m$ is the ceiling of $q^*_{EOQB}/c$. They are non-decreasing functions of $m$. Thus, we can find the lowest of $TC(q, y^*, m)$ from $q^*_{EOQM}$ where $q^*_{EOQB} \le q^*_{EOQM} \le mc$, and $m = \left\lceil \frac{q^*_{EOQB}}{c} \right\rceil$. ■

The method proposed here is sequenced as follows.

*Step 1*. Find $q^*_{EOQB}$ form the EOQB model.

*Step 2*. Set $m^* = \left\lceil \frac{q^*_{EOQB}}{c} \right\rceil$.

*Step 3*. Substitute $m^*$ into Eq. (3.20).

*Step 4*. If $q^*_{EOQM} \le cm^*$; then, $q^*_{EOQM}$ and $m^*$ are reported. Go to step 6.

*Step 5*. If $q^*_{EOQM} > cm^*$; then, $q^*_{EOQM} = cm^*$. Next, $q^*_{EOQM}$ and $m^*$ are reported. Go to step 6.

*Step 6*. $TC(q^*, y^*, m^*)$ is calculated by using Eq. (3.21).

### 3.4 The Numerical Examples

Two examples are given in this subsection in order to verify the proposed method. In our first example, we will show the impact of $f$ which imputes the order quantity to be more than $q^*_{EOQB}$. In our second example, we will show that, with a relatively high $f$, the minimum total cost will never be more than $m^*$ containers. Additionally, we will compare the basic EOQ results, the EOQB results, and the results from the optimization solver (Excel solver), which is deployed to solve problem P2.

*Example 1:* Suppose that $\delta = 100$, $K = 25$, $h = 5$, $p = 10$, $f = 3$, $c = 25$. Accordingly, to the basic EOQ and EOQB formulae, we find

$$q^*_{EOQ} = \sqrt{\frac{2K\delta}{h}} = \sqrt{\frac{2\times25\times100}{5}} \cong 32$$

$$q^*_{EOQB} = \sqrt{\frac{2K\delta(h+p)}{hp}} = \sqrt{\frac{2\times25\times100\times(5+10)}{5\times10}} \cong 39$$

However, as Eq. (3.20) of this study, we find

$$m^* = \left\lceil \frac{q^*_{EOQB}}{c} \right\rceil = \left\lceil \frac{39}{25} \right\rceil = 2$$

$$q^*_{EOQM} = \sqrt{\frac{2[25+2\times3]\times100\times(5+10)}{5\times10}} \cong 43$$

Based on the total cost function Eq. (3.21), the total cost of all alternatives are shown in Table 1.

**Table 1.** Comparison of optimal order quantity for example 1.

| Model | $q^*$ | Total cost (based on P2) |
|---|---|---|
| Basic EOQ | 32 | 150.21 |
| EOQB | 39 | 144.50 |
| Excel solver | 43 | 143.75 |
| This study | 43 | 143.75 |

From Table 1, the optimal order quantity of this study yields the minimum cost when the cost of FCL is accounted. In addition, the solution of this study is the same as the Excel solver's solution.

*Example 2:* Suppose that $\delta = 100$, $K = 25$, $h = 5$, $p = 10$, $f = 20$, $c = 25$. Example 2 has changed only $f$ and it has not affected $q^*_{EOQ}$ and $q^*_{EOQB}$; thus, they are the same at 32 and 39 units, respectively. Likewise, $m^* = 2$ as in example 1. Nevertheless, as Eq. (3.20) of this study, we find

$$q^*_{EOQM} = \sqrt{\frac{2[25+2\times20]\times100\times(5+10)}{5\times10}} \cong 63$$

As described in step 5 of the proposed method, $q^*_{EOQM} > cm^* = 25 \times 2 = 50$. We will set $q^*_{EOQM} = cm^* = 50$ units, instead. The total costs of all alternatives are shown in Table 2.

**Table 2.** Comparison of optimal order quantity for example 2.

| Model | $q^*$ | Total cost (based on P2) |
|---|---|---|
| Basic EOQ | 32 | 256.56 |
| EOQB | 39 | 231.67 |
| Excel solver | 50 | 213.33 |
| This study | 50 | 213.33 |

From Table 2, the optimal order quantity of this study yields the minimum cost when the cost of FCL is accounted. Again, its solution is the same as the Excel solver's solution.

Alternatively, if we would like to order the same quantity as the first calculated $q^*_{EOQM}$, 63 units per order, we have to use 3 containers. As a result, the total cost is

$$TC(q, y^*, m) = \frac{hp}{h+p}\frac{q}{2} + \frac{[K+mf]\delta}{q}$$

$$= \frac{5\times10}{5+10}\left(\frac{63}{2}\right) + \frac{[25+3\times20]\times100}{63}$$

$$\cong 240$$

As can be seen, it is much higher than the result from the proposed method.

## 4. The Proposed EOQ Model for Multiple Products

In this section, the model of multiple products is proposed. We made use of common number of orders per year concept as described in [13].

### 4.1 Notation

$\mathcal{N}$ = set of product ($i = 1, .., n$)
$\delta_i$ = demand rate of product $i$ (units / year)
$K$ = fixed ordering cost ($ / order)
$q_i$ = order quantity of product $i$ (units / order)
$y_i$ = fraction of demand that is backordered ($0 \le y_i < 1$)
$f$ = fixed FCL cost ($ / container)
$h_i$ = holding cost of product $i$ ($ / unit / year)
$p_i$ = backorder penalty cost of product $i$ ($ / unit / year)
$m$ = number of containers used per order (containers)
$c$ = capacity of the container (units / container)
$v_i$ = volume of a unit of product $i$, $i \in \mathcal{N}$

### 4.2 The Model for Multiple Products

The total cost of the proposed EOQ model with planned backorders and FCL costs is expressed as Eq. (4.1) – Eq. (4.4) which we will call P3.

$$\text{P3: } TC(q_i, y^*, m) = \sum_{i=1}^{n} \frac{h_i p_i q_i}{2(h_i+p_i)} + [K + mf]\frac{\sum_{i=1}^{n}\delta_i}{\sum_{i=1}^{n} q_i} \qquad (4.1)$$

Subject to

$$\sum_{i=1}^{n} v_i q_i \le mc \qquad (4.2)$$

$$m \in I^+ \qquad (4.3)$$

$$q_i \ge 0, i \in \mathcal{N} \qquad (4.4)$$

In the case of P3, we introduce the Lagrange multiplier $\lambda$, and the problem P4 is now to find $q_1, q_2, \dots, q_n$, and $\lambda$ to solve the unconstrained problem:

$$\text{P4: } L = \sum_{i=1}^{n} \frac{h_i p_i q_i}{2(h_i+p_i)} + [K + mf] \frac{\sum_{i=1}^{n} \delta_i}{\sum_{i=1}^{n} q_i} + \lambda\left[\sum_{i=1}^{n} v_i q_i - mc\right] \quad (4.5)$$

Necessary conditions for optimality are that

$$\frac{\partial L}{\partial q_i} = 0, \quad \forall i \in \mathcal{N}$$

$$\frac{\partial L}{\partial m} = 0$$

and

$$\frac{\partial L}{\partial \lambda} = 0$$

The first $n$ conditions give

$$\frac{1}{2}\frac{h_i p_i}{h_i + p_i} - \frac{\delta_i}{q_i^2}[K + mf] + \lambda v_i = 0$$

Rearranging terms, we get

$$q_i^* = \sqrt{\frac{2\delta_i[K+mf]}{\frac{h_i p_i}{h_i+p_i} + 2\lambda v_i}} \quad (4.6)$$

and we also have the conditions,

$$f\frac{\sum_{i=1}^{n} \delta_i}{\sum_{i=1}^{n} q_i} = \lambda c \quad (4.7)$$

$$\sum_{i=1}^{n} v_i q_i = mc \quad (4.8)$$

Please note that we have known that $N = \sum \delta_i / \sum q_i$, $M = \sum v_i \delta_i / c$, and $m = M/N$ ($N$ is the number of orders per year, $M$ is the number of containers per year). There are a number of methods to solve this kind of problem, such as the KKT conditions and subgradient optimization [14, 15].

### 4.3 The Approaching Method

The method proposed here is sequenced as follows.

*Step 1*. Calculate $q_i, \forall i \in \mathcal{N}$ by using the method for single product as in section 3. Set $t = 1$.

*Step 2*. Calculate $\lambda^t$, by using Eq. (4.7)

*Step 3* Calculate $m$ where $m = \frac{\sum q_i}{\sum \delta_i} \frac{\sum v_i \delta_i}{c}$.

*Step 4*. Calculate $q_i, \forall i \in \mathcal{N}$ by using Eq.(4.6).

*Step 5*. If $\sum_{i=1}^{n} v_i q_i^* \le mc$; then, $q_i^*$ and $m$ are reported. Go to step 8.

*Step 6*. If $\sum_{i=1}^{n} v_i q_i^* > mc$; then, $\lambda^{t+1} = \lambda^t + \Delta$ where

$$\Delta = \frac{\alpha \lambda^t}{\sqrt{\left(\sum_{i=1}^{n} v_i q_i - mc\right)}}$$

where $\alpha = \frac{\sum_{i=1}^{n} v_i \delta_i}{mc}$
and $t = t + 1$.

*Step 7*. If $\sum_{i=1}^{n} v_i q_i^* \le mc$; then, $q_i^*$ and $m$ are reported and go to step 8. Else, go to step 6

*Step 8*. $TC(q_i, y^*, m)$ is calculated by using Eq. (4.1)

Please note that $\Delta$ of step 5 looks like the subgradient method (see [15] for details); however, it is much easier than the original one. Additionally, we do not guarantee a local optimal solution here.

### 4.4 The Numerical Examples

To evaluate the performance of the proposed method for the multi-product inventory problem, we randomly generated 12 benchmark problems. There were 3 problems for each $i = 2, 3, 4$, and 5. All parameters were random and uniformly distributed as follows: $\delta_i \in [50, 250]$, $h_i \in \{1, 2\}$, $p_i \in \{3, 4\}$, $v_i \in \{1, 2, 3\}$, and $c \in \{25, 50, 75\}$.

**Table 3.** Comparison of solutions for benchmark problems.

| Prob. No. | No. of products | Total cost (based on P3) | | | |
|---|---|---|---|---|---|
| | | Multi-product model | Excel solver | Single-product model | % Diff |
| 1 | 2 | 619.64 | 617.32 | 705.19 | 0.35 |
| 2 | 2 | 589.63 | 589.15 | 632.23 | 0.08 |
| 3 | 2 | 673.72 | 658.33 | 700.54 | 2.31 |
| 4 | 3 | 1021.98 | 1013.15 | 1096.75 | 0.86 |
| 5 | 3 | 963.55 | 958.04 | 1041.39 | 0.57 |
| 6 | 3 | 927.47 | 915.89 | 1087.69 | 1.25 |
| 7 | 4 | 1031.34 | 1018.29 | 1193.77 | 1.27 |
| 8 | 4 | 1474.99 | 1470.15 | 1513.36 | 0.32 |
| 9 | 4 | 1218.46 | 1198.03 | 1512.43 | 1.69 |
| 10 | 5 | 1810.94 | 1787.41 | 2015.99 | 1.30 |
| 11 | 5 | 1772.21 | 1768.79 | 2003.70 | 0.19 |
| 12 | 5 | 1694.55 | 1612.48 | 2017.59 | 4.96 |

Table 3 shows the results for the benchmark problems. Column 2 indicates the problem size by the number of products. Column 3 shows the results of the proposed method in section 4.3 (for multi-product problems). Column 4 gives the results from Excel solver; and, the fifth column is the results from the proposed method of single product model in section 3.3 The last column is the percentage difference between two solutions which can be calculated by using Eq. (4.9).

$$\% \, Difference = \frac{|TC_1 - TC_2|}{(TC_1 + TC_2)/2} \times 100 \qquad (4.9)$$

where $TC_1$ is total cost of the proposed method (for multi-product problems) and $TC_2$ is total cost of the Excel solver.

From Table 3, the proposed method yields acceptable solutions when compared to the results from Excel solver. The percentage difference ranges from 0.08-4.96. Fig. 4 shows the average percentage difference of the different number of products.

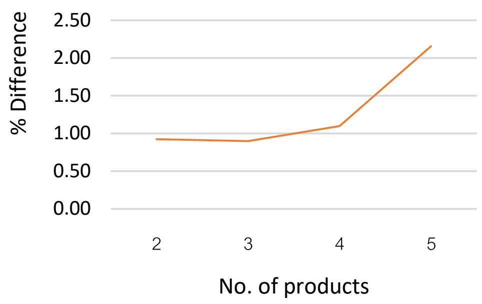


**Fig. 4.** Average percentage difference of the benchmark problem set

From Fig. 4, it shows that the proposed method works well on small number of products. The average percentage difference is 0.92 when there are two products in the model. However, when the number of products is 5, the average percentage difference is up to 2.15.

Table 3 also shows that we can save some money by using the multi-product model instead of the single-product model. As is shown, for all problems, the total costs of the multi-product model is lower than the total costs of the single-product model. Accordingly, we can have benefit by ordering them in multiple-product ordering policy.

## 5. Conclusion

The contribution and importance of this study is to simplify the problem of economic order quantity calculation in which the costs of planned backorders and full container load are included. The model of single product ordering EOQ was proposed. It is a constrained optimization problem; however, we proposed a simple method to determine the optimal solution. The new method yielded optimal solutions comparable to Excel solver's solutions. Next, we considered the model of multiple products. The Lagrange multiplier and subgradient - like method were introduced. Theoretically, the proposed method could not yield the optimal solution; however, it yielded acceptable near- optimal solutions. Furthermore, we showed that the multi-product ordering model yielded lower total costs than the single-product ordering model. The company could gain the benefit from this clue if the company's fixed ordering cost is relatively high. It is worth noting that the proposed method for multi-product problems is quite complicated. We recommend to use a spreadsheet for the proposed algorithm.

In the future, we are interested extending this study by including some realistic situations; such as, the case of quantity discounts, slow- moving products, and the products with the risk of obsolescence.

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