

# Efficient Variable Air Volume Air Conditioning System Based on Fuzzy Logic Control for Buildings

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## Abstract

A variable air volume (VAV) system is highly preferred to be an energy efficient air conditioning scheme in modern heating, ventilation and air conditioning (HVAC) applications. Based on the energy consumption characteristics, VAV systems are fast replacing constant air volume (CAV) systems and are capable of maintaining the thermal comfort for varying load conditions. Fuzzy logic controllers (FLC) are highly preferred rather than conventional controllers since FLC exhibits reduced peak overshoot that is observed under transient conditions of the system. FLC has the capability of controlling the system precisely with the set points defined. This paper describes the thermal comfort and energy conservation potential of the VAV system utilizing a fuzzy logic controller (FLC) that enhances the system performance substantially. A simple VAV building model was developed and the energy utilization of the VAV system has been experimentally investigated. Input data for fuzzy logic are zone temperature and duct static pressure and the output is supply air fan speed. Experimental results show that the energy saving potential of the VAV system was 27% at part load conditions, compared with the CAV system. Experimental results express that the required thermal comfort was achieved using FLC.

**Keywords:** Energy conservation; Fuzzy logic controller, Thermal comfort, Transient response, VAV

## 1. Introduction

Constant air volume systems (CAV) consume more power as they run throughout the life span, without change in air volume even at part load conditions. A Variable air volume (VAV) air conditioning system operates by varying the supply air volume delivered into the conditioned, space by varying the fan speed and dampers angles. A considerable quantity of energy can be conserved. Conventional Proportional, Integral and Derivative (PID) controllers are used to control the air volume in VAV systems. The PID con-

trollers are sluggish in response and are characterized by high overshoot. They are also limited by SISO (single input single output). Fuzzy logic control overcomes the problem of high overshoot and has quicker response. FLC can operate with MIMO (multi input multi output). Precise control can be obtained using FLC. A numerical model was developed to simulate a space controller for a VAV system [1]. Studies on VAV air conditioning system applied to office buildings were developed that reveal the utilization of outdoor air to obtain building zone comfort, especially during part load conditions [2]. Research on

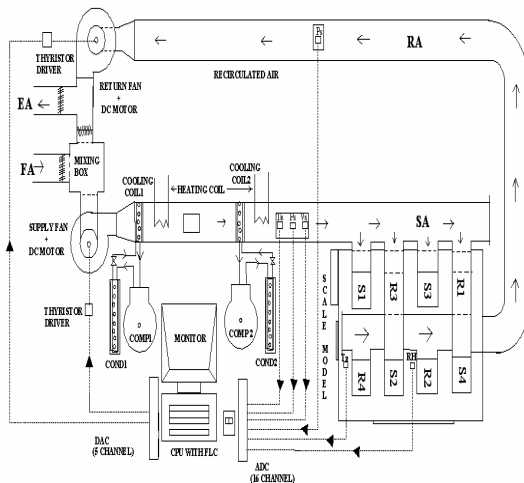
varying the airflow rate to individual zones of a building was done having control over a diffuser outlet, preventing excessive drafts that allow selecting a wide range of supply air temperatures [3]. PID controller tuning methods are useful in simulation and a tuning method for PID controller utilizing optimization subject to constraints on control input was developed [4]. A case study on variable frequency drives installed in an office building was presented that analyzed the fan input power consumption as a function of airflow rate supplied into the conditioned space [5]. The study also infers that upon having reduced supply duct static pressure, the energy savings potential can be increased. A notable work was done on a VAV system that utilized direct digital control (DDC) terminal boxes to achieve occupant comfort conditions, making use of a feedback control loop technique [6]. Energy conservation studies in VAV systems incorporated with variable speed drive (VSD) fan units was determined by modeling fan power as a function of outside temperature [7]. The performance curves obtained was used to estimate the energy savings of VSD over variable inlet vanes for the same air-handling units considered. The energy savings were calculated using a least square best-fit model technique. Control functions of energy management for effective control of HVAC systems in buildings have been developed that utilize a dynamic simulation technique [8]. Optimal strategy for outdoor air control using a system approach based on prediction to minimize energy consumption was developed using an ARMA model and the energy-increment equation was formed to involve the real-time variations of an air handling unit (AHU) load and energy use of reheaters of VAV terminals [9]. Based on EMCS control strategy, the dynamic models of VAV systems and building characteristics were developed and tested [10]. Feed-forward and feedback control strategies greatly influence the control strategies

of a controller. By means of a number of computer simulations based on an identified state-space model of a small building, it was shown that use of feed-forward control strategy reduce the energy use as well as improve the controller performance [11]. The Fuzzy logic control technique has a greater potential in heating, ventilation, air conditioning and refrigeration (HVAC&R) applications. Fuzzy logic control of compressor speed in refrigeration has been investigated for a cold store application [12]. Building thermal analysis was described using theoretical and experimental modeling that is based on a fuzzy model represented by non-linear relations between input and output variables obtained by least-squares optimization [13].

## 2. System description

A software laboratory located at Anna University, India is used to study the VAV system characteristics based on FLC for controlling the fan speed. The building dimensions are given as 33m x 8.5m x 2.9m. The building has seven windows on each side and a door with dimensions 0.91m x 1.83m and 0.91m x 2.13m. The construction materials and properties are selected according to the ASHRAE handbook. The zone has 45 computers on each side with an occupancy level of 95. Lighting loads are selected as per the ASHRAE standards. The load pattern is noted on an hourly basis and it accounts for the load variation with time. Depending upon the required criteria, a scale model is built in the refrigeration and air conditioning department laboratory, Anna University, India in 1:25 ratio with that of the original model. The system shown in Figure 1 is the scale model experimental set up for the real building. It consists of an air handling unit which incorporates two compressors each with 0.5 TR capacity, two cooling coils, steam cooker, two heating coils, DC motor with a circulating supply fan and a return fan, the

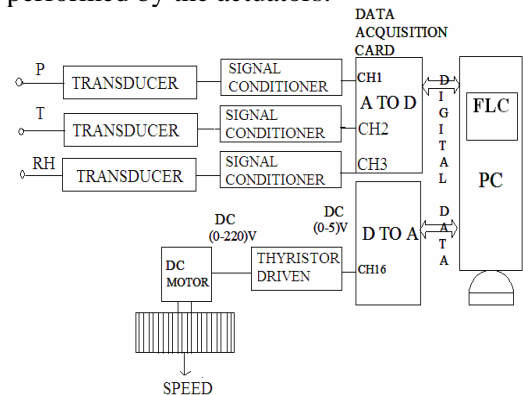
thermally insulated building space, connecting duct work, fresh, exhaust and return air dampers, mixing air components, thyristor based driver electronic circuitry (0-5 V DC) with signal conditioners, ADC (Analog to Digital converter), DAC (Digital to Analog Converter), PCI card, temperature sensor (Range: 0-100°C, Accuracy:  $\pm 0.5^\circ\text{C}$ ), Static pressure sensor, air velocity sensor, RH sensor and the computer to link the system with the fuzzy logic controller.



**Figure 1.** Single zone VAV system with FLC

Sensors are placed in the system to detect the temperature in the space, air velocity and the static pressure at the supply and the return duct. The analog signal from the sensors are digitized and fed to the FLC. The output of the controllers which happens to be the voltage to drive the fan is digital. A DAC card was used to convert the signal to an analog signal, and was fed to the thyristor based variable speed drive. The fuzzy controller interference circuit is shown in Figure 2. Interfacing of fuzzy controller is needed to obtain the required controlled operation of the VAV system. The signal from sensors located at various positions is processed by the fuzzy con-

troller and the desired control actions are performed by the actuators.



**Figure 2.** Fuzzy controller interface

### 3. Fuzzy logic controller design

The fuzzy set is characterized by a membership function whose value ranges from 0 to 1. The fuzzy logic controller consists of three steps of problem salvation.

#### 3.1 Fuzzification

#### 3.2 Linguistic description

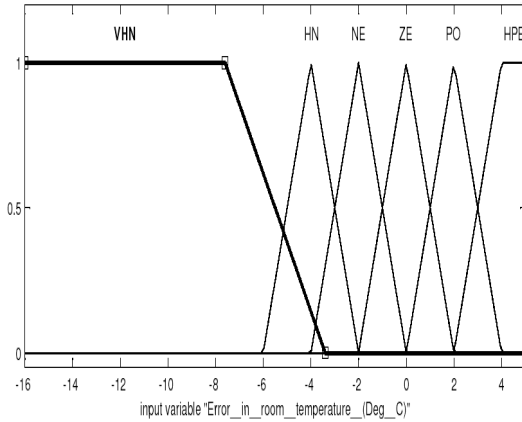
#### 3.3 Defuzzification

In The fuzzification step, input variables are defined in the form of membership functions. The input variables considered in this work are temperature and static pressure. The range of the membership functions is selected according to the maximum error and the change of error values which occurred. Therefore, these ranges are in the interval of  $[-16^\circ\text{C}, 4^\circ\text{C}]$  for error in temperature,  $[-16^\circ\text{C}, 4^\circ\text{C}]$  for change in error in temperature. For duct static pressure, the ranges are  $[0.30\text{kPa}, 0.70\text{kPa}]$  and for the change of fan speed control signal the ranges are  $[1400\text{ rpm}, 3000\text{ rpm}]$ . The input and output variables are represented in Figures 3-6 respectively. Based upon the input variables selected, human knowledge based rules are developed with respect to the required output. The defuzzification process transforms the fuzzy output into a defined value. The fuzzy results obtained from the fuzzy inference

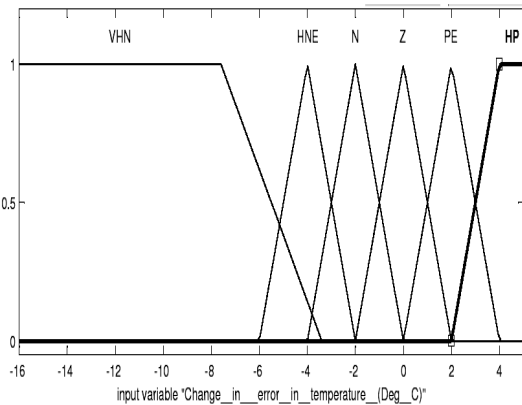
process are defuzzified and converted into meaningful crisp values. For this purpose, the centroid method, proposed by Mamdani is used and it is expressed as:

$$Z^* = \frac{\sum_i \mu_i(du) - (du)}{\sum_i \mu_i(du)} \quad (1)$$

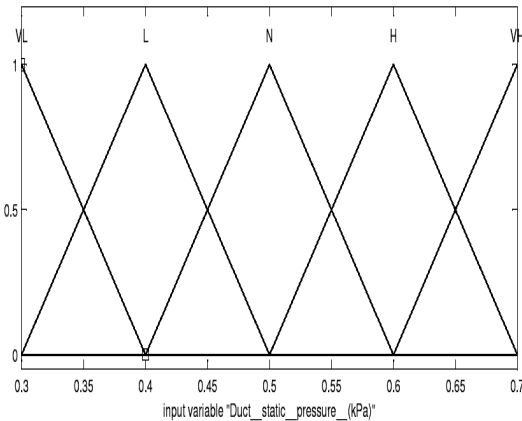
where  $\mu_i(du)$  is the membership degree of  $du$ .



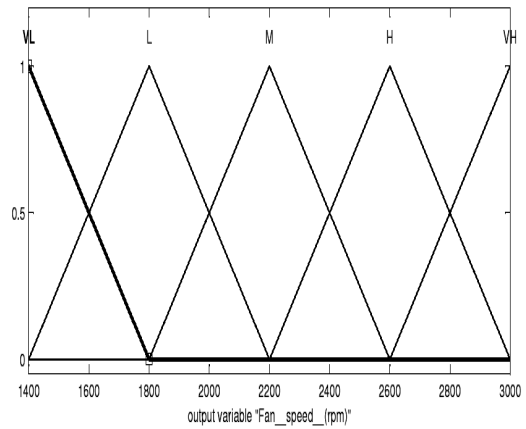
**Figure 3.** Error in temperature



**Figure 4.** Change error in temperature



**Figure 5.** Static pressure range



**Figure 6.** Fan speed variation

## 4. HVAC system simulation

The MATLAB-SIMULINK software package is used for the simulation. The outdoor temperature variation is taken as per the meteorological department for the month of May, since May has the maximum average temperature throughout the year. The outdoor temperature variation, occupancy load, equipment load and lighting load are represented in Figure 7. The load pattern is noted for the real building and the pattern is drawn in terms of percentage, so that the pattern can be followed for both building and scale models.

### 4.1. HVAC system and sensing system model

As described earlier, the HVAC system consists of duct, supply fan, cooling coil, thermostat etc. All these components can be modeled using a first order approximation with difference in time constants and gain.

#### 4.1.1. Duct model

The duct transfer function is modeled to be a simple time delay due to the transportation lag time, which is equal to the length of the duct divided by the air velocity. The duct time constant is usually less than 10s.

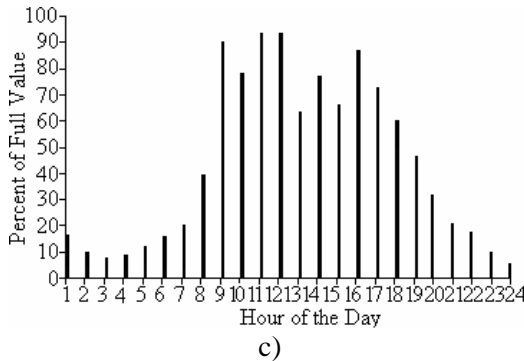
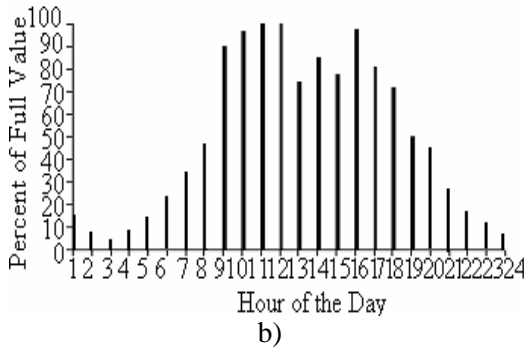
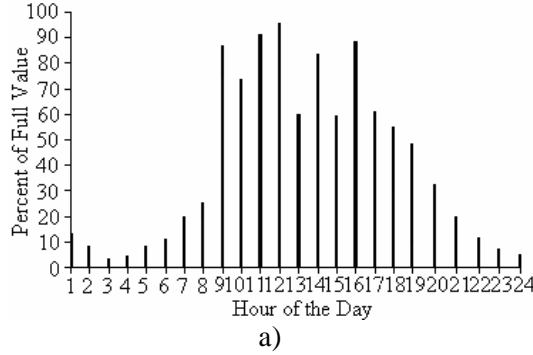
$$G_d(s) = e^{-sT_d} \quad (2)$$

#### 4.1.2. Fan model

The fan motor is an induction motor and the transfer function model of is given by:

$$G_f(s) = K / (1+sT) \quad (3)$$

The time constant for this is less than 5s.



**Figure7.** Inputs for the building thermal model (a) Occupancy (b) Light-ing (c) Equipment (d) Outside air temperature

#### 4.1.3. Temperature sensor model

The thermostat can be modeled as a first order transfer function.

$$G_t(s) = K_p / (1+sT) \quad (4)$$

where  $K_p$  is the proportional gain, and  $T$  is the time constant of thermostat.

#### 4.1.4. PID controller model

The transfer function of the proportional integral derivative controller is modeled as:

$$G_c(s) = K_p + (1/s) K_i + sK_d \quad (5)$$

The values for  $K_p$ ,  $K_i$ ,  $K_d$  are dependent upon the tuning method of the controller.

The building under study is rectangular in floor plan with dimensions of  $12.8 \times 27.8 \times 3.67 \text{ m}^3$  and six windows and one door with dimensions of  $0.91 \times 1.83 \text{ m}^2$  and  $0.91 \times 2.13 \text{ m}^2$ . The construction materials and properties are selected according to ASHRAE handbook (1990) [14]. The inputs for the state space model are internal load (occupancy, lighting, equipment) and outside temperature.

#### 4.2. Mathematical equations for building model

Mathematical Energy balance equations are framed for all the heat load components in the zones. The overall heat transfer coefficient,  $U$  and thermal capacitance,  $C$  is taken from the building standards as per ASHRAE. The area for the components wall, floor, ceiling and glass are calculated based on the dimensions for both the building and the scale model. All the energy balance equations are simplified in order to have the equations in matrix form. Thus, the formed matrix is framed in the form of state space notation. Other parameters in the equation are calculated using a cooling load calculation.

Energy Balance on Walls:

$$dT_w/dt = A_w/C_w[U_{wo}(T_{ao}-T_w) - U_{wi}(T_w - T_{ai})] \quad (6)$$

Energy Balance on Floor:

$$dT_f/dt = A_f/C_f[Q_s/A_f + U_f(T_{ai}-T_f)] \quad (7)$$

Energy balance on ceiling:

$$dT_c/dt = A_c/C_c[U_c(T_{ai}-T_c)] \quad (8)$$

Energy Balance on Indoor Air:

$$dT_{ai}/dt = 1/C_a[[Q_p+Q_i+(A_g U_g)(T_{ao}-T_{ai})]+[A_w U_{wi}(T_w-T_{ai}) + A_f U_f(T_f-T_{ai})]+[A_c U_c(T_c-T_{ai})]] \quad (9)$$

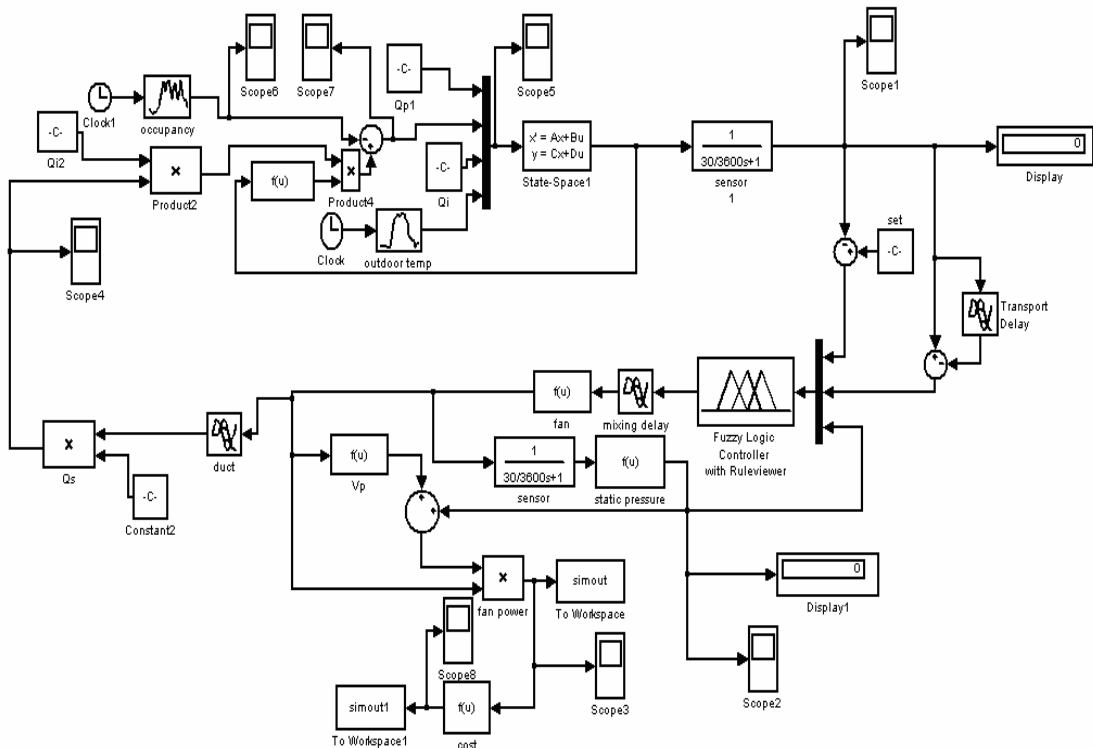
Using the STATE-SPACE notation:

$$dT/dt = AT + Bu \quad (10)$$

where  $A$ ,  $B$  are matrices of coefficients,  $u$  is the input vector,  $T$  is the matrix of temperatures. The inputs for the model are internal heat gain ( $Q_i$ ), solar heat gain ( $Q_s$ ), plant load ( $Q_p$ ) and outside air temperature ( $T_{ao}$ ). The output can be viewed either by scope or by gathering the output in the workspace in the MATLAB-SIMULINK software.

$$\begin{bmatrix} dT_w/dt \\ dT_f/dt \\ dT_c/dt \\ dT_{ai}/dt \end{bmatrix} = \begin{bmatrix} -A_w(U_{wi}+U_{wo})/C_w & 0 & 0 & A_w U_{wo}/C_w \\ 0 & -A_f U_f/C_f & 0 & A_f U_f/C_f \\ 0 & 0 & -A_c U_c/C_c & A_c U_c/C_c \\ A_w U_{wi}/C_a & A_f U_f/C_a & A_c U_c/C_a & -(1/C_a)(A_g U_g + A_w U_{wi} + A_f U_f + A_c U_c) \end{bmatrix} \begin{bmatrix} T_w \\ T_f \\ T_c \\ T_{ai} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & A_w U_{wo}/C_w \\ 0 & 0 & 1/C_f & 0 \\ 0 & 0 & 0 & 0 \\ 1/C_a & 1/C_a & 0 & A_g U_g/C_a \end{bmatrix} \begin{bmatrix} Q_p \\ Q_i \\ Q_s \\ T_{ao} \end{bmatrix} \quad (11)$$

The plant load and the supply temperature are assumed to be constant throughout the operation. Based on the outdoor temperature variation, occupancy load pattern and cooling load pattern, the simulation work is carried out. The schematic representation of the Simulink model of VAV system for the scale model developed is depicted in Figure 8.



**Figure 8.** Simulation of the model

## 5. Experimental methodology

The photographic view of the experimental set up is depicted in Figure 9. Experimentation is done with the scale model. The load is given in watts equivalent to the occupancy pattern in percentage on an hourly basis corresponding to the thermal load. The sensor output is fed to the corresponding interfacing unit with a PCI card. The analog signal is converted into a digital signal and is processed by the fuzzy logic controller. Corresponding to the input, output signals are regulated. The signal is given to the DAC where the analog signal from the interfacing unit is 0-5 V and it is amplified to 0-230 V and the output AC voltage is converted into 0-230 DC. The DC voltage is then fed to the thyristor driven variable speed drive and the varied voltage correspondingly varies the fan speed according to the load variation existing in the conditioned space.

## 6. Results and discussion

The inherent operational characteristics of the variable air volume air conditioning system equipped with FLC are discussed in this section. A series of experimental results are presented for the performance comparison of the VAV system in this section. The simulation of the HVAC system with building thermal model was carried out in the Matlab-Simulink environment. The tuning of the PID controller was done using classical control theory. The values for the proportional gain are  $K_p=2$ , integral gain  $K_i=0.01$ , and derivative gain  $K_d=0$ . The results are discussed under the following categories:

- 6.1 Transient Analysis
- 6.2 Steady-state Analysis
- 6.3 System performance in achieving thermal comfort

#### 6.4 Energy consumption comparison with the Two-Position, PID and Fuzzy controllers

### 6.1 Transient analysis

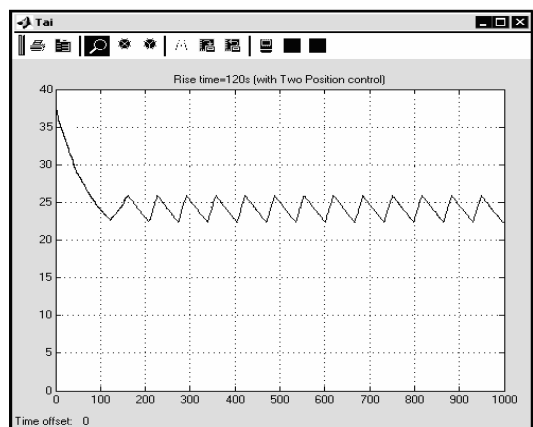
The transient behavior of the system for the three types of controllers is shown in Figures 10 - 12. Table 1 represents the transient response of the HVAC system with Two-Position, PID and Fuzzy controllers.



**Figure 9.** Photographic view of experimental set up

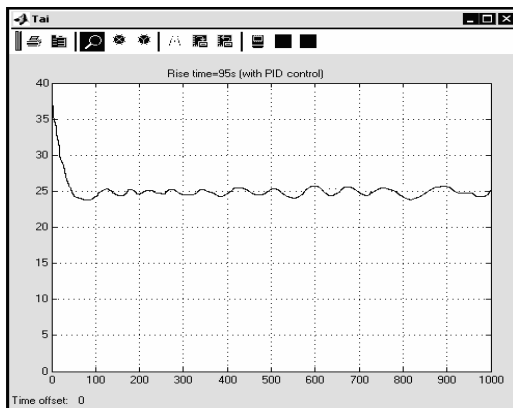
**Table 1.** Transient performance of Two-Position, PID and Fuzzy controllers

Control	Rise Time (s)	Peak overshoot (%)
Two-Position	120	5.42
PID	95	4.58
Fuzzy	55	0.83

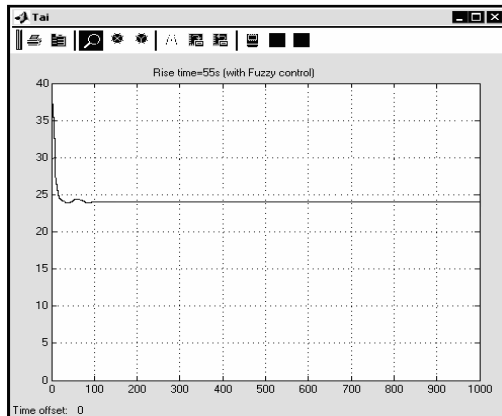


**Figure 10.** Transient response of the system with two-position controller





**Figure 11.** Transient response of the system with PID controller

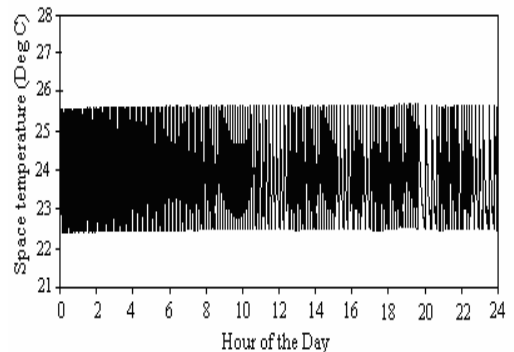


**Figure 12.** Transient response of the system with fuzzy controller

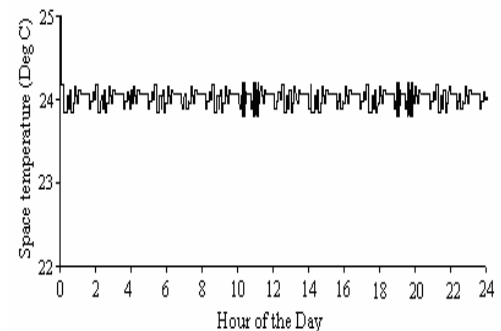
From Table 1 it is inferred that the rise time of the Fuzzy controller is 25s. This means that to attain the set point temperature inside the room, the Fuzzy controller takes 25s, whereas the Two-Position and PID controllers take 100s and 55s, respectively. Peak overshoot for the Fuzzy controller is 0.83% which is low which are compared to Two-Position and PID controllers of 5.42% and 4.58%, respectively. So, the Fuzzy controllers have very good transient performance compared to the other two conventional controllers.

## 6.2. Steady state analysis

The steady-state analysis is carried out for the system and the results obtained for the HVAC system are represented in Figures 13-15. Table 2 shows the steady-state response of the HVAC system with Two-Position, PID and Fuzzy controllers.



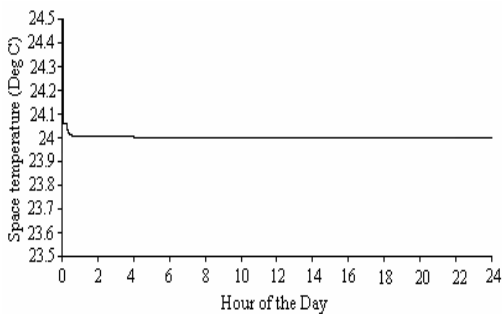
**Figure 13.** Steady state temperature variation inside room with two-position controller



**Figure 14.** Steady state temperature variation inside room with PID controller

From Table 2 it is observed that the steady-state error for the Fuzzy controller was zero. It means that even if the load inside the room changes, the Fuzzy controller maintains constant temperature, whereas the conventional controllers are unable to maintain the desired temperature inside the room. The value of IAE that is a measure of controller error for the three controllers is shown in Table

2. It is inferred that Fuzzy controllers have less error compared to conventional controllers.



**Figure 15.** Steady state temperature variation inside room with fuzzy controller

**Table 2.** Steady-state performance of Two-position, PID and Fuzzy controllers

Control	Steady-state error	IAE
Two-Position	Not settled	23.84
PID	Not settled	11.68
Fuzzy	0	2.32

### 6.3 System performance in achieving thermal comfort

The performance of the system can be observed by considering the space temperature in the conditioned space in the building zone during all working hours. The set point temperature inside the conditioned room is 24°C. The set point temperature is based on ASHRAE standard 55-1992. The onset and offset temperatures are 22.5°C and 25.5°C. This selection is based on ASHRAE recommended comfort zone for the summer season. The Table 3 shows the steady state temperature variation of the conditioned space with variation in internal load and outside weather temperature variation.

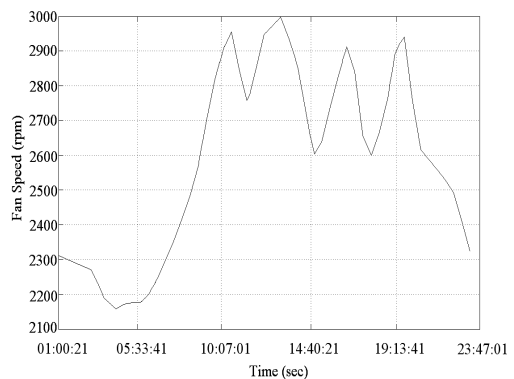
**Table 3.** Variation of space temperature with Two-Position, PID and Fuzzy controllers

Control	Temperature variation (°C)
Two-Position	22.2-25.5
PID	23.8-24.2
Fuzzy	24.0-24.05

Figures 13-15 represent the steady state temperature variation in the conditioned space with variation in internal load (occupancy, lighting, equipment) and outside air temperature. The space temperatures with two-position control fluctuate from 22.5°C to 25.5°C. With the PID control the variation of space temperatures are in between 23.8°C and 24.2°C, whereas the Fuzzy control maintains the temperature exactly at 24°C with fast response compared to the other two controllers.

### 6.4 Energy consumption comparison of two-position, PID and fuzzy controllers

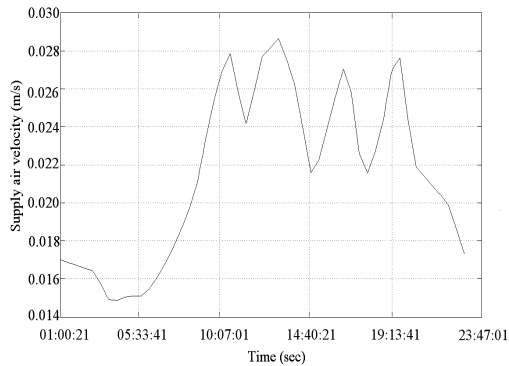
Figure 16 illustrates the experimental result of variation of fan speed obtained corresponding to the cooling load persisting in the conditioned space.



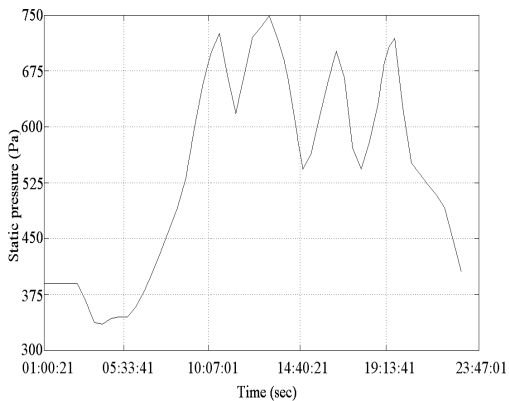
**Figure 16.** Variation of fan speed

The fan speed varies between 2150 rpm to 3000 rpm. The reason for having the supply air fan speed variation using a fuzzy logic controller is that for the proposed VAV A/C system, the fuzzy

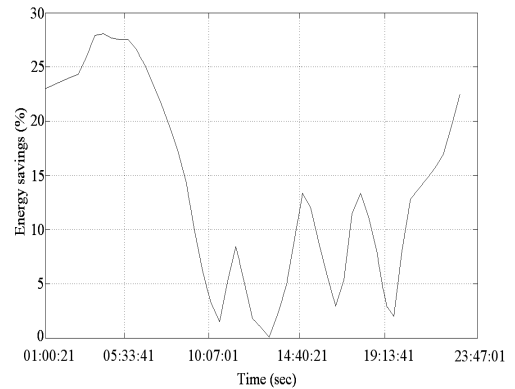
controller maintains the zone temperature around the set point precisely by delivering the actual required supply air quantity for the corresponding cooling load condition, and hence helps reduce energy spent for cooling the building zone substantially. Figure 17 and Figure 18 represents the variation of supply air velocity and duct static pressure relative to the change in cooling load existing inside the conditioned space respectively.



**Figure 17.** Variation of supply air velocity

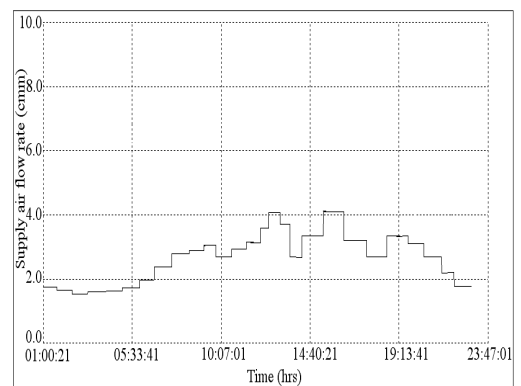


**Figure 18.** Variation of static pressure

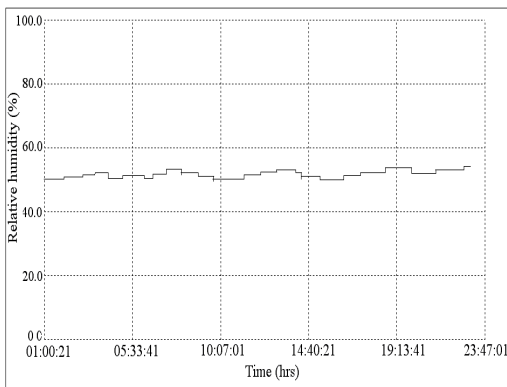


**Figure 19.** Energy savings vs time

The supply air velocity varies between 0.015m/s to 0.029m/s and the duct static pressure varies between 350Pa to 750Pa. Varying the supply air velocity for varying load conditions provides a better thermal comfort into the conditioned space. Having the fan speed control based on the duct static pressure enhances both the system performance and energy conservation. Also the static pressure in the duct is observed to be nominal. Based on the test result shown in Figure 19, for the VAV air conditioning system, the energy savings potential is expected to achieve 27%.



**Figure 20.** Supply airflow rate variation



**Figure 21.** Relative humidity variation

This yields a conclusion that compared to CAV systems, VAV systems possibly save a considerable quantity of energy and the system performance is also enhanced. By using the FLC technique, based on the experimental result, Figure 20 show that the supply airflow rate of the conditioned space is maintained precisely for varying load conditions occurring in the conditioned space.

The experimental result shown in Figure 21 represents the variation of relative humidity in the conditioned space. The relative humidity in the zone is also maintained within 50% to 55% which is obtained by the implementation of an effective fuzzy logic control strategy. Based on the experimental results, it is concluded that the required thermal comfort is achieved in the zone. As the fan speed is modulated based on the thermal load existing inside the zone, thermal energy wastage is avoided.

## 7. Conclusion

This paper describes the inherent operational characteristics of a VAV air conditioning system and presents the experimental results focused to obtain the thermal comfort and energy savings potential by utilizing a fuzzy logic control strategy. Fuzzy logic control is considered to be a promising control strategy that

enhances the system performance much better than conventional two-position and PID controllers. Fuzzy logic controllers under transient conditions have very low peak overshoot compared with than conventional controllers, which allows FLC to control the system effectively. Based on the experimental result, it is evident that by controlling the fan speed in accordance with the load, the fan power consumption is reduced considerably by about 27%, compared to a CAV air conditioning system. Since fan power consumption has a greater influence over total energy consumption, control of fan speed forms a promising strategy to enhance energy conservation. The experimental result suggests that by implementing a fuzzy logic controller, quick and precise control of room temperature, supply airflow rate, which is directly related to supply air velocity and relative humidity inside the conditioned space, are achieved. The thermal comfort conditions are maintained accurately by FLC when compared to conventional controllers. Thus FLC based VAV system ensures power saving without a compromise on thermal comfort and air distribution.

## 8. Acknowledgements

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