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Hierarchy Analysis Process Under Fuzzy Environment: A Positive Way of Selecting Stocks into Portfolio Under Ambiguity in Stock Information

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ABSTRACT

This study uses a hierarchy analysis process under a fuzzy environment to analyze financial factors for stock selection into a portfolio, especially when the decisive factors are too ambiguous to make a straightforward decision. The outcomes of this hierarchy analysis process are weights of each factor that can be used for stock analysis. The score assigned to a stock according to each factor is constructed from the past five years of financial information (2013-2017). When the current data of the interesting stocks are input and processed, the output will be an overall score that can be used to rank these stocks. An investor can select some stocks into his or her portfolio according to these scores. The stocks that are worth the investment will have a high overall score. This study includes an example of the application of this process to several stocks in Thailand's stock market (SET50) as a case study.

Keywords: Financial ratio; Fuzzy analytical hierarchy process (FAHP); Stock analysis; Stock exchange of Thailand

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1. Introduction

Investing in stocks can provide a good return on an investor's money. However, fluctuation in stock price entails a high risk of losing money. Investors should analyze stock information carefully in their selection of stocks into their portfolio in order to minimize the risk. A popular tool for stock analysis is the various financial ratios of the companies such as Return on Asset (ROA%), Return on Equity (ROE%), Price to Earnings Ratio(P/E), and Price to Book value Ratio (P/BV). The use of this tool, however, has an underlying issue. Each kind of financial ratio has certain advantages and disadvantages; hence, to rely on a single financial ratio for stock selection is not sensible. For this reason, investors usually use several financial ratios together to rank stocks which raises another issue: how to decide on an analysis result when the stock rankings from different financial ratios conflict among themselves. More specifically, if one wants to compare the performances between stocks from two different companies by considering several financial ratios, how can a comparison and a conclusion be made? On top of that, there are other factors involved in stock selection such as the industrial group that the stock belongs to as well as the reputation and stability of the company. Fortunately, all of these issues can be settled by employing an Analytical Hierarchy Process (AHP) in stock analysis.

AHP was invented and developed in 1980 by Saaty [11]. It is a technique that systematically analyzes different criteria or factors in order to reach a good decision [1]. The applied criteria or factors can be either qualitative or quantitative. Nevertheless, investors may encounter another issue when the variables of interest are language variables such as a stock with a high, moderate, or low P/E, the comparison and interpretation become obscure since the differences between variables are still unclear. Fuzzy set and fuzzy logic were developed to deal with vague defined variables and have been useful in different areas of application as in [12], [13], [14], [16]. Fuzzy techniques are also applied to improve AHP in [2], [6], [7], [9], [10]. We propose to tackle the issue of ambiguous and hard-to-interpret variables by applying fuzzy logic to AHP analysis.

In brief, this study associates the application of AHP with fuzzy logic to analyze various financial factors for properly selecting stocks into a portfolio. The conceptual framework of this study is shown in Fig. 1.

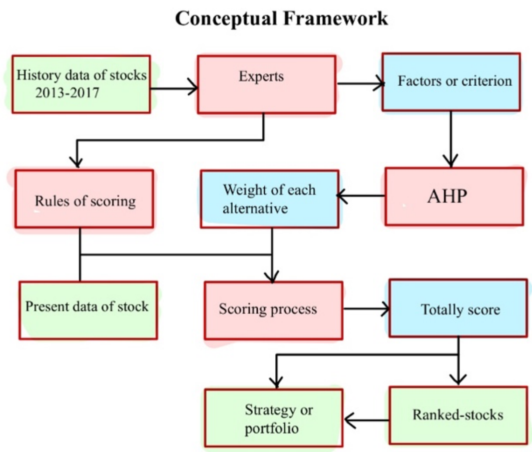

Fig. 1. The conceptual framework of the study.

2. Materials and Methods

This section provides essential concepts in fuzzy logic, Analysis Hierarchy Process (AHP), Fuzzy Analytic Hierarchy Process (FAHP), and financial ratio calculation.

2.1 Fuzzy Set and Fuzzy Relation

In 1965, Zadeh [15] proposed a definition of fuzzy set that is in harmony with the original set A (a crisp set) under a relative universe U by employing a characteristic function $\mu : U \to \{0,1\}$ where $\mu_A(x) = \begin{cases} 1\,; x \in A \\ 0\,; x \notin A \end{cases}$. Therefore, A can be expressed in terms of membership as $A=\{(x,\mu_A(x))\}$. According to this definition, $\mathcal{A}$ is a fuzzy set on the original set A where $\mathcal{A}=\{(x,\mu_A(x))\mid x\in A\}$, and $\mu_A(x):A\to[0,1]$ is called a membership function. In addition, if B is another original set, a fuzzy set $\tilde{\gamma}$ on set $A\times B$ with a membership function $\mu_{\tilde{\gamma}}:A\times B\to[0,1]$ is called a fuzzy relation from set A to set B.

2.2 Fuzzy Analytical Hierarchy Process

Analytical hierarchy process (AHP) is a decision-making process with capability of performing Multiple Criteria Decision Making (MCDM) [5], [8]. The criteria can be both qualitative and quantitative. Construction of a criteria hierarchy is shown in Fig. 2.

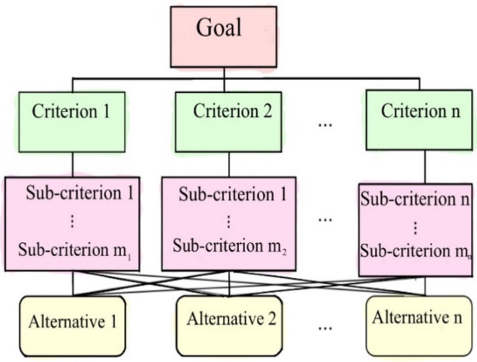

Fig. 2. The structure of a hierarchy of decisions.

The construction results in a reciprocal comparison matrix $A=\left[a_{ij}\right]_{n\times n}$ formulated from Saaty's preference levels with consistency property (i.e., there exists a weight vector $W=\langle w_1,\ldots,w_n\rangle$ where $\sum_{i=1}^{n}w_i=1$ and $a_{ij}=\frac{w_i}{w_j}$ for every $i,j\in\{1,\ldots,n\}$).

Saaty's preference levels are specified as $\frac{1}{9},\frac{1}{8},\ldots\frac{1}{2},1,2,\ldots,9$. The fuzzy analytical hierarchy process (FAHP) which is a multiple criteria decision-making process under ambiguous conditions is defined slightly differently from that of the original AHP by the fuzzy comparison matrix discussed in [3], [4] as follows:

Definition 2.1 Let $X=\{x_1,x_2,x_3,\ldots x_n\}$ and $\tilde{\gamma}$ be a fuzzy relation on $X\times X$ under a membership function $\mu_{\tilde{\gamma}}:X\times X\to[0,1]$, matrix $R=\left[r_{ij}\right]_{n\times n}$ is a fuzzy comparison matrix if $r_{ij}=\mu_{\tilde{\gamma}}(x_j,x_i)$ for every $i,j=1,\ldots,n$.

Definition 2.2 Let $R=\left[r_{ij}\right]_{n\times n}$ be a fuzzy comparison matrix, R is a Fuzzy-reciprocal matrix if $r_{ij}+r_{ji}=1$ for every $i,j=1,\ldots,n$.

Definition 2.3 Let $R=\left[r_{ij}\right]_{n\times n}$ be a fuzzy reciprocal matrix, R is a Fuzzy consistency matrix if there exists a weight vector $W=\langle w_1,w_2,w_3,\ldots w_n\rangle$ where $\sum_{i=1}^{n}w_i=1$ and $r_{ij}=\frac{w_i}{w_i+w_j}$ for every $i,j=1,..,n$, that is,

$$R=\begin{bmatrix}\frac{w_1}{w_1+w_1} & \frac{w_1}{w_1+w_2}\cdots & \frac{w_1}{w_1+w_n}\\ \frac{w_2}{w_2+w_1} & \frac{w_2}{w_2+w_2}\cdots & \frac{w_2}{w_2+w_n}\\ \vdots & \vdots & \vdots\\ \frac{w_1}{w_n+w_1} & \frac{w_n}{w_n+w_2}\cdots & \frac{w_n}{w_n+w_n}\end{bmatrix} \quad (2.1)$$

Theorem 2.4 [1] Let $R=\left[r_{ij}\right]_{n\times n}$ be a fuzzy reciprocal matrix, R is also a fuzzy consistency matrix if and only if

$$\frac{1}{r_{ij}}-1=\left(\frac{1}{r_{ik}}-1\right)\times\left(\frac{1}{r_{ki}}-1\right). \quad (2.2)$$

In this study, a fuzzy reciprocal matrix $R=\left[r_{ij}\right]_{n\times n}$ was constructed where the meanings of levels of preference in the matrix are as shown in Table 1.

Table 1. The meanings of the results from paired fuzzy comparison.

Level of significance r_{ij}	The meanings of the results from paired comparison
0.5	Equal significance
(0.5, 0.6)	Slightly more significance
[0.6, 0.7)	Significance
[0.7, 0.8)	Moderately more significant
[0.8 0.9)	Fairly more significance
[0.9, 1.0]	Most significance

2.3 Weight of Importance Calculation and Consistency Check

As a fuzzy reciprocal matrix $R=\left[r_{ij}\right]_{n\times n}$ has been constructed following Eq. (2.2), a weight vector $W=\langle w_1,\ldots,w_n\rangle$ can be constructed by

$$w_i=\frac{1}{n}\sum_{j=1}^{n}\frac{a_{ij}}{\sum_{k=1}^{n}a_{kj}} \text{ and } a_{ij}=\frac{r_{ij}}{1-r_{ij}}. \quad (2.3)$$

Since we want to construct a fuzzy comparison matrix that has a consistency property, we need to review the consistency of the fuzzy reciprocal matrix with a consistency index (CI) which is calculated by the following formula,

$$CI=\frac{\lambda_{\max}-n}{n-1} \quad (2.4)$$

where

$$\lambda_{\max}=\sum_{k=1}^{n}\left(w_k\sum_{i=1}^{n}a_{ik}\right) \quad a_{ij}=\frac{r_{ij}}{1-r_{ij}} \text{ and } R=[r_{ij}]_{n\times n}.$$

A fuzzy comparison matrix satisfies a consistency property if and only if $CI=0$. Nevertheless, the construction of a fuzzy comparison matrix with full consistency property of size 3 x 3 or higher is fairly difficult; therefore, a fuzzy matrix with near zero CI is adequate in practice and it is called an acceptable fuzzy consistency matrix. In general, a fuzzy consistency matrix is verified by using a consistency ratio (CR) which is calculated by the following formula,

$$CR=\frac{CI}{RI} \quad (2.5)$$

where RI is a random consistency index (some values of RI are shown in Table 2). A fuzzy consistency matrix is acceptable if the CR is less than 10 %.

2.4 Financial Ratios

This section describes the financial ratios used in this study: ROA (%), ROE (%), net profit margin (%), P/E (times), and P/BV (times).

Table 2. Random consistency indices of matrices of different sizes.

Matrix size	1	2	3	4	5	6	7	8	9	10
Random consistency index (RI)	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

Return on Asset (ROA) : ROA measures the profit-making ability of the total assets that are used for business operation. A high ROA value shows a high efficiency in asset usage. The ROA of a company should not be less than 5%. It is calculated by the formula below,

$$ROA(\%)=\frac{NP}{TA}\times 100, \quad (2.6)$$

where NP is net profit, and TA is Total assets which is calculated by

Total assets = stockholder's equity + debt (Equity + Liabilities).

Return on Equity (ROE): ROE shows how much the rate of return for the proprietor is from the assets that he or she put into the business. A high ROE indicates a good profit-making ability and a good business administration ability to give a good return to the proprietor and stockholders. It is related to stock price and calculated by the following formula,

$$ROE(\%) = \frac{NP}{E} \times 100, \quad (2.7)$$

where E is equity.

Net Profit Margin (NPM): this indicator shows the level of ability to make net profit at a particular time after taxes and expenses have been deducted. A high net profit margin indicates a high profit-making ability (especially after taxes and expenses have been deducted). It is calculated as follows,

$$NPM(\%) = \frac{NPT}{NS} \times 100 \quad (2.8)$$

where NPT is net profit after taxes and NS is net sales.

Price per Book value (P/ BV): P/BV is a ratio of the market price of a common stock to the accounting or book value of the stock stated in the latest financial report issued by the company. This ratio shows how many times higher (or lower) the stock price is at that time compared to its book value. A high P/BV indicates that general stock market investors expected that the company has a high growth potential (but also a high level of risk). It is calculated by the following formula,

$$P / BV = \frac{MP}{BV}, \quad (2.9)$$

where MP is market price of the stock and BV is book value of the stock.

Price to earnings ratio (P/E): this ratio is a ratio of the market price of a common stock to the net profit that it has generated in the latest annual cycle. This ratio indicates how many times over 1 baht of net profit that the company makes investors are willing to pay for the stock. It is calculated as follows,

$$P / E = \frac{MP}{PC}, \quad (2.10)$$

where PC is profit made from the stock in the latest annual cycle.

Dividend yield (D%): Dividend yield is an indicator that shows investors how much the percentage rate of dividend return is if they buy the stock at the current market price. It is calculated by the following formula,

$$D(\%) = \frac{DV}{MP} \times 100, \quad (2.11)$$

where DV is dividend per stock.

3. Results and a Case Study

3.1. Factor Assessment and Its Analytical Steps

This study uses a fuzzy analytic hierarchy process (FAHP) to assess factors or indicators that are used as criteria for stock analysis to select stocks into a portfolio. This process involves the following steps.

Step 1) An expert specifies the factors or indicators that will be used for stock selection into the portfolio which may be from sources such as a popularity poll or a factor analysis.

Step 2) The expert constructs a hierarchy of criteria consisting of criteria and sub-criteria.

Step 3) The expert compares pairs of criteria and pairs of sub-criteria of each criterion and constructs a fuzzy reciprocal matrix for the pairs of criteria and one for the pairs of sub-criteria.

Step 4) Calculate the weight vector according to Eq. (2.3)

Step 5) Check the consistency of the fuzzy reciprocal matrix from Step 3), i.e., calculate the consistency ratio (CR) according to Eq. (2.4) and (2.5). If CR< 10%, the matrix is considered acceptable and so proceed to the next step. If not, go back to Step 3).

Step 6) Calculate the final weight of each sub-criterion by a multiplication method,

Step 7) Design a stock scoring scheme according to the specified criteria.
Step 8) Input the information of the stocks to be analyzed and assigned scores.
Step 9) Ranks stocks according to the calculated scores. High ranking stocks that achieve acceptably high scores are selected into portfolio.

3.2 A Case Study: An Analysis of Stocks in Thailand's Stock Market

This section describes an analysis of stocks in Thailand's stock market, following the steps described in Section 3.

3.2.1 Design a stock scoring scheme

Step 1-2) Specifying factors and indicators as criteria for selection of stocks into portfolio. Here, the criteria and sub-criteria were specified according to their popularity in technical analysis for stock trading shown in Table 3. The hierarchical structure of these criteria is shown in Fig. 3.

Step 3) Assessing the criteria and sub-criteria, comparing them in pairs.

It is assumed that an expert has assessed both the criteria and sub-criteria and achieved a fuzzy reciprocal matrix of paired criteria and sub-criteria using a scale of [0,1] as shown in Table 4-9.

Table 3. Criteria and sub-criteria for stock analysis.

Criteria	Sub-criteria
Index group	SET50+SETHD, SET50-SETHD, SET100+SETHD, SET100-SETHD, SETHD-SET100
Financial ratio	ROE (%), ROA (%), Net profit margin (%)
Statistics	P/E, P/BV
Other benefits	Dividend yield (%), Price difference
Industrial group	PROPCON, FINCIAL, RESOUR, SURVICE, INDUS, CONSUMP, TECH, AGRO

Step 4) Calculating the weight vector of each criterion specified in Step 3)

Using Eq. (2.3), weight vectors were calculated. They are shown in the last column of Table 4-9.

Step 5) Checking the consistency of the fuzzy reciprocal matrix

The values were calculated according to Eq. (2.4) and (2.5). They are shown in the last row of Table 4-9. It can be seen that the value of every criterion was less than 10%, hence it was acceptable to use the comparison matrix further in the next step.

Step 6) Calculating the final weight of each sub-criterion by a multiplication method. The final weight would be the normal of the multiplied criterion and sub-criterion as shown in Table 10.

Step 7) Designing of stock scoring according to the specified criteria.

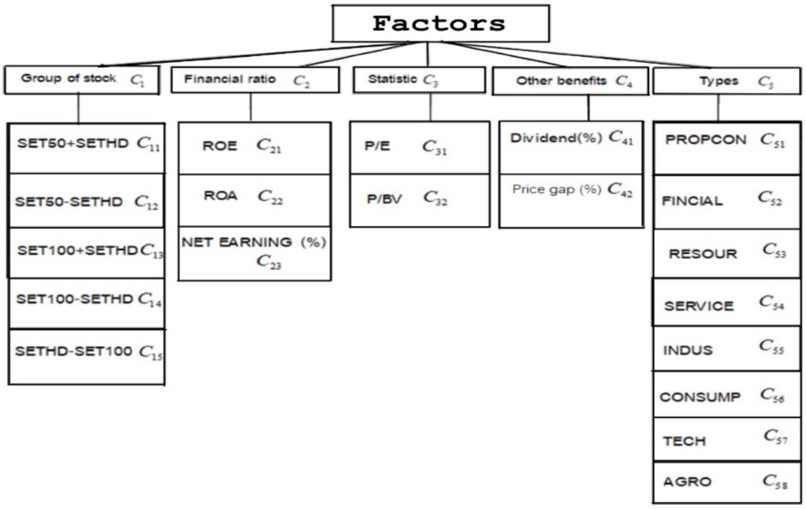

Fig. 3. The structure of a hierarchy of factor assessment of a stock analysis.

Table 4. Matrix of paired criteria comparison.

Criterion	C_1	C_2	C_3	C_4	C_5	W^C
C_1	0.5	0.6	0.8	0.75	0.9	0.44
C_2	0.40	0.5	0.65	0.55	0.8	0.23
C_3	0.2	0.35	0.5	0.4	0.67	0.11
C_4	0.25	0.45	0.60	0.5	0.8	0.17
C_5	0.1	0.2	0.33	0.2	0.5	0.05
					CR=	0.009265<0.1

Table 5. Matrix of paired sub-criteria comparison (of the index group).

Index group C_1	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	W^{C_1}
C_{11}	0.5	0.65	0.73	0.8	0.9	0.64
C_{12}	0.35	0.5	0.67	0.75	0.83	0.15
C_{13}	0.27	0.33	0.5	0.8	0.88	0.11
C_{14}	0.2	0.25	0.2	0.5	0.86	0.05
C_{15}	0.1	0.17	0.12	0.14	0.5	0.04
					CR=	0.0968713<0.1

Table 6. Matrix of paired sub-criteria comparison (of the financial ratios).

Financial ratio C_2	C_{21}	C_{22}	C_{23}	W^{C_2}
C_{21}	0.5	0.67	0.83	0.57
C_{22}	0.33	0.5	0.8	0.33
C_{23}	0.17	0.20	0.5	0.10
			CR=	0.032876<0.1

Table 7. Matrix of paired sub-criteria comparison (of the statistics).

Statistics C_3	C_{31}	C_{32}	W^{C_3}
C_{31}	0.50	0.67	0.67
C_{32}	0.33	0.50	0.33
		CR=	0<0.1

Table 8. Matrix of paired sub-criteria comparison (of the other benefits).

Other benefits C_4	C_{41}	C_{42}	W^{C_4}
C_{41}	0.50	0.20	0.2
C_{42}	0.80	0.50	0.8
		CR=	0<0.1

Table 9. Matrix of paired sub-criteria comparison (of the industrial group).

Industrial group C_5	C_{51}	C_{52}	C_{53}	C_{54}	C_{55}	C_{56}	C_{57}	C_{58}	W^{C_5}
C_{51}	0.5	0.67	0.75	0.8	0.83	0.86	0.88	0.89	0.33
C_{52}	0.33	0.5	0.67	0.75	0.8	0.83	0.86	0.88	0.23
C_{53}	0.25	0.33	0.5	0.67	0.75	0.8	0.83	0.86	0.16
C_{54}	0.2	0.25	0.33	0.5	0.67	0.75	0.8	0.83	0.11
C_{55}	0.17	0.2	0.25	0.33	0.5	0.67	0.75	0.8	0.07
C_{56}	0.14	0.17	0.2	0.25	0.33	0.5	0.67	0.75	0.05
C_{57}	0.12	0.14	0.17	0.2	0.25	0.33	0.5	0.8	0.04
C_{58}	0.11	0.12	0.14	0.17	0.2	0.25	0.2	0.5	0.02
								CR=	0.054559<1

Table 10. Final weights of sub-criteria.

Criteria	Sub-criteria	Final weight
C_1	C_{11}	0.1748533
	C_{12}	0.1085062
	C_{13}	0.0906828
	C_{14}	0.0466764
	C_{15}	0.0150397
C_2	C_{21}	0.1287391
	C_{22}	0.075618
	C_{23}	0.0225444
C_3	C_{31}	0.0750345
	C_{32}	0.0369573
C_4	C_{41}	0.0349
	C_{42}	0.1396002
C_5	C_{51}	0.0166396
	C_{52}	0.0115331
	C_{53}	0.0079076
	C_{54}	0.0054015
	C_{55}	0.0037008
	C_{56}	0.0024978
	C_{57}	0.0020185
	C_{58}	0.0011491
	Sum	1

3.2.2 Rules and Scoring Process

Criterion C_1 (indicator group)

If A was in any group, a score of 1 was assigned to it (it had to be in one and only one group), and the scores of the other groups would be 0.

Criterion C_2 (financial ratio group)

For each sub-criterion C_{21} (ROE), C_{22} (ROA) and C_{23} (net profit), a score was assigned according to the value of $C_{2i}, i = 1,2,3$ as shown in Table 11.

Criterion C_3 (statistics)

For each sub-criterion C_{31} (P/E) and C_{32} (P/BV), a score was assigned according to the value of C_{3i} , $i = 1,2$ as shown in Table 12.

Criterion C_4 (other benefits)

For each sub-criterion C_{41} (dividend rate) and C_{42} (profit from price difference), a score was assigned according to the value of C_{4i} , $i = 1,2$ as shown in Table 13.

Criterion C_5 (industrial group)

If stock A is in any of the industrial groups, a score of 1 was assigned (it had to be in one and only one group) while a score of 0 was assigned for all of the other groups.

Table 11. Specifications of the levels of ROE(%), ROA (%) and Net profit margin (%) and their assigned scores.

ROE(%)	[0,10)	[10,12)	[12,17)	[17,24)	[24,40]
ROA (%)	[0,3)	[3,7)	[7,10)	[10,13)	[13,30]
Net profit margin (%)	[0,4)	[4,9)	[9,19)	[19,26)	[26,40]
Level	Very low	Low	Medium	High	Very high
Score	0.2	0.4	0.6	0.8	1

Table 12. Specifications of the levels of P/E (times) and P/BV (times) and their assigned scores.

P/E (times)	[0,12)	[12,16)	[16,29)	[29,38)	[38,50]
P/BV (times)	[0,1)	[1,2)	[2,4)	[4,6)	[6,20]
Level	Very low	Low	Medium	High	Very high
Score	1	0.8	0.6	0.4	0.2

Table 13. Specifications of the levels of Dividend rate (%) and Profit from price difference (%) and their assigned scores.

Dividend rate (%)	[0,1)	[1,2)	[2,3)	[3,4)	[4,10]
Profit from price difference (%)	[0,5)	[5,8)	[8,15)	[15,21)	[21,40]
Level	Very low	Low	Medium	High	Very high
Score	0.2	0.4	0.6	0.8	1

Note: this study used the decile of all of the past-5-year SET50 stock information (2013-2017) for specifying the level of each sub-criterion C_{21}, C_{22}, C_{23}, C_{31}, C_{32}, C_{41}, and C_{42} as follows,

C_{ij}	$[0, D_2]$	$[D_2, D_4]$	$[D_4, D_6]$	$[D_6, D_8]$	$[D_8, b_{ij}]$
Level	Very low	Low	Medium	High	Very high

where D_n is the n^th^-decile and for some $b_{ij} > \max$ of each certain sub-criterion.

An example of the scoring scheme: the scoring of CPF stock is shown in Table 14. The calculated scores and ranks of all stocks in SET50 in the studied period are shown in Table 15.

We gave an example of stock portfolio allotment with 10 top-ranked stocks from Table 15, which is shown in column 4 of Table 16. We tested the performance of this procedure by using the portfolio in Table 16 with a buy-sell simulation. The period of holding was supposed to be approximately 1 year, which is shown in column 5-6 of Table 16. The percentage gains (or losses) are shown in column 7 and in the portfolio. The total percentage gain was about 29.84%.

4. Conclusion

This paper presents a mathematically sound way to perform stock selection into a portfolio under ambiguous stock information. We use a fuzzy analytical hierarchy process to calculate the weight of each factor of stock information. The factors analyzed in this work (shown in Table 3) were selected according to the authors' experience and consideration of their popularity in stock analysis, but in actual use, a user may need to add a factor selection step by doing a survey in combination with a statistical analysis of the relationships between factors such as a factor analysis. In this paper, the authors chose to use decile to specify the level of each sub-criterion—C_{21}, C_{22}, C_{23}, C_{31}, C_{32}, C_{41}, and C_{42}—to be very low, low, medium, high, and very high. A user may use other specifications of these levels. Moreover, the criteria comparison matrix and scoring scheme for the selected criteria should be carefully devised by an expert in the organization.

Table 14. An example of score calculation according to the specified criteria.

Sub-criteria	Imported data	Score	Final weight	Final score
C_{11}	1	1	0.1748533	0.1749
C_{12}	0	0	0.1085062	0.0000
C_{13}	0	0	0.0906828	0.0000
C_{14}	0	0	0.0466764	0.0000
C_{15}	0	0	0.0150397	0.0000
C_{21}	10.13	0.4	0.1287391	0.0515
C_{22}	5.86	0.4	0.075618	0.0302
C_{23}	2.92	0.2	0.0225444	0.0045
C_{31}	13.85	0.6	0.0750345	0.0450
C_{32}	1.2	0.8	0.0369573	0.0296
C_{41}	3.1	0.8	0.0349	0.0279
C_{42}	3.70	0.2	0.1396002	0.0279
C_{51}	0	0	0.0166396	0.0000
C_{52}	0	0	0.0115331	0.0000
C_{53}	0	0	0.0079076	0.0000
C_{54}	0	0	0.0054015	0.0000
C_{55}	0	0	0.0037008	0.0000
C_{56}	1	1	0.0024978	0.0025
C_{57}	0	0	0.0020185	0.0000
C_{58}	0	0	0.0011491	0.0000
Sum			1	0.3940

Table 15. Calculated scores and ranks of all stocks in SET50 at the specified time.

Stock	Sum-score	Normal-score	Rank	Stock	Sum-score	Normal-score	Rank
PTT	0.6016	0.0275	1	BCP	0.4388	0.0200	26
TISCO	0.5942	0.0271	2	ROBINS	0.4380	0.0200	27
PTTGC	0.5765	0.0263	3	CBG	0.4348	0.0199	28
LH	0.5610	0.0256	4	WHA	0.4340	0.0198	29
SPRC	0.5552	0.0254	5	TMB	0.4328	0.0198	30
TOP	0.5552	0.0254	6	KTB	0.4085	0.0187	31
KKP	0.5524	0.0252	7	CPF	0.4076	0.0186	32
EGCO	0.5372	0.0245	8	PSH	0.4007	0.0183	33
SCC	0.5366	0.0245	9	CENTEL	0.3981	0.0182	34
ADVANC	0.5308	0.0242	10	BPP	0.3926	0.0179	35
SAWAD	0.5307	0.0242	11	KBANK	0.3920	0.0179	36
CPN	0.5300	0.0242	12	TU	0.3875	0.0177	37
BEAUTY	0.5236	0.0239	13	BDMS	0.3776	0.0172	38
INTUCH	0.5189	0.0237	14	GPSC	0.3766	0.0172	39
MTLS	0.5007	0.0229	15	PTTEP	0.3700	0.0169	40
HMPRO	0.4950	0.0226	16	BANPU	0.3548	0.0162	41
TCAP	0.4898	0.0224	17	TPIPP	0.3530	0.0161	42
CPALL	0.4730	0.0216	18	GLOBAL	0.3474	0.0159	43
KCE	0.4718	0.0215	19	AOT	0.3256	0.0149	44
EA	0.4692	0.0214	20	MINT	0.3175	0.0145	45
IVL	0.4646	0.0212	21	BJC	0.3023	0.0138	46
IRPC	0.4644	0.0212	22	BTS	0.2599	0.0119	47
SCB	0.4571	0.0209	23	BEM	0.2390	0.0109	48
BH	0.4564	0.0208	24	DTAC	0.2356	0.0108	49
BBL	0.4465	0.0204	25	TRUE	0.1817	0.0083	50
				sum-w	21.8990	1.0000	

As a case study, this work analyzed the stocks in Thailand's SET50 stock market. All of the stocks were analyzed and ranked, and the results are shown in Table 15. Investors can use these ranks and scores to formulate their investment strategy. Stocks with high scores at several highest ranks are most interesting for investment.

Table 16. Investment weights of the first ten highest ranking stocks for 20th December 2016.

PTT	0.6016	0.1074	10.74	364.00	440.00	2.24%
TISCO	0.5942	0.1061	10.61	54.75	88.50	6.54%
PTTGC	0.5765	0.1029	10.29	63.25	85.00	3.54%
LH	0.5610	0.1002	10.02	9.25	10.50	1.35%
SPRC	0.5552	0.0991	9.91	12.10	17.10	4.10%
TOP	0.5552	0.0991	9.91	72.25	103.50	4.29%
KKP	0.5524	0.0986	9.86	57.50	79.25	3.73%
EGCO	0.5372	0.0959	9.59	195.50	216.00	1.01%
SCC	0.5366	0.0958	9.58	482.00	484.00	0.04%
ADVANC	0.5308	0.0948	9.48	145.00	191.00	3.01%
Total weight	5.6007	1	100.00	Earning of portfolio %		29.84%

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