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An Approach to Identify Significant Parameters in Blood Flow Through Human Arteries

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ABSTRACT

In this work, we attempt to identify parameters that are significant in blood flow through human arteries. For this, we developed a mathematical model and carried out simulations using data on human arteries and then performed a statistical analysis of the data generated. The mathematical model considers blood as a Newtonian fluid with its constitutive equation comprising two parameters termed as fluid parameters, namely the density and viscosity. Arteries are modelled as tapered and elastic circular pipes wherein the radius of the artery is in terms of the material parameters, termed taper fraction, expanding/contracting parameter and aspect ratio $\left(= \frac{\text{radius of the artery}}{\text{length of the artery}}\right)$. Blood flow is described as pulsatile with the frequency of oscillations as the flow parameter monitored. The resulting system of partial differential equations is solved using the Homotopy Analysis Method (HAM). Data on human arteries (healthy) has been taken from literature, and we worked on different age groups ranging from 15 years to 65 years, both on male and female populations. The two physical quantities of interest i.e. the wall shear stress (WSS) and the volumetric flow rate have been computed and analysis has been carried out to identify significant parameters using a multiple linear regression model.

Keywords: Blood flow; HAM; Human artery; Mathematical model; Newtonian fluid; Statistical analysis; Wall shear stress

1. Introduction

Understanding blood flow in the human arterial system is important owing to its significance not only in diagnosing diseases related to heart and circulatory system but also in the design of efficient med-

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ical devices. A literature survey shows that researchers have used different methodologies, namely lumped parameter modelling, mathematical modelling and clinical methods to unveil the mechanism governing this system. Although each method has its advantages and limitations, the present work is on applying mathematical modelling to the problem of identifying parameters to be monitored in blood flow through human arteries.

In general, construction of mathematical models for the human arterial system requires appropriate mathematical description of the salient features of this system, i.e. (i) flow of blood is due to pulsatile pressure gradient induced by the heart, (ii) blood is a complex fluid that is a suspension of particles, and (iii) the arteries are elastic/ muscular with tapering and branching. Hence, mathematical models comprise three sets of parameters, i.e. flow parameters, fluid parameters and mechanical parameters to describe each of these features, and thus one has to handle three sources of non-linearities, namely geometric, structural (or mechanical) and fluid related. As observed by Jiyauan et al. [1], the human circulatory system has self-repairing mechanisms and possibly it is not required to introduce all three sources of non-linearities into a mathematical model. However, not much work has been reported in the direction of identifying the appropriate non-linearity/non-linearities to be included in a model for different arteries in the human arterial system. Thus, this study aims to develop a mathematical model for blood flow in arteries with its radius varying from *mm* to *μm* and use statistical tools to identify significant parameters to be monitored in individuals

2. Statement of the Problem

A mathematical description of the features of the circulatory system described above includes flow parameters to describe the flow of blood, fluid parameters by fluid constitutive equations to describe the complex nature of blood, and mechanical parameters for describing the mechanical properties of arteries. As mentioned above, mathematical models should include non-linearities, namely geometric non-linearity as the flow domain is unknown, fluid constitutive nonlinearity due to the complex nature of fluid, i.e. blood and structure constitutive inequality as the arterial vessel is elastic with stress-strain response varying along with the system [2]. Thus, a model incorporating all these three non-linearities would result in a computationally expensive one and is not often desirable. Hence, it is necessary to understand the mechanism of a system to explore the possibility of reducing this complexity and still be able to achieve accuracy. Thus, an attempt made in this direction for the human arterial system resulted in a finding that this system has self-repair mechanisms where, in some arteries, the arterial wall can remodel itself by increasing its external diameter to accommodate plaque without narrowing the lumen, and hence, it is understood that not all non-linearities or parameters in the model may have significant effect on the flow variables [1]. Therefore, in this work, we propose to develop a mathematical model with a Newtonian fluid model to describe blood and arteries as tapered and elastic circular pipes. This model is then applied to carry out simulations using data on human arteries, and statistical analysis has been performed to identify the most significant parameters in the model developed.

3. Formulation of the Problem

A mathematical model has been constructed using the Navier Stokes equations for the fluid flow [3]. The fluid, i.e. blood, is taken to be Newtonian and, for the behaviour of the artery, models have been chosen as described in [4, 5]. Furthermore, we assume that the flow is axisymmetric and the fluid is incompressible, so that the velocity vector and the pressure, denoted by $\overline{q}$ and p respectively, are functions of r, z and time t only. We assume that the non-vanishing components of the velocity vector are in radial and axial directions so that the velocity vector is

$$\overline{q}(r,z,t) = (u(r,z,t), 0, w(r,z,t)).$$

Also, let the thermodynamic pressure p be $p(r,z,t)$.

Now, the continuity equation and momentum equations take the form

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{3.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{1}{\rho}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{\partial}{\partial z}(\tau_{rz})\right), \tag{3.2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{1}{\rho}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{\partial}{\partial z}(\tau_{zz})\right). \tag{3.3}$$

Here, ρ is the density of the blood, and τ_{ij} are the components of the deviatoric stress tensor [3].

Under the assumption that the radial flow velocity and the convective acceleration terms are respectively of a smaller order of magnitude with respect to the axial flow velocity and the local acceleration terms, the radial momentum Eq. (3.2) reduces to

$$-\frac{\partial p}{\partial r} = 0. \tag{3.4}$$

Thus, it is clear that the pressure is independent of r. Since the fluid is considered Newtonian, its constitutive equation is

$$\tau_{ij} = -p\delta_{ij} + 2\mu e_{ij} \tag{3.5}$$

where p is the thermodynamic pressure, μ is the viscosity and e_{ij} is the rate of deformation tensor [3].

Using the constitutive Eq. (3.5), Eq. (3.1), Eq. (3.2) and Eq. (3.3) take the form

$$\frac{\partial u}{\partial r} + \frac{1}{r}u + \frac{\partial w}{\partial z} = 0, \tag{3.6}$$

$$-\frac{\partial P}{\partial r} = 0, \tag{3.7}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2}\right). \tag{3.8}$$

Since the pipe is considered tapered with contracting/expanding nature, it is modelled as

$$R(z,t) = R_0\left(1 - f\frac{z}{L}\right)(1-\beta t)^{0.5} \tag{3.9}$$

where R_0 is the radius of the pipe at the inlet, f is the fraction of tapering, β is the contracting/expanding parameter, and L is the length of the pipe [4, 5].

As the fluid (blood) flow in the pipe (arteries) is due to the pumping of the heart which is pulsatile, the pressure gradient is,

$$-\frac{\partial P}{\partial z} = a_0 + a_1\cos\omega t, \tag{3.10}$$

where a_0 is the average blood pressure, a_1 is the pulse difference and ω is the frequency of oscillations, i.e. the heart rate per minute.

Now, the aim is to solve the coupled system of partial differential equations given by Eq. (3.6) to Eq. (3.8) together with the boundary conditions given by

$$u = \frac{\partial R}{\partial t}(z,t) \text{ on } r = R(z,t)$$

(fluid velocity matches with the rate of displacement of the wall),

$$w = 0 \text{ on } r = R(z,t)$$

(no-slip condition),

$$u = 0,\ \frac{\partial w}{\partial r} = 0 \text{ at } r = 0 \tag{3.11}$$

(velocity is finite at the center of the pipe).

4. Solution to the Problem

Introducing the following non-dimensional scheme,

$$r^* = \frac{r}{R_0},\ z^* = \frac{z}{L},\ t^* = \omega t,$$

$$u^* = \frac{u}{R_0\omega},\ w^* = \frac{w}{L\omega} \tag{4.1}$$

Eq. (3.6), Eq. (3.8), after dropping '*' transform to

$$\frac{\partial u}{\partial r} + \frac{1}{r}u + \frac{\partial w}{\partial z} = 0 \tag{4.2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = P^* - \frac{1}{\alpha^2}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \delta^2\frac{\partial^2 w}{\partial z^2}\right), \tag{4.3}$$

where

$$P^* = -\frac{1}{\rho\omega^2 L^2}\frac{\partial P}{\partial z}$$

is the non-dimensional pressure gradient, $\delta = \frac{R_0}{L}$ is the aspect ratio and $\alpha = \sqrt{\frac{\rho\omega R_0^2}{\mu}}$ is the Womersley number.

And the boundary conditions in the non-dimensional form are

$$u = \frac{\partial R^*}{\partial t}(z,t),\ w = 0 \text{ on } r = R^*(z,t),$$

$$u = 0,\ \frac{\partial w}{\partial r} = 0 \text{ at } r = 0, \tag{4.4}$$

where

$$R^*(z,t) = (1 - fz)(1 - \beta t)^{0.5}. \tag{4.5}$$

It may be noted that hereafter while mentioning the expression for the radius given in Eq. (4.5), '*' will be dropped from the same.

Since the above mathematical model given by Eq. (4.2), Eq. (4.3) is a coupled system of non-linear partial differential equations and finding analytical solutions is almost not impossible; we implemented the Homotopy Analysis Method (HAM) for finding semi-analytical solutions [6–11]. For this, we identified the homotopy for Eq. (4.2) as

$$h_1(p) = (1-p)\left(\frac{\partial^2 u}{\partial r^2}\right) + hp\left(\frac{\partial u}{\partial r} + \frac{1}{r}u + \frac{\partial w}{\partial z}\right) \tag{4.6}$$

and for Eq. (4.3), the homotopy is taken as

$$\begin{aligned} h_2(p) &= (1-p)\left(\frac{\partial^2 w}{\partial r^2}\right) \\ &\quad + kp\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} + P \right. \\ &\quad \left. - \frac{1}{\alpha^2}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{x}\frac{\partial w}{\partial r} + \delta^2\frac{\partial^2 w}{\partial z^2}\right)\right), \end{aligned} \tag{4.7}$$

where h and k are auxiliary parameters. Following the method described by Liao [7,8], let us take

$$u(r,z,t) = u_0(r,z,t) + u_1(r,z,t)p$$

$$+u_2(r,z,t)\,p^2 + u_3(r,z,t)\,p^3 + \ldots \qquad (4.8)$$

and

$$w(r,z,t) = w_0(r,z,t) + w_1(r,z,t)\,p + w_2(r,z,t)\,p^2 + w_3(r,z,t)\,p^3 + \ldots \qquad (4.9)$$

Let the initial solution be $u_0(r,z,t) = 0$, $w_0(r,z,t) = 0$. Now, substituting the expressions for $u(r,z,t)$ and $w(r,z,t)$ from Eq. (4.8), Eq. (4.9) in Eq. (4.6), Eq. (4.7), collecting the coefficients of p^1 and equating them to zero, we have

$$\frac{\partial^2 u_1}{\partial r^2} = 0, \qquad (4.10)$$

$$k\frac{a_0 + a_1 \cos t}{\rho\omega^2 L^2} + \frac{\partial^2 w_1}{\partial r^2} = 0. \qquad (4.11)$$

The above equations are solved (using MATHEMATICA) along with the boundary conditions,

$$u_1 = \frac{\partial R(z,t)}{\partial t},\ w_1 = 0 \text{ on } r = R(z,t)$$

$$u_1 = 0,\ \frac{\partial w_1}{\partial r} = 0 \text{ at } r = 0 \qquad (4.12)$$

and the expression for the first approximation of the velocity components has been obtained. We, then, collect the coefficient of p^2 from RHS of Eq. (4.6), and Eq. (4.7) and equate them to zero to obtain the equations governing the second approximations of the velocity components as shown below.

$$-\frac{h\beta}{1-\beta t} - hkf(1-fz)(1-\beta t)\frac{a_0 + a_1\cos t}{\rho\omega^2 L^2} + \frac{\partial^2 u_2}{\partial r^2} = 0, \qquad (4.13)$$

$$\left(k + \frac{k^2}{\alpha} + \frac{k^2}{x\alpha} - \frac{\delta^2 k^2 f^2(1-\beta t)}{\alpha} - \frac{k^2\beta(1-fz)^2}{2}\right)\frac{a_0 + a_1\cos t}{\rho\omega^2 L^2} + \left(k^2x^2 - k^2(1-fz)^2(1-\beta t)\right)\frac{a_1 \sin t}{2\,\rho\omega^2 L^2} + \frac{\partial^2 w_2}{\partial r^2} = 0. \qquad (4.14)$$

These equations are again solved using the boundary conditions given by

$$u_2 = 0,\ w_2 = 0 \text{ on } r = R(z,t),$$

$$u_2 = 0,\ \frac{\partial w_2}{\partial r} = 0 \text{ at } r = 0 \qquad (4.15)$$

to get the second approximation to the velocity components. Subsequent approximations are obtained by collecting the next higher powers of p and equating them to zero to find the system of equations governing the next higher approximations to velocity components. The resultant systems are solved together with homogenous boundary conditions similar to the ones given in Eq. (4.15) and thus we obtain expressions for $u(r,z,t)$ and $w(r,z,t)$ in terms of the auxiliary parameters h and k. As observed by Liao, these parameters affect the convergence region and also the rate of convergence and hence, are to be determined using a certain class of curves called h-curves. Since the model that we are working with has two auxiliary parameters and finding both of them using h-curves is difficult, we implemented the following method to identify the region of convergence.

Initially, we fixed the value of 'h' at -0.00007 and, for this value, we determined the range of 'k' using h-curves. Fig. 1 depicts h-curves to identify the range of the auxiliary parameter 'k'. From these graphs, we identified the range for 'k', and the value -0.06 has been taken for the study (see in Fig. 2).

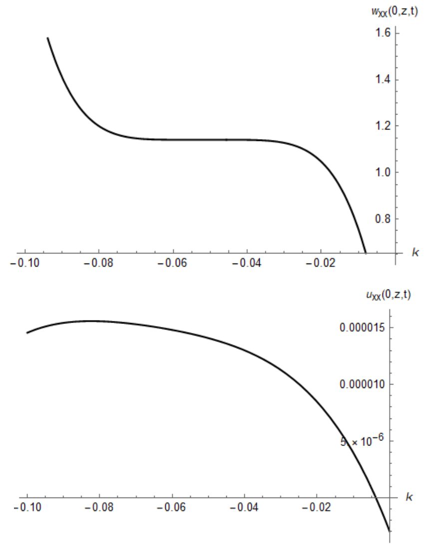

Fig. 1. *h*-curves to identify the range for the auxiliary parameter 'k'

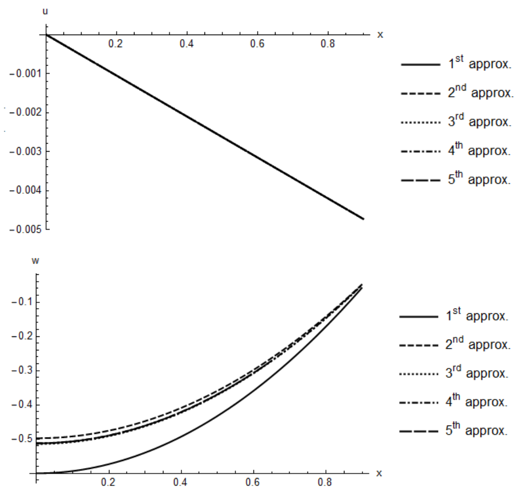

Fig. 2. Graphs of $u(r,z,t)$ and $w(r,z,t)$ for $k = -0.06$

5. Results and Discussions

To carry out simulations using the developed model, we considered data on human arteries [12, 13] as shown below in Tables 1, 2 and 3.

Table 1 shows different age groups and the average blood pressure (systolic/diastolic) in $mmHg$ (for healthy individuals) in that group.

Table 1. Avg Blood Pressure (mmHg)

Age Group	Female	Male
15 – 18	117/77	120/85
19 – 24	120/79	120/79
25 – 29	120/80	121/80
30 – 35	122/81	123/82
36 – 39	123/82	124/83
40 – 45	124/83	125/83
46 – 49	126/84	127/84
50 – 55	129/85	128/85
56 – 59	130/86	131/87
≥ 60	134/84	135/88

Table 2 presents the range for heart-rate in healthy individuals of different age groups in males and females. This table also shows the data we considered to carry out simulations using the mathematical model.

Table 3 presents other parameters in the model, and the values taken to carry out simulations. The values of density and viscosity are taken from data on human blood, whereas the other mathematical parameters take values that describe the different states of the artery. For example, taper fraction with value 0.01 describes the case where the radius (outlet) of the artery reduces to 99% of the inlet radius. Similarly, the value 0.1 describes the case where the outlet radius reduces to 90% of the inlet radius. Negative values of the parameter β indicate the state where the artery contracts with time and positive values describe the state where the artery expands. The value of $\delta = 0.001$ models an artery of radius $1mm$ and the value $\delta = 10^{-5}$ models that of radius $1\mu m$.

We then applied the multiple regression techniques, with WSS and volumetric flow rate, taken to be dependent on the dif-

Table 2. Heart Rate per minute

Age Group	Heart Rate (Female)		Heart Rate (Male)	
	Data from literature [13]	Values considered	Data from literature [13]	Values considered
$18-25$	$70-73$	$70, 72, 73$	$70-73$	$70, 72, 73$
$26-35$	$73-76$	$73, 75, 76$	$71-74$	$71, 73, 74$
$36-45$	$74-78$	$74, 76, 78$	$71-75$	$71, 73, 75$
$46-55$	$74-77$	$74, 76, 77$	$72-76$	$72, 74, 76$
$56-65$	$74-77$	$74, 76, 77$	$72-75$	$72, 74, 75$
> 65	$73-76$	$73, 75, 76$	$70-73$	$70, 72, 73$

Table 3. Other parameters and their values

Density (ρ) (in $kg.m^{-3}$)	1055
	1058.1
	1061.2
Viscosity (μ) (in $Pa.s$)	0.00552
	0.00619
	0.00686
Taper Fraction (f)	0.01
	0.03
	0.05
	0.07
	0.09
	0.1
Contracting/ Expanding Parameter (β)	−0.05
	−0.01
	0
	0.01
	0.05
Aspect Ratio (δ)	0.01
	0.001
	0.0001
	0.00001

ferent parameters of our model and noted the p-values.

Table 4 shows the p-values for WSS, and volumetric flow rate for different age groups in the female population and Table 5 presents the same for the male population.

From Tables 4 and 5, we see that among the fluid parameters, i.e., viscosity and density, viscosity has a significant effect on both WSS and flow rate whereas density has a significant effect only on the volumetric flow rate in all the age groups for both male and female populations. In the case of material parameters, we see that taper fraction has no significant effect on the physical quantities among all ages and in both genders while the expanding/contracting parameter exhibits it. We also observe that the frequency of oscillations (mentioned as heart rate in the table), which is the flow parameter, has a significant effect on both WSS and flow rate in all age groups for both male and female population.

Table 4. *p*-values for WSS and Volumetric flow rate in Female Population

WSS	*p*-values					
Age Group	Heart Rate Rate	Aspect Ratio Ratio	Taper fraction	Contracting Expanding Parameter	Density	Viscosity
19 – 24	4.11×10^{-5}	2×10^{-16}	0.5275	0.0017	0.6449	2×10^{-16}
25 – 29	8.27×10^{-5}	2×10^{-16}	0.52718	0.00169	0.64467	2×10^{-16}
30 – 35	8.27×10^{-5}	2×10^{-16}	0.52718	0.00169	0.64467	2×10^{-16}
36 – 39	8.62×10^{-7}	2×10^{-16}	0.52901	0.00177	0.64613	2×10^{-16}
40 – 45	8.62×10^{-7}	2×10^{-16}	0.52901	0.00177	0.64613	2×10^{-16}
46 – 49	0.000103	2×10^{-16}	0.527099	0.001686	0.644601	2×10^{-16}
50 – 55	0.000103	2×10^{-16}	0.527099	0.001686	0.644601	2×10^{-16}
56 – 60	0.000897	2×10^{-16}	0.588984	0.007268	0.693526	2×10^{-16}
> 60	8.27×10^{-5}	2×10^{-16}	0.52718	0.00169	0.64467	2×10^{-16}
Vol. Flow rate	*p*-values					
Age Group	Heart Rate Rate	Aspect Ratio Ratio	Taper fraction	Contracting Expanding Parameter	Density	Viscosity
19 – 24	0.0295	2×10^{-16}	0.1923	1.60×10^{-10}	2×10^{-16}	3.78×10^{-11}
25 – 29	0.0367	2×10^{-16}	0.192	1.56×10^{-10}	2×10^{-16}	3.67×10^{-12}
30 – 35	0.0367	2×10^{-16}	0.192	1.56×10^{-10}	2×10^{-16}	3.67×10^{-12}
36 – 39	0.00893	2×10^{-16}	0.19383	1.85×10^{-10}	2×10^{-16}	4.47×10^{-12}
40 – 45	0.00893	2×10^{-16}	0.19383	1.85×10^{-10}	2×10^{-16}	4.47×10^{-12}
46 – 49	0.0392	2×10^{-16}	0.1919	1.55×10^{-10}	2×10^{-16}	3.64×10^{-12}
50 – 55	0.0392	2×10^{-16}	0.1919	1.55×10^{-10}	2×10^{-16}	3.64×10^{-12}
56 – 60	0.0392	2×10^{-16}	0.1919	1.55×10^{-10}	2×10^{-16}	3.64×10^{-12}
> 60	0.0367	2×10^{-16}	0.192	1.56×10^{-10}	2×10^{-16}	3.67×10^{-12}

Table 5. p-values for WSS and Volumetric flow rate in Male Population

WSS	p-values					
Age Group	Heart Rate Rate	Aspect Ratio Ratio	Taper fraction	Contracting Expanding Parameter	Density	Viscosity
19 – 24	4.11×10^{-5}	2×10^{-16}	0.5275	0.0017	0.6449	2×10^{-16}
25 – 29	5.24×10^{-5}	2×10^{-16}	0.5274	0.0017	0.6448	2×10^{-16}
30 – 35	5.24×10^{-5}	2×10^{-16}	0.5274	0.0017	0.6448	2×10^{-16}
36 – 39	3.13×10^{-7}	2×10^{-16}	0.52943	0.00179	0.64646	2×10^{-16}
40 – 45	3.13×10^{-7}	2×10^{-16}	0.52943	0.00179	0.64646	2×10^{-16}
46 – 49	4.44×10^{-7}	2×10^{-16}	0.52928	0.00178	0.64635	2×10^{-16}
50 – 55	4.44×10^{-7}	2×10^{-16}	0.52928	0.00178	0.64635	2×10^{-16}
56 – 60	6.61×10^{-5}	2×10^{-16}	0.52727	0.00169	0.64474	2×10^{-16}
> 60	4.11×10^{-5}	2×10^{-16}	0.5275	0.0017	0.6449	2×10^{-16}
Vol. Flow rate	p-values					
Age Group	Heart Rate Rate	Aspect Ratio Ratio	Taper fraction	Contracting Expanding Parameter	Density	Viscosity
19 – 24	0.0295	2×10^{-16}	0.1923	1.60×10^{-10}	2×10^{-16}	3.78×10^{-11}
25 – 29	0.0318	2×10^{-16}	0.1922	1.59×10^{-10}	2×10^{-16}	3.74×10^{-12}
30 – 35	0.0318	2×10^{-16}	0.1922	1.59×10^{-10}	2×10^{-16}	3.74×10^{-12}
36 – 39	0.0655	2×10^{-16}	0.19424	1.92×10^{-10}	2×10^{-16}	4.67×10^{-12}
40 – 45	0.0655	2×10^{-16}	0.19424	1.92×10^{-10}	2×10^{-16}	4.67×10^{-12}
46 – 49	0.00729	2×10^{-16}	0.1941	1.9×10^{-10}	2×10^{-16}	4.6×10^{-12}
50 – 55	0.00729	2×10^{-16}	0.1941	1.9×10^{-10}	2×10^{-16}	4.6×10^{-12}
56 – 60	0.0342	2×10^{-16}	0.1921	1.58×10^{-10}	2×10^{-16}	3.71×10^{-12}
> 60	0.0295	2×10^{-16}	0.1923	1.60×10^{-10}	2×10^{-16}	3.78×10^{-11}

6. Conclusion

In this paper, the work is on identifying parameters that are significant in blood flow in human arteries through a mathematical modelling approach and statistical analysis. Two dimensional Navier Stokes equations for fluid flow are considered to construct the mathematical model and data on human arteries (healthy) has been taken from literature to simulate the model. By analysing the data generated using statistical tools, the following conclusions were derived:

1. Density and viscosity: Viscosity of the fluid, i.e. blood, has a significant effect on both WSS and flow rate whereas the density of the blood has a significant effect only on the volumetric flow rate in all age groups for both male and female population.

2. Taper fraction, expanding/contracting characteristics: Taper fraction did not show any significant effect on the physical quantities of all ages and gender whereas the expanding/contracting parameter has significant effect indicating that the elastic nature of the artery or arterial stiffness parameter is an important parameter that helps in understanding the abnormal states (anomalies) of human arterial system.

3. Frequency of oscillation: It also has been observed that frequency of oscillations has a significant effect on both WSS and flow rate in all age groups for both male and female population strongly suggesting the heart rate as another important parameter that has to be monitored.

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