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Original research article

Comparative Analysis and Validation of Selected Explicit Equation Models for Determination of Darcy Friction Factor to Estimate Major Head Loss for a Pressurized Flow System

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ABSTRACT

Fluid flow through pipelines has a huge number of applications. The pipelines design aims to deliver the fluid in a cost-effective manner with the necessary flow rate and required head. The analysis of the network design for the fluid flow is highly important in order to evaluate its performance. The essential and integral part of the network analysis during the motion of fluid through the pipelines is the estimation of head loss which includes the determination of friction factor. The choosing of the appropriate, optimum and economic conduit involves numerous hydraulic computations. The commonly used, well-established methods universally adopted are the Darcy-Weisbach equation and the Hazen-Williams formula for the determination of major friction loss of the fluid flow. A number of approximate explicit equations have been developed based on the friction factor of the Darcy-Weisbach equation. The study involves selecting an appropriate explicit equation for the determination of the Darcy friction factor and estimating the major head loss existing in the newly modeled pipe network for the farm located at Hamelmalo Agricultural College in Eritrea. Eleven different explicit equations were chosen for the determination of friction factor based on different selection criteria, and the pressure loss was determined in each case and was compared with the standard Colebrook-White equation. Statistical methodologies were used to evaluate and obtain the best fit equation for the network.

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1. Introduction

Fluid substances are those in which deformation of surface occurs permanently and continuously when subjected to a number of forces which are governed by Newton's second law of motion [1]. These forces acting on the fluid are unequal in magnitude and direction. The motion of the fluid is impacted by the resistance offered to flow [2]. The hydraulic resistance offered during the conveyance of fluid flow is contributed by the roughness of pipes at the walls. The resistance offered in turn affects the rate of flow and distribution of velocity of process fluid in the pipes [3]. In a given network of a pipeline system, the flow conveyance through the network is governed by a number of basic physical equations. For a given incompressible flow involving Eulerian description of motion of fluid particles, which assumes a uniform velocity and a constant density, the law of conservation of mass assumes the most simplified form of the Continuity Equation. The final integrated form of this results in Bernoulli's Equation [4]. The factor proportional to the ratio between the momentum of a flowing fluid and its viscosity governs the behavior of the flow of the fluid and is termed as the Reynolds number (Re) given in Eq. (1.1) [5].

$$Re = \frac{vd}{\nu}, \tag{1.1}$$

where Re is the Reynolds number, a dimensionless value, v is the velocity of liquid flow in m/s, d is the diameter of the pipe through which the liquid flows in m, and ν is the kinematic viscosity in m^2/s. The inertial forces of the Reynolds flow equation contribute to the disorderliness while the viscous forces contribute to the damping factor of the instabilities caused in the flow, owing to which low Reynolds number recommends a laminar flow while higher values of Reynolds number indicate turbulence. The turbulence can result in either a transitional zone or a fully turbulent zone within the flow. The Reynolds number at which the flow changes from laminar to turbulent is termed as the critical Reynolds number.

The major frictional head loss for the rate of flow of liquid through a given network is related by the Darcy-Weisbach equation which is given by Eq. (1.2).

$$h_f = \frac{flv^2}{2gd}, \tag{1.2}$$

where h_f is the head loss due to friction in units of energy per unit length in m, l represents the length of the pipe in m, g denotes the gravitational acceleration in m/s^2 and f represents the coefficient of surface resistance or the Darcy friction factor which is dimensionless in nature [6]. It relates the head loss due to friction along a given length of pipe to the average velocity of the fluid flow for an incompressible fluid. The equation is considered a rational formula. However, the friction factor ' f ' is a complex function of various variables which include pipe roughness, diameter, kinematic viscosity and velocity of flow. The complexity in friction factor ' f ' led to the development of several dimensionally inhomogeneous, irrational empirical formulas [7]. The laminar flow is a part of the region in which the relative roughness of the pipe has no influence on the factor f. The head loss due to friction of the laminar flow region where the Reynolds number is usually less than

2000 is obtained using Hagen-Poiseulle's Equation given by Eq. (1.3) [8].

$$h_f = \left(\frac{Re}{64} \right). \tag{1.3}$$

The friction factor in the transitional region of the fluid flow where the Reynolds number ranges from 2000 to a maximum of 4000 and turbulent regions where the *Re* accounts for a value greater than 4000 of the fluid flow is determined using the implicit Colebrook-White equation given by Eq. (1.4)

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{\varepsilon}{3.71d} + \frac{2.51}{R\sqrt{f}} \right], \tag{1.4}$$

where $\frac{\varepsilon}{d}$ denotes the relative pipe roughness. Initially, the equation deduced an iterative solution for the determination of the friction factor 'f' using the Moody's Chart or using the principles of numerical methods. Further, owing to the development of empirical explicit equations and technological software, the determination of the overall major frictional loss has become much simpler and more accurate.

In spite of the advances in technology and formulations of explicit equations, the Darcy-Weisbach based on Colebrook-White equation is much favored as compared to the limited and less accurate Hazen-Williams equation for the determination of major head loss. The reluctance of engineers to apply the Colebrook-White formula as compared to Hazen-Williams equation is the available database for Hazen-Williams coefficient C_{HW} as compared to the relatively small database of the equivalent sand roughness values of the Colebrook-White equation [9]. Most of the pipe system networks consist of a number of components such as elbows, valves, and inlets along with sudden contraction and expansion of the flow network. These additional components add to the overall head loss of the systems. These losses are termed as minor losses or local losses. The general formula to determine these head losses is given by Eq. (1.5).

$$h_L = K_L \left(\frac{v^2}{2g} \right), \tag{1.5}$$

where h_L denotes the minor losses in the network corresponding to each component, K_L denotes the coefficient of minor loss. These minor losses add up to the major friction losses. If the length of the pipe used in networks is quite long, then the local losses can be eliminated. If in case the length of the pipes used in the network is less than 100 m diameter, the local losses can exceed the major friction losses [4]. Topographic survey methods such as tacheometry and grid survey aim at preparation of the contour of the given area which would serve as a basis for any type of irrigation scheme [10]. The surveying of the farm area had to be recommended in order to estimate the elevation losses of the network. The overall pressure drop during the flow of fluid through the pipes is contributed by the total sum of major friction losses, local losses of various components in the network and the elevation of the network system.

Explicit equations have been proposed by various researchers in order to determine the Darcy friction factor of the implicit Colebrook-White equation. These equations have been proposed based on various constraints. [11] investigated 33 explicit formulas and assessed their accuracy based on maximum deviations, range in Moody's diagram and simplicity of the formula which had been proposed for the Colebrook-White equation since 1947. [9] discussed the implicit solution of Hazen-Williams roughness coefficient and deduced an explicit approximate solution to convert the roughness coefficient to equivalent sand roughness. [12] elaborates about application of AI techniques in order to predict the Darcy friction factor and the performance of the methods were compared with explicit equations. [13] details the estimation of friction factor using the shifted Lambert W-function. The friction factor estimated was accurate with a relative error of not more than 0.0096%. [14] developed four explicit equations to deduce the friction factor without iterations. The authors developed the formulas based on Colebrook-White and Konakov apud Nekrasov equations.

This case-study aims at comparing 11 different explicit equations proposed by various researchers which were chosen based on different selection criteria available in literature such as the simplicity of the equation, accuracy of the equation, explicit equations based on mathematical fundamental reference equations and algorithms, explicit equations which don't depend upon the absolute roughness of pipes and the artificial intelligence techniques based explicit equations in order to estimate the Darcy friction factor and determine the friction losses in the new modeled pipe flow network located at Hamelmalo Agricultural College (HAC) farm at Eritrea. The estimated values were further compared with the standard reference Colebrook-White equation and further validated with statistical indices. The best fit explicit equation which was most approximate with the standard equation is considered to perform relatively better than the rest of the other explicit equations and can be recommended for its performance in this particular case-study.

The rest of this paper is organized as follows. Section 2 describes the materials and methodology where the overall description of the case-study is discussed. Furthermore, the various explicit equations to determine the friction factor in order to deduce the major friction losses are discussed in detail. Section 3 gives a vivid description of the statistical performance and error criterion indices techniques adopted for the purpose of comparison and validation. Section 4 accounts for the analyses on the results and discussion of the various explicit equations based on statistical indices. Section 5 provides the conclusion of the case study.

2. Materials and Methods

2.1 Topographical details of Hamelmalo Agricultural College (HAC) Farm

The case-study involves the estimation of Darcy friction factor and major head loss for the new modeled design of pipe flow network located at Hamelmalo Agricultural College farm in Eritrea. The farm area was divided into various sub-plots according to the departments functioning in the college and administrative conveniences. HAC is geographically positioned at 15°52'35''N latitude and 38°27'45" E longitude, at an elevation of about 1264 m above mean sea level. The college is 12 km north of Keren along the Keren-Nakfa Road near Hamelmalo village. The college premises comprise a total area estimated around 76.3 hectare. The farm area is estimated to be around 16.3 hectare. The network existing at HAC has a linear type framework. The source of water to the network is supplied from two wells, whose location is in close proximity of the Anseba River which ensures the availability of water throughout the year. One of the two is a hand-dug well and supplies water at a rate of 8.3 l/s. The other is a tube well which has a yield of 18 l/s. The network initiates from these two wells to meet at a point of junction. The network is so designed in order to follow a precautionary measure, as even if one well fails to produce the required output as a result of failure in pumping units or other defects such as drought, the other well would act as a substitute and would supply the crops with sufficient water. The pipeline network assists in providing the necessary water to each sub-plot from the wells. The network comprises of a mainline, sub mains, manifolds, gate valves and emission devices. The path of conveyance of water from the wells to the various sub-plots was studied and the piping arrangement was modeled. The pipes laid in the existing network were manufactured using galvanized iron (G.I.) and PVC (Poly Vinyl Chloride). However, in the new model of the network the PVC pipes were recommended to be installed replacing the galvanized iron due to its numerous advantages [15]. A theodolite and GPS were used to study the elevation and location of the existing network available at HAC. The location of the HAC premises with the farm along with the existing pipe network is as shown in Fig. 1 and Fig. 2 respectively [16]. The contour map of the chosen study area also adapted from [16] is shown in Fig. 3.

Fig. 1. Location of HAC Farm chosen for study.

Reference [17] elaborates about the modeling of pipe diameter for the pressurized flow network using the velocity method of pipe design, where the minimum and maximum velocity of optimum pipe design technique were used to design optimum pipe diameters and were compared among themselves. However, a number of explicit methods are available in literature to design the appropriate diameter of the pipe network system.

Fig. 2. Existing Pipe network of the farm located at HAC.

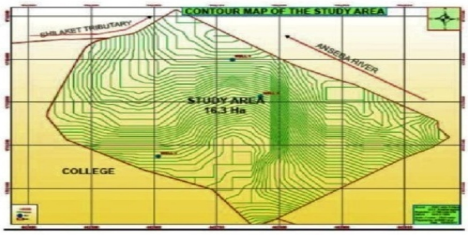


Fig. 3. Contour map of the chosen study area.

The field is located at about 600 m from the wells. Usually the tube well (well 1) with a submersible pump is used for irrigating the whole farm. The pump has a capacity to discharge 18 l/s indicated by the triangular symbol in Fig. 4. The flow is scheduled for two intervals based on the design of the existing network located at HAC farm which comprises 15 major links as shown in the Hydraulic Analysis chart in Fig.4. The links are namely AB, B1, BC, C2, CD, D3, DE, E4 and E5 in which the conveyance of water takes place during the primary schedule. During the secondary schedule water is allowed to be conveyed through EF, FG, G6, GH, HI and H7 links of the network, respectively.

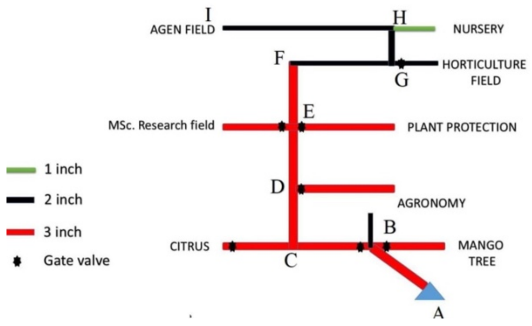


Fig. 4. Hydraulic Analysis Chart of HAC Farm.

2.2 Discussion on Explicit Equations for determination of Darcy friction factor

The flow through a pipeline depends on a number of factors. The increase in flow rate causes a rise in velocity of the flow. The viscosity of the flow of fluid also rises subsequently. The head loss is related to the square of the velocity, owing to which the losses also increase with respect to it. The velocity of the fluid through the pipe has a relationship with pressure gradient, shape and geometry of the pipeline system. The external factors such as height differential owing to gravity also play a role in the velocity of flow rate. Factors such as temperature, density of the fluid, roughness of the pipe walls, hydraulic diameter and length of the pipes also affect the flow rate of fluid. The head or energy loss effects are due to frictional losses and viscosity. When the inside diameter is made larger, the flow area increases and the velocity of the liquid at a given flow rate is reduced. When the fluid velocity is reduced there is a reduction in the head loss due to friction in the pipes. The scale deposits and corrosion increase the roughness of the pipe walls. Scale deposition has an added disadvantage of reducing the inside diameter of the pipe.

The head loss due to friction in a fluid flow can be contributed either by a laminar flow or a turbulent flow. The flow profile through the pipe is usually parabolic in nature in case of a laminar flow whereas it flattens up as the flow changes to a turbulent nature [18]. The head loss due to friction for turbulent flow can be estimated using an empirical equation named after Henry Darcy and Julius Weisbach. The Darcy-Weisbach equation relates the loss of pressure due to friction along the given length of pipe to the average velocity of the incompressible fluid flow. The Darcy friction factor ‘ f ’ can be determined using the implicit Colebrook-White equation given by Eq. (2.1) [19, 20]

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{\varepsilon}{3.71d} + \frac{2.51}{R\sqrt{f}}\right]. \quad (2.1)$$

The Darcy’s factor ‘ f ’ is determined using a chart known as the Moody’s Diagram. It is a family of curves that relates the friction factor to Reynold’s number and the relative roughness for a given pipe of a certain material. Since the solution of Eq. (2.1) involves either the iterative procedures which are time consuming or the graphical solutions which might not be accurate, various explicit equations have been proposed by researchers in order to overcome the complex mathematical calculations [11]. White gave approximations to the logarithmic smooth turbulent element in the Colebrook-White equation where the value of Reynolds number is considered as a substitution for R√f, which is given by [19] in Eq. (2.2).

$$\frac{1}{\sqrt{f}} \sim 1.8\log_{10}\left(\frac{Re}{6.8}\right) = 2\log_{10}\left(\frac{Re^{0.9}}{5.614}\right). \quad (2.2)$$

The above equation is considered as a standard reference in estimating the friction factor of flow through the pipes.

The 11 explicit equations for the determination of Darcy friction factor of the liquid flow through the pipe network at HAC farm are discussed below in detail. The explicit equations were chosen from the various available literatures based on the selection criteria mentioned below in this case-study:

(a) Explicit equations based on simplicity criteria in estimating the Darcy friction factor: Haaland's equation, Morrison's equation, Gallardo et.al equation.

(b) Explicit equations based on criteria of accuracy in estimating Darcy friction factor: Swamee-Jain equation, Churchill's equation.

(c) Explicit equations derived based on mathematical theory analysis and algorithms:

(i) Diniz and Souza equation: based on Colebrook-White and Konakov apud Nekrasov equations.

(ii) Genić and Jaćimović equation: based on improvisation of the permanently cited and well-known friction factor equations using statistical theory analysis criteria and establishing correlations.

(iii) Brkić and Praks equation: Based on shifted Lambert W-function.

(d) Explicit equations based on non-existence of absolute pipe roughness'ε' parameter:

(i) Blasius equation

(ii) Modified Hazen-Williams equation.

(e) AI based explicit equations:

(i) Gomes et.al equation.

The case-study aims at analyzing and comparing all 11 equations and validating them with the reference standard benchmark Colebrook-White equation. The best performing explicit equation was close and accurate with the reference equation.

2.2.1 Blasius equation (1913)

Paul Blasius, a student of Ludwig Prandtl, proposed the simplest empirical equation in 1913 for determining the Darcy's friction factor [7]. The equation is generally preferable for smooth pipes. Usually, it has an acceptable range of Reynolds number of a value 10^5 [21]. The Blasius equation is given by Eq. (2.3).

$$f = \left(\frac{0.316}{Re^{0.25}}\right), \tag{2.3}$$

where the constant 0.316 is termed as the coefficient and the constant 0.25 is termed as the exponent in Blasius forms [22]. The insertion of the Blasius estimated friction factor in the Darcy- Weisbach equation has numerous advantages. The combined equation is dimensionally homogeneous and has a strong theoretical fundamental base. The equation is highly accurate for plastic pipe when Reynolds number is less than 10^5. The correction factors can be easily implemented for viscosity changes in the fluid flow [23].

2.2.2 Churchill equation (1973)

The correlation proposed in 1973 by Churchill is generally applicable for turbulent regime of the fluid flow [24]. The applicable range of *Re* and relative roughness of the equation is not specified for any fluid regimes. The equation is given by Eq. (2.4).

$$f = -2\log\left[\frac{\frac{\varepsilon}{d}}{3.7} + \frac{7}{Re^{0.9}}\right]^{-2}. \tag{2.4}$$

The estimated error of Churchill approximation is up to a value of 2.81% compared to the Colebrook equation [25].

2.2.3 Swamee-Jain equation (1976)

For a given Reynolds number, Swamee and Jain (S J) (1976) gave an approximation of the friction factor. The first approximation of this equation is further used in the implicit Colebrook-White equation to obtain the second approximation of the friction factor. For *Re* lying between the laminar, transition and turbulent zones of the fluid flow, the friction factor f can be estimated using Eq. (2.5).

$$f = 0.25\left[\log\left(\frac{\varepsilon/d}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^{-2}. \quad (2.5)$$

By proper choice of variables in formulating the non dimensional groups and using suitable curve fitting techniques it has been possible to obtain explicit equation for the pipe diameter. The use of the above equation along with the empirical Darcy-Weisbach formula provides a direct accurate result in an explicit form for the head loss [26]. The applicable range of this equation is given in the interval of Re ranging from 5000-10^8 and relative pipe roughness ranging from 0.000001-0.05 [19]. The S J equation is considered to be one of the best explicit approximations of Colebrook's-White equation [11].

2.2.4 Haaland equation (1983)

Haaland proposed an equation in the year 1983 which is an approximation of Colebrook and White equation. It does not involve iterative procedures for the determination of Darcy friction factor [21]. The Haaland's equation for the determination of Darcy's friction factor f is given by Eq. (2.6).

$$f = \left\{-1.81\log\left[\left(\frac{\varepsilon/d}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]\right\}^{-2}. \quad (2.6)$$

The accuracy range of the Darcy factor obtained from this equation is about 2% which can either be positive or negative. The applicable range of Reynolds number ranges from 4000 to a value of 10^8. The range of relative roughness of pipes is estimated to be in the interval of 10^{-6} to 0.05 [11]. Haaland's equation has been considered to provide good results with respect to statistical parameters [24]. The approximation established in this equation with respect to Colebrook-White equation is simple and accurate [25]. This was the equation which was designed for the calculation of Darcy's factor which includes both liquids and gases [25, 27].

2.2.5 Modified Hazen-Williams equation (2000)

The Hazen-Williams (HW) formula is an empirical equation which was introduced in 1902. The equation is widely used among hydraulic engineers due to various reasons. The formula is mathematically simple and involves a direct solution as compared to Darcy-Weisbach equation which depends on a number of factors for estimation of the friction factor. The formula is applicable in transitional turbulent flow regimes [28]. Reference [29] reports the drawbacks of using the HW equation and concludes that it would result in errors as high as 40% when applied beyond the recommended ranges of Reynolds number, coefficient C_{HW} and diameters of pipes. Reference [30] focuses on the inconveniences of calculations which occur due to the dimensionally nonhomogeneous method of determining head losses. The HW equation also does not take into consideration the changes in viscosity and temperature of the fluid flow. HW equation is generally preferable to be implemented only in the ordinary moderate temperature ranges approximately from 40 to 75 °F. The HW equation also exhibits a significant error in applications with hot water due to variation with viscosity [31]. The variable C_{HW} is a friction loss coefficient which is a dimensionless value. Three categories of relationships for converting C_{HW} to equivalent Darcy friction factor ‘ f ’ are discussed in [32]. The first category includes the parameters, namely Re and pipe diameter, for conversion. The second category requires only the pipe diameter's information, whereas the third category relates both C_{HW} to the absolute roughness of the pipes which doesn't require the details of diameter.

In this case-study, Locher's equivalent conversion of C_{HW} to f is considered under standard water conditions [32, 9] given by Eq. (2.7) and comes under the first category.

$$f = 1{,}016.610 C_{HW}^{-1.85} d^{-0.0185} Re^{-0.148}. \quad (2.7)$$

The above equation is considered as the Modified Hazen Williams equation in this case-study [9, 32-33]. The Hazen-Williams constant was assumed to be 150, since the model is based on smooth PVC pipes [32].

2.2.6 Diniz and Souza equation (2009)

This explicit equation is derived based on the reference Colebrook-White equation and Konakov apud Nekrasov equation. Using various mathematical calculations and curve fitting techniques, the equation has been deduced. There are no iterations involved in this technique which is considered one of the advantages as compared to the traditional techniques. The authors have expressed the equation based on S J Equation with certain modifications using curve fitting techniques. The explicit equation proposed by the authors is given by Eq. (2.8) [14]

$$f = \left\{ \left(\frac{64}{Re} \right)^8 + 9.5 \left[\left(\frac{\varepsilon}{3.7d} + \frac{5.80}{Re^9/10} \right) - \left(\frac{2500}{Re} \right)^6 \right]^{-16} \right\}^{0.125}. \quad (2.8)$$

2.2.7 Morrison correlation equation (2013)

F.A. Morrison recommends expressing the data correlation of friction factor with the entire range of Reynolds number. The correlation deduced covers the three regimes of *Re*. The data correlation has been developed by the researcher for smooth pipes and expressed with much simpler form explicitly expressed in terms of friction factor. The equation is suggested to follow the Prandtl correlation, the original source of the reference standard Colebrook equation and the smooth-pipe equivalent at high *Re*. The correlation is not suggestible for any values of *Re* greater than or equal to 10^6. The correlation equation proposed is given by Eq. (2.9)

$$C_f = \left(\frac{0.0076 \left(\frac{3170}{Re} \right)^{0.165}}{1 + \left(\frac{3170}{Re} \right)^7} \right) + \frac{16}{Re}, \quad (2.9)$$

where C_f denotes the Fanning friction factor [34, 35].

2.2.8 Gomes et.al equation (2016)

Gomes et.al has proposed an explicit equation which approximates the reference equation using the principle of artificial intelligence in Eureqa Analyzer Software. The authors propose that the suggested equation did not obtain the best approximation but has a maximum relative error of less than 1%. The suggested equation is given by Eq. (2.10) [36].

$$f = 1.348 \left\{ 1.342 - \ln \left[\varepsilon + \frac{22}{Z} \right] \right\}^{-2} - 0.0001548, \quad (2.10)$$

where the parameter *Z* is given by Eq. (2.11)

$$Z = \left(393 + Re + 35118\varepsilon + \varepsilon \left(Re - 2966 \right)^{1.42 - 3.7944\varepsilon} \right)^{0.9011}. \quad (2.11)$$

2.2.9 Brkić and Praks equation (2019)

This equation is recommended to estimate the Darcy friction factor using the shifted Lambert W-function, generally termed as the Wright ω- function. The authors have suggested this equation in order to overcome the drawbacks of the Lambert W-function. The solution estimated is an accurate explicit approximation of the reference standard Colebrook equation with a relative error of not more than 0.0096%. The explicit equation suggested by Brkić and Praks is given by Eq. (2.12) [13]

$$\frac{1}{\sqrt{f}} \approx \left[0.8686B - C + \frac{1.0119C}{(B+A)} + C - \frac{2.3849}{(B+A)^2}\right], \tag{2.12}$$

where $A \cong \left[\frac{Re \times \varepsilon}{8.0878}\right]$, $B \cong \left[\frac{Re \times \ln 10}{2 \times 2.51}\right]$, $C \cong \ln\left[B + A\right]$.

2.2.10 Genić and Jaćimović equation (2019)

Genić and Jaćimović reviewed an isothermal single phase flow process based on various friction factor correlations and experimental data. The authors have proved statistically that certain popular friction factor equations can be improved. The authors have suggested a relation which covers the whole turbulent zone regime which is given by Eq. (2.13)

$$f = \left\{-1.8\log_{10}\left[\frac{7.35 - 1200(\varepsilon/d)^{1.25}}{Re} + \left(\frac{\varepsilon/d}{3.15}\right)^{1.15}\right]\right\}^{-2}. \tag{2.13}$$

It has been recommended that the range of Re for Eq. (2.13) covers the range from 4000 to 35.5×10^6. The relative roughness covers the range between 0 and 0.0333 [37].

2.2.11 Gallardo et.al equation (2021)

An explicit formulation of correlation of the Darcy friction factor in cylindrical pipes under turbulent flow conditions has been proposed by Gallardo et.al. The relation proposed includes the regimes of high and low turbulence. The authors have suggested that the best performance of roughness range covered by this equation lies from 10^{-2} to 0.005. The equation is recommended to have simple mathematical operations and simpler structure and implies a maximum relative error of 1.60% with reference to the standard equation. The equation proposed is given by Eq. (2.14) [38]

$$f = \left[-2\log\left(\frac{4.859}{Re^{-0.888}} + \frac{\varepsilon/d}{3.7}\right)\right]^{-2}. \tag{2.14}$$

3. Statistical Analyses and Evaluation of Performances of the Various Explicit Equations

In order to have a rigorous comparison between all the methods an extended statistical analysis was done using different indices. Quantitative approaches were applied in order to evaluate the explicit equations. The various explicit equations discussed above were compared using various statistical indices and error criterion techniques and the results obtained were analyzed. The statistical indices and error criterion techniques are represented from Eq. (3.1) to Eq. (3.9) [24, 39-43].

The Willmott's index of agreement (d):

$$d = 1 - \frac{\sum_{i=1}^{n}(P_i - O_i)^2}{\sum_{i=1}^{n}\left(\left|P_i - \bar{O}\right| + \left|O_i - \bar{O}\right|\right)^2}. \tag{3.1}$$

Root Mean Square Error:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(P_i - O_i)^2}{n}}. \tag{3.2}$$

Relative Error:

$$RE = \frac{RMSE}{\bar{O}}. \tag{3.3}$$

Mean Bias Error:

$$MBE = \frac{\sum_{i=1}^{n}(P_i - O_i)}{n}. \tag{3.4}$$

Mean Absolute Error:

$$MAE = \frac{\sum_{i=1}^{n} | P_i - O_i |}{n}. \qquad (3.5)$$

Variance of distribution of differences:

$$S_d^{\,2} = \sum_{i=1}^{n} \frac{(P_i - O_i - MBE)^2}{(n-1)}. \qquad (3.6)$$

Correlation Ratio:

$$\eta = \sqrt{1 - \frac{\sum_{i=1}^{n} (O_i - P_i)^2}{\sum_{i=1}^{n} (O_i - \bar{O})^2}}. \qquad (3.7)$$

Nash and Sutcliffe coefficient of efficiency index:

$$E = \sqrt{1 - \frac{\sum_{i=1}^{n} (O_i - P_i)^2}{\sum_{i=1}^{n} (O_i - \bar{O})^2}}. \qquad (3.8)$$

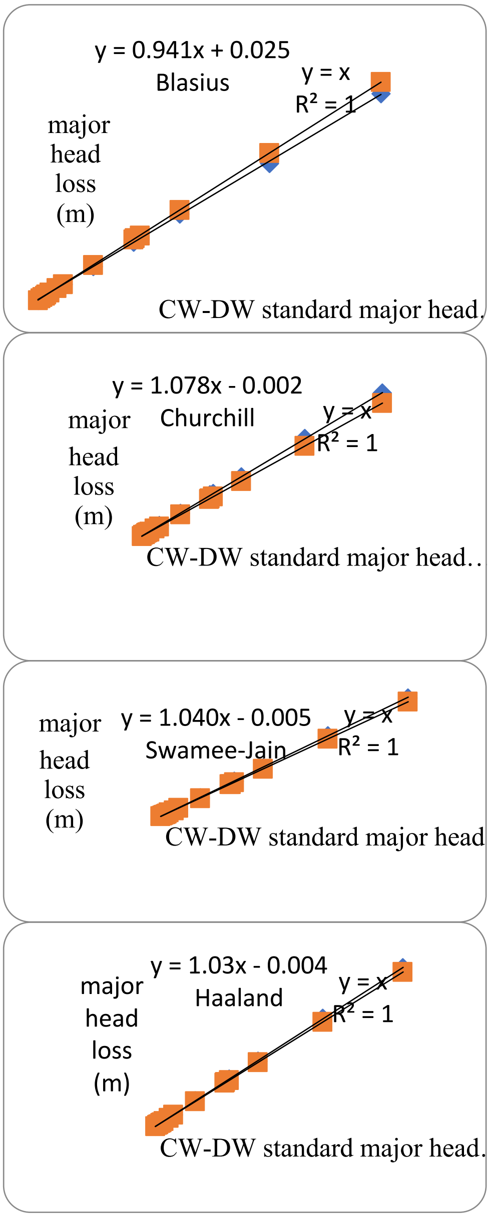
Standard Deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} \left(\frac{Oi - Pi}{Oi} \right)^2}{n}}, \qquad (3.9)$$

where O_i refers to standard observed value of major friction loss using the reference Colebrook-White formula, *Pi* represents the predicted value of the major friction loss using the various explicit equations, $\bar{O}$ denotes the arithmetic mean of the observed values, *n* corresponds to the number of observations.

4. Results and Discussion

The comparisons of the major frictional losses were made between the standard reference Colebrook-White equation and the explicit equations formulated by various researchers for the proposed case-study. Colebrook White equation was selected as a benchmark for comparison with various explicit equations as it is a globally accepted model, used under a variety of reference conditions.



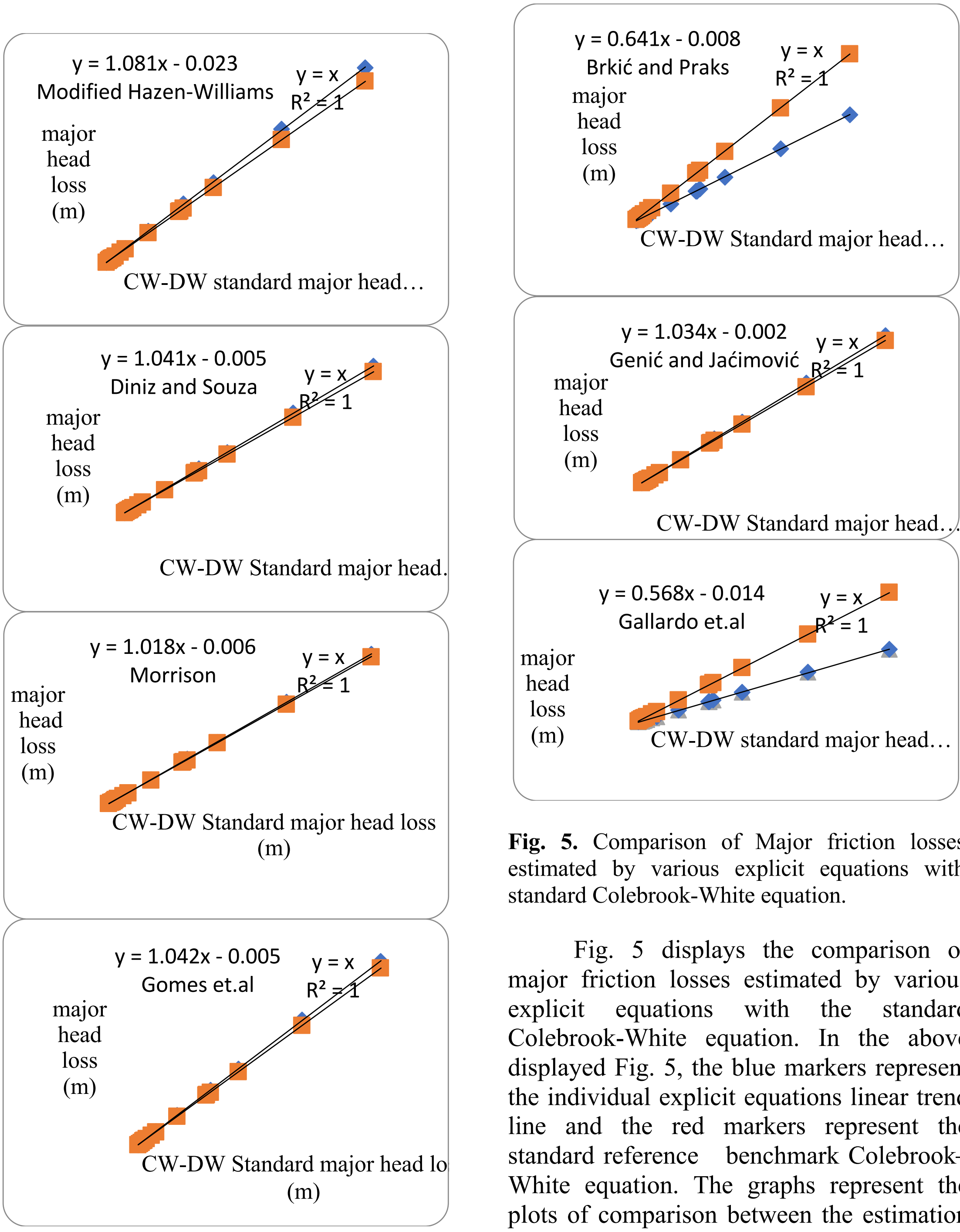


Fig. 5. Comparison of Major friction losses estimated by various explicit equations with standard Colebrook-White equation.

Fig. 5 displays the comparison of major friction losses estimated by various explicit equations with the standard Colebrook-White equation. In the above displayed Fig. 5, the blue markers represent the individual explicit equations linear trend line and the red markers represent the standard reference benchmark Colebrook–White equation. The graphs represent the plots of comparison between the estimation of major friction losses by individual explicit equations and the standard Colebrook-White equation which were estimated at the 15 links at HAC chosen for case study. The relationships between the explicit and benchmark equations were represented in the form of regression equations as shown in Fig. 5. With regard to the regression equations, the Morrison's equation resulted in a value of slope close to unity (1.01) giving the best predicted values with respect to the standard Colebrook-White equation. The second-best values were those obtained by the Haaland equation resulting in a slope value of (1.03). Followed by the above equations, the Genić and Jaćimović equation with an estimated slope value of (1.034) performed better. The Gallardo's et.al equation systematically evaluated an underestimated slope with a value of (0.56). Followed by this, Brkić and Praks equation deduced an underestimated slope with a value of (0.64). On the other hand, Diniz and Souza and Gomes et.al equation resulted in an almost uniform value of slope with a value of (1.04).

The statistical indices and error criterion techniques were used in order to analyze the performance of the explicit equations in the estimation of the major frictional loss with the standard reference equation. Table 1 lists the various components available in the existing flow network of the HAC farm. Table 2 presents the determination of major friction losses by the standard reference benchmark Colebrook–White equation. Table 3 displays the determination of Darcy friction factor by the various explicit equations. Table 4 presents the evaluation of Darcy-Weisbach major friction losses estimated using the friction factor of explicit equations determined in Table 3. Table 5 illustrates the comparison of the eleven explicit equations with the standard Colebrook-White equation in terms of the performance indices and error criterion techniques

Table 1. Components available in the existing flow network available at HAC Farm.

S.No.	Components Available in flow network	Parameters available regarding the Component
1.	Well1	Location: 15°52'30''N, 38°27'50"E Elevation: 1268.5 m Yield: 18 l/s Depth: 40 m Pump used: Submersible pump [16]
2.	Well 2	Location: 15°40'16.47''N, 39°31.6'86"E Elevation: 1269 m Yield: 8.3 l/s Depth: 7 m Pump used: Centrifugal pump [16]
3.	Minor fittings	Gate valves, Long turn elbow, elbow, sudden contraction, union, Tee,
4.	Elevation of network	Measured using Electronic type theodolite, GPS- Garmin Oregon 550
5.	Pipe Parameters	Flow discharge : Bucket and Stop watch Method Pipe Length: Odometer and tape

Table 2. Determination of Colebrook- White friction factor (f) and Colebrook White Darcy Friction Losses (h_f).

Links	Length(m)	Diameter(m)	Velocity(m/s)	Discharge(m^3/s)	Reynolds Number(Re)	Colebrook-White friction factor (f)	Colebrook-White Darcy-Weisbach Loss (h_f) (m)
AB	70	0.0762	3.946	0.0179	375907.8	0.0137	5.5918
BC	42.3	0.0762	3.362	0.0153	320303.2	0.0141	2.5268
B1	38	0.0508	0.986	0.0019	62651.32	0.0196	0.1810
C2	43.3	0.0762	0.877	0.0039	83535.1	0.0184	0.2297
CD	177.4	0.0762	2.921	0.013	278304	0.0145	8.2136
D3	89.7	0.0762	0.657	0.0029	62651.25	0.0196	0.2848
DE	61.2	0.0762	2.655	0.0121	252889.8	0.0147	2.3824
E4	106.2	0.0762	0.877	0.0039	83535.1	0.0184	0.5634
E5	74	0.0762	0.877	0.0039	83535.1	0.0184	0.3926
EF	116.3	0.0762	2.289	0.0104	218096	0.0152	3.4640
FG	53.5	0.0762	2.15	0.0098	204861.7	0.0153	1.4231
G6	12	0.0508	1.479	0.0029	93976.95	0.0180	0.1179
GH	73	0.0762	1.231	0.0056	117262.4	0.0171	0.7110
H7	33	0.0508	1.479	0.0029	93976.95	0.0180	0.3242
HI	65.5	0.0381	3.508	0.0039	167070.1	0.0160	2.4120

The components and various parameters related to the components in the flow network are listed in detail in Table 1. In Table 2, the standard reference Colebrook-White based Darcy friction losses were estimated. The bucket and stop watch method was used to determine the discharge of water during the experiment. The pipe diameter of the existing pipe network was initially determined in order to model and formulate the new network. The pipes newly modeled were assumed to be with the same existing diameter but manufactured using the PVC material due to its numerous benefits. The kinematic viscosity of the water flow was assumed to be 8×10^{-7} stoke. The roughness coefficient of PVC pipe walls was assumed to be 0.003334 mm [44, 45]. The Reynolds number was estimated using (1.1) and the flow was observed to be completely turbulent in the network comprising of 15 links at the HAC farm. Based on the determination of Re, the Darcy friction factor and Darcy-Weisbach major friction losses were determined subsequently. Table 3 displays the Darcy friction factor determined using the various explicit equations. Table 4 lists the estimated Darcy-Weisbach major friction losses calculated using the Darcy friction factor estimated in Table 3 for each case as shown below.

Table 3. Determination of Darcy Friction Factor using the various Explicit equations.

Links	Explicit Equation Numbers										
	(2.3)	(2.4)	(2.5)	(2.6)	(2.7)	(2.8)	(2.9)	(2.10)	(2.12)	(2.13)	(2.14)
AB	0.0127	0.0148	0.0143	0.0141	0.0149	0.0143	0.0139	0.0143	0.0088	0.0142	0.0078
BC	0.0132	0.0152	0.0147	0.0145	0.0152	0.0147	0.0143	0.0147	0.0090	0.0146	0.0080
B1	0.0199	0.0210	0.0200	0.0199	0.0195	0.0200	0.0196	0.0201	0.0120	0.0201	0.0102
C2	0.0185	0.0196	0.0187	0.0186	0.0186	0.0188	0.0184	0.0188	0.0114	0.0188	0.0097
CD	0.0137	0.0156	0.0150	0.0149	0.0156	0.0150	0.0147	0.0150	0.0092	0.0149	0.0082
D3	0.0199	0.0209	0.0199	0.0198	0.0194	0.0199	0.0196	0.0200	0.0120	0.0200	0.0102
DE	0.0140	0.0158	0.0152	0.0151	0.0158	0.0153	0.0150	0.0153	0.0094	0.0152	0.0083
E4	0.0185	0.0196	0.0187	0.0186	0.0186	0.0188	0.0184	0.0188	0.0114	0.0188	0.0097
E5	0.0185	0.0196	0.0187	0.0186	0.0186	0.0188	0.0184	0.0188	0.0114	0.0188	0.0097
EF	0.0146	0.0163	0.0156	0.0155	0.0161	0.0157	0.0154	0.0157	0.0096	0.0156	0.0084
FG	0.0148	0.0164	0.0158	0.0157	0.0163	0.0158	0.0155	0.0158	0.0097	0.0158	0.0085
G6	0.0180	0.0193	0.0184	0.0183	0.0184	0.0185	0.0180	0.0185	0.0111	0.0184	0.0096
GH	0.0170	0.0183	0.0175	0.0174	0.0177	0.0176	0.0173	0.0176	0.0107	0.0176	0.0092
H7	0.0180	0.0193	0.0184	0.0183	0.0184	0.0185	0.0180	0.0185	0.0111	0.0184	0.0096
HI	0.0156	0.0174	0.0167	0.0165	0.0170	0.0168	0.0161	0.0168	0.0101	0.0166	0.0088

Table 4. Determination of Darcy Friction Major Losses (m) for the various Explicit Equations.

Links	Explicit Equation Numbers										
	(2.3)	**(2.4)**	**(2.5)**	**(2.6)**	**(2.7)**	**(2.8)**	**(2.9)**	**(2.10)**	**(2.12)**	**(2.13)**	**(2.14)**
AB	5.2004	6.0541	5.8466	5.7854	6.0822	5.8549	5.7029	5.8533	3.5989	5.8044	3.2057
BC	2.3747	2.7271	2.6304	2.6041	2.7324	2.6343	2.5735	2.6343	1.6201	2.6157	1.4378
B1	0.1841	0.1936	0.1847	0.1834	0.1805	0.1851	0.1806	0.1854	0.1111	0.1855	0.0941
C2	0.2313	0.2448	0.2339	0.2324	0.2321	0.2343	0.2300	0.2348	0.1421	0.2350	0.1216
CD	7.7877	8.8433	8.5206	8.4391	8.8332	8.5338	8.3537	8.5359	5.2484	8.4845	4.6421
D3	0.2897	0.3034	0.2893	0.2876	0.2822	0.2899	0.2843	0.2905	0.1747	0.2911	0.1482
DE	2.2721	2.5613	2.4661	2.4432	2.5520	2.4700	2.4206	2.4710	1.5188	2.4578	1.3401
E4	0.5674	0.6004	0.5737	0.5700	0.5692	0.5748	0.5643	0.5759	0.3487	0.5764	0.2984
E5	0.3954	0.4183	0.3997	0.3972	0.3966	0.4005	0.3932	0.4013	0.2430	0.4016	0.2079
EF	3.3324	3.7167	3.5747	3.5431	3.6869	3.5806	3.5136	3.5830	2.2002	3.5674	1.9342
FG	1.3739	1.5257	1.4668	1.4541	1.5104	1.4692	1.4424	1.4704	0.9024	1.4645	0.7921
G6	0.1181	0.1264	0.1210	0.1200	0.1207	0.1212	0.1182	0.1213	0.0732	0.1211	0.0628
GH	0.7061	0.7587	0.7266	0.7214	0.7333	0.7280	0.7154	0.7291	0.4443	0.7285	0.3841
H7	0.3250	0.3476	0.3327	0.3300	0.3321	0.3333	0.3252	0.3338	0.2014	0.3331	0.1729
HI	2.3544	2.6252	2.5274	2.4992	2.5630	2.5314	2.4383	2.5315	1.5235	2.5101	1.3283

Table 5. Ranking based on Comparison of Explicit Equations based on Statistical Performance Indices and Error Criterion Estimation.

Explicit Equation No.	Statistical Performance and Error Criterion Estimation									
	MBE (m)	**MAE (m)**	**S_d^2**	**RMSE (m)**	**Slope**	**d**	**RE**	**E**	**σ**	**η**
(2.3)	-0.087 (7)	0.089 (7)	0.019 (7)	0.161 (7)	0.94 (7)	0.9986 (7)	0.084 (7)	0.9948 (7)	0.034 (7)	0.9974 (7)
(2.4)	0.148 (9)	0.148 (9)	0.034 (8)	0.232 (9)	1.07 (8)	0.9975 (9)	0.120 (9)	0.9894 (9)	0.072 (9)	0.9947 (9)
(2.5)	0.071 (4)	0.071 (4)	0.009 (4)	0.116 (4)	1.04 (4)	0.9993 (4)	0.060 (4)	0.9973 (4)	0.030 (4)	0.9986 (4)
(2.6)	0.052 (2)	0.052 (2)	0.005 (2)	0.086 (2)	1.03 (2)	0.9996 (2)	0.045 (2)	0.9985 (2)	0.022 (2)	0.9992 (2)
(2.7)	0.132 (8)	0.132 (8)	0.036 (9)	0.227 (8)	1.08 (9)	0.9976 (8)	0.118 (8)	0.9898 (8)	0.051 (8)	0.9949 (8)
(2.8)	0.074 (5)	0.074 (5)	0.009 (5)	0.121 (5)	1.04 (5)	0.9993 (5)	0.063 (5)	0.9971 (5)	0.032 (5)	0.9985 (5)
(2.9)	0.029 (1)	0.029 (1)	0.001 (1)	0.051 (1)	1.01 (1)	0.9998 (1)	0.026 (1)	0.9994 (1)	0.011 (1)	0.9997 (1)
(2.10)	0.075 (6)	0.075 (6)	0.009 (6)	0.121 (6)	1.04 (6)	0.9993 (6)	0.063 (6)	0.9970 (6)	0.033 (6)	0.9985 (6)
(2.12)	-0.697 (10)	0.697 (10)	0.704 (10)	1.069 (10)	0.64 (10)	0.9203 (10)	0.556 (10)	0.7756 (10)	0.372 (10)	0.8807 (10)
(2.13)	0.063 (3)	0.063 (3)	0.006 (3)	0.101 (3)	1.03 (3)	0.9995 (3)	0.052 (3)	0.9979 (3)	0.029 (3)	0.9989 (3)
(2.14)	-0.843 (11)	0.843 (11)	1.017 (11)	1.288 (11)	0.56 (11)	0.8769 (11)	0.670 (11)	0.6745 (11)	0.455 (11)	0.8212 (11)

In Table 4, the various explicit equations and their rankings based on the comparison of the statistical indices and performance error criteria are displayed when compared with the standard reference equation. The (*) represents the subsequent ranking of each and every model based on indices when compared among themselves with respect to the benchmark standard Colebrook-White Darcy Weisbach major friction loss. On comparison, the Morrison's equation (2.9) under the selection criteria of simplicity in estimating Darcy friction factor performed the best among all the other explicit equations when compared with the standard reference Colebrook-White equation, as it approximates the standard equation relatively much better as compared with other explicit equations. Although it outperformed the other equations, the equation is suggested to be implemented for smooth pipes and the equation is not recommended for *Re*=10^6 or beyond which are its limitations. The Haaland's equation (2.6) which also comes under the simplicity factor in estimating Darcy friction factor criteria estimates the Darcy friction factor and in combination with Darcy-Weisbach equation estimates the major loss in flow followed the Morrison's equation and outperformed the rest of the nine equations. The obtained result supports [24] which suggest that Haaland's equation generally provides good statistical parameters when compared with many of the existing models and requires less computation as compared to many explicit models to estimate the major head loss. The Genić and Jaćimović equation (2.13) ranked the third best model among the eleven equations based on the comparison with the explicit equations on the criteria of all the statistical indices. This equation comes under the selection category of the equation derived based on mathematical theory analysis and algorithms. The explicit models rank as the top three in this particular case study and support the results obtained by [37], where the authors recommend that the specific model's statistical parameters result in good correlation with the experimental results and its estimation of friction factor is fairly accurate. The Gallardo's et.al equation (2.14) under the selection criteria of simplicity in estimating Darcy friction factor gave the least accurate estimate for all the indices among all the tested models when compared with the standard equation. The authors [38] have reported that the explicit model approximates the standard equation and involves few computations and results in an error percentage of 1.6%. The Swamee-Jain equation (2.5) under the selection criteria of accuracy factor, although considered as more accurate and the best approximation in many studies [11, 21], was found to rank the fourth among the eleven models in this particular study. Diniz and Souza's model (2.8) [14] based on explicit equation under mathematical theory analysis and Gomes et.al model (2.10) [36] based upon AI based model almost resulted in a similar value while computing the various statistical indices and error criterion techniques. The authors Diniz and Souza also recommend that the various equations proposed by them for the determination of Darcy friction factor are also applicable for estimation of diameter, head losses and discharge in the hydraulic networks without the iteration process being necessary. The Gomes et.al explicit equation suggests that the equation proposed did not meet the exact criteria of approximation of the reference equation, although the relative error produced by it was less than 1%. In this case-study, the Gomes et.al equation resulted in values almost similar to the Diniz and Souza's explicit equation and performed much better than the remaining five explicit equations in the order of ranking.

Although Blasius equation (2.3) under the criteria of equations based on absence of absolute pipe roughness'ε' parameter is suggested to perform relatively better in case of smooth pipes, the equation ranked only 7th among the explicit equations in this case-study. The results obtained in this case study supported the results as suggested by [21] where Blasius equation does not perform much better than Swamee-Jain and Haaland's equation. Moreover, Blasius equation is not much preferable in rough pipes, whereas that's not the case with the other two equations. The modified Hazen-Williams explicit model (2.7) under the category of absence of absolute roughness'ε' parameter, ranked 8th among the explicit models. The limitation of this model is that it includes the Hazen-Williams constant which has to be precisely known in order to evaluate the exact Darcy friction factor, which includes appropriate testing for a range of *Re* and diameters available. Churchill's model (2.4) under the selection category of accuracy, determines the friction factor and friction losses which were not very accurate in this specific case-study and the model ranked 9th among all the models. The Brkić and Praks model (2.12) under the category of explicit equations derived based on mathematical analysis theory ranked last and comparatively resulted in the least accurate values for the determination of the friction factor.

5. Conclusion

The objective of this study was to estimate the major friction losses for the newly modeled pipe network located at Hamelmalo Agricultural College in Eritrea. Eleven explicit equations selected based on different criteria chosen from various available literatures were evaluated relative to Colebrook-White equation which is generally considered as the standard reference model. The selection criteria were based on five different factors namely: equations based on simplicity factor, equations based on accuracy factor, equations based on mathematical theories and algorithms, Eequations with the absence of absolute pipe roughness and lastly the equations based on AI. Based on the selection criteria, the explicit model which outperformed the rest of the other models when compared with the standard equation in this specific case-study is the Morrison's model, which is suggested to have a simple structure and covers the entire flow regime. The explicit equations based on AI, namely the Gomes et.al model and Diniz and Souza model based on mathematical theory and algorithms, performed almost similar in this case–study ranking in the 5th and 6th positions. The explicit equation based on shifted Lambert's W-function was relatively in-accurate as far as this case-study is concerned. Results obtained from Haaland's equation and S-J equation in this study almost supported the recommendations as suggested in many case-studies regarding their accuracy providing a better approximation of the reference Colebrook-White equation. Based on the selection criteria and the results obtained from the process of comparison and validation, the explicit equations based on the factor of simplicity criteria performed relatively better than the other criteria in this specific case-study. It has to be considered that under the simplicity criteria, although out of the three chosen explicit equation models two performed almost the most accurate with the standard equation among the 11 explicit models, the third explicit model performed the least accurate of all the models.

Various other existing explicit models can be chosen based on different selection criteria and can be validated for the modeled network at HAC. A number of statistical performance indices other than the ones mentioned in the case-study can be used to check the best performing explicit equations in the future.

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