P-ISSN 2586-9000
E-ISSN 2586-9027
Homepage : https://tci-thaijo.org/index.php/SciTechAsia
Science & Technology Asia
Vol. 26 No.4 October - December 2021
Page: [48-58]
Original research article

New Expansion Scheme to Solitary Wave Solutions for a Model of Wave-Wave Interactions in Plasma

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Received 25 August 2021; Received in revised form 11 November 2021
Accepted xx Month 201x; Available online 30 December 2021

ABSTRACT

In this article, a new expansion technique is executed for travelling wave solutions of Generalized Zakharov System (GZS) as a model of wave-wave interaction in plasma. New Solitary and periodic wave solutions with arbitrary parameters are obtained and expressed in terms of the hyperbolic, trigonometric, and rational functions. Some of the acquisition solutions are graphically sketched.

Keywords: Generalized Zakharov System (GZS); Periodic wave; Solitary wave solution; solution; Travelling wave solution

1. Introduction

Nonlinearity is a mesmerizing component of nature, Almost, in all the scientific and engineering fields nonlinear wave phenomena appears in one or other ways. There are of more practical use of it if one can find the travelling wave solutions for nonlinear evolution equations (NLEEs). The wave-wave interactions can also be explained by these solutions. The model of wave-wave interaction in plasma may be written as [1-3]

$$iE_t + E_{xx} - 2\beta|E|^2 E + 2\upsilon E = 0,$$
$$\upsilon_{tt} - \upsilon_{xx} + |E|^2_{xx} = 0, \qquad (1.1)$$

where, $E(x,t)$ and $\upsilon(x,t)$ are the envelope of the high-frequency electric field and plasma density measured from its equilibrium value respectively. Eq. (1.1) reduced to the classical Zakharov equations [4, 5] of plasma physics whenever $\beta = 0$. In the past three decades, several methods explored which includes-the homogeneous balance method [6], Tanh-coth scheme [5, 7], sine-cosine method [8] and so on [9-21]. Here in this article the study involving to the New Expansion Scheme for solving the generalized Zakharov system (GZS) is presented. Solitary wave propagation is also discussed to explain the wave interactions phenomenon arising in NLEEs.

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doi: 10.14456/scitechasia.2021.65

2. Explanation of Scheme

Let us consider nonlinear partial differential equation (PDE) as

$$\Phi(v, v_t, v_x, v_{xx}, v_{tt}, v_{tx}, ...) = 0. \quad (2.1)$$

This equation involving Φ is a polynomial of unknown function $v(x,t)$ with its derivatives and nonlinear terms.

Step 1: We introduce a variable $\eta = x \pm V\,t$,

$$v(x,t) = v(\eta), \quad (2.2)$$

where travelling wave speed is V. So, Eq. (2.1) will be

$$\Psi(v, v', v'', v''', ...) = 0, \quad (2.3)$$

where Ψ is a polynomial of unknown function, its derivatives with respect to η and nonlinear terms.

Step 2: let the solution of Eq. (2.3) may be written as :

$$v(\eta) = \sum_{i=0}^{N} \alpha_i (d+M)^i + \sum_{i=1}^{N} \beta_i (d+M)^{-i}, \quad (2.4)$$

where $\alpha_i (i = 0,1,2,....,N)$, d and $\beta_i (i = 1,2,....,N)$ are constant may be determined and $M(\eta)$ is as

$$M(\eta) = (G'/G), \quad (2.5)$$

where $G = G(\eta)$ solution of auxiliary nonlinear equation:

$$AGG'' - BGG' - EG^2 - C(G')^2 = 0, \quad (2.6)$$

where *A, B, C* and *E* are parameters will be determined.

The General solution of Eq. (2.6) put in Eq. (2.4) with the value of N find by homogeneous balance between nonlinear term and highest order derivative term in Eq. (2.3), so we get solutions of Eq. (2.3) and we may derive the value of $M(\eta)$ for Eq. (2.5):

Family 1: When $B \neq 0$, $\omega = A - C$ and $\Omega = B^2 + 4E(A-C) > 0$,

$$M(\eta) = \frac{B}{2\omega} + \frac{\sqrt{\Omega}}{2\omega} \frac{C_1 \sinh\left(\frac{\sqrt{\Omega}}{2A}\eta\right) + C_2 \cosh\left(\frac{\sqrt{\Omega}}{2A}\eta\right)}{C_1 \cosh\left(\frac{\sqrt{\Omega}}{2A}\eta\right) + C_2 \sinh\left(\frac{\sqrt{\Omega}}{2A}\eta\right)}. \quad (2.7)$$

Family 2: When $B \neq 0$, $A - C = \omega$ and $B^2 + 4E(A-C) = \Omega < 0$,

$$M(\eta) = \frac{B}{2\omega} + \frac{\sqrt{-\Omega}}{2\omega} \frac{-C_1 \sin\left(\frac{\sqrt{-\Omega}}{2A}\eta\right) + C_2 \cosh\left(\frac{\sqrt{-\Omega}}{2A}\eta\right)}{C_1 \cos\left(\frac{\sqrt{-\Omega}}{2A}\eta\right) + C_2 \sinh\left(\frac{\sqrt{-\Omega}}{2A}\eta\right)}. \quad (2.8)$$

Family 3: $B \neq 0$, $A - C = \omega$ and $B^2 + 4E(A-C) = \Omega = 0$,

$$M(\eta) = \frac{B}{2\omega} + \frac{C_2}{C_1 + C_2\eta}. \quad (2.9)$$

Family 4: $B = 0$, $A - C = \omega$ and $\Delta = \omega E > 0$,

$$M(\eta) = \frac{\sqrt{\Delta}}{\omega} \frac{C_1 \sinh(\frac{\sqrt{\Delta}}{A}\eta) + C_2 \cosh(\frac{\sqrt{\Delta}}{A}\eta)}{C_1 \cosh(\frac{\sqrt{\Delta}}{A}\eta) + C_2 \sinh(\frac{\sqrt{\Delta}}{A}\eta)}. \quad (2.10)$$

Family 5: When $B = 0$, $A - C = \omega$ and $\Delta = \omega E < 0$,

$$M(\eta) = \frac{\sqrt{-\Delta}}{\omega} \frac{C_1 \sin(\frac{\sqrt{-\Delta}}{A}\eta) + C_2 \cos(\frac{\sqrt{\Delta}}{A}\eta)}{C_1 \cos(\frac{\sqrt{-\Delta}}{A}\eta) + C_2 \sin(\frac{\sqrt{-\Delta}}{A}\eta)}. \quad (2.11)$$

3. Application of Scheme to Model of Wave-Wave Interactions in Plasma

Let for Eq. (1.1)

$$E(t,x) = u(\eta)e^{i\theta}, \; \upsilon(t,x) = \psi(\eta),$$

$$\theta = kx + qt,\ \eta = \left(x - 2kt\right)p.$$

We get

$$u(k^2 + q) - p^2u'' + 2\beta u^3 - 2u\psi = 0, \quad (3.1)$$

$$(4k^2 - 1)\psi'' + u'' = 0. \quad (3.2)$$

Twice time integral of Eq. (3.2) with first integration constant zero, we get

$$\psi(\eta) = \frac{c - u^2}{4k^2 - 1}. \quad (3.3)$$

Its use in eq. (3.1), we get

$$au - p^2u'' + 2bu^3 = 0, \quad (3.4)$$

where

$$a = \left(k^2 + q - \frac{2c}{4k^2 - 1}\right), \quad b = \left(\beta + \frac{1}{4k^2 - 1}\right),$$

the value of N by homogeneous balance between nonlinear term u^3 and highest order derivative term u'' in Eq. (3.4) such that $3N = N + 2$ i.e., $N = 1$. So, we get from Eq. (2.4):

$$u(\eta) = \alpha_0 + \alpha_1(d + M) + \beta_1(d + M)^{-1} \quad (3.5)$$

It put with Eqs. (2.5) and (2.6) into Eq. (3.4), we yields a set of simultaneous algebraic equations, if we equate each coefficient of resultant polynomial to zero. Solve these algebraic equations and got three set of solutions:

Set 1:

$$\alpha_0 = \frac{Aa - 4bhd}{2Ab}, \quad \alpha_1 = 0, \ d = d,$$

$$\beta_1 = -\frac{p^2}{bA}(d^2\omega + Bd - E), \quad (3.6)$$

where $A - C = \omega$, $h = -\frac{p^2}{2b}$ other parameters are unrestricted.

Set 2:

$$\alpha_0 = \frac{1}{2b} - \frac{hB + 2hd\omega}{A},$$

$$\beta_1 = 0, \alpha_1 = \frac{2h\omega}{A}, d = d. \quad (3.7)$$

where $A - C = \omega,\ h = -\frac{p^2}{2b}$ other parameters are unrestricted.

Set 3:

$$\alpha_0 = \frac{a}{2b}, \ \alpha_1 = \frac{p^2\omega}{bA}, \ d = -\frac{B}{2\omega},$$

$$\beta_1 = \frac{p^2}{2bA\omega}(4E\omega + B^2), \quad (3.8)$$

where $A - C = \omega$, other parameters are unrestricted.

Solution for set 1 with Family-1

If $C_1 = 0$ but $C_2 \neq 0$,

$$u_{1_1}(\eta) = \frac{Aa - 4bhd}{2Ab} - \frac{p^2}{bA}(d^2\omega + Bd - E)\left(d + \frac{B}{2\omega} + \frac{\sqrt{\Omega}}{2\omega}\coth(\frac{\sqrt{\Omega}}{2A}\eta)\right)^{-1}.$$

If $C_2 = 0$ but $C_1 \neq 0$,

$$u_{1_2}(\eta) = \frac{Aa - 4bhd}{2Ab} - \frac{p^2}{bA}(d^2\omega + Bd - E)\left(d + \frac{B}{2\omega} + \frac{\sqrt{\Omega}}{2\omega}\tanh(\frac{\sqrt{\Omega}}{2A}\eta)\right)^{-1}.$$

Solution for set 1 with Family-2

If $C_1 = 0$ but $C_2 \neq 0$,

$$u_{1_3}(\eta)=\frac{Aa-4bhd}{2Ab}-\frac{p^2}{bA}(d^2\omega+Bd-E)\times\left(d+\frac{B}{2\omega}+\frac{\sqrt{-\Omega}}{2\omega}\coth(\frac{\sqrt{-\Omega}}{2A}\eta)\right)^{-1}.$$

If $C_2=0$ but $C_1\neq 0$,

$$u_{1_4}(\eta)=\frac{Aa-4bhd}{2Ab}-\frac{p^2}{bA}(d^2\omega+Bd-E)\times\left(d+\frac{B}{2\omega}+\frac{\sqrt{-\Omega}}{2\omega}\tan(\frac{\sqrt{\Omega}}{2A}\eta)\right)^{-1}.$$

Solution for set 1 with Family-3

$$u_{1_5}(\eta)=\frac{Aa-4bhd}{2Ab}-\frac{p^2}{bA}(d^2\omega+Bd-E)\left(d+\frac{B}{2\omega}+\frac{C_2}{C_1+C_2\eta}\right)^{-1}.$$

Solution For set 1 with Family-4

If $C_1=0$ but $C_2\neq 0$;

$$u_{1_6}(\eta)=\frac{Aa-4bhd}{2Ab}-\frac{p^2}{bA}(d^2\omega+Bd-E)\times(d+\frac{\sqrt{\Delta}}{\omega}\coth(\frac{\sqrt{\Delta}}{A}\eta))^{-1}.$$

If $C_2=0$ but $C_1\neq 0$

$$u_{1_7}(\eta)=\frac{Aa-4bhd}{2Ab}-\frac{p^2}{bA}(d^2\omega+Bd-E)\times(d+\frac{\sqrt{\Delta}}{\omega}\tanh(\frac{\sqrt{\Delta}}{A}\eta))^{-1}.$$

Solution For set 1 with Family-5

If $C_1\neq 0, C_2=0$,

$$u_{1_8}(\eta)=\frac{Aa-4bhd}{2Ab}-\frac{p^2}{bA}(d^2\omega+Bd-E)\times(d+\frac{\sqrt{-\Delta}}{\omega}\cot(\frac{\sqrt{-\Delta}}{A}\eta))^{-1},$$

$$u_{1_9}(\eta)=\frac{Aa-4bhd}{2Ab}-\frac{p^2}{bA}(d^2\omega+Bd-E)\times(d-\frac{\sqrt{-\Delta}}{\omega}\tan(\frac{\sqrt{-\Delta}}{A}\eta))^{-1}.$$

Similarly, Solution For set 2 with Family-1

If $C_1=0$ but $C_2\neq 0$, $u_{2_1}(\eta)=\frac{1}{2b}+\frac{1}{A}h\sqrt{\Omega}\coth(\frac{\sqrt{\Omega}}{2A}\eta).$

If $C_2=0$ but $C_1\neq 0$, $u_{2_2}(\eta)=\frac{1}{2b}+\frac{1}{A}h\sqrt{\Omega}\tanh(\frac{\sqrt{\Omega}}{2A}\eta)).$

Solution For set 2 with Family-2

If $C_1=0$ but $C_2\neq 0$, $u_{2_3}(\eta)=\frac{1}{2b}+\frac{1}{A}h\sqrt{\Omega}\coth(\frac{\sqrt{-\Omega}}{2A}\eta)).$

If $C_2=0$ but $C_1\neq 0$, $u_{2_4}(\eta)=\frac{1}{2b}+\frac{1}{A}h\sqrt{\Omega}\tan(\frac{\sqrt{-\Omega}}{2A}\eta).$

Solution For set 2 with Family-3

$$u_{2_5}(\eta)=\frac{1}{2b}-\frac{2h\omega}{A}\left(\frac{C_2}{C_1+C_2\eta}\right).$$

Solution For set 2 with Family-4

If $C_1=0$ but $C_2\neq 0$, $u_{2_6}(\eta)=\frac{1}{2b}-\frac{h}{A}\left(B-2\sqrt{\Delta}\coth(\frac{\sqrt{\Delta}}{A}n)\right).$

If $C_2=0$ but $C_1\neq 0$, $w_{2_7}(\eta)=\frac{1}{2b}-\frac{h}{A}\left(B-2\sqrt{\Delta}\tanh(\frac{\sqrt{\Delta}}{A}n)\right).$

Solution For set 2 with Family-5

If $C_1=0$ but $C_2\neq 0$, $u_{2_8}(\eta)=\frac{1}{2b}-\frac{h}{A}\left(B-2i\sqrt{\Delta}\cot(\frac{\sqrt{-\Delta}}{A}n)\right).$

If $C_2=0$ but $C_1\neq 0$, $u_{2_9}(\eta)=\frac{1}{2b}-\frac{h}{A}\left(B-2i\sqrt{\Delta}\tan(\frac{\sqrt{-\Delta}}{A}n)\right).$

Similarly, we can find the **Solution For set 3 with Family-1**

If $C_1=0$ but $C_2\neq 0$,

$$u_{3_1}(\eta)=\frac{a}{2b}+\frac{p^2\sqrt{\Omega}}{2bA}\times\coth(\frac{\sqrt{\Omega}}{2A}\eta)+\frac{p^2}{bA\sqrt{\Omega}}(4E\omega+B^2)\times\coth(\frac{\sqrt{\Omega}}{2A}\eta).$$

If $C_2=0$ but $C_1\neq 0$,

$$u_{3_2}(\eta)=\frac{a}{2b}+\frac{p^2\sqrt{\Omega}}{2bA}\times\tanh(\frac{\sqrt{\Omega}}{2A}\eta)+\frac{p^2}{bA\sqrt{\Omega}}(4E\omega+B^2)\times\tanh(\frac{\sqrt{\Omega}}{2A}\eta)).$$

Solution For set 3 with Family-2

If $C_1=0$ but $C_2\neq 0$,

$$u_{3_3}(\eta)=\frac{a}{2b}+i\frac{p^2\sqrt{\Omega}}{2bA}\coth(\frac{\sqrt{-\Omega}}{2A}\eta)-i\frac{p^2}{bA\sqrt{\Omega}}(4E\omega+B^2)\cot(\frac{\sqrt{-\Omega}}{2A}\eta).$$

If $C_2=0$ but $C_1\neq 0$,

$$u_{3_4}(\eta)=\frac{a}{2b}-i\frac{p^2\sqrt{\Omega}}{2bA}\tan(\frac{\sqrt{-\Omega}}{2A}\eta)+i\frac{p^2}{bA\sqrt{\Omega}}(4E\omega+B^2)\tanh(\frac{\sqrt{-\Omega}}{2A}\eta).$$

Solution For set 3 with Family-3

$$u_{3_5}(\eta)=\frac{a}{2b}+\frac{p^2\omega}{bA}\left(\frac{C_2}{C_1+C_2\eta}\right)+\frac{p^2}{2bA\omega}(4E\omega+B^2)\left(\frac{C_2}{C_1+C_2\eta}\right)^{-1}.$$

Solution For set 3 with Family-4

If $C_1=0$ but $C_2\neq 0,$

$$u_{3_6}(\eta)=\frac{a}{2b}+\frac{p^2}{bA}\left(\frac{-B}{2}+\sqrt{\Delta}\coth(\frac{\sqrt{\Delta}}{A}n)\right)+\frac{p^2}{4bA}(4E\omega+B^2)\left(-B+2\sqrt{\Delta}\coth(\frac{\sqrt{\Delta}}{A}n)\right)^{-1}.$$

If $C_2=0$ but $C_1\neq 0,$

$$u_{3_7}(\eta)=\frac{a}{2b}+\frac{p^2}{bA}\left(\frac{-B}{2}+\sqrt{\Delta}\tanh(\frac{\sqrt{\Delta}}{A}n)\right)+\frac{p^2}{4bA}(4E\omega+B^2)\left(-B+2\sqrt{\Delta}\tanh(\frac{\sqrt{\Delta}}{A}n)\right)^{-1}.$$

Solution For set 3 with Family-5

If $C_1=0$ but $C_2\neq 0,$

$$u_{3_8}(\eta)=\frac{a}{2b}+\frac{p^2}{2bA}\left(-B+2i\sqrt{\Delta}\cot(\frac{\sqrt{-\Delta}}{A}n)\right)+\frac{p^2}{4bA}(4E\omega+B^2)\left(-B+2i\sqrt{\Delta}\cot(\frac{\sqrt{\Delta}}{A}\eta)\right)^{-1}.$$

If $C_2=0$ but $C_1\neq 0,$

$$u_{3_9}(\eta)=\frac{a}{2b}+\frac{p^2}{2bA}\left(-B+2i\sqrt{\Delta}\tan(\frac{\sqrt{-\Delta}}{A}n)\right)+\frac{p^2}{4bA}(4E\omega+B^2)\left(-B+2i\sqrt{\Delta}\tan(\frac{\sqrt{\Delta}}{A}\eta)\right)^{-1}.$$

4. Results and Discussion

It is worth announcement that the some of our possessed solutions are in good approximation with already done results which are relevant as following:

(1) With $\left(B^2+4E\right)>0$, $C=0$, $\omega=A=1$ $a=\left(k^2+q-\frac{2c}{4k^2-1}\right),b=\left(\beta+\frac{1}{4k^2-1}\right),$

$$u_{3_1}(\eta)=\frac{a}{2b}+\frac{3p^2}{b}\sqrt{\left(B^2+4E\right)}\coth\left(\frac{\sqrt{\left(B^2+4E\right)}}{2}p(x-2kt)\right),$$

then

$$E(x,t)=\left\{\frac{a}{2b}+\frac{3p^2}{b}\sqrt{\left(B^2+4E\right)}\coth\left(\frac{\sqrt{\left(B^2+4E\right)}}{2}p(x-2kt)\right)\right\}e^{i(kx+qt)},$$

$$v(x,t)=\frac{c}{4k^2-1}-\frac{1}{4k^2-1}\left\{\frac{a}{2b}+\frac{3p^2}{b}\sqrt{\left(B^2+4E\right)}\coth\left(\frac{\sqrt{\left(B^2+4E\right)}}{2}p(x-2kt)\right)\right\}^2.$$

(2) With $B=0$, $d=0$ $AE>0$, $C=0$, $\omega=A$,

$$a=\left(k^2+q-\frac{2c}{4k^2-1}\right),b=\left(\beta+\frac{1}{4k^2-1}\right),u_{1_6}(\eta)=\frac{a}{2b}+\frac{p^2}{b}\sqrt{\frac{E}{A}}\tanh\left(\sqrt{\frac{E}{A}}p(x-2kt)\right).$$

then

$$E(x,t)=\left\{\frac{a}{2b}+\frac{p^2}{b}\sqrt{\frac{E}{A}}\tanh\left(\sqrt{\frac{E}{A}}p(x-2kt)\right)\right\}e^{i(kx+qt)},$$

$$v(x,t)=\frac{c}{4k^2-1}-\frac{1}{4k^2-1}\left\{\frac{a}{2b}+\frac{p^2}{b}\sqrt{\frac{E}{A}}\tanh\left(\sqrt{\frac{E}{A}}p(x-2kt)\right)\right\}^2.$$

(3) With $B=0$, $d=0$ $AE<0$, $C=0$, $\omega=A$,

$$a=\left(k^2+q-\frac{2c}{4k^2-1}\right),b=\left(\beta+\frac{1}{4k^2-1}\right),\ u_{2_9}(\eta)=\frac{1}{2b}-\frac{p^2}{b}\sqrt{\frac{-E}{A}}\tan\left(\sqrt{\frac{-E}{A}}p(x-2kt)\right),$$

then

$$E(x,t)=\left\{\frac{1}{2b}-\frac{p^2}{b}\sqrt{\frac{-E}{A}}\tan\left(\sqrt{\frac{-E}{A}}p(x-2kt)\right)\right\}e^{i(kx+qt)},$$

$$v(x,t)=\frac{c}{4k^2-1}-\frac{1}{4k^2-1}\left\{\frac{1}{2b}-\frac{p^2}{b}\sqrt{\frac{-E}{A}}\tan\left(\sqrt{\frac{-E}{A}}p(x-2kt)\right)\right\}^2.$$

(4) With $B=0$, $\Delta=\omega E=AE>0$, $C=0$, $\omega=A$,

$$a=\left(k^2+q-\frac{2c}{4k^2-1}\right),b=\left(\beta+\frac{1}{4k^2-1}\right),$$

$$u_{3_7}(\eta)=\frac{a}{2b}+\frac{p^2\sqrt{\Delta}}{bA}\left(2\tanh(\frac{\sqrt{\Delta}}{A}n)+\coth(\frac{\sqrt{\Delta}}{A}n)\right).$$

then

$$E(x,t)=\left\{\frac{a}{2b}+\frac{p^2\sqrt{\Delta}}{bA}\left(2\tanh(\frac{\sqrt{\Delta}}{A}n)+\coth(\frac{\sqrt{\Delta}}{A}n)\right)\right\}e^{i(kx+qt)},$$

$$v(x,t)=\frac{c}{4k^2-1}-\frac{1}{4k^2-1}\left\{\frac{a}{2b}+\frac{p^2\sqrt{\Delta}}{bA}\left(2\tanh(\frac{\sqrt{\Delta}}{A}n)+\coth(\frac{\sqrt{\Delta}}{A}n)\right)\right\}^2.$$

(5) With $B=0$, $d=0$, $\Delta=\omega E=AE<0$, $C=0$, $\omega=A$,

$$a=\left(k^2+q-\frac{2c}{4k^2-1}\right), b=\left(\beta+\frac{1}{4k^2-1}\right),$$

$$u_{3_9}(\eta)=\frac{1}{2b}-\frac{ip^2\sqrt{\Delta}}{2Ab}\left\{2\tan\left(\frac{\sqrt{-\Delta}}{A}p(x-2kt)\right)-\cot\left(\frac{\sqrt{-\Delta}}{A}p(x-2kt)\right)\right\},$$

then

$$E(x,t)=\left\{\frac{1}{2b}-\frac{ip^2\sqrt{\Delta}}{2Ab}\left\{2\tan\left(\frac{\sqrt{-\Delta}}{A}p(x-2kt)\right)-\cot\left(\frac{\sqrt{-\Delta}}{A}p(x-2kt)\right)\right\}\right\}e^{i(kx+qt)},$$

$$v(x,t)=\frac{c}{4k^2-1}-\frac{1}{4k^2-1}\left\{\frac{1}{2b}-\frac{ip^2\sqrt{\Delta}}{2Ab}\left\{2\tan\left(\frac{\sqrt{-\Delta}}{A}p(x-2kt)\right)-\cot\left(\frac{\sqrt{-\Delta}}{A}p(x-2kt)\right)\right\}\right\}^2.$$

Besides this enumeration, we may gain more new solutions (travelling and solitary wave solutions) like $u_{2_2}(\eta), u_{2_4}(\eta), u_{2_5}(\eta)-u_{2_9}(\eta)$, $u_{1_1}(\eta)-u_{1_9}(\eta), u_{3_2}(\eta)-u_{3_6}(\eta)$, which not established in the prevalence literature. But $u_{3_1}(\eta)$, $u_{1_6}(\eta)$ and $u_{2_6}(\eta)$ are as same the solution of [5].

4.1. Graphically sketched of the solution

The u_{2_2} plotting to explore the soliton solutions for the model Eq. (1.1).

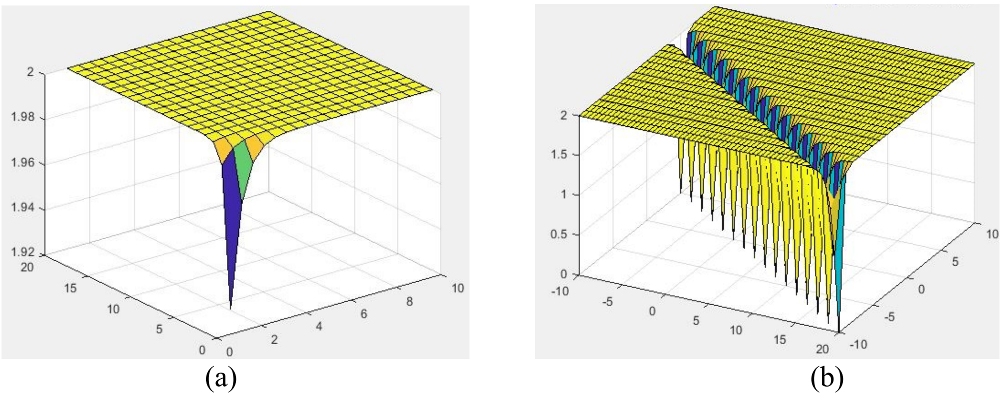

Fig. 1. (a) The one-solitary wave propagation (b) More parallel solitons plot for Eq. (1.1) with $-15\le x\le 15,\ -5\le t\le 5,\ \beta\ne 0,\ -q=d=-1/2, A=B=1,\ C=0,\ c=0, k=p=1/\sqrt{2}$.

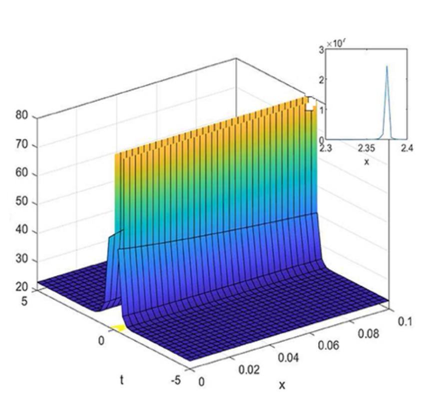

Fig. 2. Modular in 3D, 2D plot of Kink wave Shape of $u_{1_2}(\eta)$ when $A=B=1,\ C=0,\ c=0,$ $k=p=1/\sqrt{2},\ -q=d=-1/2,$ $-5\le t\le 5,\ 0\le x\le 0.1,\ \beta\ne 0$.

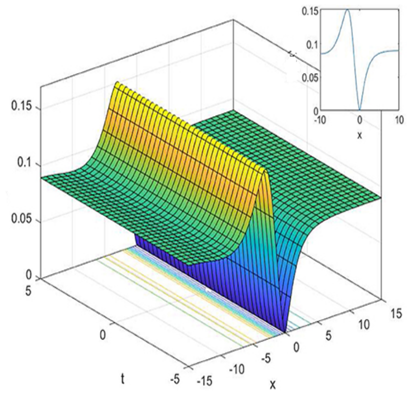

Fig. 3. Modular plot of Kink wave Shape of $u_{3_7}(\eta)$ when $A=E=1,\ B=0,\ C=0,\ d=0,\ c=0,$ $k=p=1/\sqrt{2},\ -q=d=-1/2,$ $-15\le x\le 15,\ -5\le t\le 5,\ \beta\ne 0$.

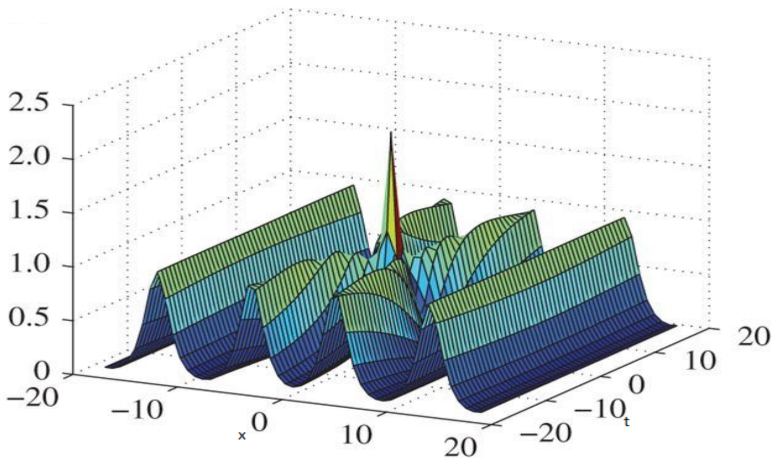

Fig. 4. Mesh 3D graph for solitons interaction $\left\{|E|^2+\nu\right\}$ for $\beta=-1$.

5. Conclusion

A new expansion scheme is exhibited to locate solitary wave solution of the model of wave-wave interactions in plasma for generalized Zakharov equations (GZE). The solitary wave solution Figs. 1, 2, 3 and its interaction in Fig. 4 of this system are exist. Therefore, we learn that this method well emulates development of many exact travelling wave solutions are gotten which contain the new soliton-like hyperbolic function solutions $u_{3_7}(\eta)$, $u_{2_2}(\eta)$ trigonometric function solutions $u_{2_9}(\eta)$, $u_{3_9}(\eta)$ and rational solutions $u_{2_5}(\eta)$ $u_{3_5}(\eta)$. Solitons are caused by a cancellation of nonlinear and dispersive effects in wave interactions. The term $\left\{|E|^2+\nu\right\}$ is plotted for $\beta=-1$ for the results mentioned in results and discussion section. The given plot is of the last results where E and v both have iota and tan & cot terms. In calculating the $\left\{|E|^2+\nu\right\}$, the *iota* terms are neutralized but the tan and cot terms together form the solitons in travelling wave. The terms under *cot* function in $u_{2_8}(\eta)$ and under *tan* function in $u_{2_9}(\eta)$ create the instability. So, it is estimated that there may be the possibility of modulational instability due to this nature, but it is the further topic of research. This scheme has shown the way for a long-playing computing truthfulness.

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