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Multi-Objective Fixed-Charge Linear Fractional Transportation Problem Using Rough Programming

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ABSTRACT

This paper uses rough programming to discuss the Multi-Objective Fixed-Charge Linear Fractional Transportation Problem (MOFCLFTP). In real life, market globalization, uncontrollable factors, and other factors may prevent you from accurately defining variables for the MOFCLFTP. Therefore, variables that use MOFCLFTP are roughly spaced. The proposed form uses an expected value operator that translates into a rough MOFCLFTP deterministic MOFCLFTP. Three different methods are used: fuzzy programming, weighted sum, and global criterion to get the best compromise for the proposed model. The MOFCLFTP example has included the theory basis.

Keywords: Fuzzy programming; Global criterion method; Multi-objective fixed-charge linear fractional transportation problem; Rough variables; Weighted-sum method

1. Introduction

It is the most important area of supply chain management and has the potential to reduce shipping costs and other factors. This is called a transportation problem. Supply chain management is becoming increasingly important in understanding the global economy and transportation systems. Information about the transportation system is essential to effectively and economically manage corporate responsibility.

Traditional transportation problems are well-known optimization problems in operations research that take into account objective functions, but in practice, two or more criteria are more important than one. When similar products are shipped from the source to the destination, the decision-maker (DM) analyzes various aspects such as shipping costs, fixed opening charges, product delivery times, and product deterioration rates. By default, transportation problems deal with several objective functions at the same time. In this case, the traditional transport problem has become a multi-objective transportation problem.

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In addition to the variable transportation costs in many practical problems, if the variable transportation costs are positive, the transportation problems are usually associated with fixed costs. In addition to shipping costs, fixed costs also generate fixed-charge. This is a so-called fixed-charge transportation problem (FCTP). There may be fixed costs suitable for car rental, license fees, tolls, etc. One form of the MOTP extends to the MOFCTP.

Midya et al. [1] is discussed the problem using rough programming of the MOFCTP. Convert rough multi-objective fixed-charge transportation problem to deterministic MOFCTP by expected value operator. Than get Pareto-solution solution from deterministic MOFCTP using by fuzzy programming method (FPM) and linear weighted-sum method (WSM). After that, take out the method to realize the comparative study between the solutions. Roy et al. [2] believes that when the supply and demand parameters are rough variables, the parameters of the objective function are random rough variables (RRV) of MOFCTP. Due to the globalization of the bazaar, unmanageable factors, etc., it may not be possible to accurately define the parameters of MOFCTP. It is recommended to use MOFCTP in rough and random rough environments. The proposed model uses expected value operators to deal with roughness and random roughness parameters. Therefore, a program was developed to convert the uncertain MOFCTP into a deterministic type and then to solve the deterministic model. There are global criteria, fuzzy programming, and ϵ-constraint methods respectively. Compare the best solutions extracted from different methods to provide the best solution to the problem raised. The ε-constraint method can now derive a set of optimal results and generate an accurate Pareto solution. Finally, the applicability and feasibility of the model are proven.

Tao et al. [3] planned a rough multi-objective planning form for solid transportation problems. The characteristics of viable and effective solutions for rough multi-objective planning problems are being studied. Utilize the interactive fuzzy satisfaction method to obtain a compromise. In sort to solve the multi-objective rough transportation problem, a genetic algorithm based on rough simulation is proposed, and the rough simulation is incorporated into the genetic algorithm. Bharati [4] In this case, the fractional transportation problem (FTP) plays a full role, not the usual transportation problem. The literature provides some limited methods of using accurate or fuzzy parameters to solve fractional transport problems, and due to inaccurate information, existing methods are uncertain due to uncertain factors FTP Cannot be resolved properly. Here, he introduced the concept of intuitive fuzzy expectations of trapezoidal intuitive fuzzy numbers and proposed some related theorems. In addition, he proposed a TIFFTP problem-solving solution based on the concept of trapezoidal intuitive fuzzy fractional transport problem (TIFFTP) and trapezoidal intuitive fuzzy number expectations. Sheng et al. [5] discussed the fixed FCTP with uncertain variables. There are two kinds of fixed cost transportation problems: direct costs and fixed costs. Direct costs are the costs connected with each source-destination pair. For any transportation activities in the corresponding source-destination pair, a fixed charge will be charged. Based on the uncertainty theory, the uncertain fixed charge transportation problem is modeled. The inverse uncertainty distribution allows the model to be transportation into a deterministic form. Safi et al. [6] planned the problem of FCTP under uncertainty, especially when the parameters are given in interval form. In this case, both cost and constraint parameters are expected to reach the number of intervals. Two solution procedures have been developed to get the optimal solution for the interval-fixed charge transportation problem, taking into account two different order relationships of the number of intervals. You can better understand the difference between these two orders by comparing them. Joshi et al. [7-8] discussed about the multi-objective transportation problem using fuzzy and more for less paradox.

This paper is prepared as follows: some basic definition of rough set and intervals in sec 2. Rough MOFCLFTP and deterministic MOFCLFTP are discussed in sec 3. Solving multi-objective programming problems from proposes three methods in sec 4. Take an example on the rough MOFCLFTP in sec 5. Discussed the results of proposed methods are in sec 6. In last section declared to conclusion.

2. Definition

2.1 Rough space and rough set

Let Λ be a non-empty set, $\mathcal{A}$ be a σ algebra of subset of Λ, Δ be an element in $\mathcal{A}$, and π be the union function of non-negative real values. In that case, $(\Lambda, \Delta, \mathcal{A}, \pi)$ is called a rough space [9].

If the bounding area of X is empty, set X is crisp (exact to R).
If the bounding area of X is not empty, set X is rough (not affected by R).
The equivalence class of R is determined by the element x and is represented by $R(x)$. In a sense, indistinguishable relations represent an inadequate understanding of our universe. Equivalence classes of indistinguishable relations (called particles produced by R) represent a fundamental part of knowledge [10].

R-lower approximation of X as

$$\underline{R}(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}.$$

R-upper approximation of X as

$$\bar{R}(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \varnothing\}.$$

R-boundary region of X as

$$BN_R(X) = \bar{R}(X) - \underline{R}(X).$$

2.2 Rough intervals

The method used to solve complex parameter problems is called "Rough intervals". The upper and lower bound approximations of the interval concept meet the mathematical definition of the upper and lower bound approximations of the rough set applicable to the interval. The concept of coarse sections was first proposed by Rebolledo [11]. Rough set theory reduces conceptual ambiguity to uncertain boundaries. Within these uncertain areas, it is not possible to draw clear conclusions about the problem at rough intervals (such as rough settings). Therefore, a rough membership value should also be defined for a rough interval.

The qualitative value RI is called rough interval. If people can assign two closed intervals $\underline{RI}$ and $\overline{RI}$ on R to it where $\underline{RI} \subseteq \overline{RI}$ and the following criteria are followed:

If $x \in \underline{RI}$ then RI of course takes x.

If $x \in \overline{RI}$ then RI can takes x.

If $x \notin \overline{RI}$ then RI certainly does not takes x.

2.3 Expected value on rough interval

Probability theory corresponds to the expected interval operator with the expected rough interval parameter in the expected interval form.

1. Let α be a rough variable on the rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$. The expected value of α is defined as follow

$$E[\alpha] = \int_0^{\infty} Tr\{\alpha \geq r\} dr - \int_{-\infty}^{0} Tr\{\alpha \leq r\} dr.$$

Suppose you have at least one integral exists, E is the operator of the expected value and the short form Tr signify trust measure [9].

Theorem 2.1: Assuming that

$$\alpha = \left[(\theta,\phi),(\chi,\psi)\right](\chi \le \theta \le \phi \le \psi),$$

is a rough interval, the expected value of α is

$$E[\alpha] = \frac{1}{2}\left[\eta(\theta+\phi)+(1-\eta)(\chi+\psi)\right],$$

where a parameter $\eta \in (0,1)$ is [10].

Theorem 2.2: Let the rough variable with finite expected values are

$$\alpha = \left[(x_1,x_2),(x_3,x_4)\right],$$

and

$$\beta = \left[(y_1,y_2),(y_3,y_4)\right].$$

Then, we get $E[a\alpha + b\alpha] = aE[\alpha] + bE[\alpha]$ for any real numbers a and b [9].

Theorem 2.3: If $\tilde{c}_{ij}$ is defined as rough interval of $\tilde{c}_{ij} = \left[(c_{ij1},c_{ij2}),(c_{ij3},c_{ij4})\right]$ where $c_{ij3} \le c_{ij1} \le c_{ij2} \le c_{ij4}$, then

$$E\left[\tilde{c}_{ij}\right] = \frac{1}{2}\left[\eta\left(c_{ij1}+c_{ij2}\right)+(1-\eta)\left(c_{ij3}+c_{ij4}\right)\right], \forall i,j. \quad (1)$$

Among them $\eta \in (0,1)$ is a limitation determined by the decision maker [10].

3. Mathematics Formulation

Consider three objective functions whereas first is represent the fixed cost of shipping at each starting point, the second is consider the shipping time of the goods, and the third is represent the rate of deterioration of the goods respectively. Suppose you have m sources $(i = 1,2,\ldots,m)$ and product is sourced from this source to n destinations $(j = 1,2,\ldots,n)$. The problem is to determine the unknown quantity x_{ij} (coefficient of determination) that is transported from the i^{th} source to the j^{th} target in order to minimize the values of the three objective functions. y_{ij} is binary number if i^{th} source is used '1' or not '0'. All the parameters of MOFCLFTP can be regarded as rough intervals without losing simplicity.

The MOFCLFTP mathematical model has a rough interval factor that can be expressed as:

[P1]

$$Min \tilde{Z}_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{\tilde{c}_{ij}x_{ij}}{\tilde{c}'_{ij}x_{ij}} + \frac{\tilde{f}_{ij}y_{ij}}{\tilde{f}'_{ij}y_{ij}}\right),$$

$$Min \tilde{Z}_2 = \left[\max \frac{\tilde{t}_{ij}}{\tilde{t}'_{ij}} : x_{ij} > 0, \forall i,j\right],$$

$$Min \tilde{Z}_3 = \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\tilde{d}_{ij}x_{ij}}{\tilde{d}'_{ij}x_{ij}},$$

subject to

$$\sum_{j=1}^{n} x_{ij} \le \tilde{a}_i,\ i = 1,2,\ldots,m,$$

$$\sum_{i=1}^{m} x_{ij} \le \tilde{b}_i,\ j = 1,2,\ldots,n,$$

$$x_{ij} \ge 0,\ \forall i,j,$$

$$y_{ij} = 0 \text{ if } x_{ij} = 0,$$

$$y_{ij} = 1 \text{ if } x_{ij} > 0.$$

The feasibility condition of model [P1] is

$$\sum_{j=1}^{n}\tilde{a}_i \ge \sum_{i=1}^{m}\tilde{b}_i.$$

Equivalent deterministic model of model [P1].

[P2]

$$Min\ E\left(\tilde{Z}_1\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} E\left(\frac{\tilde{c}_{ij}x_{ij}}{\tilde{c}'_{ij}x_{ij}} + \frac{\tilde{f}_{ij}y_{ij}}{\tilde{f}'_{ij}y_{ij}}\right),$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{E\left(\tilde{c}_{ij}\right)x_{ij}}{E\left(\tilde{c}'_{ij}\right)x_{ij}} + \frac{E\left(\tilde{f}_{ij}\right)y_{ij}}{E\left(\tilde{f}'_{ij}\right)y_{ij}}\right), \quad (2)$$

$$Min \quad E\left(\tilde{Z}_2\right) = \left[\max \frac{E\left[\tilde{t}_{ij}\right]}{E\left[\tilde{t}'_{ij}\right]} : x_{ij} > 0, \forall i, j \right], \tag{3}$$

$$\begin{aligned} Min \quad E\left(\tilde{Z}_3\right) &= \sum_{i=1}^{m} \sum_{j=1}^{n} E\left[\frac{\tilde{d}_{ij} x_{ij}}{\tilde{d}'_{ij} x_{ij}}\right] \\ &= \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{E\left(\tilde{d}_{ij}\right) x_{ij}}{E\left(\tilde{d}'_{ij}\right) x_{ij}}, \end{aligned} \tag{4}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \le E\left(\tilde{a}_i\right), \quad \mathrm{i} = 1,2,...,m, \tag{5}$$

$$\sum_{i=1}^{m} x_{ij} \ge E\left(\tilde{b}_i\right), \quad j = 1,2,...,n, \tag{6}$$

$$x_{ij} \ge 0, \ \forall i, j, \tag{7}$$

$$y_{ij} = 0 \ \text{ if } \ x_{ij} = 0, \tag{8}$$

$$y_{ij} = 1 \ \text{ if } \ x_{ij} > 0. \tag{9}$$

The feasibility condition of model [P2] is

$$\sum_{i=1}^{m} E\left(\tilde{a}_i\right) \ge \sum_{j=1}^{n} E\left(\tilde{b}_i\right).$$

4. Solution Methodology

For some fractional multi-objective optimization problems, not all objective functions always have the best solution due to the conflict between impractical functions and objective functions. For one objective function, the solution may be the best, and for another objective function, the solution may be the worst. Therefore, there is usually a series of multipurpose cases that are not easy to compare to each other. This section introduces three ways to solve the deterministic MOFC LFTP as follows:

1. Fuzzy programming method (FPM).
2. Linear weighted-sum method (LWSM).
3. Global criterion method (GCM).

4.1 FPM (Fuzzy programming method)

The FPM first finds the lower bound L^k and the upper bound U^k of the k^{th} objective function Z_k. Where L^k is the target level to achieve the k^{th} objective function U^k is the highest allowable realization level of the k^{th} objective function, and $d^k = \left[U^k - L^k\right]$ is the degradation tolerance of the k^{th} objective function. After specifying the suction level and degradation tolerance for each objective function, a fuzzy model is formed and the fuzzy model is converted into a crisp model. The deterministic solution of MOFCLFTP can be obtained by the following process

Step 1: Only one objective function is used at a time, the other objective functions are ignored, and MOFCLFTP is considered a single objective FCLFTP.

Step 2: Determine the corresponding value for each solution obtained for each objective function from the results of step 1,

Step 3: For each objective function, get the best $\left(L^k\right)$ and the worst $\left(U^k\right)$ values for the solution set from step 2. It can then write an initial fuzzy model according to the expected level of each objective function, as shown below:

Find x_{ij}, that satisfies $Z_k \le L^k : k = 1,2,3$ the constraints are (5) to (9). For the MOFCLFTP, the membership function $\mu_k\left(x\right)$ corresponding to k^{th} objective function is defined as follows:

$$\mu_k\left(x\right) = \begin{cases} 1, & \text{if } Z_k \le L^k, \\ 1 - \left(\dfrac{Z_k - L^k}{U^k - L^k}\right), & \text{if } L^k \le Z_k, k = 1,2,3, \\ 0, & \text{if } Z_k \ge U^k. \end{cases}$$

The vector minimum problem of an equivalent LPP can be written as:

Max λ,

subject to

$$\lambda \le \left(\frac{U^k - Z_k}{U^k - L^k}\right), k = 1,2,3,$$

the constraints (5) to (9) $0 \le \lambda \le 1$. Here $\lambda = \min\{\mu_k(x) : k = 1,2,3\}$.

This is a LPP, which can be simplified in advance as follows:

[P3] $Max \quad \lambda$,

subject to $Z_k + \lambda(U^k - L^k) \le U^k, \; k = 1,2,3,$

the constraints (5) to (9) $0 \le \lambda \le 1$. For model [P3] if x^* is the optimal result, it is also the non-dominated solution for model [P2]. Therefore no other feasible solution x such that $Z_k(x) \le Z_k(x^*), \; \forall \, k = 1,2,3$ and $Z_k(x) < Z_k(x^*)$ for at least one k.

4.2 LWSM (Linear weighted-sum method)

By multiplying each objective function by the weight assigned to it and combining multiple objective functions, LWSM transforms multi-objective optimization into an objective optimization problem. Here, the weight $\omega_k (k = 1,2,3)$ corresponds to each objective function $Z_k (k = 1,2,3)$. Here, ω_k can be estimated as the relative importance or value of the objective function evaluated with other objective functions. In other words, the weight can be interpreted as indicating the priority relative to the objective function. The higher the weight ω_k, the more significant the objective function Z_k is. The minor the weight ω_k is, the smaller the importance of the objective function Z_k is. Then combine them into an objective function $\sum_{k=1}^{3} \omega_k Z_k$ and $\sum_{k=1}^{3} \omega_k = 1$. Because of its characteristics, this method is called LWSM . The process of the LWSM [12] is summarized as follows:

Step 1 First, according to the importance of the objective function in the model [P2], select the weighting coefficients ω_1, ω_2 and ω_3 to the objective functions $(Z_k, k = 1,2,3)$ corresponding to the objective function. Must be $\omega_k > 0, \; k = 1,2,3$ and $\sum_{k=1}^{3} \omega_k = 1$.

Step 2 If the objective function is the WSM of all objective functions, it is solved to solve a single objective problem. The single objective problem equivalent to [P2] can be expressed as:

[P4] $\quad Min \; \sum_{k=1}^{3} \omega_k \, \mathrm{E}\left[Z_k(x_{ij}) \right],$

subject to the constraints (5) to (9).

4.3 GCM (Global criterion method)

GCM is used to solve MO optimization problems and provide a compromise solution for multi-objective optimization. GCM [13] minimizes the distance between a specific reference point and the feasible objective region. The problems with MOFCLFTP are following as:

$Min \quad \left[Z_1(x_1), Z_2(x_2), Z_3(x_3) \right],$

subject to the constraints (5) to (9).

We can get the solution to the MOFCLFTP problem by following these steps:

Step 1: MOFCLFTP solves the problem of fixed-charge transportation for a single objective. Only single objective function is used at a time, and other objective functions are ignored.

Step 2: Determine the ideal objective point according to the result of step 1, such as

$$\left(Z_1^{\min}(x_1), Z_2^{\min}(x_2), Z_3^{\min}(x_3) \right)$$

and the corresponding value

$$\left(Z_1^{\min}(x_1), Z_2^{\min}(x_2), Z_3^{\min}(x_3) \right).$$

Step 3: Formulate the following problem as follows:

[P5] $$Min\ F(x) = \left[\sum_{k=1}^{3} \left(\frac{Z_k(x) - Z_k^{\min}}{Z_k^{\max} - Z_k^{\min}} \right) \right],$$

subject to the constraints (5) to (9)

5. Numerical Example

We solve the example of MOFCLFTP using rough programming from three methods. We solve this example using Lingo 17.0 software. Let us consider the MOFCLFTP with three objectives as discussed in Table 1-4.

Table 1. Table of rough transportation cost charge (c_{ij}) represent.

		D_1	D_2	D_3	D_4	Supply
S_1	c_{ij}	[(2,5),(1,6)]	[(8,12),(7,13)]	[(7,9),(6,10)]	[(3,6),(2,7)]	[(10,15),(9,16)]
	c'_{ij}	[(3,6),(2,7)]	[(5,8),(4,9)]	[(3,9),(2,10])	[(4,6),(3,7)]	
S_2	c_{ij}	[(8,11),(7,12)]	[(7,10),(6,11)]	[(3,8),(2,9)]	[(5,12),(4,13)]	[(16,20),(15,21)]
	c'_{ij}	[(9,12),(8,13)]	[(11,13),(10,14)]	[(3,8),(2,9)]	[(7,11),(6,12)]	
S_3	c_{ij}	[(3,12),(2,13)]	[(5,9),(4,10)]	[(6,12),(5,13)]	[(8,12),(7,13)]	[(14,23),(13,24)]
	c'_{ij}	[(8,11),(7,12)]	[(13,15),(12,16)]	[(10,13),(9,14)]	[(3,7),(2,8)]	
Demand		[8,13],[7,14]	[16,21],[15,22]	[9,12],[8,13]	[5,8],[4,9]	

Table 2. Table of rough transportation fixed charge (f_{ij}) represent.

		D_1	D_2	D_3	D_4	Supply
S_1	f_{ij}	[(3,6),(2,7)]	[(5,8),(4,9)]	[(3,9),(2,10)]	[(5,8),(4,9)]	[(10,15),(9,16)]
	f'_{ij}	[(11,15),(10,16)]	[(12,13),(11,14)]	[(13,18),(12,19)]	[(2,10),(1,11)]	
S_2	f_{ij}	[(11,14),(10,15)]	[(10,15),(9,16)]	[(5,7),(4,8)]	[(3,8),(2,9)]	[(16,20),(15,21)]
	f'_{ij}	[(5,9),(4,10)]	[(6,10),(5,11)]	[(3,8),(2,9)]	[(8,13),(7,14)]	
S_3	f_{ij}	[(5,9),(6,10)]	[(11,13),(10,14)]	[(4,9),(3,10)]	[(8,11),(7,12)]	[(14,23),(13,24)]
	f'_{ij}	[(4,12),(3,13)]	[(8,11),(7,12)]	[(5,8),(4,9)]	[(6,9),(5,10)]	
Demand		[(8,13),(7,14)]	[(16,21),(15,22)]	[(9,12),(8,13)]	[(5,8),(4,9)]	

Table 3. Table of rough transporting time (t_{ij}).

		D_1	D_2	D_3	D_4	Supply
S_1	t_{ij}	[(12,16),(11,17)]	[(14,16),(13,17)]	[(16,18),(15,19)]	[(13,18),(12,19)]	[(10,15),(9,16)]
	t'_{ij}	[(8,13),(7,14)]	[(9,11),(8,12)]	[(13,18),(12,19)]	[(15,16),(14,17)]	
S_2	t_{ij}	[(12,18),(11,19)]	[(15,20),(14,21)]	[(18,22),(17,23)]	[(13,16),(12,17)]	[(16,20),(15,21)]
	t'_{ij}	[(9,13),(8,14)]	[(11,15),(10,16)]	[(8,12),(7,13)]	[(12,17),(11,18)]	
S_3	t_{ij}	[(18,23),(17,24)]	[(21,24),(20,25)]	[(18,21),(17,22)]	[(19,23),(18,24)]	[(14,23),(13,24)]

	t'_{ij}	[(7,14),(6,15)]	[(15,18),(14,19)]	[(16,19),(15,20)]	[(13,19),(12,20)]	
Demand		[(8,13),(7,14)]	[(16,21),(15,22)]	[(9,12),(8,13)]	[(5,8),(4,9)]	

Table 4. Table of rough deterioration of goods (d_{ij}).

		D_1	D_2	D_3	D_4	Supply
S_1	d_{ij}	[(7,10),(6,11)]	[(3,8),(2,9)]	[(8,10),(7,11)]	[(11,13),(10,14)]	[(10,15),(9,16)]
	d'_{ij}	[(5,9),(4,10)]	[(7,10),(6,11)]	[(8,12),(7,13)]`	[(13,15),(12,16)]	
S_2	d_{ij}	[(8,11),(7,12)]	[(5,8),(4,9)]	[(7,11),(6,12)]	[(3,5),(2,6)]	[(16,20),(15,21)]
	d'_{ij}	[(3,8),(2,9)]	[(9,12),(8,13)]	[(5,8),(4,9)]	[(5,9),(4,10)]	
S_3	d_{ij}	[(6,8),(5,9)]	[(7,11),(6,12)]	[(8,12),(7,11)]	[(8,13),(7,14)]	[(14,23),(13,24)]
	d'_{ij}	[(8,11),(7,12)]	[(6,11),(5,12)]	[(8,13),(7,14)]	[(6,10),(5,11)]	
Demand		[(8,13),(7,14)]	[(16,21),(15,22)]	[(9,12),(8,13)]	[(5,8),(4,9)]	

Solved table 1,2,3 and 4 used (1) to obtain in table 5 and 6 crisp form as.

Table 5: Transportation cost and fixed-charge $\left(c_{ij}, f_{ij}\right)$ in Crisp form.

		D_1	D_2	D_3	D_4
S_1	$E\left(\tilde{c}_{ij}\right), E\left(\tilde{f}_{ij}\right)$	3.5,4.5	10,6.5	9,6	4.5,6.5
	$E\left(\tilde{c}'_{ij}\right), E\left(\tilde{f}'_{ij}\right)$	4.5,13	6.5,12.5	6,15.5	5,6
S_2	$E\left(\tilde{c}_{ij}\right), E\left(\tilde{f}_{ij}\right)$	9.5,12.5	8.5,12.5	5.5,6	8.5,5.5
	$E\left(\tilde{c}'_{ij}\right), E\left(\tilde{f}'_{ij}\right)$	10.5,7	12,8	5.5,5.5	9,10.5
S_3	$E\left(\tilde{c}_{ij}\right), E\left(\tilde{f}_{ij}\right)$	7.5,7	7,12	9,6.5	10,9.5
	$E\left(\tilde{c}'_{ij}\right), E\left(\tilde{f}'_{ij}\right)$	9.5,8	14,9.5	11.5,6.5	5,7.5

Table 6: Transportation time and deterioration rate of goods $\left(t_{ij}, d_{ij}\right)$ in Crisp form.

		D_1	D_2	D_3	D_4
S_1	$E\left(\tilde{t}_{ij}\right), E\left(\tilde{d}_{ij}\right)$	14,8.5	15,5.5	17,9	15.5,12
	$E\left(\tilde{t}'_{ij}\right), E\left(\tilde{d}'_{ij}\right)$	10.5,7	10,8.5	15.5,10	15,14
S_2	$E\left(\tilde{t}_{ij}\right), E\left(\tilde{d}_{ij}\right)$	15,9.5	17.5,10.5	20,9	14.5,4
	$E\left(\tilde{t}'_{ij}\right), E\left(\tilde{d}'_{ij}\right)$	11,5.5	13,10.5	10,6.5	14.5,7
S_3	$E\left(\tilde{t}_{ij}\right), E\left(\tilde{d}_{ij}\right)$	20.5,7	22.5,9	19.5,10	21,10.5
	$E\left(\tilde{t}'_{ij}\right), E\left(\tilde{d}'_{ij}\right)$	10.5,9.5	16.5,8.5	17.5,10.5	21,8

6. Results

This section describes the best results for the corresponding crisp model [P2] extracted from the FPM in subsection 6.1, LWSM in subsection 6.2, & GCM in subsection 6.3 using LINGO 17.0 software.

The process described in subsection 4.1 using the crisp data in Eq. [P2] to calculate for every rough interval the expected values of $\eta = 0.5$ (DM selection) in tables 5 and 6. And do the following: The results are shown in table 7.

6.1 FPM (Fuzzy programming method)

Table 7. The Pareto-optimal solution of the expression MOFCLFTP is performed using the fuzzy programming method.

Value of τ	Z_1	Z_2	Z_3
0.7274343	1.644639	1.952381	0.823706

6.2 LWSM (Linear weighted-sum method)

In tables 5 and 6, calculate the crisp data of expected value for each rough interval of $\eta = 0.5$ (DM selection) and apply the process described in subsection 4.2 using the bright data in the formulated [P2]. It will be as follows results as shown in table 8.

Table 8. The Pareto-optimal solution of the expression MOFCLFTP is performed using the weighted sum method.

Case	w_1	w_2	w_3	Z_1	Z_2	Z_3
1	0.8	0.1	0.1	1.636512	1.033333	0.877164
2	0.7	0.1	0.2	1.636512	1.033333	0.877164
3	0.7	0.2	0.1	1.636512	1.033333	0.877164
4	0.6	0.3	0.1	1.636512	1.033333	0.877164
5	0.6	0.2	0.2	1.636512	1.033333	0.877164
6	0.6	0.1	0.3	1.734186	1.000000	0.844926
7	0.5	0.2	0.3	1.647548	1.952381	0.873114
8	0.5	0.3	0.2	1.636512	1.033333	0.877164
9	0.5	0.4	0.1	1.595875*	1.033333*	0.88041*
10	0.4	0.4	0.2	1.595875	1.033333	0.88041
11	0.4	0.5	0.1	1.595875	1.033333	0.88041
12	0.4	0.2	0.4	1.870159	1.363636	0.798957
13	0.3	0.3	0.4	1.831328	1.000000	0.809331
14	0.3	0.4	0.3	1.595875	1.033333	0.88041
15	0.2	0.2	0.6	1.906643	1.500000	0.758638

6.3 GCM (Global criterion method)

Calculate the crisp data of expected values for each rough interval in tables 5 and 6, and apply the process described in subsection 4.3 using the clear data in the formulated [P2]. The results obtained are shown in table 9.

Table 9. Compromise solution of the suggested MOFCLFTP by the global criterion method.

Z_1	Z_2	Z_3
1.638521	1.952381	0.829762

Table 10. Proposed MOFCLFTP Pareto-optimal solutions by three methods.

Applied Method	Z_1	Z_2	Z_3
Fuzzy programming method	1.644639	1.952381	0.823706
Linear weighted-sum method	1.595875	1.033333	0.88041
Global criterion method	1.638521	1.952381	0.829762

It can be concluded from table 10 that the best compromise of MOFCLFTP extracted from the linear weighted sum method is better than the fuzzy programming method and the global criterion method. In addition, it has been observed that the best compromises extracted from the linear weighted sum method, the global criterion method, and the fuzzy programming methods are different values of the three objective functions. The decision maker can choose the best solution based on the parity check of the objective function. As shown in subsection 6.2, Z_1 is more important in the other two objective functions.

7. Conclusion

In this study, we mainly studied the FCLFTP that is supported at the rough programming intervals in multi-objective environment. In the proposed MOFCLFTP, the parameters of the objective function (transportation cost and fixed cost, goods transportation time and goods deterioration rate, supply and demand parameters, etc.) are used as rough variables. This paper proposes a new concept of MOFCLFTP. It is based on the representation and processing of ambiguous and uncertain information at rough intervals in reality. Rough programming can be used to convert roughness-based MOFCLFTP to deterministic MOFCLFTP. It then solves the deterministic MOFCLFTP using fuzzy programming, linear weighted sum, and global criterion method, and uses LINGO 17.0 software to extract compromises from three methods. By comparing the above solutions in table 10, we conclude that the proposed rough MOFCLFTP best solution is provided for the LWSM. The contents of this article can open a new dimension to create in linear plus linear fractional multi-objective transportation problem and linear fractional multi-level multi-objective transportation problem using above methods.

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