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Original research article

Stochastic Analysis of Redundant System with Partial Failure and with Correlated Life-Time

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ABSTRACT

In paper, we analyze two identical units redundant system. First unit undergoes two failure types, partial failure and complete failure while second unit cannot have partial failure. Practical examples of such systems can be seen in electronics industries. Repairman having knowledge of two repair types with time distribution of repair as general is always with the system. Breakdown of system occurs if both units are in complete failure mode. Analysis of system is done to find all measures of reliability. Taking time distribution of failure as exponential and general distribution of time for others, the various reliability measures are obtained which gives the effectiveness of the system. Average Time taken by System for Failure, Availability (uptime) of System, Number of Repairman's visits, Busy Period Analysis of Repairman for both partially failed unit and completely failed unit, Expected Profit obtained. The conclusions about MTSF and profit from the system are carried out by Graphical studies. Main emphasis is on correlation between repair time and failure time.
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Keywords: Average time of failure; BVE; Correlated lifetime; MTSF; Uptime

1. Introduction

In reliability field, many researchers have studied redundant system of 2 units. Goel et al. [1] and Sehgal [2] analysed system having standby with concept of partial failure, repair types. Mahmoud [3] studied redundant model of 2 units having 2 types of failures and concept of preventive maintenance. Mokaddis and El-Said [4] compared two models of dissimilar units with concept of partial failure. Singh and Poonia [5] used regenerative point technique to assess two unit parallel system. Tuteja et al. [6-7] applied inspection randomly and explained profit evaluation of

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systemconsidering partial failure mode on single unit model.Pandey et al. [8] did profit determination of model of 2 units, used two different pairs of repairs. Rizwan and Taneja [9] performed profit evaluation with partial/complete modes of failures. Yusuf et al. [10] studied communication network system having transmitter with partial and complete failure. Siwach et al. [11] did profit analysis with instructions and accidents of a two-unit cold standby system. Gupta et al. [12] discussed different operative modes and repair policies of redundant system. Kumar et al. [13-15] did pioneer work on complex redundant systems with correlated lifetime by considering two failure types and repair rates. Kakkar et al. [16] analysed two unit oil system with correlated lifetime concept. In models [1-12], the failure and repair times are taken as not correlated, independent distribution is taken for random variables. In most of these models, simple analysis without correlation is done. The aim of this paper is to study a system in which failure times correlated with repair times. Here first unit has two types of failures, Partial Failure, in which system is working with reduced capacity and complete failure while second unit has only complete failure. Goel, Gupta and other (1993) firstly introduced concept of correlation in reliability.

PDF of Bivariate Exponential suggested by Paulson, in 1973,

$$f(x,y)=\alpha\beta(1-r)e^{-(\alpha x+\beta y)}I_0\left(2\sqrt{\alpha\beta xy}\right)$$

$\alpha,\beta,x,y>0, 0\le r<1,$

where

$$I_0(z)=\sum_{0}^{\infty}\frac{\left(\frac{z}{2}\right)^j}{(j!)^2}.$$

2. Assumptions

1. System has 2 units. In the beginning, 1st unit is in operative mode, 2nd is in cold standby mode.
2. The first unit has 3 modes i.e. operative, partial failure and complete failure.
3. Second unit has 2 modes i.e. operative and complete failure.
4. Time distribution of failure is exponential and time distribution of repair is general.
5. Repairman is always there with the system.
6. Unit works perfectly after repair.
7. System breakdown occurs when both of the units are under repair.
8. Variation in one variable does not affect other.

3. Transition Diagram

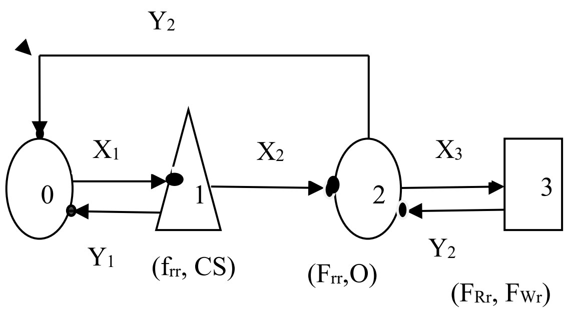
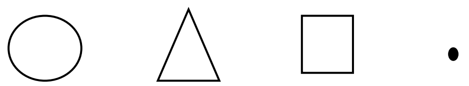


Fig. 1. Transition Diagram-The Model.



4. Notations for System States

E: Regenerative states set $\{S_j; j=0,1,2\}$.

$\bar{E}$: Non regenerative state set $\{S_3\}$.

X_1, X_2, X_3: Variables(random) for the partial failure, complete failure times of 1st component and complete failure time of 2nd component with $\alpha_1,\alpha_2,\alpha_3$ as rates of failure.

Y_1, Y_2: Variables(random) for repair times of 1st component and 2nd component with β_1,β_2 as rates of repair.

$f_i(X,Y)$: Joint PDF of $(X_i,Y_i); i=1,2,3,$ $Y_2=Y_3=\alpha_i\beta_i(1-r_i)e^{-(\alpha_i x+\beta_i y)}I_0\left(2\sqrt{\alpha_i\beta_i r_i xy}\right),$

$X, Y, \alpha_i, \beta_i > 0, 0 \le r_i < 1,$
where

$$I_0\left(2\sqrt{\alpha_i \beta_i r_i XY}\right) = \sum_{j=0}^{\infty} \frac{\left(\alpha_i \beta_i r_i XY\right)^j}{\left(j!\right)^2}.$$

P_{ij} : Transition probability from state S_i to S_j.

$P_{i,j}^{(k)}$:Transition probability from state S_i to S_j via state S_k.

μ_i : Average time in state S_i.

$\oplus / \otimes$: Laplace Convolution / Laplace stiejels Convolution.
*/**: Laplace Transformation/Laplace Stiejels Transformation.

5. States Discription

(O, CS): 1st unit is in operative mode and second is cold stand by.

(f_{rr}, CS): 1st unit is partially failed, under repair and 2nd unit is cold stand by.

(F_{Rr}, CS): 1st unit is failed, and repairman is repairing it and 2nd unit is in operative mode.

(F_{Rr}, F_{Wr}): 1st unit is failed, and repairman is repairing it and 2nd unit is failed and waiting for repair.

6.Probabilities of Transition and Average Sojourn Time

Direct non conditional probabilities of transition from S_i to S_j are given as follows

$$Q_{0,1}(t) = 1 - e^{-\alpha_1(1-r_1)t}. \quad (6.1)$$

Direct conditional probabilities $Q_{ij|x}(t)$ of transition are:

$$Q_{1,0|x}(t) = \beta_1 e^{-(\alpha_1 r_1 x)} \sum_{j=0}^{\infty} \frac{\alpha_1 \beta_1 r_1 x}{(j!)^2} \int_0^t e^{-(\beta_1 + \alpha_2(1-r_2))u} u^j du, \quad (6.2)$$

$$Q_{1,2|x}(t) = \beta_i e^{-(\alpha_1 r_1 x)} \sum_{j=0}^{\infty} \frac{(\alpha_1 \beta_1 r_1 x)^j}{(j!)^2} \int_0^t e^{-(\beta_1 v)} v^j \left[1 - e^{-(\alpha_2(1-r_2))v}\right] dv, \quad (6.3)$$

$$Q_{2,0|x}(t) = \beta_2 e^{-(\alpha_2 r_2 x)} \sum_{j=0}^{\infty} \frac{\alpha_2 \beta_2 r_2 x}{(j!)^2} \int_0^t e^{-(\beta_2 + \alpha_3(1-r_3))u} u^j du, \quad (6.4)$$

$$Q_{2,3|x}(t) = \beta_2 e^{-(\alpha_2 r_2 x)} \sum_{j=0}^{\infty} \frac{(\alpha_2 \beta_2 r_2 x)^j}{(j!)^2} \int_0^t e^{-(\beta_2 v)} v^j \left[1 - e^{-(\alpha_3(1-r_3))v}\right] dv, \quad (6.5)$$

The two steps conditional transition probability $Q_{ij|x}^{(k)}(t)$ can be given as:

$$Q_{2,2|x}^{(3)}(t) = \beta_2 e^{-(\alpha_2 r_2 x)} \sum_{j=0}^{\infty} \frac{\alpha_2 \beta_2 r_2 x}{(j!)^2} \int_0^t e^{-(\beta_2 v)} \left[1 - e^{-(\alpha_3(1-r_3))v}\right] dv, \quad (6.6)$$

Non-zero unconditional probabilities $P_{i,j}$:

$$P_{0,1} = 1. \quad (6.7)$$

Non-zero unconditional probabilities $P_{ij|x}$ are:

$$P_{0,1} = \beta_1' e^{-\alpha_1 r_1 (1-\beta_1')x},$$

where

$$\beta_1' = \frac{\beta_1}{\beta_1 + \alpha_2(1-r_2)}, \quad (6.8)$$

$$P_{1,2|x} = 1 - \beta_1' e^{-\alpha_1 r_1 (1-\beta_1')x}, \quad (6.9)$$

$$P_{2,0|x} = \beta_2' e^{-\alpha_2 r_2 (1-\beta_2')x}, \quad (6.10)$$

where

$$\beta_2' = \frac{\beta_2}{\beta_2 + \alpha_3(1-r_3)}, \quad (6.11)$$

$$P_{2,3|x} = 1 - \beta_2' e^{-\alpha_2 r_2 (1-\beta_2')x}.$$

The 2 steps non-zero conditional probability $P_{i,j|x}^{(k)}$ is:

$$P_{i,j|x}^{(k)} = \lim_{t \to \infty} Q_{i,j|x}^{(k)}(t), \quad (6.12)$$
$$P_{2,2|x}^{(3)} = 1 - \beta_2' e^{-\alpha_2 r_2 (1-\beta_2')x}.$$

From above, unconditional probabilities with correlated coefficient:

$$P_{1,0|x} = \frac{\beta_1'(1-r_1)}{1 - r_1 \beta_1'}, \quad (6.13)$$

$$P_{1,2|x} = 1 - \frac{\beta_1'(1-r_1)}{1-r_1\beta_1}, \quad (6.14)$$

$$P_{2,0|x} = \frac{\beta_2'(1-r_2)}{1-r_2\beta_2'}, \quad (6.15)$$

$$P_{2,3|x} = P_{2,2}^{(3)} = 1 - \frac{\beta_2'(1-r_2)}{1-r_2\beta_2'}, \quad (6.16)$$

$$P_{0,1} = 1,$$

$$P_{0,1} + P_{1,2} = 1,$$

$$P_{2,0} + P_{2,3} = 1,$$

$$P_{2,0} + P_{2,2}^{(3)} = 1.$$

Average Sojourn Time (μ_i):

$$\mu_i = \lim_{t\to\infty}\int_0^t P(t)[t;0<t<T]\,dt$$

$$\mu_0 = \frac{1}{\alpha_1(1-r_1)}. \quad (6.17)$$

Conditional average sojourn time $\mu_{i|x}$ are as follows:

$$\mu_{1|x} = \frac{1}{\alpha_2(1-r_2)}\left\{1-\beta_1' e^{-\alpha_1 r_1(1-\beta_1')x}\right\}, \quad (6.18)$$

$$\mu_{2|x} = \frac{1}{\alpha_3(1-r_3)}\left\{1-\beta_2' e^{-\alpha_2 r_2(1-\beta_2')x}\right\}. \quad (6.19)$$

Thus,

$$\mu_1 = \frac{1}{\alpha_2(1-r_2)}\left\{1-\frac{\beta_1'(1-r_1)}{1-r_1\beta_1'}\right\},$$

$$\mu_2 = \frac{1}{\alpha_3(1-r_3)}\left\{1-\frac{\beta_2'(1-r_2)}{1-r_2\beta_2'}\right\}.$$

Non-conditional $m_{i,j}$,

$$m_{0,1} = \mu_0. \quad (6.20)$$

Conditional $m_{i,j|x}$,

$$m_{1,0|x} = \int_0^\infty t e^{-\alpha_1(1-r_1)t} k_1(t\,|\,x)\,dt, \quad (6.21)$$

$$m_{1,2|x} = \int_0^\infty t\alpha_2(1-r_2) e^{-\alpha_2(1-r_2)t} \bar{K}_1(u\,|\,x)\,dt, \quad (6.22)$$

$$m_{2,0|x} = \int_0^\infty t e^{-\alpha_3(1-r_3)t} k_2(t\,|\,x)\,dt, \quad (6.23)$$

$$m_{2,3|x} = \int_0^\infty t\alpha_3(1-r_3) e^{-\alpha_3(1-r_3)t} \bar{K}_2(u\,|\,x)\,dt, \quad (6.24)$$

$$m_{2,2|x}^{(3)} = \int_0^\infty t\left(e^{-\alpha_3(1-r_3)t} \oplus 1\right) k_2(t\,|\,x)\,dt. \quad (6.25)$$

So that

$$m_{1,0|x} + m_{1,2|x} = \mu_{1|x},$$

$$m_{2,0|x} + m_{2,3|x} = \mu_{2|x},$$

$$m_{2,0|x} + m_{2,2|x}^{(3)} = K_{1|x}.$$

Now we have

$$m_{1,0} + m_{1,2} = \mu_1,$$

$$m_{2,0} + m_{2,3} = \mu_2,$$

$$m_{2,0} + m_{2,2}^{(3)} = \frac{1}{\beta_2(1-r_2)} = K'.$$

7. Analysis of System Performance

Different measures of the system performance are obtained by solving above probabilities and recursive relations obtained. Average time taken by system for failure $(\boldsymbol{T_0}) = \boldsymbol{N}/\boldsymbol{D}$,

Uptime/Availability $(\boldsymbol{A_0}) = \boldsymbol{N}_1/\boldsymbol{D}_1$,

Busy period of repairman $(\boldsymbol{B_0}) = \boldsymbol{N}_2/\boldsymbol{D}_1$,

Number of Repairman's visits $(\boldsymbol{V_0}) = \boldsymbol{N}_3/\boldsymbol{D}_1$,

where

$$N = \mu_1 + P_{1,2}\mu_2 + \mu_0,$$

$$D = P_{1,2}P_{2,3},$$

$$N_1 = P_{2,0}\mu_0 + P_{2,0}\mu_1 + P_{1,2}\mu_2,$$

$$D_1 = P_{2,0}\mu_0 + P_{2,0}K + P_{1,2}K',$$

$$N_2 = P_{2,0}K + P_{1,2}K', K = -g_1^{*'}(0), K' = g_2^{*'}(0),$$

$$N_3 = P_{0,1}P_{0,2} = P_{2,0}.$$

8. Cost-Profit Analysis

The expected profit is

$$P = C_0A_0 - C_1B_0 - C_2V_0, \quad (6.26)$$

where

$C_0 =$ revenue per unit availability of system,

$C_1 =$ cost per unit busy time of repairman,

C_2 = cost per repairman's visit.

9. Numerical Study and Graphical Analysis

Fig. 2 is graph of MTSF (T_0) versus failure rate (α_1) for different fixed values of repair rate (β_1) and correlation coefficient (ρ_2). This graph reveals that MTSF decreases for higher values of (α_1) and is higher for high values of (β_1) and (ρ_2).

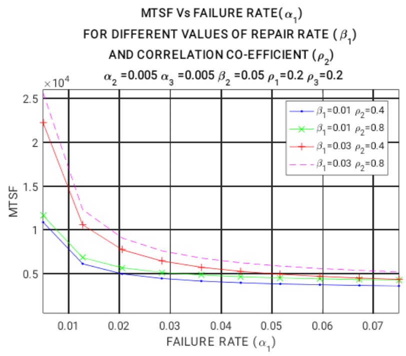


Fig 2. MTSF Versus Failure Rate.

Fig. 3, showing graph of availability (A_0) versus repair rate (β_1). Availability (A_0) increases as values of (β_1) increases for different fixed values of (α_1) and is higher for high values of correlation coefficient (ρ_2).

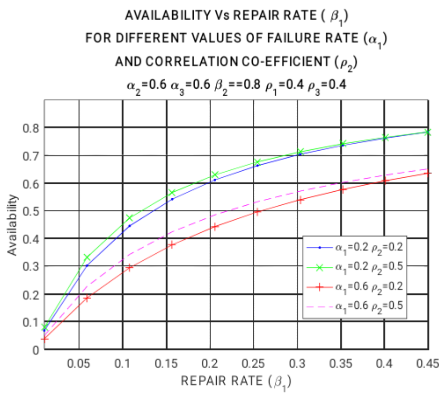


Fig. 3. Availability Versus Repair Rate.

Fig. 4 is pattern of profit w.r.t repair rate (β_1) for different values of failure rate (α_1) and correlation co-efficient (ρ_2). The profit increases for high values of (β_1). Profit is lower for high values of (α_1) but higher for high values of (ρ_2).

Thus, we get following conclusions

(i) For $\alpha_1 = 0.2, \rho_2 = 0.2, P_3 >$ or $=$ or < 0 as $\beta_1 >$ or $=$ or < 0.065. Hence system is profitable if $\beta_1 > 0.06255$.

(ii) For $\alpha_1 = 0.2, \rho_2 = 0.4, P_3 <$ or $=$ or > 0 as $\beta_1 <$ or $=$ or > 0.0556. Hence system is profitable if $\beta_1 > 0.0556$.

(iii) For $\alpha_1 = 0.4, \rho_2 = 0.2, P_3 <$ or $=$ or > 0 as $\beta_1 <$ or $=$ or > 0.0943. Hence system is profitable if $\beta_1 > 0.0943$.

(iv) For $\alpha_1 = 0.4, \rho_2 = 0.4, P_3 <$ or $=$ or > 0 as $\beta_1 <$ or $=$ or > 0.0808. System is profitable if $\beta_1 > 0.0808$.

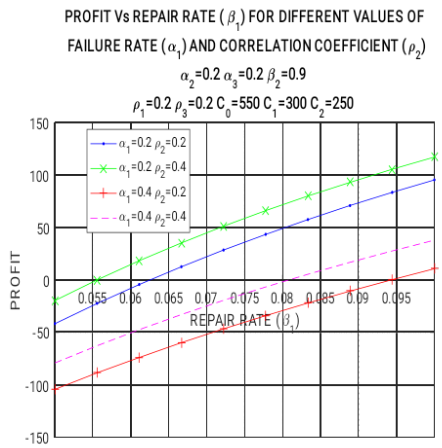


Fig. 4. Profit Versus Repair Rate.

10. Observations and Discussion

From analysis of graphs, main observations are MTSF & Profit decreases as failure rate increases. Thus, it can be concluded that higher the rate of failure, reliability and profit become less. The cut off points are also obtained. This would help to decide how much be the revenue to have positive profit from the system.

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