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Original research article

Oscillation of Full and Partial Ring Pendulum: Physics Laboratory Experiment

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ABSTRACT

In this study, we construct a piece of equipment and high-speed video analysis technique for the investigation of the ring pendulum in physics laboratory experiment. The five sets of the full and partial ring pendula are investigated and analyzed by using high-speed video at a rate of 120 frames per second and Tracker Video Analysis software. Two experiments are performed from the full ring pendula. They are the calculation of *g* and the study of the ring's moment of inertia using different methods. Another experiment is performed from the partial ring pendula, which is the measuring of the period of the partial ring oscillations, which are compared to the theoretical value. The result verifies that regardless of the length of the circumference, the period is still the same with the full ring pendulum. As a result, this technique can help students and teachers to visualize simple physics phenomena and relate them to the physics principles learned in the classroom.

Keywords: High-speed video; Partial ring pendulum; Physical pendulum; Ring pendulum; Tracker

1. Introduction

An oscillating physical pendulum is a simple experiment in a physics laboratory for undergraduate students. This experiment can give extensive information about a rigid body, especially a meter stick. The general objective is the calculation of the acceleration of gravity (g) from the oscillating period by using a digital stopwatch or photogate sensor, etc. However, many students realize that the concept of the physical pendulum is difficult, especially the oscillating non-homogeneous mass distribution, ring, etc. These lead to one of the serious problems in terms of students' perspective in learning physics. Therefore, some physics educators have attempted to assist students to learn and teachers to teach physics more effectively. The previous experiments have studied the physical

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pendulum using different methods. Examples include a study of the physical pendulum with non-homogeneous mass distribution, fidget spinner, ring pendulum, the floating eagle, etc. [1-9], using the physical pendulum in the teaching laboratory [1-3, 10, 11], using a personal computer connected to an interface system, accelerometer, photogate sensor, video analysis, and smartphone [1-3, 12-15], and using low-cost materials [3, 11, 14, 16]. Recently, the high-speed video analysis technique has been successfully used to study complicated physics, e.g., three cases of damped harmonic oscillation and their energy dissipation [17], precession rate and angular speed of the rolling disc [17], the linear coefficient of rotational friction of rotational cylindrical plate [18] and the rolling cylinders on an inclined plane [19]. It is clear that this technique can help educators, teachers, and students to see actual motions. Then, they can link the actual motion in physics laboratory with the physics principle in the classroom [20-22].

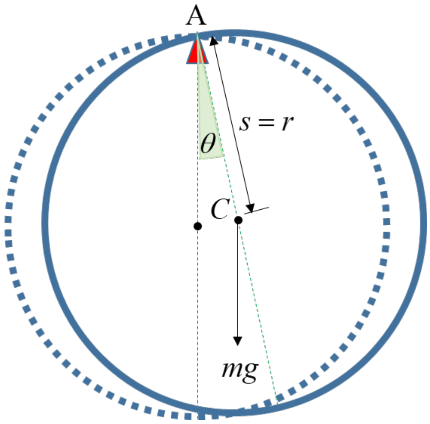
In this work, we intend to investigate the empirical behavior of the full and a set of partial ring pendula by using the high-speed video analysis technique. The interesting aspect of the set of partial ring pendula is its period of oscillation. According to the theoretical calculation, the period is independent of the ring's shapes with the same radius of curvature. Several studies have investigated the set of partial ring pendula's period with different methods both theoretically and experimentally [6-9]. However, none of them has been studied by the high-speed video analysis technique, which can record and analyze a real motion.

Therefore, the objectives of this study are to illustrate simple experiments that will allow students to: 1) calculate the value of g; 2) to study the ring's moment of inertia using different methods; and 3) to determine the period of the partial ring pendula which will be compared to the theoretical value.

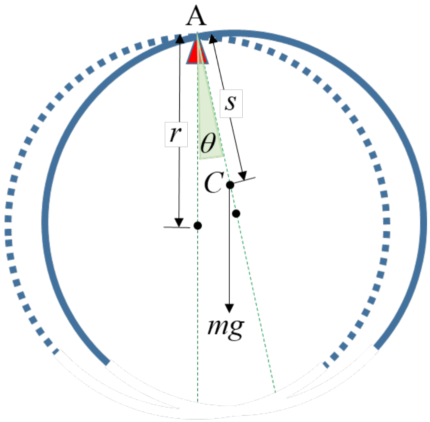
2. Materials and Methods

2.1 Theoretical background

The ring (mass m and radius r) is suspended at point A as shown in Fig. 1. The distance from point A away from the center of mass (CM, point C) is equal to s; $s = r$ for the full ring in Fig. 1(a), and $s < r$ for the partial ring in Fig. 1(b).



(a)



(b)

Fig. 1. The ring pendula; (a) the full ring, (b) the partial ring.

The ring's moment of inertia at point A is I_A. An angle out of the equilibrium of the ring is θ. The torque about the pivot point A is given by

$$I_A \frac{d^2\theta}{dt^2} = -mgs\sin\theta . \quad (2.1)$$

When the angle of oscillation is small, we may use the small-angle approximation, $\sin\theta \cong \theta$. Thus,

$$\frac{d^2\theta}{dt^2} = -\frac{mgs}{I_A}\theta . \quad (2.2)$$

The general equation for the angle $\theta(t)$ is given by

$$\theta(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t), \quad (2.3)$$

where the angular frequency, ω_0, is given by

$$\omega_0 = \sqrt{\frac{mgs}{I_A}} . \quad (2.4)$$

Then, the period T is

$$T = \frac{2\pi}{\omega_0}, \quad (2.5)$$

or

$$T = 2\pi\sqrt{\frac{I_A}{mgs}} . \quad (2.6)$$

Then, the moment of inertia at point A (I_A) of the full ring ($s = r$) oscillation can be written as

$$I_A = \frac{mgr}{\omega_0^2} . \quad (2.7)$$

Another way to obtain I_A from the parallel axis theorem is

$$\begin{aligned} I_A &= I_C + ms^2 \\ &= 2mr^2, \end{aligned} \quad (2.8)$$

where I_C is the moment of inertia at the CM that is equal to mr^2.

Thus, the angular frequency and the period of the full ring (T_{FR}) about the pivot point A in Eq. (2.4) and Eq. (2.6), respectively, can be written as follows:

$$\omega_0 = \sqrt{\frac{g}{2r}} \text{ or } \omega_0^2 = \frac{g}{2r}, \quad (2.9)$$

and

$$T_{FR} = 2\pi\sqrt{\frac{2mr^2}{mgr}} = 2\pi\sqrt{\frac{2r}{g}} . \quad (2.10)$$

In the case of the partial ring as shown in Fig. 1(b). Thus, I_A is given by

$$\begin{aligned} I_A &= I_C + ms^2 \\ &= mr^2 - m(r-s)^2 + ms^2 \\ &= 2mrs , \end{aligned} \quad (2.11)$$

where I_C is equal to $mr^2 - m(r-s)^2$.

Then, the period (T_{PR}) can be

$$T_{PR} = 2\pi\sqrt{\frac{2mrs}{mgs}} = 2\pi\sqrt{\frac{2r}{g}} . \quad (2.12)$$

Then, the period of the full ring in Eq. (2.10) is equal to the partial ring in Eq. (2.12). Thus, the full ring and the partial ring with the same radius of curvature have the same period.

2.2 Experimental setup and materials

The high video analysis technique was used to capture and analyze the oscillation. The technique involved: (1) the high-speed (HS) video camera that had high frame rates at 120 frames per second (fps) for capturing the oscillation of a full and partial ring pendulum. Each oscillation was repeated 10 times and averaged in the analyzing step; (2) the video analysis software, called Tracker 6.0.7 [23] that was used to analyze the video of motions. The positions x, y, and times of the oscillation can be obtained. Then, the angular frequency of the oscillation can be further analyzed from the Data tool application in Tracker; and (3) the spreadsheet software.

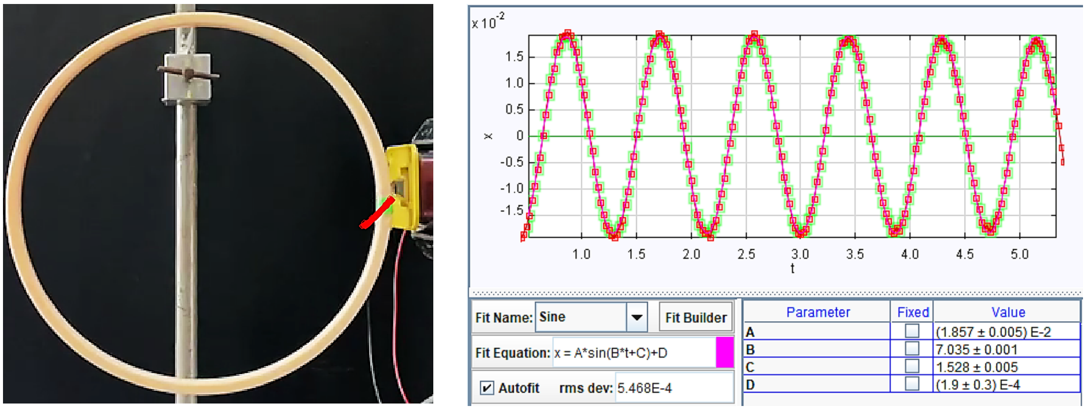
The experimental setup is shown in Fig. 2. It consists of the HS video camera (Casio EX-FH100) and the five sets of the ring pendula with radii 5.00, 7.00, 8.00, 10.00, and 12.00 centimeters. The ring suffered a knife-edge suspension at a pivot. Each set contained the five partial ring pendula (one is a full ring). They had the same radius of curvature. Their shapes were a full ring, 7/8, 3/4, 5/8, and 1/2 ring. We fixed the experiment by using a solenoid controller. Each ring was attached to a piece of thin metal sheet and was released at the same position (3 o'clock position), by using a solenoid controller. At this starting angle, the ring swung through a small angle of less than 5 degrees. Two experiments were performed from the full ring pendula. They were the calculations of g and the study of the ring's moment of inertia, using different methods. The other experiment was carried out from the partial ring pendula, which was the measuring of the period (T) of the partial ring oscillations, compared to the theoretical value. For the partial ring, this was the experiment done to verify that no matter the length of the circumference, the period was still the same, as proved in the previous section.

Fig. 2. The experimental set-up of the ring's oscillation.

3. Results and Discussion

3.1 The calculation of *g*

The HS-video analysis technique was used to capture the position of the marker on the full ring (Fig. 3(a)) and the oscillation amplitude was under a few degrees. The ring position change was fitted as a sinusoid curve with time, written as $x = A\sin Bt + C$. Then, the parameters A, B and C were evaluated (Fig. 3(b)). Angular frequency was determined from the coefficient B $(\omega = B)$ of the curve fitting and was compared with the equation of $x = A\sin \omega t + C$. The angular frequency of the five full ring oscillations for different radii is shown in Table 1. As you can see, while the ring's radius is increasing, the angular frequency is decreasing.



(a) **(b)**

Fig. 3. Using Tracker to analyze the ring's oscillation. (a) Tracking the marker on the ring and (b) the curve fitting of position versus time graph with $x = A\sin Bt + C$.

Table 1. Properties and analyzed data of the full ring pendula used in this investigation.

Radius, r ($\times 10^{-2}$ m)	**Mass, m ($\times 10^{-3}$kg)**	**Angular frequency, ω (rad s^{-1})**	**Moment of inertia, I ($\times 10^{-4}$ kg m^2)**		**% difference**
			$I_{\text{Tracker}} = \dfrac{mgr}{\omega_0^2}$	$I_{\text{Theory}} = 2mr^2$	
5.00	13.33	9.801	0.680±0.006	0.67±0.02	1.48
7.00	18.67	8.427	1.803±0.002	1.83±0.04	1.49
8.00	21.33	7.788	2.758±0.003	2.73±0.03	1.02
10.00	26.67	7.101	5.183±0.014	5.33±0.04	2.23
12.00	32.00	6.411	9.156±0.002	9.22±0.07	0.70

According to Eq. (2.9), the inverse relationship between squared angular frequency $\left(\omega_0^2\right)$ and diameter $\left(2r\right)$ or the ω_0^2 is linearly proportional to $\dfrac{1}{2r}$. Our empirical findings also indicated that the same linear relationship between our experimental values of ω_0^2 and $\dfrac{1}{2r}$ are shown as the solid markers in Fig. 4. The acceleration of gravity (g), was determined from the slope of the dotted line (which was generated from experimental values) and equaled to 9.77 ms^{-2} with about 0. 31% deviation from the real value, 9.80 ms^{-2} (the solid line, which was generated by $\omega_0^2 = \dfrac{9.80}{2r}$).

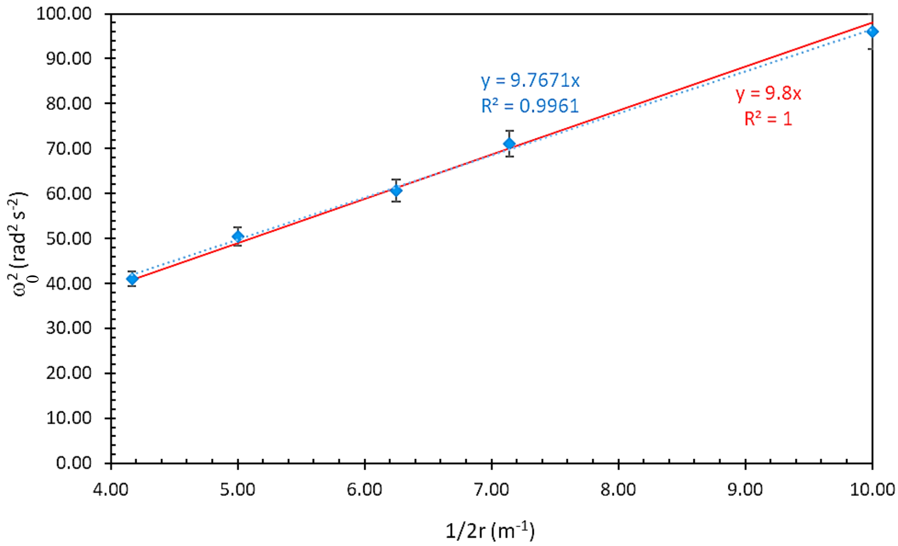


Fig. 4. Linear relation between ω_0^2 and $\frac{1}{2r}$. The solid markers (♦) are the data obtained from Tracker for different radii. The solid line is generated by $\omega_0^2 = \frac{9.80}{2r}$. Both slopes bring forth the acceleration of gravity (g).

3.2 The ring's moment of inertia

This experimental analysis is a comparison between the two methods of which the full ring's moment of inertia was analyzed. In Eq. (2.7), it is denoted as I_{Tracker} versus in Eq. (2.8), where it is denoted as I_{Theory}. The experimental data obtained from video analysis (ω_0), the direct measuring of the ring's masses (m), and the radii (r) were used to calculate the ring's moment of inertia in $I_{\text{Tracker}} = \frac{mgr}{\omega_0^2}$ and $I_{\text{Theory}} = 2mr^2$. The results based on both equations are given in Table 1. The resulting moment of inertia from the Tracker is about (0.680±0.006 to 9.156±0.002×10^{-4} kg m^2 and from the theory is about (0.67±0.02 to 9.22±0.07)×10^{-4} kg m^2 for the radii 5.00-12.00 centimeters. This illustrates that the larger the radius of the ring, the greater the moment of inertia. It is clearly indicated that the percentage difference is only about 0.70 – 2.23% as shown in the last column of Table 1. Then, the ring's moment of inertia from $\frac{mgr}{\omega_0^2}$ was plotted in comparison with $2mr^2$, as shown in Fig. 5, and the linear relationship was plotted as a solid line. The experimental data followed a linear trend within the margin of error (R=1). The average percentage difference is approximately 1.38%.

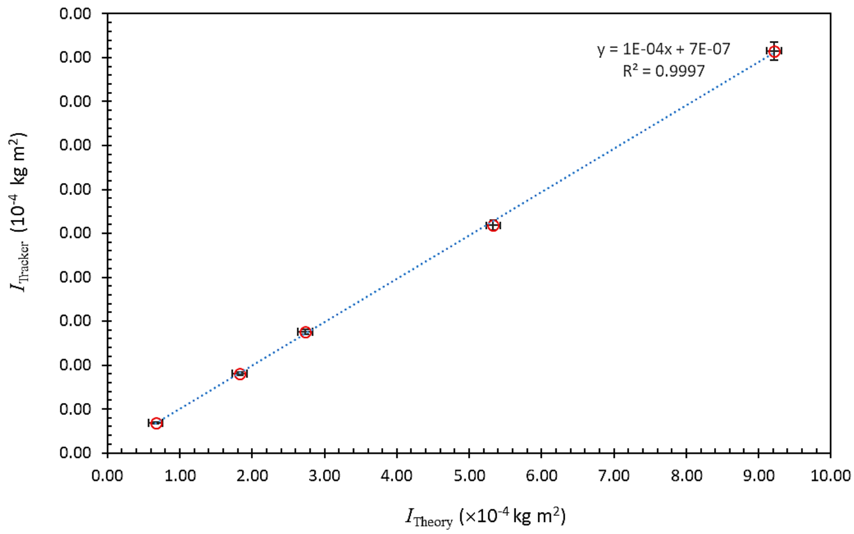


Fig. 5. Comparison plot between I_{Tracker} and I_{Theory} of the full ring pendula. The dotted lines in both graphs represent the linear relationship of I_{Tracker} that is equal to I_{Theory}.

3.3 The period of the partial ring

The five sets of partial ring pendula (one is the full ring) were used in this experiment. They were the ring with different radii: 5.00, 7.00, 8.00, 10.00, and 12.00 centimeters and their shapes; the full ring (1), 7/8, 3/4, 5/8, and 1/2 ring. The example was one of the five partial ring sets (which were the same radius of curvature), as shown in Fig. 6. The marker at the 3 o'clock position on each partial ring was tracked. Then, the average position change with time was further analyzed and compared to the others in the Excel spreadsheet, as shown in Fig. 7. These curves were generated by experimental data (positions x and times) from the Tracker. As you can see, all five curves from the partial ring with the same radius of curvature (8.00 centimeters) contained approximately the same period.

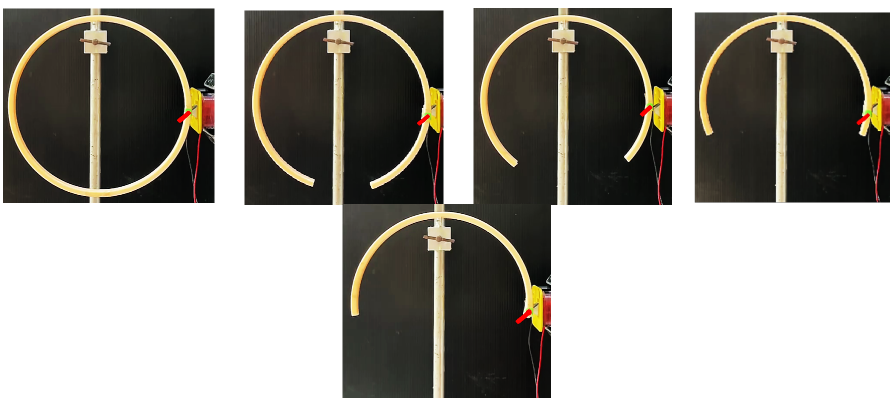


Fig. 6. The five partial ring pendula. They have the same radius of curvature (8.00 centimeters). Their shapes from left to right are as follows: a full ring (1), 7/8, 3/4, 5/8, and 1/2 ring.

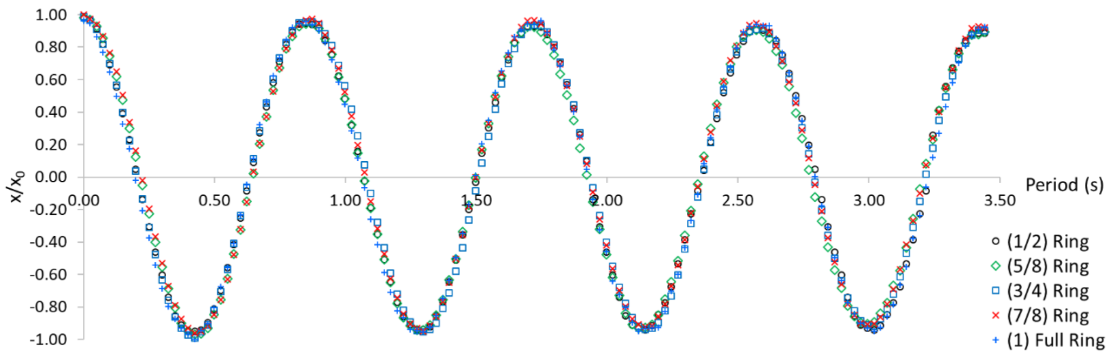


Fig. 7. Comparison plot between the average of x/x_0 versus the oscillating period of the five partial ring pendula. They have the same radius of curvature (8.00 centimeters) with the following shapes: 1/2, 5/8, 3/4, 7/8 ring, and full ring.

For another data analysis of the period based on Eq. (2.5), this oscillation period is denoted as T_{Tracker} (obtaining from ω_0 in Tracker). The period obtained from Eq. (2.10) or Eq. (2.11) is denoted as T_{Theory} (calculated from the radii of the ring). The relationship between the two methods of periods versus the ring's radii and their shapes is shown in Fig. 8.

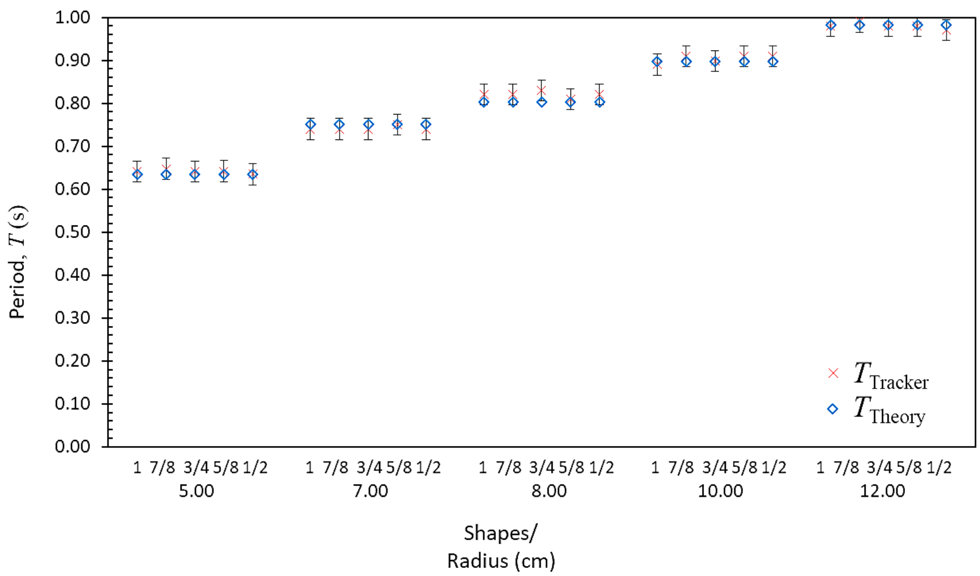


Fig. 8. Comparison plot between T_{Tracker} and T_{Theory} versus the ring's radii (5.00, 7.00, 8.00, 10.00, and 12.00 centimeters) and their shapes.

The data analysis of the relationship between both periods versus the ring's radii showed that the period depended on the ring's radius of curvatures. Those were the rings with radii: 5.00, 7.00, 8.00, 10.00, and 12.00 centimeters with the average period of the T_{Tracker} that were equal to 0.64±0.00, 0.75±0.01, 0.81±0.01, 0.88±0.01, and 0.98±0.01 seconds with about 1.00, 0.72, 0.49, 1.42, and 0.32% deviation from the T_{Theory}, respectively. However, the period was independent of the ring's shapes with the same radius of curvature, as shown in Fig. 8. Fig. 8 illustrates one radius (shapes: full ring, 7/8, 3/4, 5/8, and 1/2 ring) containing one period. As a result, the ring with shapes: full ring, 7/8, 3/4, 5/8, and 1/2 ring of the radii; 5.00, 7.00, 8.00, 10.00, and 12.00 centimeters contained the period of the T_{Tracker} in range; 0.63-0.65, 0.74-0.75, 0.80-0.82, 0.87-0.89, 0.97-0.99 seconds, respectively. This experiment verified that, regardless of the length of the circumference, the period is still the same with the full ring pendulum.

4. Conclusion

In this study, we presented the results of using a high-speed video analysis technique to investigate the set of partial ring pendula (one is a full ring) as actual motion in physics laboratory which was able to be explained by the physics principle. There were three experimental results. The first result showed that the ω_0^2 is linearly proportional to $\frac{1}{2r}$.Then, the value of g was able to obtain with a value of 9.77 ms^{-2} and about 0.31% deviation from the real value. The second result expressed the comparison between the two methods used to calculate the full ring's moment of inertia which were $I_{\text{Tracker}} = \frac{mgr}{\omega_0^2}$ and $I_{\text{Theory}} = 2mr^2$ that were acquired from the Tracker video analysis and from the direct measuring of the ring's radii, respectively. As a result, the average percentage difference is approximately 1.38%. Finally, the results showed that the oscillating periods depended on the ring's radius of curvatures but did not depend on the length of the circumference. The results from this high-speed video analysis technique can be used in physics laboratories and for teaching physical pendulum in a classroom. This will help to increase the understanding of students and teachers about this phenomenon or others, which can be set in the classroom.

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