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Original research article

Quantitative Study of the Effect of Leaf Harvesting Time on Its Edge Curvature

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ABSTRACT

Once a leaf is harvested from a plant, its water source is eliminated, and thus the process of leaf dehydration begins. The reduction in water availability induces a stress-coping mechanism in the plant, which lowers leaf water potential, thereby instigating leaf contraction. It is observed that the curvature of the leaf edge directly correlates with the time passed since harvesting. Curvature is geometrically defined as the instantaneous rate of change of direction of a moving point on the curve. In this research, we attempted to quantify such increases by determining the numerical curvature of each point on leaf edges. Experiments are performed on harvested *Magnolia* × *alba* leaves, and data on pointwise curvatures are collected. The collected data is Poisson-regressed. We then perform 3 statistical analyses: Pearson goodness-of-fit test to ensure that curvature is Poisson-distributed, difference of mean testing to ensure that the curvatures truly change, and test for randomness to assess whether curvature value is correlated with the position on the leaf edge. As expected, the average curvature for each leaf constantly increases as a function of time. Furthermore, the distribution of leaf edge curvatures is well-approximated by Poisson distribution, where the rate λ is calibrated using average curvature. Lastly, there is significant relationship of curvature between any two points on the same leaf as desired. This suggests that the increment of leaf edge curvature is a Poisson process, a finding applicable to computer graphics, leaf hydraulic system simulation, and leaf age detection.

Keywords: Leaf dehydration; Leaf edge curvature distribution; Leaf harvesting time; *Magnolia* × *alba*; Poisson distribution

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Fig. 1. From left to right: *Magnolia × alba* leaves which were subjected to heat treatment in a Memmert Universal Oven UF450 at 45°C and 50 % wind strength for varying durations of 0, 20, 40, 60, and 80 minutes.

1. Introduction

The shape of a leaf is affected by various factors. For example, temperature and humidity influence the leaf's structure and water transport [1], and thereby its shape. Another way that a leaf's structure and shape can be altered is due to the lack of water [2]. The water-deficit stress induces leaf contraction, and, in extreme cases, results in plasmolysis [3], the separation of cell membrane from cell wall, both of which transform the leaf's shape.

One way that water deficiency can occur is by heat. Heat makes the water in the leaves evaporate and hence their shape change, as can be observed in Fig. 1. Amongst other things, the change that is the most evident is in the leaf edge's curvature. We aim to quantify said change and introduce a model for the correlation between heated time and the leaf edge's curvature.

Curvature is mathematically defined point-wise. In summary, curvature is the curve quantity that quantifies the deviation of the infinitesimal curve from the straight line. The osculating circle is the best-approximating circle of the curve at the particular point. The inverse of the radius of the osculating circle is the curvature of the curve at that point [4]. Another, more tangible interpretation of curvature is the instantaneous rate of change of direction of the tangent vector in a unit-speed parameterization of the curve. The reader is referred to [5] for a more rigorous treatment on the topic.

Since the curvature of each point in a curve differs from the others, we consider the distribution of the curvatures of all sample points in the leaf edge and observe the trend of the change of the distribution. We will test whether this distribution conforms with Poisson distribution. From this research we obtain the change of curvature from its distribution, so it can help in botanical study or computer science such as predicting the amount of water in a leaf. Also, we can create a model using a computer graphic for creating a simulation of leaf growth or leaf hydration system.

The paper is organized as follows. Section 2 is the methodology of the study. Section 3 is where we performed statistical analyses of the data obtained. Lastly, Section 4 summarizes the findings.

2. Materials and Methods

2.1 Data Collection

White Champaca (*Magnolia × alba*), an angiosperm regularly cultivated in Southeast Asia and tropical regions of East Asia, was selected as the sample plant for this research. First, we gathered 10 leaves from a White Champaca tree. Next, we scanned the leaves using a Sense 3D scanner. The rotary table was used to rotate the leaf to have the sight of all directions of the leaves and the C clamp apparatus was used to fix the rotary table to the table, as shown in Fig. 2. We then used a Memmert Universal Oven UF450 to heat the leaves, artificially inducing the loss of water in the leaves. This process is akin to leaving the leaves to dry at room temperature, as it makes water evaporate from the leaves, but requires much less time. The oven's temperature was set to 45 degrees Celsius and the wind strength was set at 50%. The leaves were heated for the next 80 minutes. Every 20 minutes during the heating, we took the leaves out of the oven and built their 3D model via the same process as when it is scanned initially.

2.2 Curvature Computation

After we had 3D models of all leaves along the time, we imported the models into the SolidWorks 2019 program and used the plot function to plot 35 points with approximately equal distances for any consecutive points on both sides of the leaf, as shown in Fig. 3.

Next, we computed the curvature for each plotted point using a discretized version of the curvature formula

$$\kappa(t) = \frac{||\overrightarrow{\gamma}'(t) \times \overrightarrow{\gamma}''(t)||}{||\overrightarrow{\gamma}'(t)||^3},$$

where κ is the curvature function and $\overrightarrow{\gamma}$ is a parametrized curve.

The numerical formula can be stated as follows. For any three consecutive points $t_1 = (x_1, y_1, z_1)$, $t_2 = (x_2, y_2, z_2)$ and $t_3 = (x_3, y_3, z_3)$ (see Fig. 4), the numerical curvature κ of t_2 is given by

$$\kappa(t_2) = \frac{2\left(\sum_{\text{cyc}}(x'y'' - y'x'')^2\right)^{1/2}}{((x')^2 + (y')^2 + (z')^2)^{3/2}},$$

where $\sum_{\text{cyc}}$ denotes cyclic sum over variables x, y, and z, and their related derivatives, $x' = x_3 - x_1$ and $x'' = x_3 - 2x_2 + x_1$ and y', y'', z' and z'' are defined analogously.

In practice, we imported the data from SolidWorks to a Python program which calculated the numerical curvature from the aforementioned formula.

2.3 Statistical Analyses

We will compare our data to Poisson distribution. It is a discrete probability distribution that describes the number of events that occur in a fixed period of time, given that the occurrence of each event is independent of one another and the mean time between two events is known [6]. More elaborately, if a random variable X follows a Poisson distribution with parameter λ, the mean number of occurrences in the fixed time, has a probability mass function given by

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!},$$

for $x = 0, 1, 2, \ldots$, and where e is the Euler's constant.

After we obtain the curvature values for all periods of time in all leaves, we assign a class to each value to analyse them in a Poisson distribution. An empirical observation is that the range of curvature values is between 0 and 0.2. So, we divided the curvature values into 10 classes, with a range of 0.02 for each class. If the pointwise curvature is in the range $[0.02k, 0.02(k+1))$, then it is assigned to class k. The Poisson parameter λ is estimated from $\bar{x}/0.02$, where $\bar{x}$ is the sample mean of all pointwise curvatures for a leaf L at time T.

We then perform 3 statistical tests to analyze the data. The first is the chi-square goodness-of-fit test, which is used to determine whether the distribution of curvatures is well-approximated by a Poisson distribution. The second is the test of mean difference using Wilcoxon rank sum. It is used to compare the average curvature in different periods of time, in order to determine whether the average curvature indeed increases over time. Finally, runs test for randomness is performed to assess the randomness of the curvature data. This test helps us confirm that there is a correlation between

Fig. 2. Setup of the scanning apparatus.

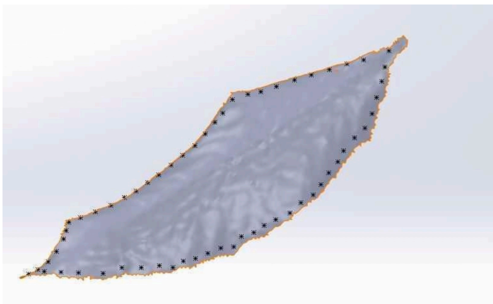

Fig. 3. Plotted leaf edges in SolidWorks 2019 program.

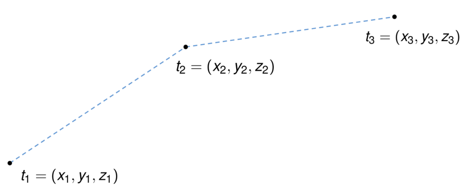

Fig. 4. The three consecutive points in the calculation of numerical curvature.

the curvature at any two points on the same leaf depending on its position.

In summary, these statistical tests provide insights into the distribution of curvatures and the relationship between curvature and time.

Table 1. Average statistics from Poisson regression at time T.

T (minutes)	Average $\hat{\lambda}$	Average R^2
0	3.685762976	0.702751266
20	3.582465405	0.741343469
40	3.608748432	0.661190418
60	3.894043737	0.671287878
80	4.295511904	0.62137327

3. Result and Discussion

The collected data is used to estimate Poisson parameter λ as described in the previous section. Table 1 summarizes the average values of λ obtained from Poisson regression for each time. A surface observation is that $\hat{\lambda}$ increases as time went on (see Fig. 5). In addition, the high R^2 statistic in comparison between the real data and expected frequency from the Poisson distribution indicates that the Poisson distribution may be fitting to the data.

Fig. 6 shows a histogram of the curvature values, with the horizontal axis representing the classes and the vertical axis representing the probability that a curvature value falls into a given class. The graph moderately fits Poisson regression.

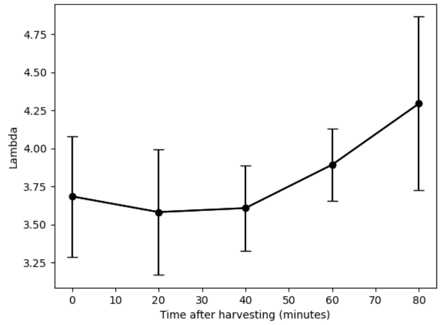

Fig. 5. Progression of $\hat{\lambda}$ with respect to time. Error bars are the range of the values.

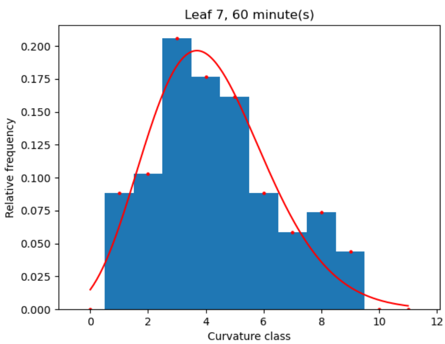

Fig. 6. An example of histogram combined with Poisson-regression curve with $R^2 = 0.895$.

The chi-squared goodness-of-fit test is performed to further investigate this relationship.

The result of the goodness-of-fit test is in Table 2. For most data, the p-value of the observed χ^2-statistic falls within 95% confidence interval. Additionally, the mean of the parameter λ is slightly increasing over time, suggesting that the curvature of leaf is growing throughout the period as the water in the leaf evaporates and causes the leaf structure to twist, resulting in more curvature.

Fig. 7 shows that the peak of the graph decreases as a function of time, indicating that the curvature increases over time. The graph also appears to be right-shifted, meaning that there are more points with high curvature as the time of harvesting increases. The drop in curvature from 80 minutes to 60 minutes is more pronounced than the drop from 0 minutes to 20 minutes. Overall, these tests support our hypothesis that the increment of leaf edge curvature is a Poisson process.

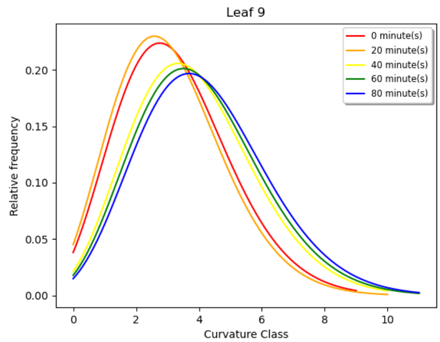

Fig. 7. The change of estimated distribution of curvature classes with respect to time in a leaf.

Next, we perform a test of difference in means to confirm that the difference in time leads to a difference in the average curvature. This is done by the Wilcoxon rank-sum test [7]. The distribution of the rank sum is assumed to be normal with mean $\frac{1}{2}n(2n+1)$ and variance $\frac{1}{12}n^2(2n+1)$, where n is the number of data points, which is 68 in this instance.

Table 3 records the result of these tests. We can see that the p-value is low (less than 0.05) for the comparisons between curvatures at times T_1 and T_2 where $T_1 \leq 20$ and $T_2 \geq 40$. This indicates that the curvature increases significantly in the period between 20 to 40 minutes. Furthermore, the curvature is increasing over time.

We will next discuss the relationship between the position of a point on the leaf edge and curvature at that point. Empirically, we can observe that the curvature at

Table 2. Result of chi-square goodness-of-fit test for a leaf at time T. ** indicates significance with 95% confidence interval.

T (minutes)	χ^2-statistic	Degrees of freedom	p-value	Stat. significance
0	5.66154637	9	0.57977654	**
20	20.33334239	10	0.009145856	
40	7.539793224	10	0.479663002	**
60	11.76114622	10	0.162183335	**
80	10.2322043	10	0.249105047	**

Table 3. Result of Wilcoxon rank-sum test comparing λ for each times T_A and T_B. Each entry is the p-value of the null hypothesis that λ is unchanged from T_A to T_B. ** indicates 95% confidence.

T_A/T_B (minutes)	0	20	40	60	80
0		0.583692	0.027105**	0.013436**	0.008793**
20	0.583692		0.022079**	0.025026**	0.010258**
40	0.027105**	0.022079**		0.741400	0.470294
60	0.013436**	0.025026**	0.741400		0.927493
80	0.008793**	0.010258**	0.470294	0.927493	

a point closer to the apex tends to be less than that at a point further from it, with the exception of the point at the apex itself. To this end, we use runs test for randomness to determine whether the curvature values are truly random or dependent on the position on the leaf edge (which determines the sequence of curvature values).

For this, we create a binary sequence by comparing consecutive values of curvature and assigning a value of 0 if the curvature increases and a value of 1 if it decreases. The expected number of runs in the binary sequence is given by $\frac{2MN}{M+N}+1$, where M and N are the number of 0s and 1s in the sequence, respectively. The empirical number of runs is the actual number of runs in the binary sequence obtained from the data.

Table 4 shows that the expected and empirical number of runs are different and p-value is low (less than 0.5), indicating that our data is not random and that there is relationship between curvatures at any two points on the same leaf, about which further observation should be made.

4. Conclusion

When leaves are deprived of water, we observed that the structure of the leaf and its shape change over time. The edges of the leaves grew rougher or, more precisely, grew in their curvature. That observation was confirmed by the increase in the mean curvatures over time in each leaf that we experimented with. We also discovered that the curvatures of points on a leaf's edge are Poisson-distributed. Summarily, the statistical tests we performed showed that leaf curvature can be modelled as a Poisson process with λ increasing over time. Moreover, from the run test for randomness, the difference in time leads to a difference in curvature, and there is significant relationship between curvatures at any two points on the same leaf.

Our work is a preliminary inves-

Table 4. Runs test for randomness statistics. Each entry is the p-value of the null hypothesis that curvatures on the same leaf are random. ** indicates 95 % confidence.

Time (minutes)	Expected no. of runs	Empirical no. of runs	p-value
0	34.4328358	47	0.001932014**
20	32.34328358	43	0.004995567**
40	34.13432836	45	0.006825628**
60	34.13541384	49	0.000214653**
80	34.43283582	48	0.000816316**

tigation into the geometry of leaf edges from a probabilistic perspective. There are many applications for this study, such as leaf hydraulic system simulation, leaf age detection, and computer graphics simulation. This study can be developed further to examine the leaf edge curvature in other classes of plants. More ambitiously, the study can be extended to the surface curvature of the leaf.

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