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Original research article

C-Class F-Contraction in C^*-Algebra Valued Metric Space

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ABSTRACT

In the present manuscript, we enlarge the class of F-contraction in the framework of C^*-algebra valued metric space. We present some results on fixed points with the help of C-class function for different types of F-contractive condition. The result is an extension and generalization of several metric space results available. Moreover, some examples are presented here to illustrate the usability of obtained results.

Keywords: C-class function; C^*-algebra valued metric space; F-contraction

1. Introduction and Preliminaries

The study of fixed point theory for self mappings has been a fascinating area of research for the last few decades. The formal theoretical approach to the fixed point originated from Picard's work. However, it was Stefen Banach [1] who underlined the idea into an abstract framework and provided a constructive tool to establish the fixed points of mapping in the metric space. In 2014, Ma et al. [2] extended the Banach contraction principle to C^*-algebra valued metric spaces by replacing the set of real numbers with the set of all positive members of unital C^*-algebra. H. Massit and M. Rosaafi [3] inspired by the work of H. Piri et al. [4], introduced the concept of (ϕ, F)-contraction in C^*-algebra valued metric space and proved some fixed point results. Later on, M. Rossafi et al. [5] generalized (ϕ, MF)-contraction and established some fixed point results on C^*-algebra valued metric space. Many researchers have obtained various results in this theory of fixed points and common fixed points (for references, see [6–17] and references therein).

Inspired by the work of Rossafi et al. [5], in this manuscript, we enlarge the class of F-contraction in the framework of C^*-algebra valued metric space. We present some results on fixed points with the help of C-class functions for different types of F-contractive conditions. The result is an extension and generalization of several met-

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ric space results available. Moreover, some examples are presented here to illustrate the usability of obtained results.

We gave some notation and definition mentioned in [2], which will be required in sequel to prove the results.

Definition 1.1 ([2])**.** Suppose X is a nonempty set. A function $d : X \times X \to \mathbb{A}$ satisfies :

(i) $d(\sigma,\rho) \geq \theta_{\mathbb{A}}$ and $d(\sigma,\rho) = \theta_{\mathbb{A}}$ if and only if $\sigma = \rho$;

(ii) $d(\sigma,\rho) = d(\rho,\sigma)$;

(iii) $d(\sigma,\rho) \preceq d(\sigma,\mu) + d(\mu,\rho)$;

for all $\sigma,\rho,\mu \in X$. Then, d is called a C^*-algebra valued metric and $(X,\mathbb{A},d)$ is called a C^*-algebra valued metric space.

Definition 1.2 ([2])**.** A sequence $\{\sigma_n\}$ in $(X,\mathbb{A},d)$ is said to be

1. (a) convergent with respect to $\mathbb{A}$, if for given $\epsilon > 0$, there exists a positive integer k such that $\|d(\sigma_n,\sigma)\| < \epsilon$, for all $n > k$;

   (b) Cauchy sequence with respect to $\mathbb{A}$ if for any $\epsilon > 0$, there exists $k \in \mathbb{N}$ such that $\|d(\sigma_n,\sigma_m)\| < \epsilon$, for all $n,m > k$.

2. $(X,\mathbb{A},d)$ is called a complete C^*-algebra valued metric space if every Cauchy sequence with respect to $\mathbb{A}$ is convergent.

Definition 1.3 ([10])**.** Let $\Psi_{\mathbb{A}}$ be the set of positive functions, $\psi_{\mathbb{A}} : \mathbb{A}^+ \to \mathbb{A}^+$ satisfying the following conditions :

(i) $\psi_{\mathbb{A}}(a)$ is continuous and nondecreasing;

(ii) $\psi_{\mathbb{A}}(a) = \theta_{\mathbb{A}}$ if and only if $a = \theta_{\mathbb{A}}$.

Definition 1.4 ([10])**.** (C- Class Function) A continuous function $F^* : \mathbb{A}^+ \times \mathbb{A}^+ \to \mathbb{A}^+$ is called a C-class function if for any $P,Q \in \mathbb{A}^+$, the following conditions hold :

(i) $F^*(P,Q) \preceq P$;

(ii) $F^*(P,Q) = P$ implies that either $P = \theta_{\mathbb{A}}$ or $Q = \theta_{\mathbb{A}}$.

An extra condition could be imposed on F^* if required such that $F^*(\theta_{\mathbb{A}},\theta_{\mathbb{A}}) = \theta_{\mathbb{A}}$.

2. Main Result

In this section, we give some fixed point results using the different types of F-contractive conditions with C-class function. We also discuss some examples to illustrate the obtained results.

Theorem 2.1. *Let $(X,\mathbb{A},d)$ be a complete C^*-algebra valued metric space. Define T be self mapping on X and there exists $F : \mathbb{A}^+ \to \mathbb{A}$ satisfying the following conditions:*

(i) *F is continuous and nondecreasing on $\mathbb{A}^+$;*

(ii) *For each sequence $\{\alpha_n\} \subseteq \mathbb{A}^+$*

$$\lim_{n\to\infty} \alpha_n = \theta_{\mathbb{A}} \text{ if and only if } \lim_{n\to\infty} F(\alpha_n) = \theta_{\mathbb{A}}; \qquad (2.1)$$

(iii) *for every $\zeta,\eta \in X$ with $\tau > 0$,*

$$\tau + F(\psi(d(T\zeta,T\eta))) \preceq F^*\Big(F\big(\psi(d(\zeta,\eta))\big), F\big(\phi(d(\zeta,\eta))\big)\Big); \qquad (2.2)$$

where F^* is C-class function and $\psi,\phi \in \Psi$. Then, T has unique fixed point.

Proof. First, let us observe that T has at most one fixed point. Indeed if

$$\zeta_1, \zeta_2 \in X; T\zeta_1 = \zeta_1 \neq \zeta_2 = T\zeta_2.$$

From Eq. (2.2), we have

$$\begin{aligned} \tau &+ F(\psi(d(T\zeta_1, T\zeta_2))) \\ &\preceq F^*\Big(F(\psi(d(\zeta_1, \zeta_2))), \\ &\quad F(\phi(d(\zeta_1, \zeta_2)))\Big) \\ &\preceq F(\psi(d(\zeta_1, \zeta_2))), \end{aligned}$$

implies that $\tau \preceq F(\psi(d(\zeta_1, \zeta_2))) - F(\psi(d(T\zeta_1, T\zeta_2))) = \theta_{\mathbb{A}}$, which is a contradiction.

In order to show that a fixed point, let $\zeta_0 \in X$. We construct a sequence $\{\zeta_n\}$ in X as follows $\zeta_{n+1} = T\zeta_n$ for all $n \in \mathbb{N}$. If $\zeta_n = \zeta_{n+1}$ for some $n \in \mathbb{N}$, then ζ_n is a fixed point of T. Now, suppose that $\zeta_n \neq \zeta_{n+1}$ for all $n \in \mathbb{N}$. Define $d_n = d(\zeta_n, \zeta_{n+1})$. Substitute $\zeta = \zeta_{n+1}$ and $\eta = \zeta_n$ in (2.2), we have

$$\begin{aligned} &F(\psi(d(\zeta_{n+1}, \zeta_n))) = F(\psi(d(T\zeta_n, T\zeta_{n-1}))) \\ &\preceq \tau + F(\psi(d(T\zeta_n, T\zeta_{n-1}))) \\ &\preceq F^*\Big(F\big(\psi(d(\zeta_n, \zeta_{n-1}))\big), \\ &F\big(\phi(d(\zeta_n, \zeta_{n-1}))\big)\Big) \preceq F\big(\psi(d(\zeta_n, \zeta_{n-1}))\big). \end{aligned}$$

Therefore,

$$F(\psi(d_n)) \preceq F(\psi(d_{n-1})). \quad (2.3)$$

Since, F is non-decreasing and so the sequence $\{\psi(d_n)\}$ is monotonically decreasing in $\mathbb{A}^+$. Hence, ψ is non-decreasing in $\mathbb{A}^+$ so the sequence $\{d_n\}$ is monotonically decreasing in $\mathbb{A}^+$. Therefore, there exists $\theta_{\mathbb{A}} \preceq t \in \mathbb{A}^+$ such that

$$d(\zeta_n, \zeta_{n+1}) \to t \quad \text{as} \quad n \to \infty.$$

From Eq. (2.3), we get $\lim_{n\to\infty} F(\psi(d_n)) = \theta_{\mathbb{A}}$. From Eq. (2.1), together we have

$$\lim_{n\to\infty} \psi(d_n) = \theta_{\mathbb{A}} \quad \text{implies} \quad \lim_{n\to\infty} d_n = \theta_{\mathbb{A}}. \quad (2.4)$$

Now, we shall show that $\{\zeta_n\}$ is a Cauchy sequence in $(X, \mathbb{A}, d)$.

Assume that $\{\zeta_n\}$ is not a Cauchy sequence in $(X, \mathbb{A}, d)$. Then, there exist $\epsilon > 0$ and subsequences (ζ_{m_k}) and (ζ_{n_k}) with $n_k > m_k > k$ such that

$$\|d(\zeta_{m_k}, \zeta_{n_k})\| \geq \epsilon. \quad (2.5)$$

Now, corresponding to m_k we can choose n_k such that it is the smallest integer with $n_k > m_k$ and satisfying Eq. (2.5). Hence,

$$\|d(\zeta_{m_k}, \zeta_{n_k-1})\| < \epsilon. \quad (2.6)$$

Using Eqs. (2.5)-(2.6), we get

$$\begin{aligned} \epsilon &\leq \|d(\zeta_{m_k}, \zeta_{n_k})\| \\ &\leq \|d(\zeta_{m_k}, \zeta_{n_k-1})\| + \|d(\zeta_{n_k-1}, \zeta_{n_k})\| \\ &\leq \epsilon + \|d(\zeta_{n_k-1}, \zeta_{n_k})\|. \end{aligned} \quad (2.7)$$

From Eq. (2.4), we have

$$\lim_{k\to\infty} \|d(\zeta_{n_k-1}, \zeta_{n_k})\| = \theta_{\mathbb{A}}. \quad (2.8)$$

Taking limit as $k \to \infty$ in Eq. (2.7) and using Eq. (2.8), we get

$$\lim_{k\to\infty} \|d(\zeta_{m_k}, \zeta_{n_k})\| = \epsilon. \quad (2.9)$$

Again,

$$\begin{aligned} &\|d(\zeta_{n_k}, \zeta_{m_k})\| \\ \leq\ &\|d(\zeta_{n_k}, \zeta_{n_k-1})\| + \|d(\zeta_{n_k-1}, \zeta_{m_k})\| \\ \leq\ &\|d(\zeta_{n_k}, \zeta_{n_k-1})\| + \|d(\zeta_{n_k-1}, \zeta_{m_k-1})\| \\ &+\|d(\zeta_{m_k-1}, \zeta_{m_k})\|. \end{aligned} \quad (2.10)$$

Also,

$$\|d(\zeta_{n_k-1}, \zeta_{m_k-1})\| \leq$$

$$\begin{aligned} &\|d(\zeta_{n_k-1},\zeta_{n_k})\| + \|d(\zeta_{n_k},\zeta_{m_k-1})\| \\ \leq\ &\|d(\zeta_{n_k-1},\zeta_{n_k})\| + \|d(\zeta_{n_k},\zeta_{m_k})\| \\ &+\|d(\zeta_{m_k},\zeta_{m_k-1})\|. \qquad (2.11) \end{aligned}$$

Taking limit as $k \to \infty$ in Eqs. (2.10)-(2.11) and using Eqs. (2.8)-(2.9), we get

$$\lim_{k\to\infty} \|d(\zeta_{n_k-1},\zeta_{m_k-1})\| = \epsilon.$$

Since, $d(\zeta_{n_k-1},\zeta_{m_k-1}), d(\zeta_{n_k},\zeta_{m_k}) \in \mathbb{A}^+$ and

$$\lim_{k\to\infty} \|d(\zeta_{n_k-1},\zeta_{m_k-1})\| = \lim_{k\to\infty} \|d(\zeta_{n_k},\zeta_{m_k})\| = \epsilon,$$

there exist $s \in \mathbb{A}^+$ with $\|s\| = \epsilon$ such that

$$\lim_{k\to\infty} \|d(\zeta_{n_k-1},\zeta_{m_k-1})\| = \lim_{k\to\infty} \|d(\zeta_{n_k},\zeta_{m_k})\| = s.$$

By Eq. (2.2), we have

$$\begin{aligned} F(\psi(s)) &= \lim_{k\to\infty} F\big(\psi\big(d(\zeta_{n_k},\zeta_{m_k})\big)\big) \\ &= \lim_{k\to\infty} F\big(\psi\big(d(T\zeta_{n_k-1},T\zeta_{m_k-1})\big)\big) \\ &\leq \tau + \lim_{k\to\infty} F\big(\psi\big(d(T\zeta_{n_k-1},T\zeta_{m_k-1})\big)\big) \\ &\leq \lim_{k\to\infty} F^*\Big(F\big(\psi(d(\zeta_{n_k-1},\zeta_{m_k-1})\big), \\ &\quad F\big(\phi(d(\zeta_{n_k-1},\zeta_{m_k-1}))\big)\Big) \\ &\leq \lim_{k\to\infty} F\big(\psi(d(\zeta_{n_k-1},\zeta_{m_k-1})\big) \\ &= F(\psi(s)). \end{aligned}$$

Therefore, $F(\psi(s)) = \theta_{\mathbb{A}}$. Hence, $\psi(s) = \theta_{\mathbb{A}}$ implies $s = \theta_{\mathbb{A}}$, which is a contradiction. Hence, we get $\{\zeta_n\}$ is a Cauchy sequence in a complete C^*-algebra valued metric space $(X, \mathbb{A}, d)$. Thus, there exists $\zeta \in X$ such that $\zeta_n \to \zeta$ as $n \to \infty$.
Consider,

$$\begin{aligned} &\|F(\psi(d(\zeta,T\zeta)))\| \leq \\ &\tau + \lim_{n\to\infty} \|F(\psi(d(\zeta_n,T\zeta_n)))\| \\ \leq\ &\lim_{n\to\infty} \Big\|F^*\Big(F\big(\psi(d(\zeta_{n-1},\zeta_n))\big), \\ &F\big(\phi(d(\zeta_{n-1},\zeta_n))\big)\Big)\Big\| \\ \leq\ &\lim_{n\to\infty} \big\|F\big(\psi(d(\zeta_{n-1},\zeta_n))\big)\big\| \\ =\ &\|F(\psi(d(\zeta,\zeta)))\| \\ =\ &\|F(\psi(\theta_{\mathbb{A}}))\| \\ =\ &\|F(\theta_{\mathbb{A}})\| = \theta_{\mathbb{A}}. \end{aligned}$$

Therefore, we get $\|F(\psi(d(\zeta,T\zeta)))\| = \theta_{\mathbb{A}}$ implies $\psi(d(\zeta,T\zeta)) = \theta_{\mathbb{A}}$. Hence, $T\zeta = \zeta$, i.e. ζ is a fixed point of T. This completes the proof. □

Taking $F^*(r,t) = r$ and $\psi(t) = t = \phi(t)$, we have the following result.

Corollary 2.2. *Let $(X, \mathbb{A}, d)$ be a complete C^*-algebra valued metric space. Define T be self mapping on X and there exists $F : \mathbb{A}^+ \to \mathbb{A}$ satisfying the following conditions:*

- (i) *F is continuous and nondecreasing on $\mathbb{A}^+$;*
- (ii) *For each sequence $\{\alpha_n\} \subseteq \mathbb{A}^+$*

$$\lim_{n\to\infty} \alpha_n = \theta_{\mathbb{A}} \text{ if and only if } \lim_{n\to\infty} F(\alpha_n) = \theta_{\mathbb{A}};$$

- (iii) *for every $\zeta, \eta \in X$ with $\tau > 0$,*

$$\tau + F(d(T\zeta,T\eta)) \leq F(d(\zeta,\eta)).$$

Then, T has a unique fixed point.

3. Remarks

In Eq. (2.2), we can extend the following version of well known results of literature for self mapping in the framework of C^*-algebra valued metric space.

(1) (Kannan type, see [18]) for every $\zeta, \eta \in X$ with $\tau > 0$,

$$\tau + F\big(\psi(d(T\zeta, T\eta))\big) \leq F^*\Bigg(F\bigg(\psi\bigg(\frac{d(\zeta, T\zeta) + d(\eta, T\eta)}{2}\bigg)\bigg), F\bigg(\phi\bigg(\frac{d(\zeta, T\zeta) + d(\eta, T\eta)}{2}\bigg)\bigg)\Bigg). \quad (3.1)$$

(2) (Chatterjea type, see [19]) for every $\zeta, \eta \in X$ with $\tau > 0$,

$$\tau + F\big(\psi(d(T\zeta, T\eta))\big) \leq F^*\Bigg(F\bigg(\psi\bigg(\frac{d(\zeta, T\eta) + d(\eta, T\zeta)}{2}\bigg)\bigg), F\bigg(\phi\bigg(\frac{d(\zeta, T\eta) + d(\eta, T\zeta)}{2}\bigg)\bigg)\Bigg).$$

(3) (Reich type, see [20]) for every $\zeta, \eta \in X$ with $\tau > 0$,

$$\tau + F(\psi(d(T\zeta, T\eta))) \leq F^*\Bigg(F\bigg(\psi\bigg(\frac{d(\zeta, \eta) + d(\zeta, T\zeta) + d(\eta, T\eta)}{3}\bigg)\bigg), F\bigg(\phi\bigg(\frac{d(\zeta, \eta) + d(\zeta, T\zeta) + d(\eta, T\eta)}{3}\bigg)\bigg)\Bigg).$$

(4) (Hardy-Roger type, see [21]) for every $\zeta, \eta \in X$ with $\tau > 0$,

$$\tau + F\big(\psi(d(T\zeta, T\eta))\big) \leq F^*\big(F\big(\psi\big(d(\zeta, \eta) + d(\zeta, T\zeta) + d(\eta, T\eta) + d(\eta, T\zeta) + d(\zeta, T\eta)\big)/5\big), F\big(\phi\big(d(\zeta, \eta) + d(\zeta, T\zeta) + d(\eta, T\eta) + d(\eta, T\zeta) + d(\zeta, T\eta)\big)/5\big)\big).$$

(5) (Weak F-contraction type, see [22]) for every $\zeta, \eta \in X$ with $\tau > 0$,

$$\tau + F(\psi(d(T\zeta, T\eta))) \leq F^*\Big(F\big(\psi(M(\zeta, \eta))\big), F\big(\phi(M(\zeta, \eta))\big)\Big),$$

where

$$M(\zeta, \eta) = \max\bigg\{d(\zeta, \eta), d(\zeta, T\zeta), d(\eta, T\eta), \frac{d(\zeta, T\eta) + d(\eta, T\zeta)}{2}\bigg\}.$$

Example 3.1. Let $X = [0, 2]$ and $\mathbb{A} = \mathbb{C}$. Let $F : \mathbb{A}^+ \to \mathbb{A}$ defined as $F(a) = 25a$ and $d : X \times X \to \mathbb{A}$ defined by

$$d(\rho, \sigma) = \begin{cases} |\rho| + |\sigma| & \text{if } \rho \neq \sigma \\ 0 & \text{if } \rho = \sigma. \end{cases}$$

Define, $T\rho = \dfrac{\rho}{150}$.

Then,

(i) $(X, \mathbb{A}, d)$ be a complete C^*-algebra valued metric space;

(ii) F is non-decreasing;

(iii) F is continuous;

(iv) $\lim_{n\to\infty} \sigma_n = 0$ and $\lim_{n\to\infty} F(\sigma_n) = 0$;

(v) for all $\sigma, \rho \in X$ with $\tau = 0.1$, $F^*(r, t) = r$, $\psi(t) = \phi(t) = 5t$, we get $d(T\rho, T\sigma) = \dfrac{\rho + \sigma}{150}$ and $\psi(d(T\rho, T\sigma)) = \dfrac{\rho + \sigma}{30}$. Hence,

$$\begin{aligned} \tau + F(\psi(d(T\rho, T\sigma))) &= 0.1 + F\bigg(\frac{\rho + \sigma}{30}\bigg) \\ &= 0.1 + \frac{5(\rho + \sigma)}{6} \\ &\leq 125(\rho + \sigma) \\ &= F\big(\psi(d(\rho, \sigma))\big) \\ &= F^*\Big(F\big(\psi(d(\rho, \sigma))\big), F\big(\phi(d(\rho, \sigma))\big)\Big). \end{aligned}$$

From Theorem (2.1), we get T has a unique fixed point. Indeed, 0 is the fixed point.

Example 3.2. Let $X = [0,2]$ and $\mathbb{A} = \mathbb{C}$. Let $F : \mathbb{A}^+ \to \mathbb{A}$ defined as $F(a) = 40a$ and $d : X \times X \to \mathbb{A}$ defined by

$$d(\rho,\sigma) = |\rho - \sigma|.$$

Define, $T\rho = \frac{\rho}{120}$.

Then,

(i) $(X, \mathbb{A}, d)$ be a complete C^*-algebra valued metric space;

(ii) F is non-decreasing;

(iii) F is continuous;

(iv) $\lim_{n\to\infty} \sigma_n = 0$ and $\lim_{n\to\infty} F(\sigma_n) = 0$;

(v) for all $\sigma, \rho \in X$ with $\tau = 0.2$, $F^*(r,t) = \frac{r}{2}$ and $\psi(t) = \phi(t) = 6t$

$$\begin{aligned}
d(T\rho,T\sigma) &= \frac{|\rho-\sigma|}{120}\\
d(\rho,T\rho) &= \frac{|119\rho|}{120}\\
d(\sigma,T\sigma) &= \frac{|119\sigma|}{120}\\
\psi(d(T\rho,T\sigma)) &= \frac{|\rho-\sigma|}{20}\\
\psi\left(\frac{d(\rho,T\rho)+d(\sigma,T\sigma)}{2}\right)\\
&= \frac{119|\rho+\sigma|}{40}.
\end{aligned}$$

Hence,

$$\begin{aligned}
&\tau + F(\psi(d(T\rho,T\sigma)))\\
&= 0.2 + F\left(\frac{|\rho-\sigma|}{20}\right)\\
&= 0.2 + 2(|\rho-\sigma|)\\
&\preceq \frac{119|\rho+\sigma|}{2}\\
&= \frac{1}{2}F\left(\psi\left(\frac{d(\rho,T\rho)+d(\sigma,T\sigma)}{2}\right)\right)\\
&= F^*\left(F\left(\psi\left(\frac{d(\rho,T\rho)+d(\sigma,T\sigma)}{2}\right)\right),\right.\\
&\qquad \left.F\left(\phi\left(\frac{d(\rho,T\rho)+d(\sigma,T\sigma)}{2}\right)\right)\right).
\end{aligned}$$

From Theorem (3.1), we get T has a unique fixed point. Indeed, 0 is the fixed point.

Example 3.3. Let $X = [0,2]$ and $\mathbb{A} = \mathbb{C}$. Let $F : \mathbb{A}^+ \to \mathbb{A}$ defined as $F(a) = 25a$ and $d : X \times X \to \mathbb{A}$ defined by

$$d(\rho,\sigma) = \begin{cases} |\rho| + |\sigma| & \text{if } \rho \neq \sigma \\ 0 & \text{if } \rho = \sigma. \end{cases}$$

Define, $T\rho = \begin{cases} 1/10, & \text{if } x \in [0,1] \\ 1/20, & \text{otherwise.} \end{cases}$.

Then,

(i) $(X, \mathbb{A}, d)$ be a complete C^*-algebra valued metric space;

(ii) F is non-decreasing;

(iii) F is continuous;

(iv) $\lim_{n\to\infty} \sigma_n = 0$ and $\lim_{n\to\infty} F(\sigma_n) = 0$;

(v) for all $\sigma, \rho \in X$ with $\tau = 0.1$, $F^*(r,t) = r$, $\psi(t) = \phi(t) = 5t$, we get $d(T\rho, T\sigma) = 0$ and $\psi(d(T\rho,T\sigma)) = 0$. Hence,

$$\begin{aligned}
&\tau + F(\psi(d(T\rho,T\sigma)))\\
&= 0.1 + F(0)\\
&= 0.1\\
&\preceq 125(|\rho| + |\sigma|)\\
&= F(\psi(d(\rho,\sigma)))\\
&= F^*\Big(F(\psi(d(\rho,\sigma))), F(\phi(d(\rho,\sigma)))\Big).
\end{aligned}$$

From Theorem (2.1), we get T has a unique fixed point. Indeed, $\frac{1}{10}$ is the fixed point.

Author Contributions

All the authors contributed equally for the preparation of the present manuscript.

Competing interests

The author declare that they do not have any competing interests.

Conflict of interest

All author declare that they have no conflict of interest.

Ethical approval

This article does not contain any studies with human participants or animal performed by any of the authors.

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