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Original research article

Average Run Length Formulas for Mixed Double Moving Average - Exponentially Weighted Moving Average Control Chart

Suganya Phantu[1], Yupaporn Areepong[2], Saowanit Sukparungsee[2,*]

[1]*Faulty of Science, Energy and Environment, King's Mongkut University of Technology North Bangkok, Rayong 21120, Thailand*
[2]*Department of Applied Statistics, Faculty of Applied Science, King's Mongkut University of Technology North Bangkok, Bangkok 10800, Thailand*

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ABSTRACT

This research aims to propose an explicit formula for average run length (ARL) for a mixed Double Moving Average - Exponentially Weighted Moving Average (DMA-EWMA) control chart. In addition, the performance of the DMA-EWMA control chart is compared with the existing control charts; Shewhart and Exponentially Weighted Moving Average (EWMA) control charts. The common criterion to measure the efficiency is the average run length, commonly measured by the out of control average run length (ARL_1). The in control average run length (ARL_0) is given to 370, and the process observations are from normal, exponential, gamma, and Laplace distributions. The numerical results found that the DMA-EWMA control chart performs better than other control charts for all magnitudes of changes (λ). Furthermore, the real applications are addressed to validate the proposed control chart.

Keywords: Explicit formulas; Mixed control chart; Mean shifts; Performance

1. Introduction

Quality control is very important in the manufacturing and service industries to develop products and services that have quality standards to satisfy consumers' needs. Otherwise, non-standard products may pose a risk to consumers. Therefore, quality monitoring systems that will reduce the risk of producing non-standard products by using Statistical Process Control (SPC) to control the quality of standard products need to be investigated and studied. Control charts, which which provide statistical quality control for repetitive production process control, are essential tools for analyzing the quality of products in the production process,. The control charts' objective is to set product standards, assist in

***Corresponding author:** saowanit.s@sci.kmutnb.ac.th **doi:** 10.14456/scitechasia.2023.8

implementing plans, and continuously improve quality.

In 1924, Shewhart [1] presented the Shewhart control chart that effectively detects significant mean changes when the magnitude of changes is large $(\delta \geq 1.5)$, but it is insensitive to detecting small changes $(\delta < 1.5)$. Subsequently, in 1959, other quality control charts were developed that were more effective in detecting small changes than the Shewhart control chart. Roberts [2] developed an Exponentially Weighted Moving Average Control Chart (EWMA) by using the principles of applying the entire data all the time observations in the process to be used to make decisions. It was found that the EWMA control chart performed better than the Shewhart control chart when the changing size of the process means was small $(\delta < 1.5)$. In 2004, Khoo [3] proposed the Moving Average control chart (MA), one of the memorized control charts. Next, Khoo and Wong [4] presented a double-moving average control chart developed from the MA chart. Recently, several authors have introduced different designs of the MA and EWMA chart designs.

In 2019, Taboran et al. [5] studied the mixed control chart, namely, the Moving Average - Exponentially Weighted Moving Average control chart (MA-EWMA), as a combination of the Moving Average (MA) and Exponentially Weighted Moving Average (EWMA) control charts in which the statistics belong to the MA and control limits are obtained from the EWMA control chart. In 2021, Wiwek et al. [6] evaluated the explicit formula of the Moving Average (MA) and Exponentially Weighted Moving Average (EWMA) control charts. The result showed that the MA-EWMA chart performs best for moderate shifts in a process.

Generally, the most widely used control chart performance is Average Run Length (ARL) which can be divided into two states: in-control Average Run Length (ARL_0) and out of control Average Run Length (ARL_1). In addition, there are several methods for calculating the Average Run Length. For example, Monte Carlo simulation (MC) is a method for approximating the Average Run Length from a simulation under a determined situation; it is a simple tool to check the accuracy of results obtained from other methods. However, there is a limitation in that it is very time-consuming to process the numerical results. Besides, the explicit formulas method takes less time to calculate but may not be able to find in all cases of studies. Several methods for evaluating ARL_0 and ARL_1 have been reported in the literature. Areepong [7] and Chananet et al. [8] used the explicit formula for the ARL to examine the performance of the MA control chart. Later, much research used the Markov chain approach for the ARL to investigate the performance of the EWMA control chart, see [9-11]. In 2019, Thitisoowaranon et al. [12] applied a mixed CUSUM-tukey control chart for detecting process dispersion. Later, Phantu and Sukparungsee [13] investigated the robustness of χ CUSUM and CP CUSUM control charts using fast initial response in detecting process dispersion.

In this research, the author extended studies from Wiwek et al. [6], which are derivative proof of explicit formula of average run length (ARL) for mixed Double Moving Average - Exponentially Weighted Moving Average control chart (DMA-EWMA). Also, the process observations have normal, exponential, gamma and Laplace distributions and are comparable to the detection performance of control charts with Shewhart, EWMA, and DMA-EWMA control charts.

2. Materials and Methods

2.1 Distributions

This research studied measuring the performance of control charts with average run length divided into four distributions as follows:

2.1.1 Normal distribution

A normal distribution is a continuous probability distribution. The general form of its probability density function is

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\ ;\ -\infty\le x\le\infty. \quad (2.1)$$

The parameter μ is the mean of the distribution. The variance of the distribution is σ^2.

2.1.2 Exponential distribution

The probability density function of an exponential distribution is

$$f(x)=-\lambda e^{-\lambda x}\ ;\ -\infty\le x\le\infty. \quad (2.2)$$

The mean of an exponentially distributed random variable X with rate parameter λ is given by $1/\lambda$. The variance of X is given by $1/\lambda^2$.

2.1.3 Gamma distribution

The probability density function of a gamma distribution is

$$f(x)=\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}\ ;\ -\infty\le x\le\infty. \quad (2.3)$$

The mean of the gamma distribution given by the product of its shape and scale parameters is α/β. The variance is α/β^2.

2.1.4 Laplace distribution

The Laplace distribution is a continuous probability distribution. A random variable has a distribution if its probability density function is

$$f(x)=\frac{1}{2b}e\left(-\frac{|x-\mu|}{b}\right);\ -\infty\le x\le\infty. \quad (2.4)$$

The mean of the distribution is μ. The variance of the distribution is σ^2.

2.2 Control charts

In this research, the author is interested in finding the explicit formulas of the Moving Average - Exponentially Weighted Moving Average control chart (MA-EWMA) and comparing average detection performance. The studied cases under symmetric distribution are the normal distribution, and the cases of asymmetric distribution are Laplace and exponential and gamma distributions. Considering the performance of the control chart is based on the average run length when the production process is out of control Average Run Length (ARL_1). The theory and related research are detailed as follows:

2.2.1 Shewhart control chart

Shewhart [1] categorizes variation into 2 types: variation from a common cause or random cause, which is random variation caused by natural causes. It is common and occurs regularly with every product of the production process. The variation from a particular cause or assignable cause that is not abnormal, unnatural, and occasionally occurs out of control of the manufacturing process. Control charts, which will help prevent and resolve quality issues, were first invented by Shewhart [1] while working for Bell Labs in the 1920s to reduce variation in industrial production. The control limits of the Shewhart control chart are shown in Eq. (5) as follows:

$$UCL/LCL=\bar{\bar{X}}\pm H_1\sigma_{\bar{X}}, \quad (2.5)$$

where $\bar{\bar{X}}$ is the mean group, $\sigma_{\bar{X}}$ is the standard deviation of the data, H_1 is the coefficient of the control limit of the Shewhart control chart equal to 3, corresponding to the $ARL_0 = 370$.

2.2.2 Double Moving Average control chart (DMA)

A Double Moving Average control chart was proposed by Khoo and Wong [4].

The observation of DMA statistics is collected as a double-moving average of MA statistics. The DMA statistic is defined by Eq. (2.6) as follows:

$$DMA_i = \begin{cases} \dfrac{MA_i + MA_{i-1} + MA_{i-2} + ...}{i} & , \ i \le w \\ \dfrac{MA_i + MA_{i-1} + ... + MA_{i-w+1}}{w}, & \text{w}<i < 2w-1 \\ \dfrac{MA_i + MA_{i-1} + ... + MA_{i-w+1}}{w}, & i \ge 2w-1. \end{cases} \tag{2.6}$$

The MA statistic [3] is a weighted moving average control chart with moving average period (w). The statistical value of the MA control chart covering the value of w and time i can be calculated as in Eq. (2.7)

$$MA_i = \begin{cases} \dfrac{X_i + X_{i-1} + X_{i-2} + ...}{i}, & i < w \\ \dfrac{X_i + X_{i-1} + ... + X_{i-w+1}}{w}, & i \ge w, \end{cases} \tag{2.7}$$

where X_i is an observed value at a time i, w is the width of the moving average, MA_i is moving average value at time i.

The control limits of Double Moving Average control chart (DMA) as in Eq. (2.8)

$$UCL/LCL = \begin{cases} \mu_0 \pm H_2\sqrt{\dfrac{\mu_0}{i^2}\displaystyle\sum_{j=1}^{i}\dfrac{1}{j}} & , \ i \le w \\ \mu_0 \pm H_2\sqrt{\dfrac{\mu_0}{w^2}\displaystyle\sum_{j=1}^{i}\dfrac{1}{j} + (\text{i}-\text{w}+1)\left(\dfrac{1}{w}\right)} & , \ \text{w}<i < 2w-1 \\ \mu_0 \pm H_2\sqrt{\dfrac{\mu_0}{w^2}} & , \ i \ge 2w-1 \end{cases} \tag{2.8}$$

where H_2 is defined as the coefficient of the control limits of DMA equal to 3 for corresponding to the $ARL_0 = 370$.

2.2.3 Exponentially Weighted Moving Average control chart (EWMA)

The EWMA control chart, invented by Roberts [2], is a control chart that can quickly detect changes of average in the process. The statistics of the EWMA control chart can be calculated as in Eq. (2.9)

$$Z_i = \lambda X_i + (1-\lambda)Z_{i-1}, \ i = 1, \ 2, \ 3, ... \tag{2.9}$$

where Z_i is the value of the EWMA statistic at the time i, and determine $Z_0 = \mu_0$. X_i is an observed value at i. λ is a weighted factor of EWMA; $0 < \lambda \le 1$.

The control limits of the Exponentially Weighted Moving Average control chart (EWMA) can be calculated as in Eq. (2.10)

$$UCL/LCL = \mu_0 \pm H_3\sqrt{\sigma^2\left(\dfrac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]}, \tag{2.10}$$

where H_3 is a coefficient of the control limit of EWMA control chart.

2.2.4 Double Moving Average-Exponentially Weighted Moving Average control chart (DMA-EWMA)

The DMA-EWMA control chart combines the DMA and the EWMA control charts. The statistical value of the DMA control chart, as in Eq. (2.6), and control chart is the use of the EWMA control chart where the control limit of the MA-EWMA control chart, as in Eq. (2.11)

$$UCL/LCL = \begin{cases} \mu_0 \pm H_4\sqrt{\dfrac{\mu_0}{i^2}\displaystyle\sum_{j=1}^{i}\dfrac{1}{j}\left(\dfrac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]} & , \ i \le w \\ \mu_0 \pm H_4\sqrt{\dfrac{\mu_0}{w^2}\displaystyle\sum_{j=1}^{i}\dfrac{1}{j} + (\text{i}-\text{w}+1)\left(\dfrac{1}{w}\right)\left(\dfrac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]}, & \text{w}<i < 2w-1 \\ \mu_0 \pm H_4\sqrt{\dfrac{\mu_0}{w^2}\left(\dfrac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]} & , \ i \ge 2w-1, \end{cases} \tag{2.11}$$

where H_4 is a coefficient of the control limit DMA-EWMA control chart.

2.3 Average Run Length (ARL)

Average Run Length (ARL) is the expected number of observations taken from an in-control process until the control charts falsely signal out-of-control, which is described by two states. When the process is in the control state denoted by ARL_0, it can be calculated as in Eq. (2.8) and when the process is out of control state characterized by ARL_1 it can be calculated as in Eq. (2.9), respectively

$$ARL_0 = \frac{1}{\alpha}, \tag{2.12}$$

where α is the probability that a process is found to be out of the control limits. When the process does not change, then

$$ARL_1 = \frac{1}{1-\beta}, \tag{2.13}$$

where β is the probability that a process is found to be in a control state when the process changes. This research studies the method of calculating the average run length by Areepong [7]. ARL of the Double Moving Average - Exponentially Weighted Moving Average control chart (DMA-EWMA) can be proved as follows.

Defined as $ARL = n$

$$
\begin{aligned}
\frac{1}{ARL} &\cong \frac{1}{n}P(\text{out of control signal at time } i \le w) \\
&+\frac{1}{n}P(\text{out of control signal at time } w < i < 2w-1) \\
&+\frac{n-(2w-2)}{n}P(\text{out of control signal at time } i \ge 2w-1) \\
&\cong \frac{1}{n}\sum_{i=1}^{w}\left(P\left(MA_i > UCL_{i\le w}\right)+P\left(MA_i < LCL_{i\le w}\right)\right) \\
&+\frac{1}{n}\sum_{j=i-w+1}^{2w-2}\left(P\left(MA_j > UCL_{w<i<2w-1}\right)+P\left(MA_j < LCL_{w<i<2w-1}\right)\right) \\
&+\frac{n-(2w-2)}{n}\left(P\left(MA_i > UCL_{i\ge 2w-1}\right)+P\left(MA_i < LCL_{i\ge 2w-1}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
&\cong \frac{1}{n}\sum_{i=1}^{w}\left(P\left(\frac{\sum_{j=1}^{i}MA_j}{i} > \mu_0 + H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}\right)\right. \\
&\left.+P\left(\frac{\sum_{j=1}^{i}MA_j}{i} < \mu_0 - H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}\right)\right) \\
&+\frac{1}{n}\sum_{j=i-w+1}^{2w-2}\left(P\left(\frac{\sum_{j=i-w+1}^{i}MA_j}{w} > \mu_0 + H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}\right)\right. \\
&\left.+P\left(\frac{\sum_{j=i-w+1}^{i}MA_j}{w} < \mu_0 - H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}\right)\right) \\
&+\frac{n-(2w-2)}{n}\left(P\left(\frac{\sum_{j=i-w+1}^{i}MA_j}{w} > \mu_0 + H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}\right)\right. \\
&\left.+P\left(\frac{\sum_{j=i-w+1}^{i}MA_j}{w} > \mu_0 - H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
&\cong \frac{1}{n}\sum_{i=1}^{w}P\left(\left(Z > \frac{\mu_0 + H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}}\right)\right. \\
&\left.+P\left(Z < \frac{\mu_0 - H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}}\right)\right) \\
&+\frac{1}{n}\sum_{j=w+1}^{2w-2}P\left(\left(Z > \frac{\mu_0 + H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\right. \\
&\left.+P\left(Z < \frac{\mu_0 - H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\right) \\
&+\frac{n-(2w-2)}{n}\left(P\left(Z > \frac{\mu_0 + H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}}\right)\right. \\
&\left.+P\left(Z < \frac{\mu_0 - H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}}\right)\right),
\end{aligned}
\tag{2.14}
$$

where μ_0 is an in-control process mean, μ_1 is an out-of-control process mean, σ^2 is a process variance, H_4 is a coefficient of the control limit, w is the width of the moving average, λ is a weighting factor, $0<\lambda\leq 1$.
From Eq. (2.13), given *A*, *B* and *C* are as follows

$$A=\sum_{i=1}^{w}\left(P\left(Z>\frac{\mu_0+H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}}\right)\right)$$

$$+P\left(Z<\frac{\mu_0-H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}}\right)$$

$$B=\sum_{j=i-w+1}^{2w-2}\left(P\left(Z>\frac{\mu_0+H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\right)$$

$$+P\left(Z<\frac{\mu_0-H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)$$

$$C=P\left(Z>\frac{\mu_0+H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}}\right)$$

$$+P\left(Z<\frac{\mu_0-H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}}\right),$$

when substituting *A*, *B,* and *C* into Eq. (2.14)

$$\frac{1}{n}\cong\frac{1}{n}(A)+\frac{1}{n}(B)+\frac{n-(2w-2)}{n}(C)$$

$$n\cong\frac{(1-A-B)}{C}+(2w-2).$$

Therefore, explicit formulas of ARL can be calculated as in Eq. (2.15)

$$ARL\cong[(1-A)-B]C^{-1}+(2w-2)$$

then,

$$ARL\cong\left\{1-\sum_{i=1}^{w}\left(P\left(Z>\frac{\mu_0+H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}}\right)\right.\right.$$

$$\left.\left.+P\left(Z<\frac{\mu_0-H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}}\right)\right)\right.$$

$$-\sum_{j=i-w+1}^{2w-2}\left(P\left(Z>\frac{\mu_0+H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\right)$$

$$\left.+P\left(Z<\frac{\mu_0-H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\right\}$$

$$\times\left\{\begin{array}{l}P\left(Z>\frac{\mu_0+H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}}\right)\\+P\left(Z<\frac{\mu_0-H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}-\mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}}\right)\end{array}\right\}^{-1}+(2w-2), \quad (2.15)$$

when the process is in a control state, defined as $\mu=\mu_0$. and when the process is out of a control state, defined as $\mu=\mu_1$.

3. Numerical Results

This research studies the evaluation method of average run length (ARL) via explicit formulas for a mixed Double Moving Average – Exponentially Weighted Moving Average control chart (DMA-EWMA). To compare the performance of control charts detection, the DMA-EWMA control chart is compared with Shewhart, Double Moving Average (DMA), and Exponentially Weighted Moving Average (EWMA), measured by the out-of-control average run length (ARL_1). The numerical results can be calculated using Mathematica® software. The in-control average run length (ARL_0) is given as 370. Weighting factors (λ) of the DMA-EWMA control charts were 0.05, 0.2, 0.25, and the width of the moving average (w) was 2, 3, 5, 10, 15, and 20 for DMA and DMA-EWMA control charts. The number of repetitions of the Shewhart and EWMA chart is 5,000 iterations, and the sample size is one group. The changing sizes of the process (δ) were 0.02, 0.04, 0.06, 0.08, 0.10, 0.15, 0.20, and 0.25, as shown in Table 1.

Table 1. ARL_1 of Shewhart, EWMA and DMA-EWMA control charts when the data are from normal (1, 1).

δ	Shewhart $H_1=3$	EWMA	DMA-EWMA $H_4=3$
		$\lambda = 0.05$	
		$H_3=1.018$	
0.02	308.426	209.875	*58.218*
0.04	200.075	167.334	*32.271*
0.06	119.665	128.976	*23.052*
0.08	71.552	101.877	*18.366*
0.10	43.895	82.573	*16.642*
0.15	14.968	56.638	*10.137*
0.20	*6.303*	42.822	7.650
0.25	*3.241*	34.347	6.587
		$\lambda = 0.2$	
		$H_3=1.264$	
0.02	308.426	291.337	*58.218*
0.04	200.075	233.187	*32.271*
0.06	119.665	189.736	*23.050*
0.08	71.552	156.773	*18.366*
0.10	43.895	131.400	*16.642*
0.15	14.968	89.326	*10.137*
0.20	*6.303*	64.901	7.650
0.25	*3.241*	49.704	6.587
		$\lambda = 0.25$	
		$H_3=1.610$	
0.02	308.426	298.570	*67.510*
0.04	200.075	243.628	*35.806*
0.06	119.665	201.152	*26.748*
0.08	71.552	167.969	*19.923*
0.10	43.895	141.780	*17.498*
0.15	14.968	96.943	*12.128*
0.20	*6.303*	70.053	8.488
0.25	*3.241*	53.043	7.148

Note: Italics number represents the lowest ARL_1.

From Table 1, a comparison ARL_1 when normal distribution (1, 1) is given $ARL_0 = 370$, $\lambda = 0.05$, $\lambda = 0.2$ and $\lambda = 0.25$. The numerical results found that the DMA-EWMA control chart has the best performance in detecting parameter changes at $\delta = 0.02, 0.04, 0.06, 0.08, 0.10$, and 0.15, but when the size changes more, the Shewhart control chart has the best performance in detecting parameter changes, as shown in Figs. 1-3.

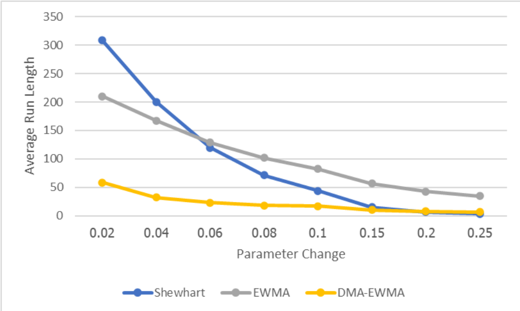

Fig. 1. Comparison of ARL_1 of DMA-EWMA, Shewhart, and EWMA for the case of normal (1, 1) and $\lambda = 0.05$.

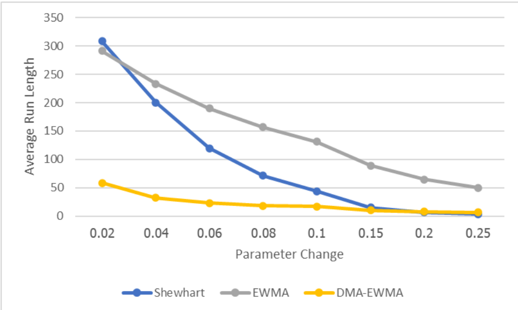

Fig. 2. Comparison of ARL_1 of DMA-EWMA, Shewhart, and EWMA for the case of normal (1, 1) and $\lambda = 0.2$.

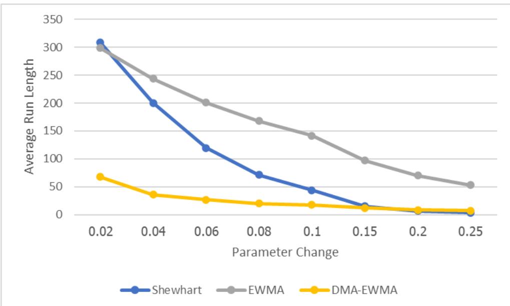

Fig. 3. Comparison of ARL_1 of DMA-EWMA, Shewhart, and EWMA for the case of normal (1, 1) and $\lambda = 0.25$.

From Table 2, a comparison ARL_1 when the data has an exponential (1) distribution, defined as $\lambda = 0.05$, $\lambda = 0.2$ and $\lambda = 0.25$ the numerical results, found that the performance of the DMA-EWMA control chart performs better than other control charts for all magnitudes of λ, as shown in Figs. 4-6.

Table 2. ARL_1 of Shewhart, EWMA, and DMA-EWMA control charts when the data are from exponential (1).

δ	Shewhart $H_1=3$	EWMA $H_3=1.143$	DMA-EWMA $H_4=3$
		$\lambda = 0.05$	
0.02	369.670	230.297	*31.393*
0.04	367.500	159.125	*17.982*
0.06	363.935	117.972	*13.328*
0.08	359.046	82.386	*8.982*
0.10	352.931	65.885	*7.433*
0.15	333.076	42.357	*4.911*
0.20	308.426	30.167	*3.616*
0.25	281.153	22.803	*2.965*
		$\lambda = 0.2$	
δ	Shewhart $H_1=3$	EWMA $H_3=1.223$	DMA-EWMA $H_4=3$
0.02	369.670	287.630	*58.218*
0.04	367.500	226.617	*32.271*
0.06	363.935	181.087	*23.052*
0.08	359.046	146.684	*18.366*
0.10	352.931	120.362	*16.642*
0.15	333.076	77.291	*10.137*
0.20	308.426	52.877	*7.650*
0.25	281.153	38.055	*6.587*
		$\lambda = 0.25$	
δ	Shewhart $H_1=3$	EWMA $H_3=1.321$	DMA-EWMA $H_4=3$
0.02	369.670	293.998	*67.510*
0.04	367.500	235.842	*35.806*
0.06	363.935	191.032	*26.748*
0.08	359.046	156.227	*19.923*
0.10	352.931	128.972	*17.498*
0.15	333.076	83.078	*12.128*
0.20	308.426	56.363	*8.488*
0.25	281.153	39.998	*7.148*

Note: Italics number represents the lowest ARL_1.

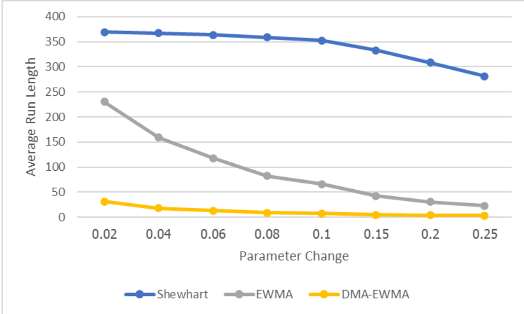

Fig. 4. Comparison of ARL_1 of DMA-EWMA, Shewhart, and EWMA for the case of exponential (1) and $\lambda = 0.05$.

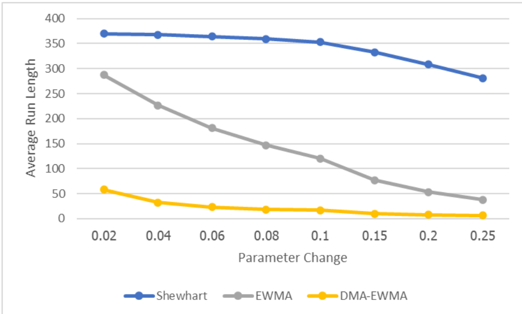

Fig. 5. Comparison of ARL_1 of DMA-EWMA, Shewhart, and EWMA for the case of exponential (1) and $\lambda = 0.2$.

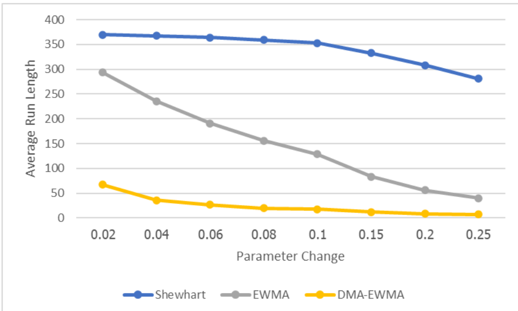

Fig. 6. Comparison of ARL_1 of DMA-EWMA, Shewhart, and EWMA for the case of exponential (1) and $\lambda = 0.25$.

Table 3 compares ARL_1 for gamma distribution (4, 1), when $\lambda = 0.05$, $\lambda = 0.2$ and $\lambda = 0.25$. The numerical results found that the performance of the DMA-EWMA control chart performs better than other control charts for all magnitudes of δ, as shown in Figs. 7-9.

Table 3. ARL_1 of Shewhart, EWMA and DMA-EWMA control charts when the data are from gamma (4,1)

δ	Shewhart H_1=3	EWMA H_3=1.182	DMA-EWMA H_4=3
	$\lambda = 0.05$		
0.02	367.500	366.153	*67.510*
0.04	359.046	362.261	*35.806*
0.06	345.706	358.456	*26.748*
0.08	328.459	354.736	*19.923*
0.10	308.426	351.096	*17.498*
0.15	253.139	342.335	*12.128*
0.20	200.075	334.025	*8.488*
0.25	155.224	326.129	*7.148*
	$\lambda = 0.2$		
δ	Shewhart H_1=3	EWMA H_3=1.387	DMA-EWMA H_4=3
0.02	367.500	337.463	*55.742*
0.04	359.046	308.839	*31.393*
0.06	345.706	283.832	*22.131*
0.08	328.459	261.902	*17.982*
0.10	308.426	242.597	*16.405*
0.15	253.139	203.493	*9.650*
0.20	200.075	174.171	*7.433*
0.25	155.224	151.716	*6.414*
	$\lambda = 0.25$		
δ	Shewhart H_1=3	EWMA H_3=2.576	DMA-EWMA H_4=3
0.02	367.500	338.284	*141.047*
0.04	359.046	310.132	*58.218*
0.06	345.706	285.169	*41.641*
0.08	328.459	262.978	*32.271*
0.10	308.426	243.203	*28.502*
0.15	253.139	202.430	*19.092*
0.20	200.075	171.225	*16.642*
0.25	155.224	146.994	*13.310*

Note: Italics number represents the lowest ARL_1.

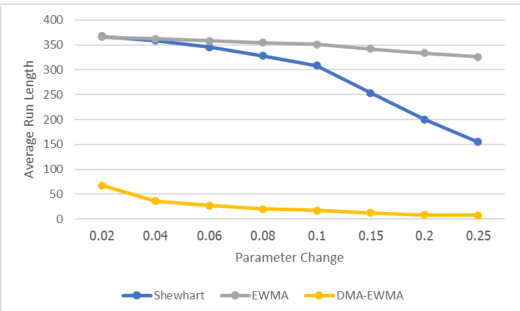

Fig. 7. Comparison of ARL_1 of DMA, DMA-EWMA, Shewhart, and EWMA for the case of gamma (4,1) and $\lambda = 0.05$.

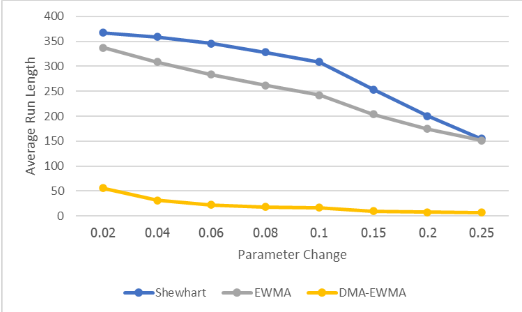

Fig. 8. Comparison of ARL_1 of DMA-EWMA, Shewhart, and EWMA for the case of gamma (4,1) and $\lambda = 0.2$.

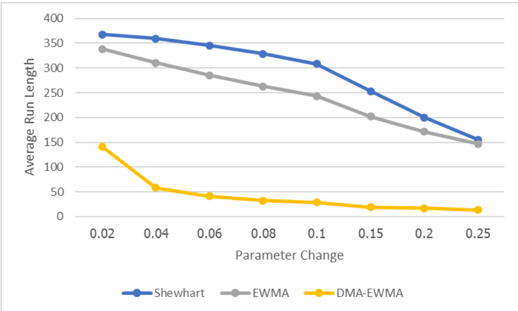

Fig. 9. Comparison of ARL_1 of DMA-EWMA, Shewhart, and EWMA for the case of gamma (4,1) and $\lambda = 0.25$.

From Table 4, a comparison of ARL_1 when Laplace distribution (1, 2) is defined as $ARL_0 = 370$, $\lambda = 0.05$, $\lambda = 0.2$ and $\lambda = 0.25$. The numerical results found that the DMA-EWMA control chart has the best performance in detecting parameter changes at δ = 0.02, 0.04, 0.06, and 0.08, but when the size changes more, the Shewhart control chart has the best performance in detecting parameter changes, as shown in Figs. 10-12.

Table 4. ARL_1 of Shewhart, EWMA and DMA-EWMA control charts when the data are from Laplace (1, 2).

δ	Shewhart $H_1=3$	EWMA	DMA-EWMA $H_4=3$
		$\lambda = 0.05$	
		$H_3=1.053$	
0.02	247.431	218.432	*42.073*
0.04	58.749	201.376	*23.526*
0.06	23.765	151.348	*17.487*
0.08	15.530	129.865	*14.790*
0.10	*9.960*	109.536	10.389
0.15	*6.245*	76.871	7.141
0.20	*4.110*	58.517	5.330
0.25	*3.110*	46.939	4.063
		$\lambda = 0.2$	
		$H_3=1.373$	
0.02	247.431	314.722	*90.006*
0.04	58.749	269.347	*43.073*
0.06	23.765	232.163	*31.014*
0.08	15.530	201.506	*24.621*
0.10	*9.960*	176.081	19.890
0.15	*6.245*	129.240	16.297
0.20	*4.110*	98.398	10.974
0.25	*3.110*	77.378	8.471
		$\lambda = 0.25$	
		$H_3=1.411$	
0.02	361.475	323.569	*106.442*
0.04	301.971	283.592	*47.076*
0.06	223.913	249.315	*33.990*
0.08	157.253	219.934	*28.528*
0.10	90.646	194.727	*22.162*
0.15	38.089	146.072	*17.070*
0.20	17.731	112.271	*13.360*
0.25	*9.180*	88.352	9.9667

Note: Italics number represents the lowest ARL_1.

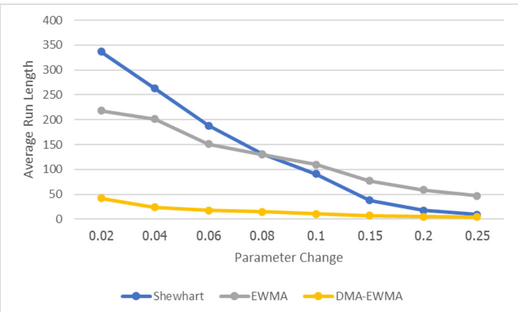

Fig. 10. Comparison of ARL_1 of DMA-EWMA, Shewhart, and EWMA for the case of Laplace (1,2) and $\lambda = 0.05$.

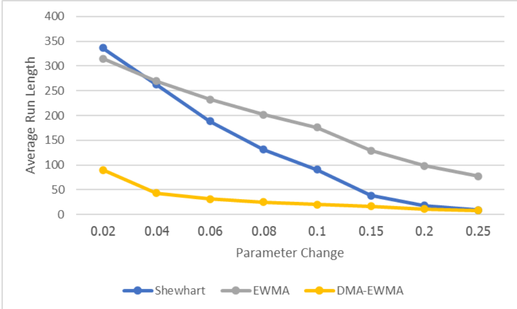

Fig. 11. Comparison of ARL_1 of DMA-EWMA, Shewhart, and EWMA for the case of Laplace (1,2) and $\lambda = 0.2$.

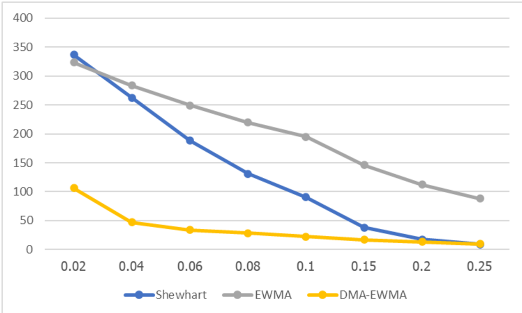

Fig. 12. Comparison of ARL_1 of DMA-EWMA, Shewhart, and EWMA for the case of Laplace (1,2) and $\lambda = 0.25$.

4. Real Application

In this section, two numerical examples of PM 2.5 and PM 10 are considered to illustrate the application of the Shewhart, EWMA, and DMA-EWMA control charts. The PM 2.5 consists of 61 observations from Jan/2015 to Jan/2020. The performance of the control chart for PM 2.5 is displayed in Figs. 13-15. The results show that the DMA-EWMA chart is most effective at detecting the change in PM 2.5. The data for the second example is related to the number of cases of PM 10, totaling 61 observations, from Jan/ 2015 to Jan/ 2020. Figs. 16-18 shows the performance of the control chart. The results show that the DMA-EWMA chart best detects the change in PM 10.

The data set of PM 2.5 is 16, 17, 18, 20, 18, 18, 17, 16, 21, 28, 27, 24, 23, 22, 26, 25, 26, 26, 21, 22, 21, 23, 27, 29, 22, 24, 24, 23, 23, 25, 27, 26, 26, 30, 33, 38, 34, 30, 33, 30, 30 31, 31, 25, 28, 30, 32, 28, 26, 21, 17, 22, 26, 29, 30, 33, 28, 21, 19, 25, 28.

The data set of PM 10 is 17, 16, 19, 33, 32, 27, 24, 29, 38, 38, 46, 27, 34, 50, 39, 39, 41, 44, 28, 30, 40, 35, 33, 39, 41, 29, 24, 20, 31, 31, 35, 34, 36, 30, 38, 51, 47, 48, 43, 39, 29, 50, 38, 42, 36, 26, 39, 44, 44, 56, 49, 59, 49, 59, 59, 55, 70, 53, 55, 34, 49, 31, 30.

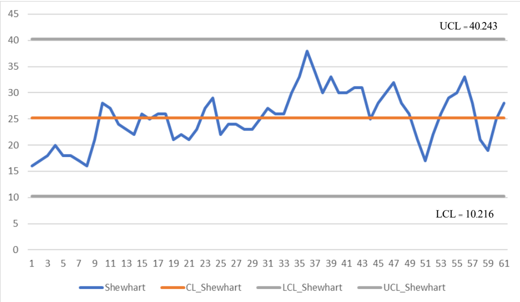

Fig. 13. Performance of Shewhart control chart for PM 2.5.

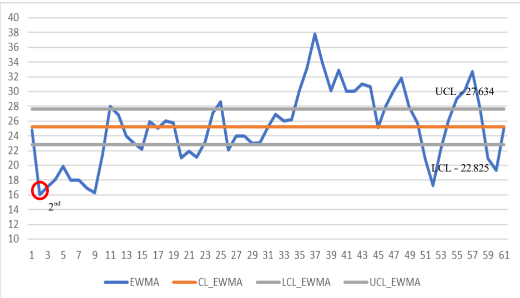

Fig. 14. Performance of EWMA control chart for PM 2.5.

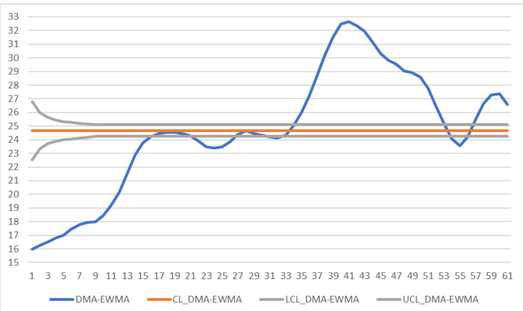

Fig. 15. Performance of DMA-EWMA control chart for PM 2.5.

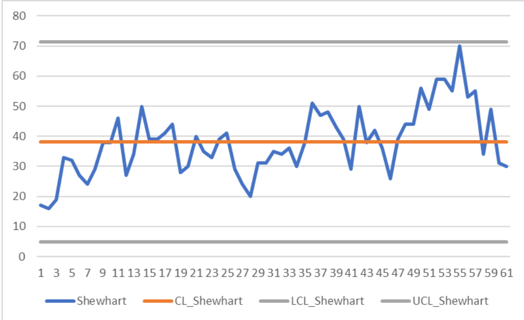

Fig. 16. Performance of Shewhart control chart for PM 10.

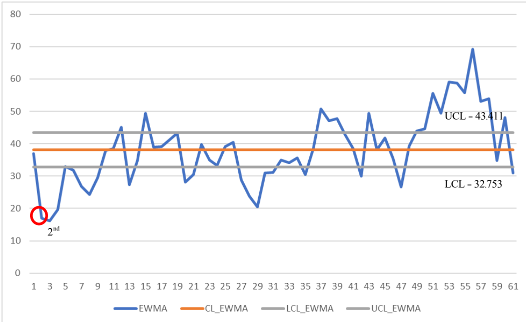

Fig. 17. Performance of EWMA control chart for PM 10.

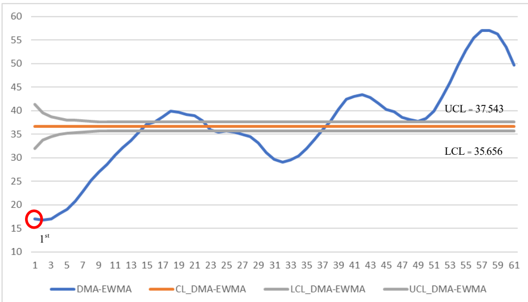

Fig. 18. Performance of DMA-EWMA control chart for PM 10.

5. Conclusion and Discussion

5.1 Conclusion

Explicit formulas of ARL_0 and ARL_1 of the Double Moving Average-Exponentially Weighted Moving Average (DMA-EWMA) control chart are presented as shown in Eqs. (16) and (17), respectively.

$$
\begin{aligned}
ARL_0 \cong \Bigg\{ 1-\sum_{i=1}^{w} \Bigg(& P\left(Z > \frac{\mu_0 + H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}} - \mu_0}{\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}} \right) \\
& + P\left(Z < \frac{\mu_0 - H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}} - \mu_0}{\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}} \right) \Bigg) \\
- \sum_{j=i-w+1}^{2w-2} \Bigg(& P\left(Z > \frac{\mu_0 + H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j} + (j-w+1)\left(\frac{1}{w}\right)} - \mu_0}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j} + (j-w+1)\left(\frac{1}{w}\right)}} \right) \\
& + P\left(Z < \frac{\mu_0 - H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j} + (j-w+1)\left(\frac{1}{w}\right)} - \mu_0}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j} + (j-w+1)\left(\frac{1}{w}\right)}} \right) \Bigg) \Bigg\} \\
\times \Bigg\{ & P\left(Z > \frac{\mu_0 + H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)} - \mu_0}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}} \right) + P\left(Z < \frac{\mu_0 - H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)} - \mu_0}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}} \right) \Bigg\}^{-1} + (2w-2),
\end{aligned}
\tag{2.16}
$$

and

$$
\begin{aligned}
ARL_1 \cong \Bigg\{ 1-\sum_{i=1}^{w} \Bigg(& P\left(Z > \frac{\mu_0 + H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}} - \mu_1}{\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}} \right) \\
& + P\left(Z < \frac{\mu_0 - H_4\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}} - \mu_1}{\sqrt{\left(\frac{\sigma^2}{i^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=1}^{i}\frac{1}{j}}} \right) \Bigg) \\
- \sum_{j=i-w+1}^{2w-2} \Bigg(& P\left(Z > \frac{\mu_0 + H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j} + (j-w+1)\left(\frac{1}{w}\right)} - \mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j} + (j-w+1)\left(\frac{1}{w}\right)}} \right) \\
& + P\left(Z < \frac{\mu_0 - H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j} + (j-w+1)\left(\frac{1}{w}\right)} - \mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\sum_{j=i-w+1}^{w-1}\frac{1}{j} + (j-w+1)\left(\frac{1}{w}\right)}} \right) \Bigg) \Bigg\} \\
\times \Bigg\{ & P\left(Z > \frac{\mu_0 + H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)} - \mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}} \right) + P\left(Z < \frac{\mu_0 - H_4\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)} - \mu_1}{\sqrt{\left(\frac{\sigma^2}{w^2}\right)\left(\frac{\lambda}{2-\lambda}\right)}} \right) \Bigg\}^{-1} + (2w-2),
\end{aligned}
\tag{2.17}
$$

where K is a coefficient of the control limit DMA-EWMA control chart.

The results of the research comparing the efficiency of the control chart showed that the DMA-EWMA control chart has the best performance in detecting parameter changes for normal distribution (1, 1), exponential distribution (1), gamma distribution (4, 1), and Laplace distribution (1, 2) for all magnitudes of change.

5.2 Discussion

From the results, the DMA-EWMA control chart has the best performance in detecting parameter changes for normal distribution (1, 1), exponential distribution (1), gamma distribution (4, 1), and Laplace distribution (1, 2) for all size changes. The DMA-EWMA is suitable for data with minor changes. In future research, the appropriate parameter of the DMA-EWMA chart can be obtained to make it convenient for the actual application.

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