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Original research article

A Dynamical and Sensitivity Analysis of the Caputo Fractional-Order Ebola Virus Model: Implications for Control Measures

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ABSTRACT

The recurrence of outbreaks in cases of Ebola virus among African countries remains one of the greatest issues of concern. Practices such as hunting or consumption of contaminated bush meat, unsafe funeral practices, and environmental contamination have all been implicated as possible contributors. This paper investigates the transmission dynamics of the Ebola virus model in the setting of a Caputo fractional-order derivative that accounts for both direct and indirect transmissions of the virus. By employing the concept of fixed theorems, we derived the existence and uniqueness results of the model. Moreover, we analyzed the forward normalized sensitivity indices to identify the critical parameters for controlling the infection and found that reducing the contact rate between infected individuals and susceptible vectors is vital to limiting the virus's spread. Comparing the proposed fractional-order model with those of the previously developed integer-order model numerically, we found that the proposed model provides more reliable information on the model's dynamics. Thus, we conclude that the Caputo fractional-order operator is a precise tool for describing the proposed model behavior and can help understand the complexities of Ebola virus disease outbreaks.

Keywords: Ebola virus; Fixed point theorems; Mathematical model; Numerical simulations; Sensitivity analysis

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1. Introduction

Ebola virus disease, one of the deadliest viral diseases, is markedly characterized by hemorrhagic fever [1]. The virus was first discovered in 1976 in the Democratic Republic of the Congo, and secondly within the same year in Sudan, some miles away from the vicinity of the first case [2] (Fig. 1). The name 'Ebola' originated from the name of a river near the village of Yambku, where the virus was first discovered. From the date of discovery to March 25, 2020, the virus has accounted for the deaths of an estimated 2,267 people, and an outbreak was reported in about 38 predominantly African countries [3].

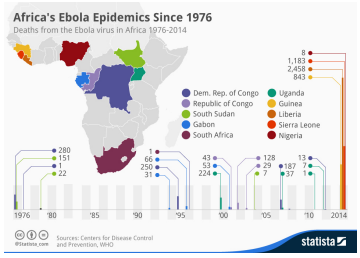

Fig. 1. Heaths from the Ebola virus in Africa from 1976 to 2014.

The Ebola virus disease remains an important health problem that poses a threat to public health. First is the high case-fatality rate associated with the disease, with an average case fatality rate of about 90 being reported in African species of the virus [4] (Fig. 2). Moreover, the absence of definitive treatment as a cure or a defined vaccine makes only supportive treatment the viable treatment option [5–8]. Lastly, there is the high transmissibility of the virus via direct contact with a patient or the patient's bodily fluids of an infected individual, thereby prompting quarantine measures with any form of contact with a confirmed infected individual [9] (Fig. 3).

The recurrence of outbreaks in cases

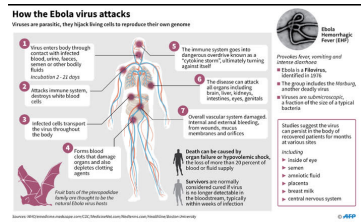

Fig. 2. A fact file on how the Ebola virus attacks. Picture: AFP.

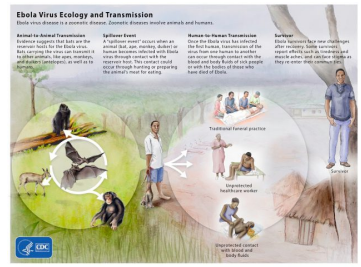

Fig. 3. Ebola-transmission-medium.

of Ebola virus among African countries remains one of the greatest issues of concern. For instance, the most recent was on September 20, 2022, when an outbreak was reported in Uganda after a suspected case was confirmed at the Uganda Virus Research Institute (UVRI). The reasons behind the recurrence and resurgence of the Ebola outbreak are not far-fetched. Practices such as hunting or consumption of contaminated bush meat, unsafe funeral practices, and environmental contamination have all been implicated as possible contributors [10–15].

A comprehensive understanding of the nature of a pandemic is essential for effectively limiting its spread and reducing infections. To gain insights into the transmission dynamics and potentially work towards eradicating the disease, numerous researchers have developed mathematical models for the spread of infectious diseases [16–18]. These models provide valuable tools for studying and analyzing the dy-

namics of transmission, predicting future trends, evaluating the impact of interventions, and developing strategies for control and prevention. By utilizing mathematical models, researchers can gain valuable insights into the mechanisms of disease transmission and identify effective measures to mitigate its impact on public health. Most commonly, mathematical models for the spread of infectious diseases are based on classical differential equations. However, fractional-order differential equations have gained prominence and outperformed standard mathematical modeling approaches. The use of fractional-order differential equations has become significant due to their wide applications across various fields such as science, engineering, finance, and epidemiology [19–22]. Recent research has demonstrated the effectiveness of fractional-order modeling in capturing complex dynamics and providing more accurate descriptions of real-world phenomena [23–31] and references cited therein. These advancements highlight the potential of fractional-order modeling to enhance our understanding of epidemiological processes and improve the accuracy of disease transmission predictions.

The paper is organized into seven sections. Section 2 presents the preliminary concepts of Caputo fractional-order derivatives. Sections 3 and 4 provide the development of the Caputo fractional-order mathematical model and analysis of the basic properties of the model, including the existence and uniqueness of solutions, positivity, and boundedness. The sensitivity analysis in relation to the basic reproduction number of the model is discussed in sections 5. In section 6, a dynamically consistent numerical scheme is utilized, and numerical simulations are presented to support the theory. Finally, in section 7, the concluding remarks on how their findings relate to existing literature and potential extensions of the model were provided.

2. Some Basic Background of Caputo Fractional-Order Derivatives

The main aim of this section is to recall some basic background and notions of the Caputo fractional-order derivative, which are key for the theoretical analysis.

Definition 2.1 ([20])**.** The fractional operator defined by

$$I_0^r f(t) = \frac{1}{\Gamma(r)} \int_0^t f(t)(t-z)^{r-1} dz, \quad (2.1)$$

is called the Riemann-Liouville fractional integral of order $r(0 < r < 1)$ of the function $f \in L^1[0,T]$ for $0 < t < T$. Moreover,

$$\Gamma(r) = \int_0^\infty z^{r-1} e^{-z} dz, r \in \mathbb{C}/\{0, -1, -2, \ldots\},$$

is the gamma function.

Definition 2.2 ([20])**.** Suppose the function $f \in C^n[0,T]$, $n \in \mathbb{N}$ and $t > 0$. The fractional operator

$${}^C D_0^r f(t) = \frac{1}{\Gamma(1-r)} \int_0^t \frac{1}{(t-z)^r} \frac{d}{dt} f(z) dz, \quad (2.2)$$

is referred to the Caputo fractional derivative of order $r(0 < r < 1)$. Note that if $r \to 1$ then ${}^C D_0^r f(t) = \frac{d}{dt} f(t)$.

Lemma 2.3 ([20])**.** *Suppose* $f \in C([0,T], \mathbb{R})$ *and any* $u \in C^1[0,T]$. *Then,* $u(t)$ *is a solution of:*

$$\begin{cases} {}^C D_0^r u(t) = f(t), \ t \in [0,T], \ 0 < r \leq 1, \\ u(0) = u_0, \end{cases}$$

if and only if $u(t)$ *satisfies the integral equation:*

$$u(t) = u_0 - \frac{1}{\Gamma(r)} \int_0^t f(t)(t-z)^{r-1} dz.$$

3. Description of the Ebola Virus Model

In the context of Caputo fractional derivatives, we investigate the dynamic transmission of the Ebola virus model proposed in [32]. The proposed model is based on the following setting:

$$
\begin{aligned}
{}^{C}D^{r}S &= \pi - (\beta_1 I + \beta_2 D + \lambda P)S - \mu S,\\
{}^{C}D^{r}I &= (\beta_1 I + \beta_2 D + \lambda P)S - (\mu + \delta + \gamma)I,\\
{}^{C}D^{r}R &= \gamma I - \mu R,\\
{}^{C}D^{r}D &= (\mu + \delta)I - bD,\\
{}^{C}D^{r}P &= \sigma + \xi I + \alpha D - \eta P.
\end{aligned}
\tag{3.1}
$$

with the initial conditions

$$
S(0) = S_0 \geq 0,\ I(0) = I_0 \geq 0,\ R(0) = R_0 \geq 0,\\
D(0) = D_0 \geq 0,\ \text{and}\ V(0) = V_0 \geq 0.
\tag{3.2}
$$

Tables 1-2 provides the meaning of each of the state variables as well as the parameters.

Table 1. States variables.

Compartment	Description
S	Susceptible human population
I	Infected human population with Ebola
R	Recovered human population
D	Infected and deceased human population
P	Ebola virus pathogens

4. Theoretical Analysis of the Ebola Virus Model

This section presents the existence and uniqueness of solutions to model (3.1) via the techniques of fixed point theorems.

Table 2. Meaning of the parameters.

Parameters	Biological Meanings
π	Recruitment rate of susceptible individuals
β_1	Infectious individuals effective contact rate
β_2	Deceased individuals effective contact
λ	Ebola virus effective contact rate
μ	Natural death rate
δ	Disease-induced death rate
γ	Recovered rate
$1/b$	Deceased individuals mean caring duration
σ	Recruitment rate of Ebola virus in the environment
ξ	Shedding rate of infectious individuals
α	Shedding rate of deceased individuals
η	Decay rate of Ebola virus in the environment

4.1 Existence and uniqueness result

In this subsection, by making use of the fixed point theory, the existence and uniqueness of solution for model (3.1) were investigated. Let denote $\mathbb{B}(J)$ the Banach space of all continuous real-valued function defined on $J = [0, T]$ with sub norm and $\mathbb{Q} = \mathbb{B}(J) \times \mathbb{B}(J) \times \mathbb{B}(J) \times \mathbb{B}(J) \times \mathbb{B}(J)$ with the norm $\|(S, I, R, D, P)\| = \|S\| + \|I\| + \|R\| + \|D\| + \|P\|$, $\|S\| = \sup_{t\in J} |S(t)|$, $\|I\| = \sup_{t\in J} |I(t)|$, $\|R\| = \sup_{t\in J} |R(t)|$, $\|D\| = \sup_{t\in J} |D(t)|$, $\|P\| = \sup_{t\in J} |P(t)|$. Applying the Caputo operator to the Ebola virus model (3.1) gives

$$
S(t) - S(0) = {}^{C}D^{r}\left[\pi - (\beta_1 I + \beta_2 D + \lambda P)S - \mu S\right],
$$

$$
I(t) - I(0) = {}^{C}D^{r}\left[(\beta_1 I + \beta_2 D + \lambda P)S - (\mu + \delta + \gamma)I\right],
$$

$$
\begin{aligned}
R(t) - R(0) &= {}^{C}D^{r}\left[\gamma I - \mu R\right], \\
D(t) - D(0) &= {}^{C}D^{r}\left[(\mu+\delta)I - bD\right], \\
P(t) - P(0) &= {}^{C}D^{r}\left[\sigma + \xi I + \alpha D - \eta P\right].
\end{aligned} \tag{4.1}
$$

Let us denote

$$
\begin{aligned}
f_1 &= \pi - (\beta_1 I + \beta_2 D + \lambda P)S - \mu S, \\
f_2 &= (\beta_1 I + \beta_2 D + \lambda P)S - (\mu + \delta + \gamma)I, \\
f_3 &= \gamma I - \mu R, \\
f_4 &= (\mu + \delta)I - bD, \\
f_5 &= \sigma + \xi I + \alpha D - \eta P.
\end{aligned} \tag{4.2}
$$

By means of the Caputo fractional operator, systems (4.1) can be express as

$$
\begin{aligned}
S(t) - S(0) &= \mathcal{M}(r)\int_0^t \frac{f_1(r,z,S(t))}{(t-z)^r}dz, \\
I(t) - I(0) &= \mathcal{M}(r)\int_0^t \frac{f_2(r,z,I(t))}{(t-z)^r}dz, \\
R(t) - R(0) &= \mathcal{M}(r)\int_0^t \frac{f_3(r,z,R(t))}{(t-z)^r}dz, \\
D(t) - D(0) &= \mathcal{M}(r)\int_0^t \frac{f_4(r,z,D(t))}{(t-z)^r}dz, \\
P(t) - P(0) &= \mathcal{M}(r)\int_0^t \frac{f_5(r,z,P(t))}{(t-z)^r}dz.
\end{aligned} \tag{4.3}
$$

It should be mentioned that $f_1(S,z)$, $f_2(I,z)$, $f_3(R,z)$, $f_4(D,z)$ and $f_5(P,z)$ obeys the Lipschitz condition if and only if $S(t)$, $I(t)$, $R(t)$, $D(t)$ and $P(t)$ have an upper bound. Let $S(t)$ and $S^*(t)$ be two functions, then

$$
\begin{aligned}
&\|f_1(r,z,S(t)) - f_1(r,z,S^*(t))\| \\
&\quad = \| - ((\beta_1 I + \beta_2 D + \lambda P) \\
&\qquad - \mu)(S(t) - S^*(t))\|.
\end{aligned} \tag{4.4}
$$

For $L_1 = \| - ((\beta_1 I + \beta_2 D + \lambda P) - \mu)\|$, we get

$$
\|f_1(r,z,S(t)) - f_1(r,z,S^*(t))\| \le L_1\|S(t) - S^*(t)\|, \tag{4.5}
$$

and likewise when $L_2 = \| - (\mu + \delta + \gamma)\|$, $L_3 = -\mu$, $L_4 = -b$ and $L_5 = -\eta$, we attain as follows:

$$
\begin{aligned}
&\|f_2(r,z,I(t)) - f_1(r,z,I^*(t))\| \\
&\le L_2\|I(t) - I^*(t)\|, \\
&\|f_3(r,z,R(t)) - f_1(r,z,R^*(t))\| \\
&\le L_3\|R(t) - R^*(t)\|, \\
&\|f_4(r,z,D(t)) - f_1(r,z,D^*(t))\| \\
&\le L_4\|D(t) - D^*(t)\|, \\
&\|f_5(r,z,P(t)) - f_1(r,z,P^*(t))\| \\
&\le L_5\|P(t) - P^*(t)\|,
\end{aligned} \tag{4.6}
$$

thus, the Lipschitz condition is achieved. Re-written Eq. (4.3) in recursive form yields:

$$
\begin{aligned}
S_n(t) &= \mathcal{M}(r)\int_0^t \frac{f_1(r,z,S_{n-1}(t))}{(t-z)^r}dz, \\
I_n(t) &= \mathcal{M}(r)\int_0^t \frac{f_2(r,z,I_{n-1}(t))}{(t-z)^r}dz, \\
R_n(t) &= \mathcal{M}(r)\int_0^t \frac{f_3(r,z,R_{n-1}(t))}{(t-z)^r}dz, \\
D_n(t) &= \mathcal{M}(r)\int_0^t \frac{f_4(r,z,D_{n-1}(t))}{(t-z)^r}dz, \\
P_n(t) &= \mathcal{M}(r)\int_0^t \frac{f_5(r,z,P_{n-1}(t))}{(t-z)^r}dz,
\end{aligned} \tag{4.7}
$$

associated with the initial conditions $S_0(t) = S(0)$, $I_0(t) = I(0)$, $R_0(t) = R(0)$, $D_0(t) = D(0)$, and $P_0(t) = P(0)$. By subtracting the successive terms, we get

$$
\begin{aligned}
\Phi_{S,n}(t) &= S_n(t) - S_{n-1}(t) \\
&= \mathcal{M}(r)\int_0^t \frac{f_1(r,z,S_{n-1}(z))}{(t-z)^r} \\
&\quad - \frac{f_1(r,z,S_{n-2}(z))}{(t-z)^r}dz, \\
\Phi_{I,n}(t) &= I_n(t) - I_{n-1}(t) \\
&= \mathcal{M}(r)\int_0^t \frac{f_2(r,z,I_{n-1}(z))}{(t-z)^r}
\end{aligned}
$$

$$
\begin{aligned}
&\quad - \frac{f_2(r,z,I_{n-2}(z))}{(t-z)^r}dz,\\
\Phi_{R,n}(t) &= R_n(t) - R_{n-1}(t)\\
&= \mathcal{M}(r)\int_0^t \frac{f_3(r,z,R_{n-1}(z))}{(t-z)^r}\\
&\quad - \frac{f_3(r,z,R_{n-2}(z))}{(t-z)^r}dz,\\
\Phi_{D,n}(t) &= D_n(t) - D_{n-1}(t)\\
&= \mathcal{M}(r)\int_0^t \frac{f_4(r,z,D_{n-1}(z))}{(t-z)^r}\\
&\quad - \frac{f_4(r,z,D_{n-2}(z))}{(t-z)^r}dz,\\
\Phi_{P,n}(t) &= P_n(t) - P_{n-1}(t)\\
&= \mathcal{M}(r)\int_0^t \frac{f_5(r,z,P_{n-1}(z))}{(t-z)^r}\\
&\quad - \frac{f_5(r,z,P_{n-2}(z))}{(t-z)^r}dz.
\end{aligned}
\tag{4.8}
$$

If we consider as

$$
\begin{aligned}
S_n(t) &= \sum_{k=0}^{n}\Phi_{S_n,k}(t),\\
I_n(t) &= \sum_{k=0}^{n}\Phi_{I_n,k}(t),\\
R_n(t) &= \sum_{k=0}^{n}\Phi_{R_n,k}(t),\\
D_n(t) &= \sum_{k=0}^{n}\Phi_{D_n,k}(t),\\
P_n(t) &= \sum_{k=0}^{n}\Phi_{P_n,k}(t),
\end{aligned}
\tag{4.9}
$$

and employing Eqs. (4.5)-(4.6) and considering:

$$
\begin{aligned}
&\Phi_{S,n-1}(t) = S_{n-1}(t) - S_{n-2}(t),\\
&\Phi_{I,n-1}(t) = I_{n-1}(t) - I_{n-2}(t),\\
&\Phi_{R,n-1}(t) = R_{n-1}(t) - R_{n-2}(t),\\
&\Phi_{D,n-1}(t) = D_{n-1}(t) - D_{n-2}(t),\\
&\Phi_{P,n-1}(t) = P_{n-1}(t) - P_{n-2}(t),
\end{aligned}
$$

we obtain the followings:

$$
\begin{aligned}
\|\Phi_{S,n}(t)\| &= \mathcal{M}(r)\int_0^t \frac{\|\Phi_{S,n-1}(z)\|}{(t-z)^r}dz,\\
\|\Phi_{I,n}(t)\| &= \mathcal{M}(r)\int_0^t \frac{\|\Phi_{I,n-1}(z)\|}{(t-z)^r}dz,\\
\|\Phi_{R,n}(t)\| &= \mathcal{M}(r)\int_0^t \frac{\|\Phi_{R,n-1}(z)\|}{(t-z)^r}dz,\\
\|\Phi_{D,n}(t)\| &= \mathcal{M}(r)\int_0^t \frac{\|\Phi_{D,n-1}(z)\|}{(t-z)^r}dz,\\
\|\Phi_{P,n}(t)\| &= \mathcal{M}(r)\int_0^t \frac{\|\Phi_{P,n-1}(z)\|}{(t-z)^r}dz.
\end{aligned}
\tag{4.10}
$$

From the results above, we prove the following theorem.

Theorem 4.1. *The Caputo fractional-order Ebola virus model (3.1) has a unique solution if*

$$
\frac{\mathcal{M}(r)}{r}T^r L_j < 1,\ j = 1,\ldots,5, \tag{4.11}
$$

is true when $t \in [0,T]$.

Proof. As we can see from the above, the functions $S(t)$, $I(t)$, $R(t)$, $D(t)$ and $P(t)$ are bounded and f_1, f_2, f_3, f_4, f_5 obeys the Lipschitz condition. So, with the help of the recursive techniques and Eq. (4.10), gives

$$
\begin{aligned}
\|\Phi_{S,n}(t)\| &\le \|S_0(t)\|\left(\frac{\mathcal{M}(r)}{r}T^r L_1\right)^n,\\
\|\Phi_{I,n}(t)\| &\le \|I_0(t)\|\left(\frac{\mathcal{M}(r)}{r}T^r L_2\right)^n,\\
\|\Phi_{R,n}(t)\| &\le \|R_0(t)\|\left(\frac{\mathcal{M}(r)}{r}T^r L_3\right)^n,\\
\|\Phi_{D,n}(t)\| &\le \|D_0(t)\|\left(\frac{\mathcal{M}(r)}{r}T^r L_4\right)^n,\\
\|\Phi_{P,n}(t)\| &\le \|P_0(t)\|\left(\frac{\mathcal{M}(r)}{r}T^r L_5\right)^n.
\end{aligned}
\tag{4.12}
$$

Thereby, it can be considered that for $n \to \infty$, $\|\Phi_{S,n}(t)\| \to 0$, $\|\Phi_{I,n}(t)\| \to$

0, $\|\Phi_{R,n}(t)\| \to 0$, $\|\Phi_{D,n}(t)\| \to 0$ and $\|\Phi_{P,n}(t)\| \to 0$. Moreover, by triangle inequality and the system (4.12) for any p, we have

$$\begin{aligned}
\|S_{n+p}(t) - S_n\| &\leq \sum_{k=n+1}^{n+p} j_1^k = \frac{j_1^{n+1} - j_1^{n+p+1}}{1 - j_1},\\
\|I_{n+p}(t) - I_n\| &\leq \sum_{k=n+1}^{n+p} j_2^k = \frac{j_2^{n+1} - j_2^{n+p+1}}{1 - j_2},\\
\|R_{n+p}(t) - R_n\| &\leq \sum_{k=n+1}^{n+p} j_3^k = \frac{j_3^{n+1} - j_3^{n+p+1}}{1 - j_3},\\
\|D_{n+p}(t) - D_n\| &\leq \sum_{k=n+1}^{n+p} j_4^k = \frac{j_4^{n+1} - j_4^{n+p+1}}{1 - j_4},\\
\|P_{n+p}(t) - P_n\| &\leq \sum_{k=n+1}^{n+p} j_5^k = \frac{j_5^{n+1} - j_5^{n+p+1}}{1 - j_5},
\end{aligned} \tag{4.13}$$

such that $\frac{\mathcal{M}(r)}{r} T^r L_j \leq 1$. Thus, S_n, I_n, R_n, D_n, P_n are Cauchy sequences in $\mathbb{B}(J)$ and hence uniformly convergent. Thus, by the hypothesis of the limit theorem, we conclude that the limit of the sequences (4.7) is the unique solution of the fractional-order Ebola virus model (3.1). □

4.2 Positivity and boundedness of solution

Positivity and the boundedness of solutions are important features of epidemiological models. To do so, it is enough to show that all state variables are nonnegative for any $t > 0$, which means that for any $t > 0$, any trajectory that begins with a positive initial condition will remain positive. Now, systems (3.1), gives

$$\begin{aligned}
{}^C D^\alpha S(t)|_{S=0} &= \pi \geq 0,\\
{}^C D^\alpha I(t)|_{I=0} &= \beta_2 DS + \lambda PS \geq 0,\\
{}^C D^\alpha R(t)|_{R=0} &= \gamma I \geq 0,\\
{}^C D^\alpha D(t)|_{D=0} &= (\mu + \delta) I \geq 0,\\
{}^C D^\alpha P(t)|_{D=0} &= \sigma + \xi I + \alpha D \geq 0.
\end{aligned} \tag{4.14}$$

Moreover, since $N(t) = S(t) + I(t) + R(t)$, by adding the first three equations of the model (3.1) gives

$${}^C D^r N(t) = \pi - \mu N - \delta I, \tag{4.15}$$

then

$$N(t) \leq \left(N(0) - \frac{\pi}{\mu}\right) E_r(-\mu t^r) + \frac{\pi}{\mu}.$$

Therefore,

$$\Omega = \left\{(S(t), I(t), R(t)) \in \mathbb{R}_+^3 : 0 \leq N(t) \leq \frac{\pi}{\mu}\right\}, \tag{4.16}$$

is the feasible region for the Caputo fractional-order model (3.1) which is positively invariant. Thus proposed fractional-order model (3.1) is both mathematically and epidemiologically well-posed.

5. Sensitivity Analysis in Respect to R_0

In this section, we use the forward sensitivity index to analyze the sensitivity of the biological parameters in the fractional-order Ebola virus model with respect to the basic reproduction number R_0, which is an important factor in determining the spread of the Ebola virus, and reducing it to less than one is critical in controlling the infection.

By using sensitivity analysis, we can identify the parameters that have the greatest impact on R_0. Parameters with a positive sensitivity index are considered highly sensitive and will increase R_0 if their values increase. Parameters with a negative sensitivity index are considered sensitive for decreasing R_0 if their values decrease. Parameters with a sensitivity index of zero are considered neutral.

The goal of this analysis is to determine the sensitivity status of each parameter and optimize the model's output. This will allow us to identify the most critical parameters and develop effective strategies for controlling the spread of the Ebola virus. The relation

$$\Gamma_{\beta_1}^{R_0} = \frac{\beta_1}{R_0} \times \frac{\partial R_0}{\partial \beta_1}, \tag{5.1}$$

denote sensitivity index of R_0 with respect to a parameter β_1 where

$$R_0 = \frac{\pi(b\eta\beta_1 + \eta\beta_2(\mu+\delta) + \lambda(b\xi + \alpha\delta + \alpha\mu)}{b\eta\mu(\mu+\delta+\gamma)}. \tag{5.2}$$

The results from the sensitivity analysis show that the effective contact rate (λ), shedding rate of deceased human individuals (α), shedding rate of infectious human individuals (ξ), effective contact rate of deceased human individuals β_2 and effective contact rate of infectious human individuals (β_1), respectively, are the most sensitive parameters that lead to the increase of the basic reproduction number R_0. This means that an increase or decrease in these parameters will increase or decrease R_0. Therefore, finding the optimal strategies for decreasing these parameters will help in controlling the spread of the virus (see Table 3).

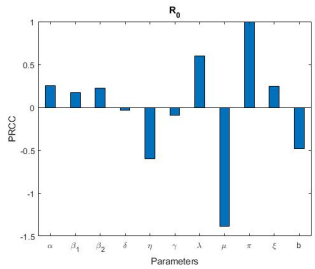

Fig. 4. Ebola-transmission-medium.

Table 3. Forward normalized sensitivity indices.

Parameters	Value	Elasticity index
π	20	1
β_1	0.0007	0.1734
β_2	0.0013	0.2277
λ	0.001	0.5989
μ	0.49	-1.3890
δ	0.86	-0.0365
γ	0.056	-0.0946
b	0.758	-0.4799
ξ	0.035	0.2477
α	0.036	0.2522
η	0.025	-0.5989

6. Numerical Simulations and Discussions

In order to capture the paths that solutions take, both classical and fractional-order models require the use of numerical schemes. The purpose of using this numerical scheme was to gain insight into the trajectories of the solutions. For more information about the accuracy, stability, and convergence of this method, see [33].

It is worth noting that the numerical scheme we have utilized in our simulations, as mentioned earlier, is not only effective, but also has several desirable properties. Specifically, it is a convergent scheme, conditionally stable, and comes equipped with error bounds. The presence of these features ensures the safe and reliable use of this method in our simulations. For conveniences, we have compiled Table 4 which presents the values of parameters used in our numerical simulations of the proposed model. This information may be useful for reproducing our results or for conducting further research using this model.

Using the numerical scheme described earlier, we obtained the dynamic behavior of each compartment in the pro-

Table 4. Parameters value of model (3.1).

Parameters	Parameters value	Source
π	20 (Variable)	Assume
β_1	0.0007 (Variable)	[32]
β_2	0.0013 (Variable)	[32]
λ	0.001 (Variable)	Assume
μ	0.49	[32]
δ	0.86 [0.4, 0.9]	[32]
γ	0.056 (0, 1)	[32]
b	0.758 (0, 1)	[32]
σ	0.001 (variable)	Assume
ξ	0.035 (0, ∞)	Assumed
α	0.036 (0, ∞)	Assume
η	0.025 (0, ∞)	[32]

posed model (3.1), for the case where the fractional-order was set to $r = 1$, as shown in Fig. 5. To further analyze the dynamics of the model and gain insight into its behavior, we varied the fractional-order value $r = 1.0, 0.8, 0.6, 0.4$ while keeping the model parameters fixed (as listed in Table 4).

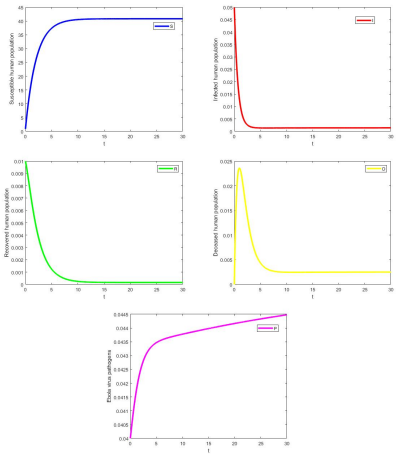

Fig. 5. Simulations of the for model (3.1) of each state variable for the classical version.

The resulting numerical simulations, shown in Fig. 6, indicate a reduction in the number of susceptible individuals and Ebola virus pathogen compartments, suggesting a decrease in the transmission of the virus. This reduction in the number of susceptible individuals can be attributed to the effectiveness of measures such as vaccination, quarantine, and contact tracing in controlling the spread of the virus.

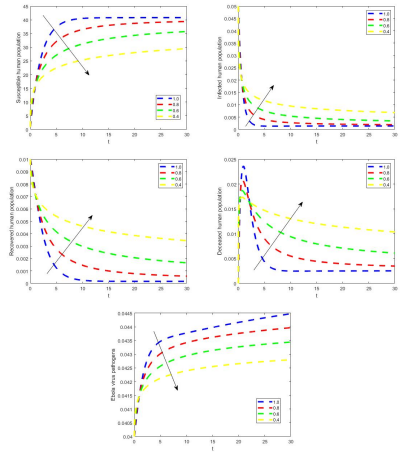

Fig. 6. Simulations of the for model (3.1) of each state variable for the fractional-order version.

In addition, the decrease in the number of individuals in the susceptible compartment can result in an increase in the number of individuals in the infected, recovered, and dead compartments, as shown in Fig. 6. This suggests that some individuals who were initially susceptible have become infected and subsequently either recovered from or died from the virus.

Therefore, the proposed fractional-order Ebola virus model (3.1), provides valuable insight into the complex dynamics of virus transmission and allows us to visualize the memory effect when varying the order of the derivative. This information can be useful in developing more effective strategies for controlling and preventing the spread of the virus in the future.

6.1 Effects of sensitive parameters

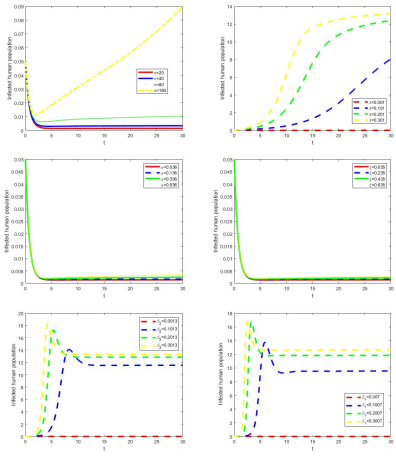

Fig. 7. Impact of sensitive parameters.

Fig. 7 shows the different dynamical phenomena when varying the most sensitive parameters that have an impact on the basic reproduction number R_0 (see Table 3 and Fig. 4), respectively. Because we are interested in the infected individuals compartment, we first vary for different values of the recruitment rate (π), and we observed that the more individuals are recruiting into the susceptible population, the number of infected individuals is getting higher. This scenario happens throughout the infected compartment when varying the remaining sensitive parameters, that is, the effective contact rate (λ), the shedding rate of deceased human individuals (α), the shedding rate of infectious human individuals (xi), the effective contact rate of deceased human individuals (β_2), and the effective contact rate of infectious human individuals (β_1), respectively. The analysis shows that controlling these sensitive parameters will lead to a decrease in the number of infected individuals, which means that there is a need for policymakers to adopt some strategies in order to control these parameters.

7. Conclusions

In this paper, we have formulated and analyzed a mathematical model based on a system of Caputo fractional-order differential equations to investigate the effect of sensitive parameters as a disease control strategy. We were able to establish a region in such a way that the model is mathematically and epidemiologically well-posed due to the fact that its solutions are positive and bounded. Fixed point results were utilized to establish the existence and uniqueness of the proposed model.

The sensitivity analysis revealed that reducing the rate of recruitment of susceptible individuals (π), effective contact rate (λ), shedding rate of deceased human individuals (α), shedding rate of infectious human individuals (ξ), effective contact rate of deceased human individuals (β_2), and effective contact rate of infectious human individuals (β_1) (as seen in Fig. 4 and Fig. 7) is crucial for reducing the basic reproduction number and mitigating disease spread. Furthermore, numerical results show the advantages of using a fractional-order model with memory effects over a classical-order model (as illustrated in Figs. 5-6).

In light of the results, we recommend that policymakers and health practitioners prioritize using effective media coverage to conduct widespread awareness campaigns on preventive measures, regardless of whether there is an ongoing epidemic or not.

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