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Original research article

Bootstrap Confidence Intervals for the Index of Dispersion of Zero-Truncated Poisson-Ishita Distribution

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ABSTRACT

The zero-truncated Poisson-Ishita distribution has been proposed for the count data without zero values, such as the number of items in a shopper's basket at a supermarket checkout line and the length of stay in hospital. However, the confidence interval estimation of the index of dispersion has not yet been examined. In this paper, confidence interval estimation based on percentile, simple, and biased-corrected and accelerated bootstrap methods was examined in terms of coverage probability and average length via Monte Carlo simulation. The results indicate that attaining the nominal confidence level using the bootstrap methods was not possible for small sample sizes regardless of the other settings. Moreover, when the sample size was large, the performances of the bootstrap methods were not substantially different. Overall, the simple bootstrap method outperformed the others, even for medium and large sample sizes. Lastly, the bootstrap methods were used to calculate the confidence interval for the index of dispersion of the zero-truncated Poisson-Ishita distribution via two numerical examples, the results of which match those from the simulation study.

Keywords: Bootstrap method; Interval estimation; Variation; Ishita distribution; Simulation

1. Introduction

The Poisson distribution is a discrete probability distribution that measures the probability of an event happening a certain number of times within a given interval of time or space [1, 2]. Data such as the number of orders a firm will receive tomorrow, the number of defects in a finished product, the number of customers arriving at a checkout counter in a supermarket from 4 to 6 p.m., etc. [3], follow a Poisson distribution.

The probability mass function (pmf) of a Poisson distribution is defined as

$$p(x;\lambda)=\frac{e^{-\lambda}\lambda^{x}}{x!},\quad x=0,1,2,\ldots,\lambda>0, \qquad (1.1)$$

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where e is a constant approximately equal to 2.71828 and λ is the mean number of events within a given interval of time or space.

This probability model can be used to analyze data containing zeros and positive values that have low occurrence probabilities within a predefined time or area range [4]. However, probability models can become truncated when a range of possible values for the variables is either disregarded or impossible to observe. Indeed, zero truncation is often enforced when one wants to analyze count data without zero. Reference [5] developed the zero-truncated (ZT) Poisson (ZTP) distribution, which has been applied to datasets of the length of stay in hospitals, the number of published journal articles in various disciplines, the number of children ever born to a sample of mothers over 40 years old, and the number of passengers in cars [6]. A ZT distribution's pmf can be derived as

$$p(x;\theta) = \frac{p_0(x;\theta)}{1 - p_0(0;\theta)}, \quad x = 1,2,3,..., \qquad (1.2)$$

where $p_0(x;\theta)$ is the pmf of the un-truncated distribution. Reference [7] defined the pmf of the Poisson-Ishita (PI) distribution as

$$p_0(x;\theta) = \frac{\theta^3}{(\theta^3 + 2)} \frac{x^2 + 3x + (\theta^3 + 2\theta^2 + \theta + 2)}{(\theta + 1)^{x+3}},$$
$$x = 0,1,2,..., \ \theta > 0. \qquad (1.3)$$

The mathematical and statistical properties of the PI distribution for modeling biological science data were established by [7]. The PI distribution arises from the Poisson distribution when parameter λ follows the Ishita distribution proposed by [8] with probability density function (pdf)

$$f(\lambda;\theta) = \frac{\theta^3}{\theta^3 + 2}\left(\theta + \lambda^2\right)e^{-\theta\lambda}, \ \ \lambda > 0, \theta > 0. \qquad (1.4)$$

Reference [8] showed that the pdf in (1.4) is a better model than the exponential, Lindley [9] and Akash [10] distributions for modeling lifetime data. Several distributions have been introduced as an alternative to the ZTP distribution for handling over-dispersion in data, such as the ZT Poisson-Lindley (ZTPL) [11], ZT Poisson-Sujatha (ZTPS) [12] and ZT Poisson-Akash (ZTPA) [13] distributions.

Reference [14] proposed the ZTPI distribution and its properties, such as the moment, coefficient of variation, skewness, kurtosis, and the index of dispersion. The method of moments and the maximum likelihood have also been derived for estimating its parameter. Furthermore, when the ZTPI distribution was applied to real data, it was more suitable than ZTP, ZTPL, ZTPS and ZTPA distributions.

This paper focus on the index of dispersion (ID) which is the ratio of the variance to the mean. To the best of our knowledge, no research has been conducted on estimating the confidence interval for the ID of the ZTPI distribution. Bootstrap methods for estimating confidence intervals provide a way of quantifying the uncertainties in statistical inferences based on a sample of data. The concept is to run a simulation study based on the actual data for estimating the likely extent of sampling error [15]. Therefore, the objective of the current study is to assess the efficiencies of three bootstrap methods, namely the percentile bootstrap (PB), simple bootstrap (SB), and bias-corrected and accelerated bootstrap (BCa) to estimate the confidence interval for the ID of the ZTPI distribution. Because a theoretical comparison is not possible, we conducted a simulation study to compare their performances and used the results to determine the best-performing method based on the coverage probability and the average length.

2. Theoretical Background

Compounding of probability distributions is a sound and innovative technique to obtain new probability distributions to fit data sets not adequately fit by common parametric distributions. Reference [7] proposed a new compounding distribution by compounding Poisson distribution with Ishita distribution, as there is a need to find more flexible model for analyzing statistical data. The pmf of the Poisson-Ishita distribution is given in (1.3).

Let X be a random variable which follow ZTPI distribution with parameter θ, it is denoted as $X \sim \text{ZTPI}(\theta)$. Using Eqs. (1.2)-(1.3), the pmf of ZTPI distribution can be obtained as

$$p(x;\theta) = \frac{\theta^3}{\theta^5 + 2\theta^4 + \theta^3 + 6\theta^2 + 6\theta + 2} \times \frac{x^2 + 3x + \left(\theta^3 + 2\theta^2 + \theta + 2\right)}{(\theta+1)^x},$$

$$x = 1, 2, 3, ..., \ \theta > 0. \quad (2.1)$$

The plots of ZTPI distribution with some specified parameter values θ shown in Fig. 1.

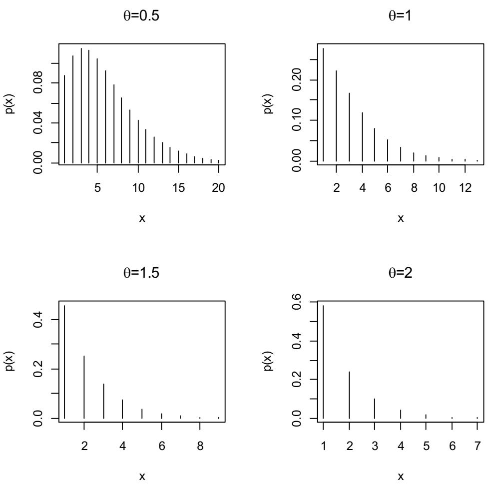

Fig. 1. The plots of the mass function of the ZTPI distribution with $\theta = 0.5, \ 1, \ 1.5$ and 2.

The expected value, the variance, the ID of X are as follows:

$$E(X) = \frac{\theta^6 + 3\theta^5 + 3\theta^4 + 7\theta^3 + 18\theta^2 + 18\theta + 6}{\theta\left(\theta^5 + 2\theta^4 + \theta^3 + 6\theta^2 + 6\theta + 2\right)}$$

$$Var(X) = \frac{(\theta+1)\begin{pmatrix} \theta^{10} + 4\theta^9 + 6\theta^8 + 27\theta^7 + \\ 69\theta^6 + 98\theta^5 + 136\theta^4 + \\ 208\theta^3 + 180\theta^2 + 72\theta + 12 \end{pmatrix}}{\theta^2\left(\theta^5 + 2\theta^4 + \theta^3 + 6\theta^2 + 6\theta + 2\right)^2},$$

and

$$ID(X) = \frac{Var(X)}{E(X)} = \kappa = \frac{\begin{pmatrix} \theta^8 + 2\theta^7 + \theta^6 + 18\theta^5 + 32\theta^4 + \\ 16\theta^3 + 72\theta^2 + 48\theta + 12 \end{pmatrix}}{\theta(\theta^3 + 6)\begin{pmatrix} \theta^5 + 2\theta^4 + \theta^3 + \\ 6\theta^2 + 12\theta + 6 \end{pmatrix}}. \quad (2.2)$$

The point estimator of θ is obtained by maximizing the log-likelihood function $\log L(x_i;\theta)$ or the logarithm of joint p.m.f. of $X_1, X_2, ..., X_n$. Thus, the maximum likelihood (ML) estimator for θ of the ZTPI distribution is derived by the following processes

$$\frac{\partial}{\partial\theta}\log L(x_i;\theta) = \frac{\partial}{\partial\theta}\left[\begin{array}{l} n\log\left(\dfrac{\theta^3}{\begin{pmatrix} \theta^5 + 2\theta^4 + \\ \theta^3 + 6\theta^2 + \\ 6\theta + 2 \end{pmatrix}}\right) - \\ \displaystyle\sum_{i=1}^{n} x_i \log(\theta+1) + \\ \displaystyle\sum_{i=1}^{n} \log\left[\begin{array}{l} x_i^2 + 3x_i + \\ \begin{pmatrix} \theta^3 + 2\theta^2 + \\ \theta + 2 \end{pmatrix} \end{array}\right] \end{array}\right]$$

$$= \frac{3n}{\theta} - \frac{n\begin{pmatrix} 5\theta^4 + 8\theta^3 + \\ 3\theta^2 + 12\theta + 6 \end{pmatrix}}{\begin{pmatrix} \theta^5 + 2\theta^4 + \theta^3 + \\ 6\theta^2 + 6\theta + 2 \end{pmatrix}} - \frac{n\bar{x}}{\theta+1} + \sum_{i=1}^{n} \frac{(3\theta^2 + 4\theta + 1)}{\begin{pmatrix} x_i^2 + 3x_i + \\ (\theta^3 + 2\theta^2 + \theta + 2) \end{pmatrix}}.$$

Solving the equation $\frac{\partial}{\partial\theta}\log L(x_i;\theta) = 0$ for $\theta,$ we have the non-linear equation

$$\frac{3n}{\theta} - \frac{n\begin{pmatrix} 5\theta^4 + 8\theta^3 + \\ 3\theta^2 + 12\theta + 6 \end{pmatrix}}{\begin{pmatrix} \theta^5 + 2\theta^4 + \theta^3 + \\ 6\theta^2 + 6\theta + 2 \end{pmatrix}} - \frac{n\bar{x}}{\theta+1} + \sum_{i=1}^{n} \frac{(3\theta^2 + 4\theta + 1)}{\begin{pmatrix} x_i^2 + 3x_i + \\ (\theta^3 + 2\theta^2 + \theta + 2) \end{pmatrix}} = 0,$$

where $\bar{x} = \sum_{i=1}^{n} x_i / n$ denotes the sample mean. Since the ML estimator for θ does not provide the closed-form solution, the non-linear equation can be solved by the numerical iteration methods such as Newton-Raphson method, bisection method and Ragula-Falsi method. In this paper, we use maxLik package [16] with Newton-Raphson method for ML estimation in the statistical software R.

The point estimator of the ID (κ) can be estimated by replacing the parameter θ with the ML estimator for θ in Eq. (2.2). Therefore, the point estimator of the ID is given by

$$\hat{\kappa} = \frac{\begin{pmatrix} \hat{\theta}^8 + 2\hat{\theta}^7 + \hat{\theta}^6 + 18\hat{\theta}^5 + 32\hat{\theta}^4 + \\ 16\hat{\theta}^3 + 72\hat{\theta}^2 + 48\hat{\theta} + 12 \end{pmatrix}}{\hat{\theta}(\hat{\theta}^3 + 6)\begin{pmatrix} \hat{\theta}^5 + 2\hat{\theta}^4 + \hat{\theta}^3 + \\ 6\hat{\theta}^2 + 12\hat{\theta} + 6 \end{pmatrix}},$$

where $\hat{\theta}$ is the ML estimator for $\theta.$

3. Bootstrap Confidence Interval Methods

In this paper, we focus on the three bootstrap confidence interval methods that are most popular in practice: percentile bootstrap, simple bootstrap, and bias-corrected and accelerated bootstrap confidence intervals.

3.1 Percentile Bootstrap (PB) method

The percentile bootstrap confidence interval is the interval between the $(\alpha/2)\times 100$ and $(1-(\alpha/2))\times 100$ percentiles of the distribution of κ estimates obtained from resampling or the distribution of $\hat{\kappa}^*$, where κ represents a parameter of interest and α is the level of significance (e.g., α=0.05 for 95% confidence intervals) [17]. A percentile bootstrap confidence interval for κ can be obtained as follows:

1) B random bootstrap samples are generated,

2) a parameter estimate $\hat{\kappa}^*$ is calculated from each bootstrap sample,

3) all B bootstrap parameter estimates are ordered from the lowest to highest, and

4) the $(1-\alpha)100\%$ percentile bootstrap confidence interval is constructed as follows:

$$CI_{PB} = \left[\hat{\kappa}^*_{(r)}, \hat{\kappa}^*_{(s)}\right], \qquad (3.1)$$

where $\hat{\kappa}^*_{(\alpha)}$ denotes the α^{th} percentile of the distribution of $\hat{\kappa}^*$ and $0 \le r < s \le 100.$ For example, a 95% percentile bootstrap confidence interval with 1000 bootstrap samples is the interval between the 2.5 percentile value and the 97.5 percentile value of the 1000 bootstrap parameter estimates.

3.2 Simple Bootstrap (SB) method

The simple bootstrap method is sometimes called the basic bootstrap method and is a method as easy to apply as the percentile bootstrap method. Suppose that the quantity of interest is κ and that the estimator of κ is $\hat{\kappa}$. The simple bootstrap method assumes that the distributions of $\hat{\kappa}-\kappa$ and $\hat{\kappa}^*-\hat{\kappa}$ are approximately the same [18]. The $(1-\alpha)100\%$ simple bootstrap confidence interval for κ is

$$CI_{SB}=\left[2\hat{\kappa}-\hat{\kappa}^*_{(s)},2\hat{\kappa}-\hat{\kappa}^*_{(r)}\right], \qquad (3.2)$$

where the quantiles $\hat{\kappa}^*_{(r)}$ and $\hat{\kappa}^*_{(s)}$ are the same percentile of empirical distribution of bootstrap estimates $\hat{\kappa}^*$ used in Eq. (3.1) for the percentile bootstrap method.

3.3 Bias-corrected and accelerated (BCa) Bootstrap method

To overcome the over coverage issues in percentile bootstrap confidence intervals [19], the BCa bootstrap method corrects for both bias and skewness of the bootstrap parameter estimates by incorporating a bias-correction factor and an acceleration factor [19, 20]. The bias-correction factor $\hat{z}_0$ is estimated as the proportion of the bootstrap estimates less than the original parameter estimate $\hat{\kappa}$,

$$\hat{z}_0=\Phi^{-1}\left(\frac{\#\{\hat{\kappa}^*\le\hat{\kappa}\}}{B}\right),$$

where Φ^{-1} is the inverse function of a standard normal cumulative distribution function (e.g., $\Phi^{-1}(0.975)\approx 1.96$). The acceleration factor $\hat{a}$ is estimated through jackknife resampling (i.e., “leave one out” resampling), which involves generating n replicates of the original sample, where n is the number of observations in the sample. The first jackknife replicate is obtained by leaving out the first case $(i=1)$ of the original sample, the second by leaving out the second case $(i=2)$, and so on, until n samples of size $n-1$ are obtained. For each of the jackknife resamples, $\hat{\kappa}_{(-i)}$ is obtained. The average of these estimates is

$$\hat{\kappa}_{(\cdot)}=\frac{\sum_{i=1}^{n}\hat{\kappa}_{(-i)}}{n}.$$

Then, the acceleration factor $\hat{a}$ is calculated as follow,

$$\hat{a}=\frac{\sum_{i=1}^{n}\left(\hat{\kappa}_{(\cdot)}-\hat{\kappa}_{(-i)}\right)^3}{6\left\{\sum_{i=1}^{n}\left(\hat{\kappa}_{(\cdot)}-\hat{\kappa}_{(-i)}\right)^2\right\}^{3/2}}.$$

With the values of $\hat{z}_0$ and $\hat{a}$, the values α_1 and α_2 are calculated,

$$\alpha_1=\Phi\left\{\hat{z}_0+\frac{\hat{z}_0+z_{\alpha/2}}{1-\hat{a}\left(\hat{z}_0+z_{\alpha/2}\right)}\right\}$$

and $$\alpha_2=\Phi\left\{\hat{z}_0+\frac{\hat{z}_0+z_{1-\alpha/2}}{1-\hat{a}\left(\hat{z}_0+z_{1-\alpha/2}\right)}\right\},$$

where $z_{\alpha/2}$ is the α quantile of the standard normal distribution (e.g. $z_{0.05/2}\approx -1.96$). Then, the $(1-\alpha)100\%$ BCa bootstrap confidence interval for κ is as follows

$$CI_{BCa}=\left[\hat{\kappa}^*_{(\alpha_1)},\hat{\kappa}^*_{(\alpha_2)}\right], \qquad (3.3)$$

where $\hat{\kappa}^*_{(\alpha)}$ denotes the α^{th} percentile of the distribution of $\hat{\kappa}^*$.

4. Simulation Study

The confidence intervals for the ID of a ZTPI distribution estimated via various bootstrap methods was conducted in this study. Because a theoretical comparison is not possible, a Monte Carlo simulation study was designed using R version 4.2.2 to cover cases with different sample sizes (n = 10, 30, 50, 100 and 200). To observe the effect of small and large variances, the true parameter (θ) was set as 0.25, 0.5, 0.75, 1, 1.5, and the population ID are 4.9269, 2.8216, 2.0487, 1.6032 and 1.0448, respectively. It shows that the ID decreases as the value of θ increases. The relationship between the values of θ and the ID shown in Fig. 2. $B = 1000$ bootstrap samples of size n were generated from the original sample and each simulation was repeated 2000 times. Without loss of generality, the confidence level $(1-\alpha)$ was set at 0.95. The performances of the bootstrap methods were compared in terms of their coverage probabilities and average lengths. The one with a coverage probability greater than or close to the nominal confidence level means that it contains the true value and can be used to precisely estimate the confidence interval for the parameter function of interest.

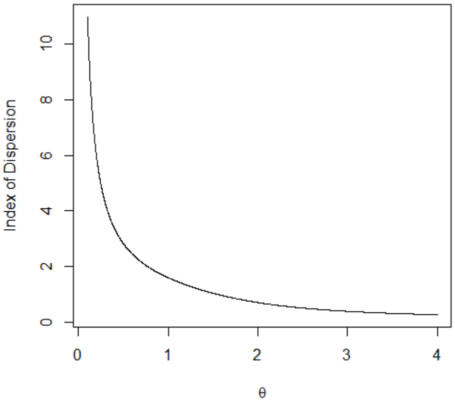

Fig. 2. The relationship between the values of θ and the ID.

The results of the study are reported in Table 1. For $n = 10$ and 30, the coverage probabilities of the three methods tended to be less than 0.93 and so did not reach the nominal confidence level. Nevertheless, the SB method had coverage probabilities close to the nominal confidence levels for medium and large sample sizes $(n \geq 50)$. Thus, as the sample size was increased, the coverage probabilities of the methods tended to increase and approach 0.95. Moreover, the average lengths of the methods decreased when the value of κ was decreased because of the relationship between the variance and κ. Unsurprisingly, as the sample size was increased, the average lengths of the three methods decreased. For small sample sizes $(n \leq 30)$, the average lengths of all methods are not compared because the coverage probabilities are lower than the nominal confidence level. When $n = 50$, the average lengths of PB and SB methods were slightly shorter than those of BCa bootstrap method. In case of large sample sizes $(n \geq 100)$, the average lengths of all bootstrap confidence intervals were not significantly different.

5. Numerical Examples

We used two real-world examples to demonstrate the applicability of the bootstrap methods for estimating confidence intervals for the ID of the ZTPI distribution.

5.1 The number of unrest events

The number of unrest events occurring in the southern border area of Thailand from July 2020 to August 2022 collected by the Southern Border Area News Summary was used for this example (the sample size was 26). The number of unrest events per month during this time period in the five southern provinces of Pattani, Yala, Narathiwat, Songkhla, and Satun provinces is reported in Table 2; the total sample size is 26. This study uses the Chi-squared goodness-of-fit test for checking whether the sample data is likely to be from a specific theoretical distribution [22]. The Chi-squared statistic was 2.5298 and the p-value was 0.9248. Thus, a ZTPI distribution with $\hat{\theta}$=0.4500 is suitable for this dataset. The point estimator of the ID is 3.0648. Table 3 reported the 95% bootstrap confidence intervals for the ID of the ZTPI distribution. The estimated parameter $\hat{\theta}$ is approximately 0.5. The results correspond with the simulation results for $n = 30$ because the average lengths of the PB and SB methods were shorter than those of the BCa bootstrap methods.

Table 1. Coverage probability and average length of the 95% bootstrap confidence intervals for the ID in the zero-truncated Poisson-Ishita distribution.

n	θ	κ	Coverage probability			Average length		
			PB	SB	BCa	PB	SB	BCa
10	0.25	4.9269	0.8850	0.8860	0.8925	2.9780	2.9790	3.0988
	0.5	2.8216	0.8885	0.8985	0.8925	1.7337	1.7335	1.7874
	0.75	2.0488	0.8780	0.9125	0.8905	1.4054	1.4060	1.4357
	1	1.6032	0.8930	0.9260	0.9135	1.2977	1.2952	1.3193
	1.5	1.0448	0.8715	0.8380	0.9255	1.1251	1.1275	1.1565
30	0.25	4.9269	0.9155	0.9135	0.9225	1.8310	1.8283	1.8659
	0.5	2.8216	0.9210	0.9270	0.9230	1.0633	1.0636	1.0813
	0.75	2.0488	0.9300	0.9435	0.9305	0.8447	0.8443	0.8555
	1	1.6032	0.9360	0.9475	0.9420	0.7721	0.7714	0.7767
	1.5	1.0448	0.9215	0.9155	0.9350	0.7197	0.7193	0.7247
50	0.25	4.9269	0.9370	0.9305	0.9380	1.4446	1.4458	1.4631
	0.5	2.8216	0.9365	0.9355	0.9340	0.8364	0.8368	0.8448
	0.75	2.0488	0.9360	0.9410	0.9360	0.6592	0.6614	0.6651
	1	1.6032	0.9405	0.9510	0.9380	0.6007	0.6011	0.6034
	1.5	1.0448	0.9355	0.9375	0.9445	0.5668	0.5665	0.5690
100	0.25	4.9269	0.9500	0.9470	0.9445	1.0289	1.0277	1.0352
	0.5	2.8216	0.9365	0.9330	0.9355	0.5990	0.5990	0.6024
	0.75	2.0488	0.9475	0.9470	0.9465	0.4681	0.4679	0.4698
	1	1.6032	0.9450	0.9495	0.9445	0.4249	0.4250	0.4265
	1.5	1.0448	0.9345	0.9355	0.9380	0.4049	0.4050	0.4050
200	0.25	4.9269	0.9500	0.9520	0.9505	0.7314	0.7312	0.7335
	0.5	2.8216	0.9435	0.9410	0.9430	0.4247	0.4245	0.4257
	0.75	2.0488	0.9455	0.9460	0.9445	0.3319	0.3317	0.3327
	1	1.6032	0.9460	0.9415	0.9435	0.3006	0.3007	0.3012
	1.5	1.0448	0.9445	0.9405	0.9435	0.2878	0.2878	0.2878

Table 2. The number of unrest events in the southern border area of Thailand.

Number of unrest events	1	2	3	4	5	6	7	≥ 8
Observed frequency	3	1	3	2	3	3	3	8
Expected frequency	1.8586	2.3890	2.6657	2.7161	2.5995	2.3772	2.1001	9.2937

Table 3. The 95% bootstrap confidence intervals and corresponding widths using all intervals for the ID in the unrest events example.

Methods	Confidence intervals	Widths
PB	(2.4693, 3.6750)	1.2057
SB	(2.4850, 3.6594)	1.1744
BCa	(2.4606, 3.6982)	1.2377

5.2 Flower heads example

The second dataset, shown in Table 4, is the number of flower heads as per the number of fly eggs reported by [23]. The total sample size is 88. For Chi-squared goodness-of-fit test [22], the Chi-squared statistic was 3.7681 and p-value was 0.7080. Thus, a ZTPI distribution with $\hat{\theta}=1.0141$ is suitable for this dataset. The point estimator of the ID is 1.5828. The 95% bootstrap confidence intervals for the ID of the ZTPI distribution were reported in Table 5. Similar to simulation results when $\kappa=1.6032$ and $n=100$, the width of all confidence intervals was around 0.39, but the largest confidence interval was from the SB method.

Table 4. The number of flower heads as per the number of fly eggs.

Number of fly eggs	1	2	3	4	5	6	≥ 7
Observed frequency	22	18	18	11	9	6	4
Expected frequency	24.9287	19.7204	14.6526	10.2922	6.9078	4.4711	7.0272

Table 5. The 95% bootstrap confidence intervals and corresponding widths using all intervals for the ID in the flower heads example.

Methods	Confidence intervals	Widths
PB	(1.3835, 1.7725)	0.3890
SB	(1.3933, 1.7893)	0.3960
BCa	(1.3864, 1.7719)	0.3856

6. Conclusions and Discussion

Herein, we propose three bootstrap methods, namely PB, SB and BCa, to estimate the confidence intervals of the ID of the ZTPI distribution. When the sample size was 10 or 30, the coverage probabilities of all three were substantially lower than 0.95. When the sample size was large enough (i.e., 50), the coverage probabilities and average lengths using three bootstrap methods were not markedly different. According to our findings, the SB method performed the best for medium and large sample sizes and parameter settings tested in both the simulation study and using real data set. Future research could focus on the other approaches to compare with the bootstrap methods.

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