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Soft Intersection Almost Tri-bi-ideals of Semigroups

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ABSTRACT

In this study, we introduce the notion of soft intersection almost tri-bi-ideals of semigroups as a generalization of nonnull soft intersection tri-bi-ideals and investigate its properties in depth. It is aimed to explore the relations of soft intersection almost tri-bi-ideals with other certain kinds of soft intersection almost ideals of semigroups. It is shown that an idempotent soft intersection almost tri-bi-ideal coincides with the soft intersection almost bi-ideal of a semigroup. It is also illustrated that every idempotent soft intersection almost tri-bi-ideal is a soft intersection almost subsemigroup. Furthermore, we propose the concepts of soft intersection prime, semiprime and strongly prime almost ideals of a semigroup and explore the relationships regarding minimality, primeness, semiprimeness, and strong primeness between almost tri-bi-ideals and soft intersection almost tri-bi-ideals by deriving a notable result that if a nonempty subset of a semigroup is an almost tri-bi-ideal, then its soft characteristic function is a soft intersection almost tri-bi-ideal, and vice versa. This enables us to construct a bridge between classical semigroup theory and soft set theory.

Keywords: Soft set; Semigroup; (almost) Tri-bi-ideals; Soft intersection (almost) tri-bi-ideals

1. Introduction

In the early 1900s, semigroups became a subject of formal study, holding significance across various branches of mathematics as they provide the abstract algebraic foundation for ”memoryless” systems, which restart at each iteration. Semigroups serve as a fundamental model for

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linear time-invariant systems in practical mathematics. The study of finite semigroups is particularly crucial in theoretical computer science, given their natural connection to finite automata. Additionally, in probability theory, semigroups are linked to Markov processes.

The study of algebraic structures and their applications necessitates the concept of ideals. Initially proposed by Dedekind to aid in the study of algebraic numbers, ideals were further developed by Noether with the introduction of associative rings. Bi-ideals for semigroups were first introduced in 1952 by Good and Hughes [1]. The concept of quasi-ideals was first introduced by Steinfeld [2] for semigroups and later extended to rings. Generalizing ideals in algebraic structures has been a focus of many mathematicians, with bi-ideals being a generalization of quasi-ideals.

The notion of almost left, right, and two-sided ideals of semigroups was introduced by Grosek and Satko [3]. Bogdanovic [4] introduced almost bi-ideals in semigroups as an extension of bi-ideals. Wattanatripop et al. [5] introduced almost quasi-ideals of semigroups. Kaopusek et al. [6] proposed almost interior ideals and weakly almost interior ideals of semigroups, using the notions of almost ideals and interior ideals of semigroups. The study of almost ideals of semigroups has garnered significant interest from researchers. Iampan et al. [7] in 2021, Chinram and Nakkhasen [8] in 2022, Gaketem [9] in 2022, and Gaketem and Chinram [10] in 2023 introduced almost subsemigroups, almost bi-quasi-interior ideals, almost bi-interior ideals, and almost bi-quasi-ideals of semigroups, respectively. Additionally, various types of almost fuzzy ideals of semigroups were investigated in [5,7-12].

To model uncertainty, Molodtsov [13] introduced the concept of a soft set in 1999, which has since garnered interest from various domains. Soft set operations, the foundational elements of the theory, were examined in [14-29]. The notion of the soft set and its operations were further developed by Cagman and Enginoglu [30]. Cagman et al. [31] also developed the idea of soft intersection groups, leading to research on several soft algebraic systems. Soft sets were also used in semigroup theory. Semigroups with soft intersection left, right, and two-sided ideals, quasi-ideals, interior ideals, and generalized bi-ideals were thoroughly examined [32, 33]. Sezgin and Orbay [34] classified various semigroups in terms of soft intersection substructures. In addition, a range of soft algebraic structures were investigated in [35-60]. Building on existing ideals, Rao [61-64] recently introduced several new types of ideals of semigroups, such as bi-interior ideals, bi-quasi-interior ideals, bi-quasi-ideals, quasi-interior, and weak interior ideals. Furthermore, Baupradist et al. [65] proposed the notion of essential ideals in semigroups.

The concept of tri-ideal as a generalization of bi-quasi-interior ideal, quasi-interior ideal, bi-interior ideal, bi-quasi ideal, quasi ideal, interior ideal, left (right) ideal, and ideal of a semigroup and semiring was introduced by Rao [66, 67]. Moreover, Rao et al. [68] proposed the concept of tri-quasi ideal as a generalization of ideals, bi-ideals, quasi-ideals, interior ideals, bi-interior ideals, tri-ideals, and bi-quasi ideals of a semigroup. In [69], soft intersection tri-bi-ideal is proposed as a generalization of some kinds of soft intersection ideals such as bi-ideal, quasi-ideal, interior ideal, bi-interior ideal, tri-ideal, and bi-quasi-ideal of a semigroup. In this study, as a further generalization of nonnull soft intersection tri-bi-ideal, we introduce the notion of a soft intersection almost tri-bi-ideal. It has been established that an idempotent soft intersection almost tri-bi-ideal coincides with a soft intersection almost bi-ideal. Moreover, it is obtained that every idempotent soft intersection almost tri-bi-ideal is a soft intersection almost subsemigroup. We posit that a semigroup can be constructed using soft intersection almost tri-bi-ideals of a semigroup with the binary operation of soft union, but not with the soft intersection operation. Additionally, several intriguing relationships concerning minimality, primeness, semiprimeness, and strongly primeness between almost tri-bi-ideals and soft intersection almost tri-bi-ideals have been derived with the important result that if a nonempty subset of a semigroup is an almost tri-bi-ideal, then its soft characteristic function is also a soft intersection almost tri-bi-ideal, and vice versa.

2. Preliminary

In this section, we first review several fundamental notions related to semigroups and then soft sets.

A semigroup S is a nonempty set with an associative binary operation, and throughout this paper, S stands for a semigroup. A non-empty subset A of S is called a subsemigroup of S if $AA \subseteq A$, is called a left-ideal (right-ideal) of S if $AS \subseteq A$ ($SA \subseteq A$), is called an ideal of S if $SA \subseteq A$ and $AS \subseteq A$, and is called an interior ideal of S if $SAS \subseteq A$. A subsemigroup A of S is called a bi-ideal of S if $ASA \subseteq A$.

Definition 2.1 ([66]). A non-empty subset A of S is called a left tri-ideal (right tri-ideal) of S if A is a subsemigroup of S and $ASAA \subseteq A$ ($AASA \subseteq A$), is called a tri-ideal of S if A is a subsemigroup of S, $SAA \subseteq A$ and $AASA \subseteq A$.

While Rao [66] introduced left (right/two-sided ideal) tri-ideals of a semigroup, left (right/two-sided ideal) tri-ideals of a semiring were also proposed by Rao [67].

Definition 2.2 ([68]). A nonempty subset A of S is called a tri-bi-ideal of S if A is a subsemigroup of S and $AASAA \subseteq A$.

Note 2.3. Rao et al. [68] terms ”tri-bi-ideal” as ”tri-quasi-ideal”; however, when we look at the definition of left tri-ideal, we see that ”SA”, representing the left ideal, is in the middle of the structure $ASAA \subseteq A$ and the other two ”A” are on the left and right of the structure, and when we look at the definition of right tri-ideal, ”AS”, representing the right ideal, is in the middle of the structure $AASA \subseteq A$ and the other two ”A” are on the left and right of the structure. As for the tri-quasi ideal, we see that ”ASA”, which is placed in the middle of the structure $AASAA \subseteq A$, evokes in the mind as the bi-ideal, and again the other two ”A” are on the left and right of the structure. Therefore, we prefer to name ”tri-bi-ideal” for ”tri-quasi ideal” throughout the paper in order to be consistent with the definition of left (right) tri-ideal.

Definition 2.4. A nonempty subset $A$of S is called an almost tri-bi-ideal of S if $AAsAA \cap A \neq \emptyset$, for all $s \in S$.

Example 2.5. Let $S = Z$ be the semigroup with the usual multiplication and $\emptyset \neq 2Z \subseteq Z$. Since $(2Z)(2Z)s(2Z)(2Z) \cap 2Z \neq \emptyset$, for all $s \in Z$, $2Z$ is an almost tri-bi ideal of S.

An almost tri-bi-ideal A of S is called a minimal almost tri-bi-ideal of S if for any almost tri-bi-ideal B of S whenever $B \subseteq A$, then $A = B$. An almost tri-bi-ideal P of S is called a prime almost tri-bi-ideal if for any almost tri-bi-ideals A and B of S such that $AB \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$. An almost tri-bi-ideal P of S is called a semiprime almost tri-bi-ideal if for any almost tri-bi-ideal A of S such that $AA \subseteq P$ implies that $A \subseteq P$. An almost tri-bi-ideal P of S is called a strongly prime almost tri-bi-ideal if for any almost tri-bi-ideals A and B of S such that $AB \cap BA \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.

Definition 2.6 ([13,30])**.** Let U be the universal set, E be the parameter set, $P(U)$ be the power set of U, and $K \subseteq E$. A soft set f_K over U is a set-valued function such that $f_K : E \to P(U)$ such that for all $x \notin K, f_K(x) = \emptyset$. A soft set over U can be represented by the set of ordered pairs

$$f_K = (x, f_K(x)) : x \in E, f_K(x) \in P(U).$$

Throughout this paper, the set of all the soft sets over U is designated by $S_E(U)$.

Definition 2.7 ([30])**.** Let $f_A \in S_E(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then f_A is called a null soft set and denoted by $\emptyset_E$.

Definition 2.8 ([30])**.** Let $f_A, f_B \in S_E(U)$. If $f_A(x) \subseteq f_B(x)$ for all $x \in E$, then f_A is a soft subset of f_B and denoted by $f_A \tilde{\subseteq} f_B$. If $f_A(x) = f_B(x)$ for all $x \in E$, then f_A is called soft equal to f_B and denoted by $f_A = f_B$.

Definition 2.9 ([30])**.** Let $f_A, f_B \in S_E(U)$. The union of f_A and f_B is the soft set $f_A \tilde{\cup} f_B$, where $(f_A \tilde{\cup} f_B)(x) = f_A(x) \cup f_B(x)$, for all $x \in E$. The intersection of f_A and f_B is the soft set $f_A \tilde{\cap} f_B$, where $(f_A \tilde{\cap} f_B)(x) = f_A(x) \cap f_B(x)$, for all $x \in E$.

Definition 2.10 ([18])**.** For a soft set f_A, the support of f_A is defined by

$$supp(f_A) = x \in A : f_A(x) \neq \emptyset.$$

It is obvious that a soft set with an empty support is a null soft set, otherwise, the soft set is nonnull.

Note 2.11 ([70])**.** If $f_A \tilde{\subseteq} f_B$, then $supp(f_A) \subseteq supp(f_B)$.

Definition 2.12 ([32])**.** Let f_S and g_S be soft sets over the common universe U. Then, soft intersection product $f_S \circ g_S$ is defined by

$$f_S \circ g_S(x) = \begin{cases} \bigcup\limits_{x=y=z} f_S(y) \cap g_S(z), & \text{if } \exists y, z \in S \\ & \text{s.t. } x = yz, \\ \emptyset, & \text{otherwise.} \end{cases}$$

Theorem 2.13 ([32])**.** *Let $f_S, g_S, h_S \in S_S(U)$. Then,*

i) $(f_S \circ g_S) \circ h_S = f_S \circ (g_S \circ h_S)$.

ii) $f_S \circ g_S \neq g_S \circ f_S$, *generally.*

iii) $f_S \circ (g_S \tilde{\cup} h_S) = (f_S \circ g_S) \tilde{\cup} (f_S \circ h_S)$ *and* $(f_S \tilde{\cup} g_S) \circ h_S = (f_S \circ h_S) \tilde{\cup} (g_S \circ h_S)$.

iv) $f_S \circ (g_S \tilde{\cap} h_S) = (f_S \circ g_S) \tilde{\cap} (f_S \circ h_S)$ *and* $(f_S \tilde{\cup} g_S) \circ h_S = (f_S \circ h_S) \tilde{\cap} (g_S \circ h_S)$.

v) If $f_S \tilde{\subseteq} g_S$, *then* $f_S \circ h_S \tilde{\subseteq} g_S \circ h_S)$ *and* $h_S \circ f_S \tilde{\subseteq} h_S \circ g_S)$.

vi) If $t_S, k_S \in S_S(U)$ *such that* $t_S \tilde{\subseteq} f_S$ *and* $k_S \tilde{\subseteq} g_S$, *then* $t_S \circ k_S \tilde{\subseteq} f_S \circ g_S$.

Definition 2.14 ([32])**.** Let A be a subset of S. We denote by S_A the soft characteristic function of A and define as

$$S_A(x) = \begin{cases} U, & \text{if } x \in A \\ \emptyset, & \text{if } x \in S/A. \end{cases}$$

The soft characteristic function of A is a soft set over U, that is, $S_A : S \to P(U)$.

Corollary 2.15 ([70])**.** $supp(S_A) = A$.

Theorem 2.16 ([32, 70]). *Let X and Y be nonempty subsets of S. Then, the following properties hold*

i) $X \subseteq Y$ *if and only if* $S_X \tilde{\subseteq} S_Y$,

ii) $S_X \tilde{\cap} S_Y = S_{(X \cap Y)}$ *and* $S_X \tilde{\cup} S_Y = S_{(X \cup Y)}$,

iii) $S_X \circ S_Y = S_{XY}$.

Definition 2.17 ([71]). Let x be an element in S. We denote by S_x the soft characteristic function of x and define as

$$S_x(y) = \begin{cases} U, & \text{if } y = x \\ \emptyset, & \text{if } y \neq x. \end{cases}$$

The soft characteristic function of x is a soft set over U, that is, $S_x : S \longrightarrow P(U)$.

Definition 2.18 ([69]). A soft set f_S over U is called a soft intersection tri-bi-ideal of S if $f_S(xyztw) \supseteq f_S(x) \cap f_S(y) \cap f_S(t) \cap f_S(w)$, for all $x, y, z, t, w \in S$. It is easy to see that if $f_S(x) = U$ for all $x \in S$, then f_S is a soft intersection tri-bi-ideal of S. We denote such a kind of tri-bi-ideal by $\tilde{\mathbb{S}}$. It is obvious that $\tilde{S} = S_S$, that is, $\tilde{\mathbb{S}}(x) = U$ for all $x \in S$.

Theorem 2.19 ([69]). *Let* f_S *be a soft set over* U. *Then,* f_S *is a soft intersection tri-bi-ideal of* S *if and only if* $(f_S \circ f_S \circ \tilde{\mathbb{S}} \circ f_S \circ f_S) \tilde{\subseteq} f_S$.

Throughout this paper, we prefer to use "SI-T-bi-ideal of S" instead of "soft intersection tri-bi-ideal".

Definition 2.20 ([70–72]). A soft set f_S is called a soft intersection almost subsemigroup of S if $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, is called a soft intersection almost left (resp. right) ideal of S if $(S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S ((f_S \circ S_x) \cap f_S \neq \emptyset_S)$, for all $x \in S$. f_S is called a soft intersection almost two-sided ideal (or briefly soft intersection almost ideal) of S if f_S is both a soft intersection almost left ideal of S and a soft intersection almost right ideal of S, is called a soft intersection almost bi-ideal of S if $(f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$.

For more about ternary semigroups characterized by spherical fuzzy bi-ideals, we refer to [73], and one may discuss the potential consequences of graph applications and network analysis for soft sets, which are characterized by the divisibility of determinants [74].

3. Results and Discussion

Definition 3.1. A soft set f_S is called a soft intersection almost tri-bi-ideal of S if

$$(f_S \circ f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$$

for all $x \in S$. Hereafter, for brevity, a soft intersection almost tri-bi-ideal of S is denoted by "SI-almost T-bi-ideal".

Example 3.2. Let $S = \mathcal{q}, \mathcal{w}$ be the semigroup with the following Cayley Table. Let

	$\mathcal{q}$	$\mathcal{w}$
$\mathcal{q}$	$\mathcal{q}$	$\mathcal{w}$
$\mathcal{w}$	$\mathcal{w}$	$\mathcal{w}$

f_S, h_S, and g_S be soft sets over $U = \mathbb{Q}$ as follows:

$$f_S = (\mathcal{q}, 1, 2), (\mathcal{w}, 0, 4)$$

$$h_S = (\mathcal{q}, -2, -1), (\mathcal{w}, -1, 1)$$

$$g_S = (\mathcal{q}, -5, -3), (\mathcal{w}, \emptyset)$$

Here, f_S and h_S are both SI-almost T-bi-ideals. Let's first show that f_S is an SI-almost T-bi-ideal, that is, $(f_S \circ f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$:
Let's start with $(f_S \circ f_S \circ S_q \circ f_S \circ f_S) \tilde{\cap} f_S$:

$$\begin{aligned} & [(f_S \circ f_S \circ S_{\mathcal{q}} \circ f_S \circ f_S) \tilde{\cap} f_S](\mathcal{q}) \\ = \; & (f_S \circ f_S \circ S_{\mathcal{q}} \circ f_S \circ f_S)(\mathcal{q}) \cap f_S(\mathcal{q}) \end{aligned}$$

$$
\begin{aligned}
&= [(f_S \circ f_S)(q) \cap (S_q \circ f_S \circ f_S)(q)] \cap f_S(q) \\
&= [(f_S(q) \cap f_S(q)) \cap (S_q(q) \cap (f_S \circ f_S)(q)] \\
&\quad \cap f_S(q) \\
&= [(f_S(q) \cap (f_S \circ f_S)(q)] \cap f_S(q) \\
&= [(f_S(q) \cap [(f_S(q) \cap f_S(q)] \cap f_S(q) \\
&= f_S(q),
\end{aligned}
$$

$$
\begin{aligned}
&[(f_S \circ f_S \circ S_q \circ f_S \circ f_S)\tilde{\cap} f_S](w) \\
&= (f_S \circ f_S \circ S_q \circ f_S \circ f_S)(w) \cap f_S(w) \\
&= [((f_S \circ f_S)(q) \cap (S_q \circ f_S \circ f_S)(w)) \\
&\quad \cup((f_S \circ f_S)(w) \cap (S_q \circ f_S \circ f_S)(q)) \\
&\quad \cup((f_S \circ f_S)(w) \cap (S_q \circ f_S \circ f_S)(w))] \cap f_S(w) \\
&= [(f_S(q) \cap f_S(w)) \cup (f_S(w) \cap f_S(q)) \\
&\quad \cup(f_S(w) \cap f_S(w))] \cap f_S(w) \\
&= f_S(w) \cap f_S(w) \\
&= f_S(w).
\end{aligned}
$$

Consequently,

$$(f_S \circ f_S \circ S_q \circ f_S \circ f_S)\tilde{\cap} f_S = \{(q, \{1,2\}), (w, \{0,4\})\} \neq \emptyset_S.$$

Similarly,

$$(f_S \circ f_S \circ S_w \circ f_S \circ f_S)\tilde{\cap} f_S = \{(q, \emptyset), (w, \{0,4\})\} \neq \emptyset_S.$$

Therefore, $(f_S \circ f_S \circ S_x \circ f_S \circ f_S)\tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$. Thus, f_S is an SI-almost T-bi-ideal.

Similarly, h_S is an SI-almost T-bi-ideal. In fact;

$$(h_S \circ h_S \circ S_q \circ h_S \circ h_S)\tilde{\cap} h_S = \{(q, \{-2,-1\}), (w, \{-1,1\})\} \neq \emptyset_S,$$

$$(h_S \circ h_S \circ S_w \circ h_S \circ h_S)\tilde{\cap} h_S = \{(q, \emptyset), (w, \{-1,1\})\} \neq \emptyset_S.$$

One can also show that g_S is not an SI-almost T-bi-ideal. In fact;

$$
\begin{aligned}
&[(g_S \circ g_S \circ S_w \circ g_S \circ g_S)\tilde{\cap} g_S](q) \\
&= (g_S \circ g_S \circ S_w \circ g_S \circ g_S)(q) \cap g_S(q) \\
&= [(g_S \circ g_S)(q) \cap (S_w \circ g_S \circ g_S)(q) \cap g_S(q) \\
&= [(g_S(q) \cap g_S(q)) \cap (S_w(q) \cap (g_S \circ g_S)(q))] \\
&\quad \cap g_S(q) \\
&= \emptyset \cap g_S(q) \\
&= \emptyset, \\
&[(g_S \circ g_S \circ S_w \circ g_S \circ g_S)\tilde{\cap} g_S](w) \\
&= (g_S \circ g_S \circ S_w \circ g_S \circ g_S)(w) \cap g_S(w) \\
&= [((g_S \circ g_S)(q) \cap (S_w \circ g_S \circ g_S)(w)) \\
&\quad \cup((g_S \circ g_S)(w) \cap (S_w \circ g_S \circ g_S)(q)) \\
&\quad \cup((g_S \circ g_S)(w) \cap (S_w \circ g_S \circ g_S)(w))] \cap g_S(w) \\
&= [(g_S(q) \cap (g_S(q) \cup g_S(w) \cup (g_S(w) \cap \emptyset) \\
&\quad \cup(g_S(w) \cap (g_S(q) \cup g_S(w)))] \cap g_S(w) \\
&= (g_S(q) \cup g_S(w)) \cap g_S(w) \\
&= g_S(w) \\
&= \emptyset.
\end{aligned}
$$

Consequently,

$$(g_S \circ g_S \circ S_w \circ g_S \circ g_S)\tilde{\cap} g_S = \{(q, \emptyset), (w, \emptyset)\} = \emptyset_S.$$

As can be seen $(g_S \circ g_S \circ S_w \circ g_S \circ g_S)\tilde{\cap} g_S = \emptyset_S$, for $w \in S$. Thus, g_S is not an SI-almost T-bi-ideal.

Proposition 3.3. *If f_S is an SI-T-bi-ideal such that $f_S \circ f_S \circ S_x \circ f_S \circ f_S \neq \emptyset_S$, then f_S is an SI-almost T-bi-ideal.*

Proof. Let f_S be an SI-T-bi-ideal such that $f_S \circ f_S \circ S_x \circ f_S \circ f_S \neq \emptyset_S$. By definition if SI-T-bi-ideal, $(f_S \circ f_S \circ \tilde{\mathbb{S}} \circ f_S \circ f_S)\tilde{\subseteq}\emptyset_S$. We need to show that

$$(f_S \circ f_S \circ S_x \circ f_S \circ f_S)\tilde{\cap} f_S \neq \emptyset_S,$$

for all $x \in S$. Since $(f_S \circ f_S \circ S_x \circ f_S \circ f_S)\tilde{\subseteq}(f_S \circ f_S \circ \tilde{\mathbb{S}} \circ f_S \circ f_S)\tilde{\subseteq} f_S$, it follows that $(f_S \circ f_S \circ S_x \circ f_S \circ f_S)\tilde{\subseteq} f_S$. Thus,

$$(f_S \circ f_S \circ S_x \circ f_S \circ f_S)\tilde{\cap} f_S = f_S \circ f_S \circ S_x \circ f_S \circ f_S \neq \emptyset_S,$$

implying that f_S is an SI-almost T-bi-ideal. Note that, $f_S \circ f_S \circ S_x \circ f_S \circ f_S \neq \emptyset_S$ implies that $f_S \neq \emptyset_S$. Moreover, it is obvious that $\emptyset_S$ is an SI-T-bi-ideal, as $(f_S \circ f_S \circ \tilde{\mathbb{S}} \circ \emptyset_S \circ \emptyset_S) = \emptyset_S \tilde{\subseteq} \emptyset_S$; but it is not SI-almost T-bi-ideal, since $(f_S \circ f_S \circ S_x \circ \emptyset_S \circ \emptyset_S)\tilde{\cap}\emptyset_S = \emptyset_S \tilde{\cap} \emptyset_S = \emptyset_S$. □

Here note that if f_S is an SI-almost T-bi-ideal, then f_S needs not be an SI-T-bi-ideal as shown in the following example:

Example 3.4. In Example 3.2, it is shown that f_S and h_S are SI-almost T-bi-ideals; however, f_S and h_S are not SI-T-bi-ideals. In fact;

$$[(f_S \circ f_S \circ \tilde{\mathbb{S}} \circ f_S \circ f_S)(w)]$$

$$\begin{aligned} &= [(f_S \circ f_S)(\wp) \cap (\tilde{\mathbb{S}} \circ f_S \circ f_S)(\omega)] \\ &\quad \cup [(f_S \circ f_S)(\omega) \cap (\tilde{\mathbb{S}} \circ f_S \circ f_S)(\wp)] \\ &\quad \cup [(f_S \circ f_S)(\omega) \cap (\tilde{\mathbb{S}} \circ f_S \circ f_S)(\omega)] \\ &= (f_S \circ f_S)(\wp) \cup (f_S \circ f_S)(\omega) \cap (f_S \circ f_S)(\wp) \\ &\quad \cup (f_S \circ f_S)(\omega) \\ &= (f_S \circ f_S)(\wp) \cup (f_S \circ f_S)(\omega) \\ &\tilde{\nsubseteq} \; f_S(\omega), \end{aligned}$$

thus, f_S is not an SI-T-bi-ideal. Similarly,

$$(h_S \circ h_S \circ \tilde{\mathbb{S}} \circ h_S \circ h_S)(\omega) = h_S(\wp) \cup h_S(\omega) \tilde{\nsubseteq} h_S(\omega),$$

thus, h_S is not an SI-T-bi-ideal.

Proposition 3.5. *Let f_S be an idempotent SI-almost T-bi-ideal. Then, f_S is an SI-almost subsemigroup.*

Proof. Assume that f_S is an idempotent SI-almost T-bi-ideal. Then, $f_S \circ f_S = f_S$ and $(f_S \circ f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$. We need to show that $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ for all $x \in S$. Since,

$$\begin{aligned} \emptyset_S &\neq (f_S \circ f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \\ &= [(f_S \circ f_S \circ S_x \circ f_S \circ f_S)] \tilde{\cap} f_S \\ &= [(f_S \circ f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} (f_S \circ f_S)] \tilde{\cap} f_S \\ &= (f_S \circ f_S) \tilde{\cap} f_S. \end{aligned}$$

f_S is an SI-almost subsemigroup. □

Proposition 3.6. *Let f_S be an idempotent soft set. Then, f_S is an SI-almost bi-ideal if and only if f_S is an SI-almost T-bi-ideal.*

Proof. Assume that f_S is an idempotent soft set. Then, $f_S \circ f_S = f_S$. By assumption, since

$$\emptyset_S \neq (f_S \circ S_x \circ f_S) \tilde{\cap} f_S = (f_S \circ f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S,$$

for all $x \in S$, the rest of the proof is obvious. □

Theorem 3.7. *If f_S is an SI-almost T-bi-ideal such that $f_S \tilde{\subseteq} h_S$, then h_S is an SI-almost T-bi-ideal.*

Proof. Assume that f_S is an SI-almost T-bi-ideal. Hence, $(f_S \circ f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$. We need to show that $(h_S \circ h_S \circ S_x \circ h_S \circ h_S) \tilde{\cap} h_S \neq \emptyset_S$. In fact,

$$(f_S \circ f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \tilde{\nsubseteq} (h_S \circ h_S \circ S_x \circ h_S \circ h_S) \tilde{\cap} h_S.$$

Since $(f_S \circ f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(h_S \circ h_S \circ S_x \circ h_S \circ h_S) \tilde{\cap} h_S \neq \emptyset_S$. This completes the proof. □

Theorem 3.8. *Let f_S and h_S be SI-almost T-bi-ideals. Then, $f_S \tilde{\cup} h_S$ is an SI-almost T-bi-ideal.*

Proof. Since f_S is an SI-almost T-bi-ideal by assumption and $f_S \tilde{\subseteq} f_S \tilde{\cup} h_S$, $f_S \tilde{\cup} h_S$ is an SI-almost T-bi-ideal by Theorem 3.7. □

Corollary 3.9. *The finite union of SI-almost T-bi-ideals is an SI-almost T-bi-ideal.*

Corollary 3.10. *Let f_S or h_S be SI-almost T-bi-ideal. Then, $f_S \tilde{\cup} h_S$ is an SI-almost T-bi-ideal.*

Here note that if f_S and h_S are SI-almost T-bi-ideals, then $f_S \tilde{\cap} h_S$ needs not be an SI-almost T-bi-ideal.

Example 3.11. Consider the SI-almost T-bi-ideals f_S and h_S in Example 3.2. Since, $f_S \tilde{\cap} h_S = \{(\wp, \emptyset), (\omega, \emptyset)\} = \emptyset_S$ $f_S \tilde{\cap} h_S$ is not an SI-almost T-bi-ideal.

Now, we give the relationship between almost T-bi-ideal and SI-almost T-bi-ideal. But first of all, we remind the following lemma in order to use it in Theorem 3.13.

Lemma 3.12. *Let $x \in S$ and Y be a nonempty subset of S. Then, $S_x \circ S_Y = S_{xY}$. If X is a nonempty subset of S and $y \in S$, then $S_X \circ S_y = S_{Xy}$.*

Theorem 3.13. *Let A be a nonempty subset of S. Then, A is an almost T-bi-ideal if and only if S_A, the soft characteristic function of A, is an SI-almost T-bi-ideal.*

Proof. Assume that $\emptyset \neq A$ is an almost T-bi-ideal. Then, $AAxAA \cap A \neq \emptyset$, for all $x \in S$, and so there exists $t \in S$ such that $t \in AAxAA \cap A$. Since,

$$((S_A \circ S_A \circ \tilde{\mathbb{S}} \circ S_A \circ S_A)\tilde{\cap} S_A)(t)$$
$$= (S_{AAxAA}\tilde{\cap} S_A)(t) = (S_{(AAxAA)\cap A})(t) = U \neq \emptyset,$$

it follows that $(S_A \circ S_A \circ S_x \circ S_A \circ S_A)\tilde{\cap} S_A \neq \emptyset$. Thus, S_A is an SI-almost T-bi-ideal.

Conversely, assume that S_A is an SI-almost T-bi-ideal. Hence, we have $(S_A \circ S_A \circ S_x \circ S_A \circ S_A)\tilde{\cap} S_A \neq \emptyset_S$, for all $x \in S$. In order to show that A is an almost T-bi-ideal, we should prove that $A \neq \emptyset$ and $AAxAA \cap A \neq \emptyset$, for all $x \in S$. $A \neq \emptyset$ is by the assumption. Now,

$$\begin{aligned}
\emptyset_S &\neq (S_A \circ S_A \circ S_x \circ S_A \circ S_A)\tilde{\cap} S_A \\
&\Rightarrow \exists n \in S; ((S_A \circ S_A \circ S_x \circ S_A \circ S_A)\tilde{\cap} S_A)(n) \\
&\quad \neq \emptyset \\
&\Rightarrow \exists n \in S; ((S_{AAxAA})\tilde{\cap} S_A)(n) \neq \emptyset \\
&\Rightarrow \exists n \in S; ((S_{(AAxAA)\cap A})\tilde{\cap} S_A)(n) \neq \emptyset \\
&\Rightarrow \exists n \in S; ((S_{(AAxAA)\cap A})\tilde{\cap} S_A)(n) = U \\
&\Rightarrow n \in AAxAA \cap A.
\end{aligned}$$

Hence, $AAxAA \cap A \neq \emptyset$. Consequently, A is an almost T-bi-ideal. □

Lemma 3.14 ([70]). *Let f_S be a soft set over U. Then, $f_S \tilde{\subseteq} S_{supp(f_S)}$.*

Theorem 3.15. . *If f_S is an SI-almost T-bi-ideal, then $supp(f_S)$ is an almost T-bi-ideal.*

Proof. Assume that f_S is an SI-almost T-bi-ideal. Thus, $(f_S \circ f_S \circ S_x \circ f_S \circ f_S)\tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$. In order to show that $supp(f_S)$ is an almost T-bi-ideal, by Theorem 3.13, it is enough to show that $S_{supp(f_S)}$ is an SI-almost T-bi-ideal. By Lemma 3.14,

$$\begin{aligned}
& (f_S \circ f_S \circ S_x \circ f_S \circ f_S)\tilde{\cap} f_S \\
\tilde{\subseteq}\ & (S_{supp(f_S)} \circ S_{supp(f_S)} \circ S_x \circ S_{supp(f_S)} \\
& \circ S_{supp(f_S)})\tilde{\cap} S_{supp(f_S)}
\end{aligned}$$

and $(f_S \circ f_S \circ S_x \circ f_S \circ f_S)\tilde{\cap} f_S \neq \emptyset_S$, it implies that

$$(S_{supp(f_S)} \circ S_{supp(f_S)} \circ S_x \circ S_{supp(f_S)} \circ S_{supp(f_S)})\tilde{\cap} S_{supp(f_S)} \neq \emptyset_S$$

□

Consequently, $S_{supp(f_S)}$ is an SI-almost T-bi-ideal and by Theorem 3.13, $S_{supp(f_S)}$ is an almost T-bi-ideal. Here, note that the converse of Theorem 3.15 is not true in general, as shown in the following example.

Example 3.16. Let $S = t, u, r$ be the semigroup with the following Cayley Table. Let

	t	u	r
t	r	t	t
u	t	r	r
r	t	r	r

l_S be soft sets over $U = \mathbb{Z}^-$ as follows:

$$l_S = \{(t, \{-1\}), (u, \{-2\}), (r, \emptyset)\}.$$

Here l_S is not an SI-almost T-bi-ideal. Let's show that:

$$\begin{aligned}
& (l_S \circ l_S \circ S_u \circ l_S \circ l_S)\tilde{\cap} l_S(t) \\
=\ & (l_S \circ l_S \circ S_u \circ l_S \circ l_S)(t) \cap l_S(t) \\
=\ & [(l_S \circ l_S)(t) \cap (S_u \circ l_S \circ l_S)(u) \\
& \cup((l_S \circ l_S)(t) \cap (S_u \circ l_S \circ l_S)(r)) \\
& \cup((l_S \circ l_S)(t) \cap (S_u \circ l_S \circ l_S)(t)) \\
& \cup((l_S \circ l_S)(r) \cap (S_u \circ l_S \circ l_S)(t))] \cap l_S(t) \\
=\ & [\emptyset \cup ((l_S \circ l_S)(t) \cap (l_S \circ l_S)(r)) \cup \emptyset \\
& \cup((l_S \circ l_S)(r) \cap (l_S \circ l_S)(t))] \cap l_S(t) \\
=\ & [(l_S \circ l_S)(t) \cap (l_S \circ l_S)(r)] \cap l_S(t) \\
=\ & [(l_S(t) \cap l_S(u) \cup (l_S(t) \cap l_S(r))] \cap l_S(t) \\
=\ & l_S(t) \cap (l_S(u) \cup l_S(r)) \\
=\ & \emptyset, \\
& (l_S \circ l_S \circ S_u \circ l_S \circ l_S)\tilde{\cap} l_S(u) \\
=\ & (l_S \circ l_S \circ S_u \circ l_S \circ l_S)(u) \cap l_S(u) \\
=\ & \emptyset, \\
& (l_S \circ l_S \circ S_u \circ l_S \circ l_S)\tilde{\cap} l_S(r) \\
=\ & (l_S \circ l_S \circ S_u \circ l_S \circ l_S)(r) \cap l_S(r) \\
=\ & [((l_S \circ l_S)(t) \cap (S_u \circ l_S \circ l_S)(t)) \\
& \cup((l_S \circ l_S)(u) \cap (S_u \circ l_S \circ l_S)(u)) \\
& \cup((l_S \circ l_S)(t) \cap (S_u \circ l_S \circ l_S)(r)) \\
& ((l_S \circ l_S)(r) \cap (S_u \circ l_S \circ l_S)(u)) \\
& ((l_S \circ l_S)(r) \cap (S_u \circ l_S \circ l_S)(r))] \cap l_S(r) \\
=\ & [((l_S \circ l_S)(t) \cap (l_S \circ l_S)(t)) \cup \emptyset \cup \emptyset \cup \emptyset \\
& \cup((l_S \circ l_S)(r) \cap ((l_S \circ l_S)(u) \cup (l_S \circ l_S)(r)))] \\
& \cap l_S(r) \\
=\ & [(l_S \circ l_S)(t) \cap (l_S \circ l_S)(r)] \cap l_S(r)
\end{aligned}$$

$$\begin{aligned} &= [l_S(t) \cup l_S(u) \cup l_S(r)] \cap l_S(r) \\ &= l_S(r) \\ &= \emptyset. \end{aligned}$$

Consequently,

$$(l_S \circ l_S \circ S_w \circ l_S \circ l_S) \tilde{\cap} l_S = \{(t, \emptyset), (u, \emptyset), (r, \emptyset)\} = \emptyset_S.$$

Thus l_S is not an SI-almost T-bi-ideal. Since $supp(l_S) = \{t, u\}$,

$$\begin{aligned} & [(supp(l_S)supp(l_S)t supp(l_S)supp(l_S)] \\ & \cap supp(l_S) \\ = & \{t, u\}\{t, u\}t\{t, u\}\{t, u\} \cap \{t, u\} \\ = & \{t\} \neq \emptyset, \\ & [(supp(l_S)supp(l_S)u supp(l_S)supp(l_S)] \\ & \cap supp(l_S) \\ = & \{t, u\}\{t, u\}t\{t, u\}\{t, u\} \cap \{t, u\} \\ = & \{t\} \neq \emptyset, \\ & [(supp(l_S)supp(l_S)r supp(l_S)supp(l_S)] \\ & \cap supp(l_S) \\ = & \{t, u\}\{t, u\}t\{t, u\}\{t, u\} \cap \{t, u\} \\ = & \{t\} \neq \emptyset. \end{aligned}$$

It is seen that

$$[(supp(l_S)supp(l_S)x supp(l_S)supp(l_S)] \neq \emptyset,$$

for all $x \in S$. That is to say, $supp(l_S)$ is an almost T-bi-ideal; although l_S is not an SI-almost T-bi-ideal.

Definition 3.17. An SI-almost T-bi-ideal f_S is called minimal if any SI-almost T-bi-ideal h_S whenever $h_S \tilde{\subseteq} f_S$, then $supp(h_S) = supp(f_S)$.

Theorem 3.18. *Let A be a nonempty subset of S. Then, A is a minimal almost T-bi-ideal if and only if S_A, the soft characteristic function of A, is a minimal SI-almost T-bi-ideal.*

Proof. Assume that A is a minimal almost T-bi-ideal. Thus, A is an almost T-bi-ideal, and so S_A is an SI-almost T-bi-ideal by Theorem 3.13. Let f_S be an SI-almost T-bi-ideal such that $f_S \tilde{\subseteq} S_A$. By Theorem 3.15, $supp(f_S)$ is an almost T-bi-ideal, and by Note 2.12 and Corollary 2.16,

$$supp(f_S) \subseteq supp(S_A) = A.$$

Since A is a minimal almost T-bi-ideal, $supp(f_S) = supp(S_A) = A$. Thus, S_A is a minimal SI-almost T-bi-ideal by Definition 3.17.

Conversely, let S_A be a minimal SI-almost T-bi-ideal. Thus, S_A is an SI-almost T-bi-ideal and A is an almost T-bi-ideal by Theorem 3.13. Let B be an almost T-bi-ideal such that $B \subseteq A$. By Theorem 3.13, S_B is an SI-almost T-bi-ideal and by Theorem 2.16 (i), $S_B \tilde{\subseteq} S_A$. Since S_A is a minimal SI-almost T-bi-ideal,

$$B = supp(S_B) = supp(S_A) = A$$

by Corollary 2.15. Thus, A is a minimal almost T-bi-ideal. □

Definition 3.19. Let f_S, g_S, and h_S be any SI-almost T-bi-ideals. If $h_S \circ g_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-prime almost T-bi-ideal.

Definition 3.20. Let f_S and h_S be any SI-almost T-bi-ideals. If $h_S \circ h_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$, then f_S is called an SI-semiprime almost T-bi-ideal.

Definition 3.21. Let f_S, g_S, and h_S be any SI-almost T-bi-ideals. If $(h_S \circ g_S) \tilde{\cap} (g_S \circ h_S) \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-strongly prime almost T-bi-ideal.

It is obvious that every SI-strongly prime almost T-bi-ideal is an SI-prime almost T-bi-ideal and every SI-prime almost T-bi-ideal is an SI-semiprime almost T-bi-ideal.

Theorem 3.22. *If S_P, the soft characteristic function of P, is an SI-prime almost T-bi-ideal, then P is a prime almost T-bi-ideal, where $\emptyset \neq P \subseteq S$.*

Proof. Assume that S_P is an SI-strongly prime almost T-bi-ideal. Thus, S_P is an SI-almost T-bi-ideal and thus, P is an almost T-bi-ideal by Theorem 3.13. Let A and B be almost T-bi-ideals such that $AB \cap BA \subseteq P$. Thus, by Theorem 3.13, S_A and S_B are SI-almost T-bi-ideals, and by Theorem 2.16,

$$\begin{aligned}(S_A \circ S_B)\tilde{\cap}(S_B \circ S_A) &= S_A B \tilde{\cap} S_B A \\ &= S_(AB \cap BA) \tilde{\subseteq} S_P.\end{aligned}$$

Since S_P is an SI-strongly prime almost T-bi-ideal and $(S_A \circ S_B)\tilde{\cap}(S_B \circ S_A)\tilde{\subseteq}S_P$, it follows that $S_A\tilde{\subseteq}S_P$ or $S_B\tilde{\subseteq}S_P$. Thus, by Theorem 2.16 (i), $A \subseteq P$ or $B \subseteq P$. Therefore, P is a strongly prime almost T-bi-ideal. □

4. Conclusion

In this study, as a generalization of a nonnull soft intersection tri-bi-ideal, we introduced the concept of a soft intersection almost tri-bi-ideal. We demonstrated that an idempotent soft intersection almost tri-bi-ideal is a soft intersection almost subsemigroup, and an idempotent soft intersection almost tri-bi-ideal coincides with a soft intersection almost bi-ideal. We also proved a theorem stating that if a nonempty subset of a semigroup is an almost tri-bi-ideal, then its soft characteristic function is also a soft intersection almost tri-bi-ideal, and vice versa. This theorem allowed us to establish relationships between soft intersection almost tri-bi-ideals of a semigroup and almost tri-bi-ideals of a semigroup in terms of minimality, primeness, semiprimeness, and strongly primeness. Moreover, we found that the binary operation of soft union can be used to construct a semigroup with the collection of soft intersection almost tri-bi-ideals, but the soft intersection operation cannot be used for this purpose. In future studies, researchers may examine various types of soft intersection almost ideals in semigroups, such as bi-quasi ideals, quasi-interior ideals, and bi-quasi-interior ideals.

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