P-ISSN 2586-9000
E-ISSN 2586-9027
Homepage : *https://tci-thaijo.org/index.php/SciTechAsia* Science & Technology Asia
Vol.29 No.2 April - June 2024 Page: [32-44]
Original research article

A Bilevel QP-PLP Approach to Demand Response Modulation between Consumers and a Single Electricity Seller

Printaporn Sanguansuttigul[1], Nattapong Chayawatto[2], Parin Chaipunya[3,4,*]

[1]*Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathum Thani 12110, Thailand*
[2]*Pilot Plant Development and Training Institute, King Mongkut's University of Technology Thonburi, Bangkok 10140, Thailand*
[3]*Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi, Bangkok 10140, Thailand*
[4]*The Joint Graduate School of Energy and Environment, King Mongkut's University of Technology Thonburi, Bangkok 10140, Thailand*

Received 15 January 2024; Received in revised form 29 February 2024
Accepted 1 April 2024; Available online 25 June 2024

ABSTRACT

During the peak hours, electricity becomes extremely expensive to produce and deliver and this causes a negative effect both to the suppliers and consumers. From the producer point-of-view, real-time pricing may be implemented to incentivize the consumers to shift their load usage to off-peak hours. That is, to adjust for a lower price during off-peak hours and for a higher price during the on-peak hours. On the other hand, the consumers would then respond optimally to the given strategic prices by means of flexible load scheduling and incorporation of smart energy management systems. We present in this paper an intermediate model that maximizes the profit of the supplier while maintaining the low expenses of the consumers by using a bilevel program of the form QP-PLP, where the upper-level problem is quadratic and the lower-level problem is a parametric linear program.

Keywords: Bilevel programming; Demand response; Demand-side management; Energy management system

1. Introduction

Demand response refers to situations where consumers could flexibly modify their schedule of load usage to minimize their payment, in reaction to some signals of the electricity sellers. The signals here

***Corresponding author:** parin.cha@kmutt.ac.th
doi: 10.14456/scitechasia.2024.23

could be price, award, or other kind of incentives offered by the supply side. Demand response has been playing an important role in modern energy management in the scale as large as a continent down to the scale of a household. To this aspect, the demand for electricity is usually high during working hours and gets lower outside of such periods. The cost per unit is often higher for a producer to produce a high volume of electricity, resulting in a shallow marginal profits. Apart from these economic aspects, shaving off high demand also helps in the environmental department. Recall that less production implies immediately a reduction of emission, or even prevents a construction of new power plants in the extreme cases. To this end, demand response is thus a very helpful tool that enhances the stability, security, flexibility, as being well as cost-effective to both the suppliers and consumers alike (see [1–4]).

While needing to maintain a reasonable price to its customers, the profit margin during the peak hours is certainly lower than, e.g., during the late night. To illustrate this controversy, let us imagine that a producer offers a static price per unit to its customers. Customer *A* has a daily demand of operating an oven for 6 hours and she is only active during the day. Meanwhile, another Customer *B* has the same requirement but she is only active during the night. Obviously, both Customers *A* and *B* pay the same bills but the producer makes more money from Customer *B* due to the larger margin. To improve the producer's profit, a dynamic pricing approach could be implemented, where a cheaper price is offered during the off-peak hours as an incentive for the customers to shift their use and (partially) lookover their inconvenience.

Knowing that consumers are able to schedule their loads, the producers could take advantage of the customers behaviors to leverage their income. To achieve the maximum profit, the producer adjusts its pricing *optimally* during different hours of the day. Typical pricing strategies include time-of-use (TOU), time-and-level-of-use (TLOU), real-time-pricing (RTP), etc. Each of these methods has its own benefits and drawbacks. For example, the TOU pricing is usually pre-set then implemented for a long timespan, i.e. months, seasons, or even for a year. Its long duration helps the consumers absorb the pricing and allows for some slow-adapted users to correctly adjust. However, its lengthy activation causes a lot inaccuracies in the consumption levels and could even lead to a shifted or expanded peak period. On the other hand, the RTP approach is more dynamic. For this program, the price is set and announced to a much shorter notice. Although this short time window allows for a producer to obtain a far more reliable demand prediction, it could be too rapid for the consumers to optimally react. In such an event, the producer would eventually lose its margin. Interested readers should refer to [5, 6], for further descriptions of each approach.

To optimally treat such complications, several mathematical modeling techniques have appeared in the recent literature both in the supply-side optimization (pricing problems) as well as the demand-side optimization (flexible loads scheduling) [7–12]. Some scholars also considered the bi-objective optimization approach that takes simultaneous care of both supply- and demand-side problems (see e.g. [13–15]).

One could notice from the above discussion a natural bilevel structure of the interactions between the producer and its customers. This structure has been taken into consideration already in the recent literature of energy optimization modeling [16–19].

In this paper, we adopt this bilevel optimization to model the situation where a choice of hourly price is signaled to the consumer, where he/she will react to the price by optimally scheduling his/her loads. The aim of the producer is therefore to *set an hourly price so it will get a maximum profit under the assumption that the customers react optimally.* The tentative model for the complete system (consisting of a producer and its consumers) then reads as

$$\begin{aligned} \text{Max} \quad & \text{Profit of the seller} \\ \text{s.t.} \quad & \text{Optimal reaction of the customers.} \end{aligned} \tag{1.1}$$

Here, the seller's marginal *profit* depends on its hourly price decision ($p_t \geq 0$) and also on the consumers' decisions on actual hourly load usage ($e_t \geq 0$), in kWh or MWh. The consumers then traditionally solve their optimization problem to get an optimal scheduling (e_t). Note that the price (p_t) will inevitably appear in the consumer's optimization problem, but as external parameters. In this paper, we propose a bilevel optimization model that describes exactly this demand response modulation between a single producer and a pack of consumers. It is particularly legitimate that the *genuine* problem for consumers will be of mixed-integer type. However, putting a mixed-integer problem into the lower-level of a bilevel programming is an absolute disaster to both theoretical and computational points of view. We thus approximate/simplify the consumers' variables as the kWh or MWh requirements to cover the target demand (possibly in other units). To escalate further the flexibility of the consumers, we assume that a solar panel and a storage unit is equipped.

Let us also discuss about the technical side of solving such a model. As one shall see in the forthcoming sections, our full model (see Eq. (2.10)) is of the form QP-PLP, i.e. the upper-level problem is a quadratic problem (QP) with a response of a parametrized linear program (PLP) with respect to the leader's decision in the lower level. Thanks to the lower-level problem being a PLP, one could unconditionally reformulate it as a parametrized KKT system. Since we deal with global solutions, the bilevel program is fully transformed into a single-level mathematical program with complementarity constraints (MPCC) with the multipliers as additional disposable variables. Note that the original bilevel program and its MPCC reformulation are not necessarily equivalent if a local solution is concerned [20]. These equivalent problems are known to be generally nonconvex and disconnected. A classical method of solving an MPCC is to adopt the big-M technique, despite the difficulty in choosing an appropriate upper bound, M, due to the unknown boundedness property of the dual variables (see [21]). In this paper, we adopt a new approach of SOS1 technique [22] (see also [23]) which branches on the complementarity pairs instead of looking at them as equations.

2. The bilevel QP-PLP model and the solution method

In this section, we present the formulation of the empirical model presented in Eq. (1.1). Recall that we regard the single producer as the leader of the bilevel problem and all consumers are grouped together as a single unit of follower. This is applicable to the case where the consumers are, e.g., the whole village sharing a solar farm, or the industries in an industrial park with some agreement contracts, etc. Now, let us first introduce in Table 1 the indices, parameters and variables that will be used across the paper.

Table 1. All the indices, parameters and variables that are used in this paper.

Indices	
T	The number of time slots in a day.
A	The number of all shiftable loads.
t	The time index which is an integer ranging from 1 to T.
a	The appliance index which is an integer ranging from 1 to A.
Parameters	
c_t	Quadratic coefficient for the electricity production cost.
d_a	The equivalent daily demand required from the operation of the load a.
κ	The energy capacity of a storage s.
ρ	The maximum charging rate of a storage s.
σ_t	The availability of the solar generated energy at the time t.
$q_{a,t}$	The monetary inconvenience cost to opearate the appliance a at the time t.
ℓ	The maximum capacity of electricity used at a time.
P	The electricity price ceiling that the producer could set.
Variables	
Producer	
p_t	The decision on electricity price at the time t.
Consumers	
$e_t^{grid\to a}$	The decision variable describing the amount of electricity from the grid used by the load a at the time t.
$e_t^{sto\to a}$	The decision variable describing the amount of electricity from the storage used by the load a at the time t.
$e_t^{grid\to sto}$	The decision variable describing the amount of electricity from the grid used to charge the storage at the time t.
$e_t^{solar\to sto}$	The decision variable describing the amount of electricity from the solar PV used to charge the storage at the time t.
e_t^{sto}	The auxiliary variable descibing the remaining charge in the storage at the time t.

2.1 The consumers' problem

In this subsection, we shall describe, step-by-step, the constraints and cost function of the lower-level problem.

2.1.1 Storage model

The state of a storage s follows a dynamic equation

$$e_1^{sto} = 0, \tag{2.1}$$

and for each $t = 2, \cdots, T$:

$$e_t^{sto} = e_{t-1}^{sto} - \sum_{a=1}^{A} e_{t-1}^{sto\to a} + e_{t-1}^{grid\to sto} + e_{t-1}^{solar\to sto}, \tag{2.2}$$

incorporating the charge-discharge of s during each time step. For this dynamics to work, an initial state to each storage has to be given. Of course, for each $t = 1, \cdots, T$, we require the following capacity constraint

$$e_t^{sto} \leq \kappa, \tag{2.3}$$

availability constraint

$$\sum_{a=1}^{A} e_t^{sto\to a} \leq e_t^{sto}, \tag{2.4}$$

and the solar availability constraint for the storage charging

$$e_t^{solar\to sto} \le \sigma_t. \tag{2.5}$$

In either case (solar or grid charging), the charge increase is limited by its maximum charging rate of

$$e_t^{grid\to sto} + e_t^{solar\to sto} \le \rho. \tag{2.6}$$

2.1.2 Load models

For each load $a \in A$, it is required that its operation during the whole day covers its daily demand d_a. This reads

$$\sum_{t=1}^{T}(e_t^{grid\to a} + e_t^{sto\to a}) = d_a. \tag{2.7}$$

Moreover, the total electricity consumption at a time cannot exceed the specifications from the consumer's side, described by

$$\sum_{a=1}^{A} e_t^{grid\to a} \le \ell. \tag{2.8}$$

2.1.3 Cost function

For the compactness of the notation we shall write e_t, for each $t = 1, \cdots, T$, as the vector of all decision variables of the consumers, namely

$$e_t := (e_t^{grid\to a}, e_t^{sto\to a}, e_t^{grid\to sto}, e_t^{solar\to sto}, e_t^{sto}). \tag{2.9}$$

The total money-equivalent cost for electricity consumers can be given as a function of their load schedule decision (e_t):

$$\psi(e_t) := \sum_{t=1}^{T}\sum_{a=1}^{A}\Big[p_t(e_t^{grid\to a} + e_t^{grid\to sto}) + q_{a,t}(e_t^{grid\to a} + e_t^{sto\to a})\Big].$$

Note that the actual payment corresponds only to the term

$$\sum_{t=1}^{T}\sum_{a=1}^{A} p_t(e_t^{grid\to a} + e_t^{grid\to sto}),$$

while the remaining term

$$\sum_{t=1}^{T}\sum_{a=1}^{A} q_{a,t}(e_t^{grid\to a} + e_t^{sto\to a}),$$

describes the equivalent monetary inconvenience cost. This inconvenience cost $q_{a,t}$ maybe interpreted similar to a shadow price, which is an equivalent amount that the consumers would happily receive in order to trade away the inconvenience.

2.1.4 Full problem of consumers

Collecting all the constraints and cost of the electricity consumers, the lower-level problem could be summarized. In the following, the notations $\lambda_*^\dagger$ and $\nu_*^\dagger$ at the end of each constriant are the corresponding Lagrange multipliers which will be used in the MPCC reformulation, Section 3. Of course, all $\nu_*^\dagger$'s must be non-negative as they are multipliers of inequality constraints. Let us now state the lower-level problem:

$$\min_{(e_t)} \quad \psi(e_t)$$

s.t. Eq. (2.1) $\quad (\lambda_1^{(1)})$

For all $t = 2, \cdots, T$:

- Eq. (2.2) $\quad (\lambda_t^{(1)})$

For all $t = 1, \cdots, T$:

- Eq. (2.3) $\quad (\nu_t^{(1)})$
- Eq. (2.4) $\quad (\nu_t^{(2)})$
- Eq. (2.5) $\quad (\nu_t^{(3)})$
- Eq. (2.6) $\quad (\nu_t^{(4)})$
- Eq. (2.8) $\quad (\nu_t^{(5)})$

For all $a = 1, \cdots, A$:

$$\left\lfloor \text{Eq. (2.7)} \qquad (\lambda_a^{(2)}) \right.$$

For all $t = 1, \cdots, T$ and $a = 1, \cdots, A$:

$$\left\lfloor \begin{array}{ll} e_t^{grid\to a} \geq 0 & (\nu_t^{grid\to a}) \\ e_t^{sto\to a} \geq 0 & (\nu_t^{sto\to a}) \\ e_t^{grid\to sto} \geq 0 & (\nu_t^{grid\to sto}) \\ e_t^{solar\to sto} \geq 0 & (\nu_t^{solar\to sto}) \\ e_t^{sto} \geq 0. & (\nu_t^{sto}) \end{array} \right.$$

With respect to the consumers' variables, the above problem is a linear program parametrized by (p_t).

2.2 Producer's margin

The profit function of the producer can be formulated directly from its margin, i.e. the difference between the gain from selling electricity and the production cost. In this paper, we assume that the production cost is momentarily a linear function of the deliverable amount. Note that this momentary linear pricing is often assumed to roughly describe the rapid increasing of cost per unit in electricity production. The margin function thus renders as

$$\mu(p_t, e_t) := \sum_{t=1}^{T} \sum_{a=1}^{A} (p_t - c_t) \left(e_t^{grid\to a} + e_t^{grid\to sto} \right).$$

It is clear from the above formulation how the lower-level response would affect the profit of the producer even though it is only in control of his decision on the selling price. In particular, the response vector (e_t) depends on the leader's decision (p_t) in the sense that (e_t) is an optimal solution of the consumers' problem parametrized by (p_t).

2.3 Bilevel model for demand response modulation

We are now ready to assemble all the pieces together and form a complete bilevel model:

$$\begin{array}{ll} \underset{(p_t),(e_t)}{\text{Max}} & \mu(p_t, e_t) \\ \text{s.t.} & \text{For all } t = 1, \cdots, T: \\ & \left\lfloor \begin{array}{l} p_t \in [0, P], \\ e_t \in \Psi(p_t), \end{array} \right. \end{array} \tag{2.10}$$

where $\Psi(p_t)$ denotes the set of all global solutions of the lower-level problem when the parameter (p_t) is provided.

The key obstacle in the model Eq. (2.10) lies in the constraint $e_t \in \Psi(p_t)$. Even though all the involved constraints are linear, the constraint set for the bilevel programming Eq. (2.10) is, in general, nonconvex. Even worse, it is usually disconnected. Due to this infavorable property, off-the-shelf solvers are usually not helpful. To actually solve this model, we need to reformulate Eq. (2.10) as a tractable single-level program which we shall discuss in the next section.

3. A reformulation of Eq. (2.10) and a solution method

In this section, we deal with a reformulation of Eq. (2.10) as a single-level Mathematical Programming with Complementarity Constraints (MPCC). Even though the reformulation reduces the bilevel problem into a single-level problem and even though the lower-level problem is a linear program (parametrized by the leader's decision), the resulting reformulation is a nonconvexly constrained quadratic program. The nonconvexity here is a known issue for MPCC for as long as its own literature. In the second part of this section, we present a solution method called SOS1 that branches on the complementarity conditions instead of treating them as continuous nonconvex equality constraints.

3.1 MPCC reformulation of Eq. (2.10)

In what follows, we split the KKT system of the lower-level problemn into two parts, namely the *stationarity* and the *complementarity* parts. Recall that we have already introduced the multipliers in Section 2.1.4. Now, the stationarity part of the parametric KKT system for the lower-level problem can be derived as:

$$\begin{aligned} p_t + q_{a,t} + \lambda_a^{(2)} + \nu_t^{(5)} - \nu_t^{grid\to a} &= 0 \\ q_{a,t} + \lambda_{t+1}^{(1)} + \nu_t^{(2)} + \lambda_a^{(2)} - \nu_t^{sto\to a} &= 0 \\ q_{a,T} + \nu_T^{(2)} + \lambda_a^{(2)} - \nu_T^{sto\to a} &= 0 \\ p_1 + \nu_1^{(4)} - \nu_1^{grid\to sto} &= 0 \\ \lambda_t^{(1)} - \lambda_{t+1}^{(1)} + \nu_t^{(1)} - \nu_t^{(2)} - \nu_t^{sto} &= 0 \\ \lambda_T^{(1)} + \nu_T^{(1)} - \nu_T^{(2)} - \nu_T^{sto} &= 0 \\ \nu_1^{(3)} + \nu_1^{(4)} - \nu_1^{solar\to sto} &= 0 \\ p_t - \lambda_t^{(1)} + \nu_t^{(4)} - \nu_t^{grid\to sto} &= 0 \\ \lambda_t^{(1)} + \nu_t^{(3)} + \nu_t^{(4)} - \nu_t^{solar\to sto} &= 0, \end{aligned} \tag{3.1}$$

and the complementarity part as:

$$\begin{aligned} \nu_t^{(1)} [e_t^{sto} - \kappa] &= 0 \\ \nu_t^{(2)} \left[\sum_{a=1}^{A} e_t^{sto\to a} - e_t^{sto} \right] &= 0 \\ \nu_t^{(3)} [e_t^{solar\to sto} - \sigma_t] &= 0 \\ \nu_t^{(4)} [e_t^{grid\to sto} + e_t^{solar\to sto} - \rho] &= 0 \\ \nu_t^{(5)} \left[\sum_{a=1}^{A} e_t^{grid\to a} - \ell \right] &= 0 \\ \nu_t^{grid\to a} e_t^{grid\to a} &= 0 \\ \nu_t^{sto\to a} e_t^{sto\to a} &= 0 \\ \nu_t^{grid\to sto} e_t^{grid\to sto} &= 0 \\ \nu_t^{solar\to sto} e_t^{solar\to sto} &= 0 \\ \nu_t^{sto} e_t^{sto} &= 0, \end{aligned} \tag{3.2}$$

where all the indices t and a range over all $\{1, \cdots, T\}$ and $\{1, \cdots, A\}$ respectively, except for the last two equations of Eq. (3.1) in which $t = 2, \cdots, T$.

To capture the full lower-level problem of Section 2.1.4, we also need to include the primal and dual feasibility, i.e. $e_t \in \Psi(p_t)$ if and only if the lower-level problem is feasible and there are multipliers of the form $\lambda_*^\dagger \in \mathbb{R}$ and $\nu_*^\dagger \geq 0$ in which both Eq. (3.1) as well as Eq. (3.2) are satisfied.

For each (p_t), let us write $K(p_t)$ the feasible set of the lower-level problem parametrized by (p_t) and $\Phi(p_t)$ the set of all tuples (e_t, λ_t, ν_t) such that $e_t \in K(p_t)$ and both Eq. (3.1) and Eq. (3.2) are satisfied at (p_t), where for each $t = 1, \cdots, T$, the tuple (e_t, λ_t, ν_t) is given by Eq. (2.9) as well as

$$\lambda_t := (\lambda_t^{(1)}, \lambda_a^{(2)}) \in \mathbb{R}^{A+1},$$

and

$$\begin{aligned} \nu_t :=& (\nu_t^{(1)}, \nu_t^{(2)}, \nu_t^{(3)}, \nu_t^{(4)}, \nu_t^{(5)}, \\ & \nu_t^{grid\to a}, \nu_t^{sto\to a}, \nu_t^{grid\to sto}, \\ & \nu_t^{solar\to sto}, \nu_t^{sto}) \in \mathbb{R}_+^{2A+8}. \end{aligned}$$

Since the lower-level problem is a parametric linear programming, it is unconditionally equivalent to its KKT system parametrized by the same (p_t). Finally, the MPCC reformulation of Eq. (2.10) reads as follows:

$$\begin{aligned} \underset{(p_t),(e_t),(\lambda_t),(\nu_t)}{\text{Max}} \quad & \mu(p_t, e_t) \\ \text{s.t.} \quad & \text{For each } t = 1, \cdots, T: \\ & \left| \begin{array}{l} p_t \in [0, P] \\ (e_t, \lambda_t, \nu_t) \in \Phi(p_t). \end{array} \right. \end{aligned} \tag{3.3}$$

We emphasize again, following the discussion in [20], that as far as the global solutions of Eq. (2.10) and its corresponding MPCC Eq. (3.3)) are concerned, both problems are equivalent at the disposal of the multipliers.

3.2 Solving MPCC using the SOS1 approach

The special-ordered set of type 1 (briefly, SOS1) method can be used to handle nonconvex equality constraint of the form $f(u)g(u) = 0$, which describes that at most one of $f(u)$ and $g(u)$ is allowed to take a nonzero value. Equivalently, at least one of $f(u)$ and $g(u)$ must take a zero value. Instead of looking at $f(u)g(u) = 0$ as it is, the SOS1 method branches on this equation instead. That is, one may proceed using a branch-and-bound-type algorithm on the scenario trees branched on each of such constraints.

Applying this technique to our model Eq. (3.3), we branch on all the complementarity part of the KKT system Eq. (3.2). Hence, at each branch, the MPCC model Eq. (3.3) is reduced into a Quadratic Programming (QP) with linear and SOS1-type constraints [22, 23].

4. Numerical simulations, Discussion and Concluding remarks

Let us recapture the snapshots again that the model Eq. (2.10) is solved, in this section, through its MPCC reformulation Eq. (3.3) where the complementarity constraints are handled using an SOS1 technique. In the implementation, the model is constructed using the `JuMP` package from `Julia` programming language. Within the `JuMP` environment, we apply `Gurobi` (licensed for academic use) as the solver.

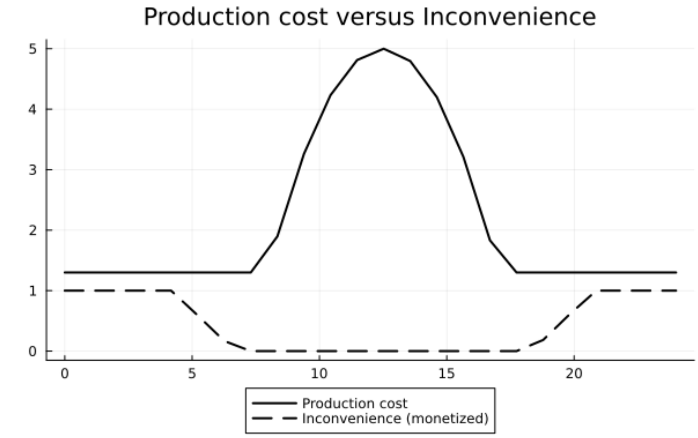
In the numerical implementation, we simulate the case where the shared solar farm has the capacity to cover 5% of the demand. Note that this is very typical in the situation in Thailand where the *enhanced single-buyer model* is adopted. We show in the following figures the numerical simulation of our model. First of all, we present the cost profile, inconvenience profile, and the solar profiles (this could be predicted from other statistical model) inputs as in Figs. 1a-1b. Note that the quadratic production cost was assumed following the idea of [17] while the quadratic inconvenience and solar profiles were adopted based on the idea of [24].

The time step is configured to 1 hour and T is set to cover 24 hours from midnight to another midnight. We consider the case with $A = 10$ consumers, each having their own randomized daily demand. Our MPCC model Eq. (3.3) carries a total of 1402 continuous variables, 264 quadratic terms and 672 SOS1 constraints to branch on.

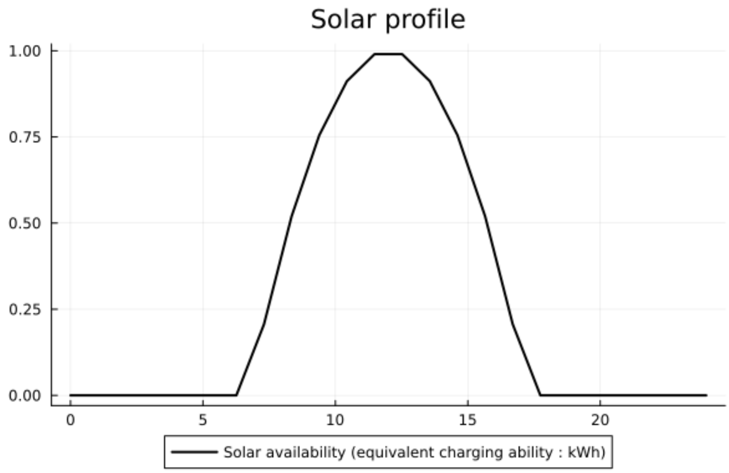
With these inputs, we solve the MPCC reformulation using SOS1 method with `Gurobi` solver with the time limit of 30 seconds. The solver was able to reduce the gap (in branch-and-bound sense) to 13.0496%. Note that beyond the first 15 seconds, we do not really see any significant improvement after another 30 minutes. The results obtained from the solver, namely the decisions of the seller on the hourly price (p_t) and of the consumers on the operations schedule (e_t) are shown in Figs. 2-3.

Looking at the optimal decisions of the seller and the consumers as depicted in Figs. 2-3, one may notice how the seller react to its production cost as well as how the consumers react to the prices set by the seller. Let us discuss this in greater detail.

- From the supply side, the seller needs to adjust to a higher price during the “peak” hours not just to regain its marginal profit, but at the same time to incentivize the consumers to shift their consumptions outside of these busy periods. Provided that the demand is fixed (or at least predetermined), if the consumption concetrates during the peak hours then



(a) Cost and monetized inconvenience profiles.

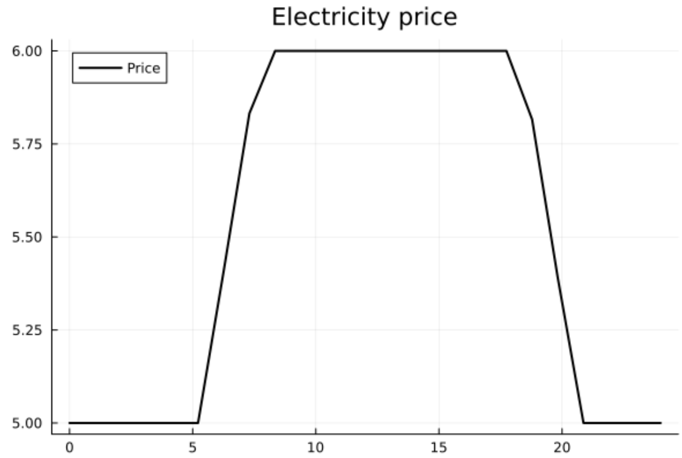


(b) Solar profile.

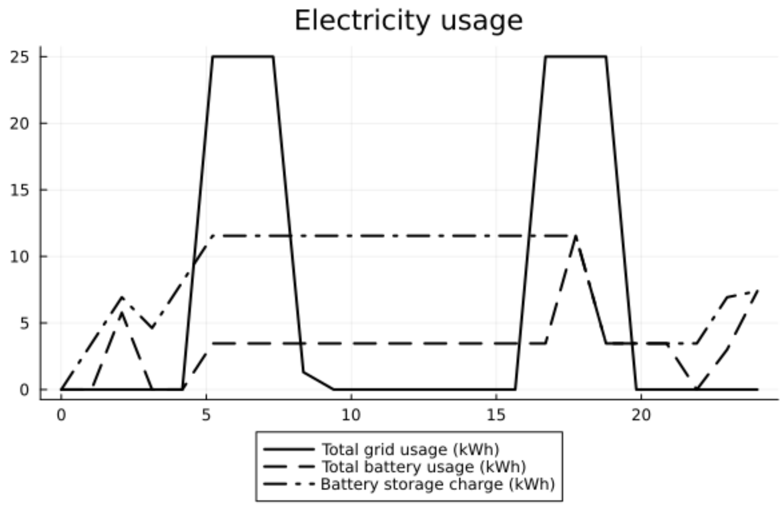
Fig. 1. Input profiles.

at the end of the day the producer will lose its profit due to the thin or even, under some circumstances, negative margin. One could compare also Figs. 1a-2a and their correlations between the production cost and the optimal selling prices. Note that the seller could have wanted to increase the price further, but the regulator's ceiling price P is in control here.

- On the other hand from the demand side, each consumer reacts to the given price by accordingly adjusting the operational period to avoid the "expensive" hours. Looking at the optimal consumption rates of each consumer (Fig. 3a) and also the total consumption of each source (Fig. 2b), one could obviously see that the consumers try their best to avoid these expensive hours. Also, note that the late night till early morning periods are also avoided due to their



(a) Optimal hourly price.

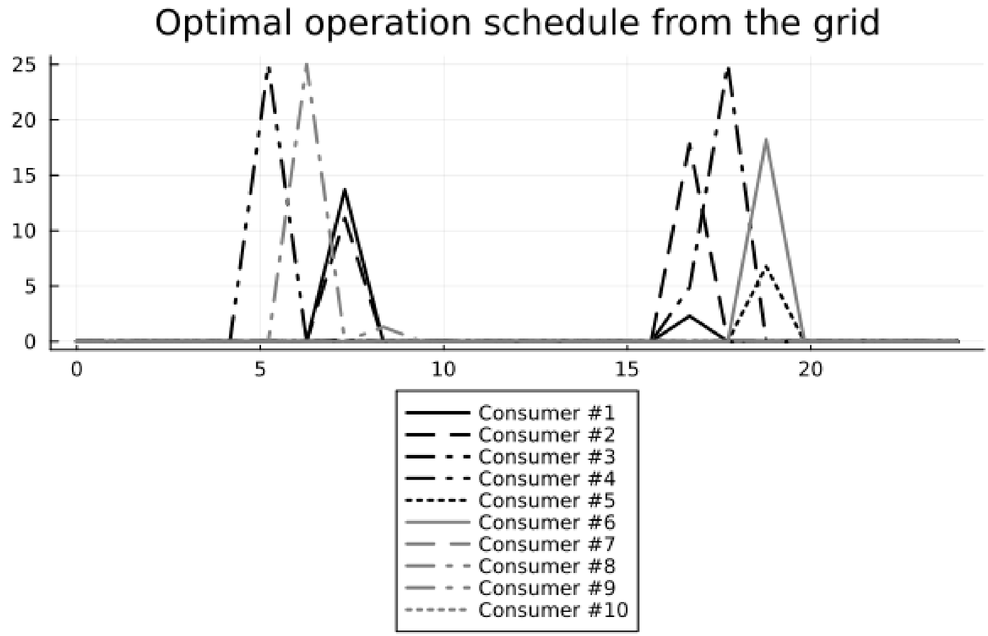


(b) Total elecricity usage under optimal planning.

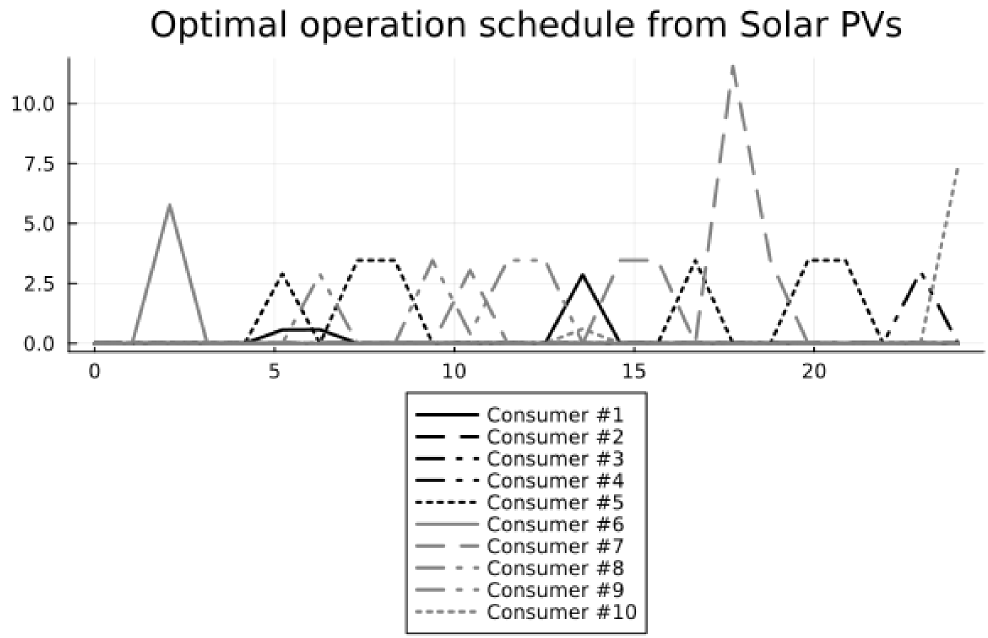
Fig. 2. Optimal solution from the model Eq. (3.3).

inconvenience as in Fig. 1a. To still fulfill its demand, the storage facility is charged during the "cheap" hours to be ready for use during the day. Likewise, the solar PV is applied as soon as the light is ready according to Figs. 1b-3b.

- Finally, we make an important economical remark for a monopoly market as we could observe. From our model Eq. (2.10), we highlight the feature that the retail prices (p_t) must not exceed the ceiling price P. Otherwise this problem Eq. (2.10) will be unbounded whenever the solar unit and energy storage facility are insufficient to cover all the demands. Indeed, due to the fact that the all the consumer's demands are unavoidably required to be fulfilled (see the constriant Eq. (2.7)), the producer could



(a) Optimal operations using grid energy.



(b) Optimal operations using PV energy.

Fig. 3. Cont. Optimal solution from the model Eq. (3.3).

raise the price as high as he wishes knowing that the consumers have no choices but to follow. This claim is generally true in practice under the current available technology. Also note that this ceiling is inevitable in the monopoly case as we adopt in this model. The price ceiling could be unnecessary if multiple producers are competing with their prices in the deregulated markets.

We make a concluding note here that distinguish between the supply-side aspect as seeing the "peak" hours as opposed to the demand-side that only sees the "expensive" hours. We could clearly observe from how our model behaves the market mechanism that occurs so naturally between the supply and demand sides. It could be seen by the bilevel structure that the supplier forces the consumers to convey to his strategy, but at the same time the consumers also convey, indirectly, to the supplier in the sense that they could react unpleasantly as a response.

Acknowledgments

This research is supported through the project "Optimizing Leader-Follower Models in Eco-Industrial Park Management" by King Mongkut's University of Technology Thonburi (KMUTT), Thailand Science Research and Innovation (TSRI) , and National Science, Research and Innovation Fund (NSRF) Fiscal year 2024.

References

[1] Siano P. Demand response and smart grids—A survey. Renewable and Sustainable Energy Reviews. 2014;30:461–478. Available from: https://www.sciencedirect.com/science/article/pii/S1364032113007211.

[2] O′Connell N, Pinson P, Madsen H, O′Malley M. Benefits and challenges of electrical demand response: A critical review. Renewable and Sustainable Energy Reviews. 2014;39:686–699. Available from: https://www.sciencedirect.com/science/article/pii/S1364032114005504.

[3] Calvillo CF, Sánchez-Miralles A, Villar J. Energy management and planning in smart cities. Renewable and Sustainable Energy Reviews. 2016;55:273–287. Available from: https://www.sciencedirect.com/science/article/pii/S1364032115012125.

[4] Beaudin M, Zareipour H. Home energy management systems: A review of modelling and complexity. Renewable and Sustainable Energy Reviews. 2015;45:318–335. Available from: https://www.sciencedirect.com/science/article/pii/S1364032115000568.

[5] Paterakis NG, Erdinç O, Catalão JPS. An overview of Demand Response: Key-elements and international experience. Renewable and Sustainable Energy Reviews. 2017;69:871–891. Available from: https://www.sciencedirect.com/science/article/pii/S1364032116308966.

[6] Panda S, Mohanty S, Rout PK, Sahu BK, Bajaj M, Zawbaa HM, et al. Residential Demand Side Management model, optimization and future perspective: A review. Energy Reports. 2022;8:3727–3766. Available from: https://www.sciencedirect.com/science/article/pii/S2352484722005479.

[7] Sangaré M, Bourreau E, Fortz B, Pachurka A, Poss M. Loads scheduling for demand response in energy communities. Comput Oper Res. 2023;160:14. Id/No 106358.

[8] Borges YGF, Schouery RCS, Miyazawa FK, Granelli F, da Fonseca NLS, Melo

LP. Smart energy pricing for demand-side management in renewable energy smart grids. Int Trans Oper Res. 2020;27(6):2760–2784.

[9] Garcés A. Mathematical programming for power systems operation. From theory to applications in Python. Hoboken, NJ: John Wiley & Sons; Piscataway, NJ: IEEE Press; 2022.

[10] Catanzaro D, Pesenti R, Ronco R. Job scheduling under time-of-use energy tariffs for sustainable manufacturing: a survey. Eur J Oper Res. 2023;308(3):1091–1109.

[11] Zhao D, Wang H, Huang J, Lin X. Time-of-use Pricing for Energy Storage Investment; 2021. Preprint, arXiv:2112.06358 [eess.SY] (2021).

[12] Cheng J, Chu F, Chu C, Xia W. Bi-objective optimization of single-machine batch scheduling under time-of-use electricity prices. RAIRO, Oper Res. 2016;50(4-5):715–732.

[13] Rossit DG, Nesmachnow S, Toutouh J, Luna F. Scheduling deferrable electric appliances in smart homes: a bi-objective stochastic optimization approach. Math Biosci Eng. 2022;19(1):34–65.

[14] Yang H, Zhang X, Chu Y, Ma Y, Zhang D, Guerrero JM. Multi-objective based demand response strategy optimization considering differential demand on reliability of power system. International Journal of Electrical Power & Energy Systems. 2023;152:109202. Available from: https://www.sciencedirect.com/science/article/pii/S0142061523002594.

[15] Capone M, Guelpa E, Verda V. Multi-objective optimization of district energy systems with demand response. Energy. 2021;227:120472. Available from: https://www.sciencedirect.com/science/article/pii/S0360544221007210.

[16] Brotcorne L, Lepaul S, von Niederhäusern L. A Rolling Horizon Approach for a Bilevel Stochastic Pricing Problem for Demand-Side Management; 2021. Preprint, arXiv:2102.13634 [math.OC] (2021).

[17] Aussel D, Brotcorne L, Lepaul S, von Niederhäusern L. A trilevel model for best response in energy demand-side management. Eur J Oper Res. 2020;281(2):299–315.

[18] Alekseeva E, Brotcorne L, Lepaul S, Montmeat A. A bilevel approach to optimize electricity prices. Yugosl J Oper Res. 2019;29(1):9–30.

[19] Hobbs BF, Nelson SK. A nonlinear bilevel model for analysis of electric utility demand-side planning issues. Ann Oper Res. 1992;34:255–274.

[20] Dempe S, Dutta J. Is bilevel programming a special case of a mathematical program with complementarity constraints? Math Program. 2012;131(1-2 (A)):37–48.

[21] Kleinert T, Labbé M, Plein F, Schmidt M. Technical note – There's no free lunch: on the hardness of choosing a correct big-M in bilevel optimization. Oper Res. 2020;68(6):1716–1721.

[22] Kleinert T, Schmidt M. Why there is no need to use a big-M in linear bilevel optimization: a computational study of two ready-to-use approaches. Comput Manag Sci. 2023;20:12. Id/No 3.

[23] Aussel D, Egea C, Schmidt M. A Tutorial on Solving Single-Leader-Multi-Follower Problems using SOS1 Reformulations. Preprint.

[24] Tsui KM, Chan SC. Demand Response Optimization for Smart Home Scheduling Under Real-Time Pricing. IEEE Transactions on Smart Grid. 2012;3(4):1812–1821.