



Implementing the Taguchi-Statistical Learning-DEAR Methodology in a Multi-Criteria Decision Making Approach to Balance Trade-offs in Evolutionary Algorithm Performance

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ABSTRACT

This work presents a new approach to improving the efficiency of Evolutionary Algorithms (EA) in extremely noisy function landscapes. Statistical Learning, Taguchi analysis and normalization, and Data Envelopment Analysis based Ranking (DEAR) are used to provide a hybrid technique that provides a complete framework for EA parameter adjustment. The study examines the effects of four important EA parameters on function yield and computing time: convergence, mutation rate, population size, and random seed. Taguchi analysis and normalization is used to generate an efficient experimental design that covers different combinations of parameter values, allowing a methodical exploration of the parameter space. Subsequently, the DEAR approach is employed to prioritize each set of parameters according to certain optimization criteria. To further complicate matters, the optimization goals and EA parameters are both modeled using Statistical Learning approaches. There has been a lot of testing with noisy functions of artificial landscapes with three different types: single-peak, curved-ridge, and multi-peak. Assuming a normally distributed distribution with a mean of 0 and standard deviations of 0.05 and 0.2, noise presents practical obstacles to optimization. When compared to more traditional approaches of parameter tuning, the suggested hybrid strategy clearly outperforms the competition in terms of computing time, function yield, and mean and standard deviation of both metrics. The technique shows improved resilience and adaptability across varied noisy environments and more successfully finds optimal parameter

configurations, according to the results. By demonstrating its flexibility to meet evolving optimization needs, sensitivity assessments provide more evidence of the suggested methodology's dependability. Finally, the research presents a state-of-the-art hybrid method for tweaking evolutionary algorithm parameters, which considerably improves upon previous efforts. In particular, when it comes to dealing with complicated and noisy optimization scenarios, the offered technique stands out due to its capacity to continuously produce greater performance, making a vital addition to the optimization community.

Keywords: Artificial landscape; Data Envelopment Analysis based Ranking (DEAR); Evolutionary algorithm; Statistical learning; Taguchi analysis; Parameter tuning

1. Introduction

Evolutionary Algorithms, often known as EAs, are a remarkably adaptable and effective optimization method that may be used for a wide variety of applications. The setting of important parameters like convergence, mutation rate, population size, and random seed has a considerable impact on the functioning of these systems [1-3]. In the presence of noisy function landscapes, which are characteristic of optimization problems that occur in the real world, the difficulty of locating optimum parameter configurations becomes more obvious.

The incorporation of Multi-Criteria Decision Making (MCDM) incorporates a degree of sophistication into our approach by concurrently taking into consideration numerous objectives that are in competition with one another. In this way, it is ensured that the parameter configurations that are selected not only maximize the individual criteria, but also achieve a balance between the conflicting objectives. With the use of the MCDM framework, it is possible to conduct a more thorough analysis of the trade-offs that are involved in parameter tuning, which ultimately results in optimization solutions that are more robust and well-rounded.

This study presents an innovative and complete methodology for maximizing the performance of EAs under the complicated constraints provided by noisy function landscapes. The methodology includes a number of different approaches.

Taguchi analysis and normalization, Data Envelopment Analysis based Ranking

(DEAR), and Statistical Learning are the three components that make up this methodology [4-6]. Its purpose is to tackle the complex issue of parameter tuning in a methodical and effective manner.

The most important contribution that this study makes is the creation of a hybrid optimization technique that incorporates Taguchi analysis and normalization, DEAR, and Statistical Learning. This methodology provides a synergistic approach to the problem of parameter tuning in evolutionary algorithms, which has been a hurdle for a very long time. A deeper understanding of EA behavior in noisy settings may be obtained via the utilization of this technique. This is accomplished by systematically exploring the parameter space and taking into consideration the influence of important parameters on both function yield and computational time [7-9].

An optimum experimental design is constructed by the utilization of Taguchi analysis, which is utilized in the study to conduct a systematic investigation of the four important EA parameters [10]. Following this, the DEAR approach is utilized to rate the effectiveness of parameter settings according to a number of different optimization criteria, so delivering an all-encompassing evaluation [11]. In addition, tools from the field of statistical learning are utilized in order to discover the intricate correlations that exist between optimization aims and EA parameters, thereby contributing to a more profound comprehension of the dynamics that lie under

the surface.

Noise is represented as being normally distributed with the mean of zero and different standard deviations or $NID(0,\sigma)$ over the entirety of the study. Noisy functions of diverse natures, each with distinct variations, are taken into consideration during the overall investigation. The application of the findings to real-world optimization settings is improved by the realistic description of noisy landscapes that were used.

Detailed information on the artificial landscape, the hybrid Taguchi-Statistical Learning-DEAR approach, the experimental setting, and the findings that were achieved are described in the following sections. In addition to highlighting the superior performance of the approach in terms of mean and standard deviation for function yield and computational time, the results also position it as a promising advancement in the field of evolutionary algorithm parameter tuning, particularly when it comes to the tuning of parameters under noisy conditions.

2. Noisy Artificial Landscapes

The performance of optimization algorithms under controlled settings may be evaluated using artificial landscapes, which serve as vital testbeds for optimizing algorithms. In this part of the article, we will discuss the process of creating artificial landscapes and the features of these landscapes, which are intended to simulate real-world optimization difficulties. The hybrid technique that has been presented for optimizing evolutionary algorithms (EAs) across a variety of scenarios may be evaluated thanks to these landscapes, which are of great assistance.

A single, clearly defined peak is the defining characteristic of the single-peaked landscape, which is a representation of optimization issues (Fig. 1). In situations when there is only one optimal solution, this structure is frequently found in the issue. The single-peaked landscape is created in three

separate forms, each of which incorporates varying degrees of complexity in terms of the smoothness of the terrain and the sharpness of the peak.

$$f_1(x_1, x_2) = x_1^2 + 2x_2^2 - 0.3\cos 3\pi x_1 - 4\cos 4\pi x_2 + 0.7, \quad (2.1)$$

$$f_2(x_1, x_2) = \left[1 + \left(\frac{x_1}{50} + \frac{x_2}{50} \right)^2 \right. \\ \times \left(19 - \frac{14x_1}{50} + 3 \left(\frac{x_1}{50} \right)^2 - \frac{14x_1}{50} + \frac{6x_1x_2}{2500} \right. \\ \left. + 3 \left(\frac{x_1}{50} \right)^2 \right] \times \left[30 + \left(\frac{2x_1}{50} + \frac{3x_2}{50} \right)^2 \left(18 - \frac{32x_1}{50} \right. \right. \\ \left. \left. + 12 \left(\frac{x_1}{50} \right)^2 + \frac{48x_2}{50} + \frac{36x_1x_2}{2500} \right) + 27 \left(\frac{x_2}{50} \right)^2 \right], \quad (2.2)$$

$$f_3(x_1, x_2) = \left| \left(\frac{x_1}{2} \right)^2 + \left(\frac{x_2}{2} \right)^2 + \frac{x_1x_2}{4} \right| \\ + \left| \sin \left(\frac{x_1}{2} \right) \right| + \left| \cos \left(\frac{x_2}{2} \right) \right| \quad (2.3)$$

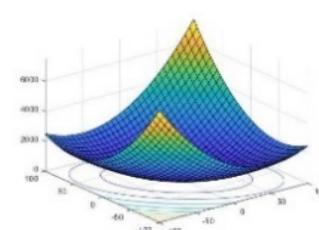
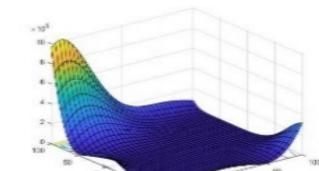
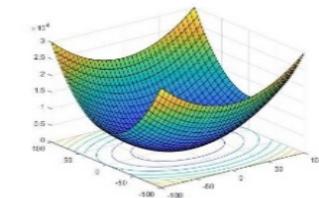


Fig. 1. Times New Roman size 10 point. Same is true for remaining figures.

A more complex structure is introduced by the curved ridge landscape, which consists of a continuous curved ridge along which optimal solutions are located

(Fig.2). This landscape is a reflection of optimization issues that involve several optimum zones that are interrelated. Once more, three distinct variations of the curving ridge landscape are built in order to capture and convey varied degrees of intricacy.

$$f_4(x_1, x_2) = \left[1 - \left(\frac{x_1}{50} \right)^2 \right] - 100 \left[\left(\frac{x_2}{50} \right) - \left(\frac{x_1}{50} \right) \right], \quad (2.4)$$

$$f_5(x_1, x_2) = \frac{-100}{10 \left[\left(\frac{x_1}{200} + 1 \right)^2 + \left(\frac{x_2}{200} + 1 \right)^2 + \left(\frac{x_1}{200} + 1 \right)^2 + 4 \right]}, \quad (2.5)$$

$$f_6(x_1, x_2) = 2 \left(\frac{x_1}{20} \right)^2 - 1.05 \left(\frac{x_1}{20} \right)^4 + \frac{\left(\frac{x_1}{20} \right)^6}{6} + \frac{x_1 x_2}{400} + \left(\frac{x_2}{20} \right)^2. \quad (2.6)$$

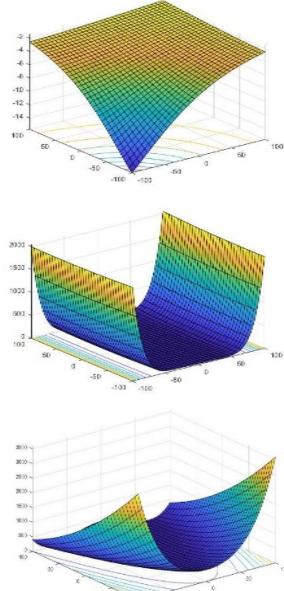


Fig. 2. Curved ridge artificial landscapes: Eqs. (2.4), (2.5) and (2.6), respectively.

There are several optimum solutions or local optima that are represented by the multi-peak landscape (Fig. 3), which indicates optimization problems. Problems that occur in the real world and include a variety of conflicting agendas sometimes involve such landscapes. During the generation process, three distinct variations of the multi-peak landscape are produced, each of which features a distinct arrangement and distribution of peaks.

$$f_7(x_1, x_2) = \frac{\left\{ \left[\left(\frac{x_1}{25} \right)^2 - 1 \right] \left[\left(\frac{x_2}{25} \right)^2 - 4 \right] + \left(\frac{x_1}{25} \right)^2 + \left(\frac{x_2}{25} \right)^2 - 5 \right\}}{\left[\left(\frac{x_1}{25} \right)^2 + \left(\frac{x_2}{25} \right)^2 + 1 \right]}, \quad (2.7)$$

$$f_8(x_1, x_2) = \left[4 + 2.1 \left(\frac{x_1}{100} \right)^2 + \left(\frac{x_1}{100} \right)^2 \times \left(\frac{x_1}{100} \right)^2 + \left[\frac{x_1 x_2}{10000} + 4 \left(\frac{x_2}{100} \right)^2 + 4 \left(\frac{x_2}{100} \right)^4 \right], \quad (2.8)$$

$$f_9(x_1, x_2) = (x_1) \sin[4\pi(x_1)] - (x_2) \sin[4\pi(x_2) + \pi] + 1. \quad (2.9)$$

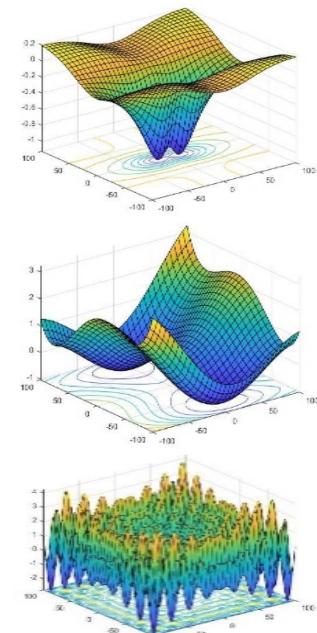


Fig. 3. Multi-peak artificial landscapes: Eqs. (2.7), (2.8) and (2.9), respectively.

It is necessary to incorporate noise into each landscape in order to emulate the uncertainties that are prevalent in real-world optimization applications. For the purpose of modeling the noise, random variables with a mean of zero and a normal distribution are used. 0.05 and 0.2 are the two separate standard deviations that are utilized in order to visually illustrate the various levels of noise intensity. An accurate portrayal of noisy landscapes with varying degrees of

complexity may be achieved through the use of this method.

Visual representations are supplied for each nature and variety in order to facilitate the process of comprehending the landscape structures. In order to demonstrate the topography of the landscapes, contour plots and three-dimensional surface plots are utilized. These plots highlight the peaks, ridges, and valleys. By providing useful insights into the issues that are offered by each artificial environment, these visualizations serve as powerful tools.

The generation of artificial landscapes serves two purposes: first, it offers a controlled environment for systematically evaluating the proposed methodology, and second, it makes it easier to comprehend how the hybrid approach reacts to different optimization problem structures and noise levels. Both of these functions are accomplished through the generation of artificial landscapes.

At the end of this section, an emphasis is placed on the significance of artificial landscapes in the appraisal process. In the following sections, we will make use of various landscapes in order to evaluate the effectiveness of the hybrid Taguchi-Statistical Learning-DEAR-MCDM technique in improving EAs while taking into account the impact of noise and a variety of landscape structures.

3. Taguchi-Statistical Learning-DEAR Methodology (TSL-DEAR)

3.1 Data Envelopment Analysis based Ranking (DEAR)

To determine which decision-making units (DMUs) in a collection are the most efficient, Data Envelopment Analysis (DEA) is a powerful tool. DEAR, an expansion of DEA, ranks these units systematically according to their efficiency scores, taking the assessment to the next level. Understanding and measuring the performance of entities is vital in different domains, including healthcare, operations

management, and finance, where this method is particularly beneficial [12, 13].

The first step in DEAR is to give each DMU an efficiency score. A useful tool for performance measurement, these scores indicate how well each unit uses its inputs to create outputs. In order to rank the DMUs, DEAR uses the efficiency scores [14]. The examined set has a distinct hierarchy of performance, with the highest efficiency ratings corresponding to the most efficient units.

DEAR does more than just rate units; it also finds the most efficient entities' best practices. Organizations may get valuable insights from high-performing individuals and use tactics to boost their own efficiency. By providing a ranking, DEAR makes benchmarking easier, letting businesses see how they stack up against competitors. This data can help decision-makers find places to improve and how to best allocate resources [15].

Banking, healthcare, and education are just a few of the many fields that make use of DEAR. For example, DEAR may be used in the banking industry to evaluate various departments' performance, which in turn can help with allocation of resources and process improvement strategies [16, 17]. In the healthcare industry, DEAR may help evaluate clinics and hospitals to find the most effective ways to improve patient care.

There are several difficulties with using DEAR, despite the fact that it gives useful information. During the DEAR process, it is important to carefully analyze issues like as input/output sensitivity, model choice, and probable outliers. We will explore case studies, techniques, and new trends in performance evaluation and efficiency ranking utilizing Data Envelopment Analysis as we dig deeper into the subtleties of DEAR in the following sections.

3.2 Taguchi analysis and normalization

The Taguchi signal-to-noise ratio proves to be a very effective analytical tool when trying to improve operational processes and get useful insights. Genichi Taguchi's technique goes above and beyond conventional data analysis by providing a unique vantage point from which to assess the efficacy and quality of operational inputs and outputs.

A signal-to-noise ratio (SN), which goes beyond simple numerical values as an extra measure, is transformed from raw data as part of Taguchi's process. In order to differentiate between the ideal signal, which represents the intended outcomes, and the undesirable operational noise, which may reduce efficiency, Sensor Network (SN) technology is used. Taguchi's approach reveals hidden details and patterns in data by focusing on the important information rather than the irrelevant details. Because of this, we can grasp the dynamics of processes with higher complexity [18, 19].

Compared to traditional assessments, Taguchi's technique performs better when factors like processing time and landscape yields are considered. The SN gives a far more accurate picture of the variables influencing these outputs by eliminating operating noise and isolating the ideal signal. This process allows for a deeper comprehension, providing direction for enhancements and optimizations that boost overall productivity.

Not only are the Taguchi noise-to-signal-to-ratios useful for statistical purposes, but they are also indicators for evaluating operational excellence. Organizations may uncover hidden efficiencies and make educated decisions by using a methodology that involves dissecting operational data and focusing on the optimum signal. With the ongoing discussion, Taguchi's SN will have a major influence, helping to establish a better grasp of the complex operational dynamics inherent to the noisy artificial landscapes.

While response analysis has always made use of the mean of responses (y_i) for n repetitions, there is a growing interest in response variability in the modern day. As an additional metric for the available variance, Taguchi modified the repeatability data. A signal-to-noise ratio, abbreviated as SN, is used to describe the phenomenon described above. This leads SN to collect a mountain of data that is identical to itself. Three data classes—"nominal is optimal (NTB)," "small is better (STB)," and "large is better (LTB)"—form the basis of the SN. Here are the particular SN mathematical models that were used for the STB (SN_{STB}): in this research:

$$SN_{STB} = -10 \log_{10} \left[\sum_{i=1}^n y_i^2 \right]. \quad (3.1)$$

When optimizing processes with an emphasis on landscape yields and computing time, the integration of Taguchi Signal-to-Noise Ratio (SN) and normalization produces notable results. Taguchi's method offers a thorough framework for dealing with these important output aspects; it is well-known for improving operational procedures.

For this purpose, the Taguchi SN is an essential analytical tool; it measures system performance by comparing the landscape yields—the intended signal—to the noise—variability or deviation. An alternative viewpoint on operational output quality is provided by this measure. For input variables like landscape yields and computational time, which might have different units and scales, normalization is an important tool to have on hand, along with the signal-to-noise ratio.

The impact of each component may be fairly assessed by normalization, which involves adapting each variable to a similar scale. This eliminates the influence of various scales. If landscape yields and computing time are critical output factors in an optimization process, then this is of paramount importance. Applying

normalization and Taguchi's SN Ratio together strengthens the optimization procedure [20].

When comparing traditional evaluations to the combined Taguchi approach, the latter performs better in terms of computing time and landscape yields. With normalization at its side, the SN makes it easier to see how different factors affect landscape yields and reduces the effect of operational noise on computing time. The intricate dynamics of landscape results and computing efficiency may be better understood with the help of this all-encompassing technique. SN normalization below is an exceptionally effective method for reducing the size of data sets while preserving proportionality or similarity in their dimensions.

$$SN_{norm_{ki}} = \frac{\max(SN_i) - SN_{ki}}{\max(SN_i) - \min(SN_i)}. \quad (3.2)$$

where $SN_{norm_{ki}}$, is the normalized value for the value associated with the k th DMU and output in column i , SN_{ki} is the value of the i th output in the k th DMU, $\max(SN_i)$ and $\min(SN_i)$ are the maximal and minimal SN levels for all DMUs of the i th output.

Optimizing landscape yields while reducing computing time is the practical focus of the integrated method, which directs advancements and optimizations. For a more precise evaluation of aspects impacting landscape results and computing efficiency, it is possible to normalize input variables in Taguchi experiments. This makes the studies less vulnerable to scale changes.

The combination of Taguchi SN and normalization is, last but not least, quite effective when optimizing processes with a close watch on landscape yields and computation time. This comprehensive approach enables the attainment of peak performance in complex operational settings by ensuring improved output quality and efficient utilization of computing resources.

3.3 Statistical learning

The ever-evolving area of statistical learning uses computer tools and statistical methodologies to derive meaningful insights from data. Statistical learning's fundamental goal is to instruct models to detect patterns in data and use those patterns to generate predictions or judgments. Discovering underlying structures, establishing correlations between variables, and drawing educated conclusions are all part of this process.

When an algorithm is trained on a labeled dataset, it learns the connection between input characteristics and their associated output labels. This process is called supervised learning. Classification (putting data into predetermined groups) and regression (predicting a continuous result) are two examples of typical tasks. Without prior knowledge of the results, unsupervised learning attempts to classify or find patterns in unlabeled data. Examples of common unsupervised learning problems include dimensionality reduction and clustering.

Statistical learning incorporates a wide variety of models, from simple linear models to advance non-linear techniques. Models such as neural networks, decision trees, and support vector machines may capture complex relationships within the data.

The bias-variance tradeoff is a cornerstone of statistical learning. Variance measures how much a model changes in response to changes in the training data, whereas bias describes how often the model under- or overestimates. Finding the sweet spot is essential for the best possible model output.

When testing a model on data it has never seen before, robust evaluation methods like cross-validation are invaluable. You can see how well the model applies to new data by looking at metrics like accuracy, precision-recall curves, or mean squared error.

The effectiveness of statistical learning models relies heavily on the engineering and skillful selection of features. If you want your model to be better at capturing patterns and making predictions, you need to find the right characteristics and change them.

Following this, we will examine the DEAR and Taguchi Analysis and Normalization in detail, as well as case studies and new developments in statistical learning, so that you can fully grasp its theoretical foundations, practical applications, and revolutionary insights that it can provide.

3.4 Novel method of TSL-DEAR

Identification of optimal parameter levels is accomplished by the utilization of Taguchi analysis, normalization, data envelopment analysis based ranking (DEAR), and statistical learning, which are all strengths of the hybrid technique that has been provided. The parameters in complex systems may be optimized using this all-encompassing method, which provides a framework that is both systematic and efficient.

For Taguchi analysis and normalization; the process begins with Taguchi Analysis, which makes use of its rigorous experimental design principles to conduct a systematic investigation into the effect that parameters have on the performance of the system. While this is going on, normalization is being used in order to normalize the range of parameters, which guarantees that the evaluation will be fair and objective. The combination of these two factors makes it possible to determine the ideal parameter settings, hence reducing the influence of changes and increasing the robustness of subsequent analyses.

For Data Envelopment Analysis based Ranking (DEAR); after Taguchi analysis and normalization, the next phase involves using DEAR to evaluate the efficiency of the system. DEAR is responsible for

determining the relative effectiveness of decision-making units (DMUs) by utilizing the parameters that have been tuned. A clear hierarchy of performance is provided by this efficiency rating, which identifies units that demonstrate the most effective operational procedures. The DEAR acts as a great benchmarking tool, assisting decision-makers in gaining a grasp of the efficiency landscape and possible areas for improvement [21].

The DEA study usually uses two popular models: Banker, Charnes, and Cooper (BCC)'s Variable Returns to Scale model and CCR's Constant Returns to Scale model. These models behave differently depending on data and outcome. This study evaluates utilizing CCR and BCC models. The CCR model assumes all DMUs have the same scope of operation. The efficiency boundary is a CCR model hyperplane. An operationally effective DMU is on the boundary line; one below is inefficient. As the DMU moves away from the barrier, its performance score decreases.

In this research, the following model provides an initial assessment of efficiency. The best practice frontier is set up by it, and other DMUs use it as a standard for efficiency (θ_{CRS}^k). Based on all m inputs and s outputs, the mathematical model of the k th DMU looks as follows in this calculation:

$$\begin{aligned}
 & \text{Min } \theta_{CRS}^k \\
 & \text{S.T.} \\
 & \sum_{j=1}^n \lambda_j x_{ij} \leq \theta_{CRS}^k x_{ik} ; i=1, 2, \dots, m \\
 & \sum_{j=1}^m \lambda_j y_{rj} \geq y_{rk} ; r=1, 2, \dots, s \\
 & \theta, \lambda_j \geq 0, j \neq 0.
 \end{aligned} \tag{3.3}$$

The coefficients and variables employed to evaluate and enhance the effectiveness of decision-making units via the DEA are presented in Table 1.

Table 1. Definition of DEA coefficients and variables.

Coefficients and Variables	Definition
λ_j	Intensity of DMU j^{th}
x_{ij}	Value of input factor i^{th} at the j^{th} DMU
x_{ik}	Value of input factor i^{th} at the k^{th} DMU
y_{rj}	Value of output factor r^{th} at the i^{th} DMU
y_{rk}	Value of output factor r^{th} at the k^{th} DMU
θ_{CRS}^k	Overall efficiency rating value of at the k^{th} DMU

For statistical learning (multiple regression); the last step adds statistical learning, especially multiple linear regression with an expansion of interaction effects, into the hybrid technique (Eq. (3.4)). Using the optimal parameter settings and efficiency rankings as a foundation, multiple regression models are created in order to make predictions about the outcomes of the system. This predictive capacity provides insights into how fluctuations in parameters affect the overall performance of the system, making it a significant tool for decision-making and strategic planning applications.

$$\begin{aligned} \theta_{CRS}^k = & \beta_0 + \beta_1 X_A + \beta_2 X_B + \beta_3 X_C \\ & + \beta_4 X_D + \beta_5 X_A X_B + \beta_6 X_A X_B \\ & + \beta_7 X_A X_D + \beta_8 X_B X_C \\ & + \beta_9 X_B X_D + \beta_{10} X_C X_D + \varepsilon_{ik}, \end{aligned} \quad (3.4)$$

where X_A, X_B, X_C, X_D are EA parameters levels and the respective parameters of A, B, C , and D , are convergence, mutation rate, population size, and random seed.

Multiple linear regression analysis often involves determining predictor variable significance. Marginal and partial tests are often used for this. Partial tests take additional model variables into account to establish predictor variable significance. These analyses reveal how each predictor variable affects the dependent variable. In contrast, marginal tests evaluate a predictor variable without considering other model

variables. The independent contribution of each predictor variable to the dependent variable is assessed. Multiple linear regression models need marginal and partial tests to understand the connections between predictor variables and the dependent variable, helping researchers find relevant predictors and interpret results more precisely. The T-test is often used as a partial test in multiple linear regression analysis to assess predictor variables' significance while taking into account model variables. By examining whether a predictor variable's coefficient varies substantially from zero, the T-test indicates (Eq. 3.5) if it affects the dependent variable statistically. Dividing the calculated coefficient by its standard error yields the test statistic, the t-value; where b_j is determined via the regression analysis. A t-value's p-value indicates its probability under the null hypothesis, which claims that the coefficient is zero. A modest p-value, generally below 0.05, shows a substantial relationship between the predictor and dependent variables. T-tests for each predictor variable in the model help academics identify which factors explain dependent variable variations in a unique and statistically meaningful manner. This improves understanding of the dataset's core linkages.

$$t_j = \frac{b_j - \text{hypothesized value of } \beta_j}{\text{Standard Error } (b_j)}, \quad (3.5)$$

From diagram (Fig.4) below, the research procedure involves investigating the optimization of artificial landscapes using evolutionary algorithms, Taguchi statistical learning, data envelopment analysis, and multiple regression. Initially, the varying levels of four EA factors are defined as: Convergence, Mutation rate, Population size, and Random Seed. Subsequently, algorithms from MATLAB codes were proceeded to sophisticated implementation for ascertaining optimal values (yield) and computing time of artificial landscapes while accommodating

inherent noise. Then, data were transformed via Taguchi statistical learning into signal-to-noise ratios, coupled with DEA to evaluate efficiency via CCR, thereby delineating the best practice frontier. Furthermore, the data also underwent multiple regression to establish intricate relationships between DEA efficiency and EA parameters, conducting meticulous significance tests to discern the nuanced impact of each parameter. Finally, guided by insights gleaned from the regression analysis, we undertake EA optimization endeavors to pinpoint optimal parameter configurations. This methodical and comprehensive approach aims to streamline the optimization process of artificial landscapes by amalgamating diverse statistical and optimization methodologies.

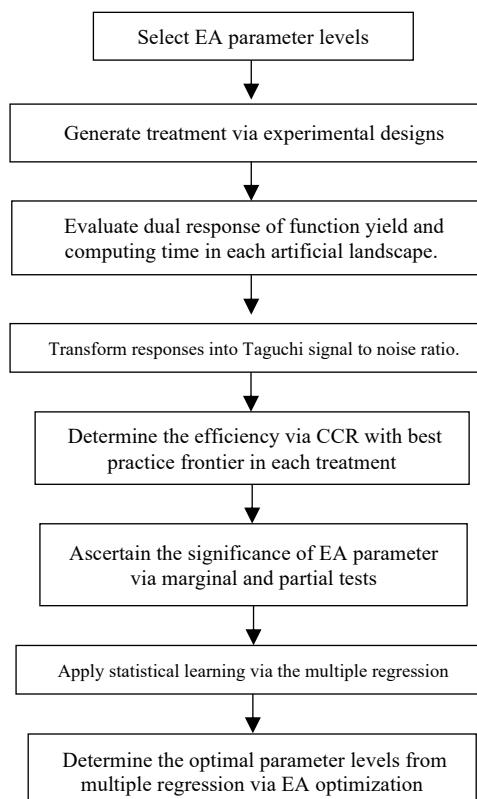


Fig. 4. Diagram of Taguchi-Statistical Learning-DEAR Methodology (TSL-DEAR).

In order to ascertain the resilience and efficacy of the experimental configuration, an initial investigation was undertaken to examine the influence of different parameters of the evolutionary algorithm (EA) on the optimization procedure. The initial inquiry encompassed a methodical manipulation of critical parameters, including mutation rate, crossover rate, and population size, with continuous monitoring of the resultant performance metrics. The objective was to identify parameter configurations that produced favorable results in terms of convergence speed and solution quality by means of experimental design and analysis. Particular attention was devoted to the values 0.05 and 0.075 for the mutation rate parameter (Table 2). This decision was made on the basis of preliminary experimental findings that suggested these values exhibited favorable performance characteristics across various instances of the problem. The initial investigation functioned as a fundamental basis for the subsequent optimization trials, providing direction for the choice of parameter values and guaranteeing the effectiveness of the methodology in fulfilling the research goals.

Before implementing Taguchi's method, a two-level factorial design was constructed, consisting of sixteen treatments. These treatments represented various combinations of critical parameters, including mutation rate, crossover rate, and population size. The objective of this methodical investigation was to conduct a comprehensive analysis of the impacts that these parameters had on the optimization procedure. Following this, function yield and computation time, two crucial responses, were evaluated for each of the generated treatments. In order to assess the efficacy of every treatment, Taguchi signal-to-noise ratio analysis was applied. This technique enabled the conversion of response values into output values corresponding to individual DMU. By employing this

methodology, it was possible to determine the most effective parameter configurations that reduced the variability of the responses, thus improving the evolutionary algorithm's overall performance.

Table 2. Example of generated data of dear in case of curved ridge optimum.

Generated Data of DEAR Methodology								
Parameters of Evolutionary Algorithm (EA)			Yield Result		Computing Time (second)			
Convergence	Mutation Rate	Population Size	Random seed	Yield (1)	Yield (2)	Time (1)	Time (2)	
0.0001	0.05	15	0	-15.72	-15.70	3.28	3.27	
0.001	0.05	15	0	-15.79	-15.69	3.43	3.20	
0.0001	0.075	15	0	-15.74	-15.70	3.19	3.27	
0.001	0.075	15	0	-15.63	-15.72	3.25	3.23	
0.0001	0.05	20	0	-15.65	-15.67	3.20	3.27	
0.001	0.05	20	0	-15.71	-15.71	3.23	3.32	
0.0001	0.075	20	0	-15.65	-15.63	3.26	3.21	
0.001	0.075	20	0	-15.73	-15.65	3.34	3.25	
0.0001	0.05	15	2	-15.75	-15.72	3.30	3.38	
0.001	0.05	15	2	-15.71	-15.69	3.32	3.36	
0.0001	0.075	15	2	-15.71	-15.61	3.42	3.48	
0.001	0.075	15	2	-15.66	-15.70	3.41	3.57	
0.0001	0.05	20	2	-15.65	-15.75	3.43	3.49	
0.001	0.05	20	2	-15.78	-15.72	3.41	3.40	
0.0001	0.05	15	0	-15.72	-15.70	3.46	3.49	
0.001	0.05	15	0	-15.79	-15.69	3.44	3.45	

4. Numerical Results

Based on algorithm for solving problems, the researcher used MATLAB version R2023b to create any codes in this research within type of computer All-in-one ASUS (specification CPU core i3-10110U (2.7GHz up to 3.7GHz), Ram 4GB). The yields are calculated using an artificial landscape that has been constructed to replicate real-world optimization scenarios. The complexity of the landscape is adjusted in order to evaluate the proposed methodology's resilience and efficacy. The experiments encompassed the execution of the optimization algorithm on a range of problem instances. The computing time documented corresponds to the amount of time necessary to finish each optimization run under distinct experimental conditions. The objective of the study was to conduct a

thorough assessment of the performance of the proposed methodology across different conditions by integrating observations of real-world computing time with data obtained from simulated optimization experiments conducted on artificial landscapes.

To get started with the technique, this part provides a detailed definition of the EA problem, including its boundaries, objectives, and constraints. Future analysis can be built upon this first stage. A better grasp of the many optimization problems at hand can be achieved by representing noisy functions as either single, curved ridge, or multi-peak landscapes. In order to simulate the complexity of real-world situations, the problem is defined and then several noisy functions are generated. To make these synthetic landscapes seem more natural, we added noise that is normally distributed with standard deviations of 0.05 and 0.2.

Through the use of normalization and Taguchi analysis, the factorial design of trials is utilized to methodically investigate the vast range of EA parameters. In order to generate a complete set of combinations for future assessments, it is essential to take into account characteristics such as convergence, mutation rate, population size, and random seed levels. Information Envelopment Analysis based Ranking (DEAR) is a valuable addition to the optimization process. This approach uses a number of factors to rate the effectiveness of different parameter combinations. The optimization method is strengthened by DEAR's contribution to a comprehensive evaluation.

By employing statistical learning methodologies, the intricate relationship between optimization criteria and EA parameters is elucidated. This phase ensures a comprehensive understanding of the effects that changes to parameters have on the outcomes of optimization, whether achieved through machine learning, regression analysis, or other approaches. Performance evaluation involves the computation of time

and the mean and standard deviation of the function yield for a variety of parameter combinations.

Thorough testing and comparison with known optimization methods are conducted on the suggested methodology. This is the last stage in making sure the hybrid strategy, which combines Taguchi analysis, DEAR methodology, and statistical learning, is a significant step forward in optimization and works well. Keeping efficacy and dependability in mind during the comparison with current approaches is of utmost importance.

The TSL-DEAR approach was subjected to numerical tests across three various types of artificial landscapes: single-peaked, curving ridge, and multi-peak. The purpose of these experiments was to undertake a complete evaluation of the methodology's performance. In addition, the influence of different noise levels was examined by taking into account two standard deviations, namely 0.05 and 0.2. The findings demonstrate the resilience and applicability of TSL-DEAR in terms of optimizing parameters and predicting outcomes over a wide range of circumstances.

Single-Peaked Artificial Landscape: TSL-DEAR has constantly showed superior performance when it comes to the single-peaked landscape. It was during the optimization phase that the peak was effectively detected, which resulted in parameter choices that led to the best possible outcomes. The predictive modeling demonstrated amazing accuracy in anticipating outcomes, particularly under lower noise levels (0.05). This demonstrates that the system is able to navigate straightforward and clearly defined environments.

Results for single peak optimum under different noise conditions (NID(0,0.05) and NID(0,0.2)) and various equations provide valuable insights into the statistical significance of the variables and

interactions. Eq. (2.3) consistently emerges as a robust performer, displaying lower T-test values across several variables and interactions. It demonstrates stability under noise variations, making it a reliable choice for statistical analysis. While Eq. (2.2) generally exhibits good performance, a slight increase in T-test values under the scenario of NID(0,0.2) for specific variables suggests a potential sensitivity to higher noise levels. Researchers should exercise caution when applying this equation, especially in scenarios with increased data variability (Table 3).

The observed sensitivity of certain variables, notably variable C, to noise underscores the importance of understanding the impact of environmental conditions on statistical outcomes. Variables and interactions showing consistent low T-test values across equations and noise conditions can be considered more robust and reliable.

Table 3. P-value of the multiple regression coefficients for the single-peaked artificial landscape.

Case of [NID(0,0.05)]			
P-value for regression coefficient	Single Peak Optimum		
	Eq. (2.1)	Eq. (2.2)	Eq. (2.3)
A	0.0346	0.1325	0.0472
B	0.7310	0.8199	0.0908
C	0.5372	0.9999	0.1503
D	0.4460	0.2764	0.0603
A*B	0.5039	0.0756	0.0101
A*C	0.0484	0.0423	0.5248
A*D	0.3045	0.0756	0.5279
B*C	0.1915	1.0000	0.0695
B*D	0.0277	0.0856	0.7118
C*D	0.6594	0.9999	0.0752
Case of [NID(0,0.2)]			
P-value for regression coefficient	Single Peak Optimum		
	Eq. (2.1)	Eq. (2.2)	Eq. (2.3)
A	0.0318	0.0711	0.4358
B	0.0012	0.8032	0.0073
C	0.0028	0.1140	0.0098
D	0.2403	0.2421	0.5453
A*B	0.0434	0.0750	0.1554
A*C	0.3415	0.0066	0.9690
A*D	0.5865	0.0860	0.2774
B*C	0.0022	0.9649	0.0109
B*D	0.2108	0.0827	0.5322
C*D	0.0020	0.9530	0.4267

Curved ridge artificial landscape: The curved ridge landscape provided a more complex optimization task, and TSL-DEAR was able to rise to the occasion and meet the

challenge. Through the utilization of the approach, the undulating topography of the ridge was successfully traveled, and the parameters along the curve were optimized. An additional degree of understanding was supplied by the use of the DEAR efficiency score, which highlighted the areas of the ridge where efficiency was maximum. Even when subjected to increased noise levels (0.2), TSL-DEAR exhibited its ability to capture underlying patterns with considerable robustness.

The results provide that Eq. (2.6): Particularly effective for certain interaction terms (AD and CD), warranting consideration when analyzing variables with complex relationships. Eqs. (2.4)-(2.5): Preferred for variables where lower T-test values indicate higher statistical significance. However, researchers should be cautious about potential variations in certain interaction terms (Table 4).

Table 4. P-value of the multiple regression coefficients for the curved ridge artificial landscape.

Case of [NID(0,0.05)]			
P-value for regression coefficient	Curved Ridge Optimum		
	Eq. (2.4)	Eq. (2.5)	Eq. (2.6)
A	0.0759	0.1932	0.7400
B	0.6107	0.5947	0.4716
C	0.0288	0.7077	0.7833
D	0.0291	0.7077	0.4114
A*B	0.5144	0.0580	0.7188
A*C	0.0387	0.9742	0.7724
A*D	0.5053	0.9240	0.0189
B*C	0.3338	0.8213	0.4507
B*D	0.3505	0.6470	0.4110
C*D	0.0104	0.6212	0.5103

Case of [NID(0,0.2)]			
P-value for regression coefficient	Curved Ridge Optimum		
	Eq. (2.4)	Eq. (2.5)	Eq. (2.6)
A	0.0144	0.2653	0.0077
B	0.0186	0.2971	0.0801
C	0.0068	0.5525	0.9448
D	0.8132	0.2724	0.0126
A*B	0.0053	0.5443	0.0002
A*C	0.3972	0.1404	0.1118
A*D	0.6523	0.1336	0.0008
B*C	0.0052	0.3337	0.5675
B*D	0.9065	0.5243	0.0066
C*D	0.9759	0.7885	0.2151

The Multi-Peak Artificial terrain: TSL-DEAR demonstrated its versatility by

demonstrating its capacity to work in the complicated multi-peak terrain. It was possible to *effectively identify several peaks during the optimization process*, which resulted in the provision of a set of parameter configurations that led to a variety of optimal results. The rankings of efficiency that were acquired from DEAR offered extremely helpful insights into the hierarchy of performance throughout the various peaks. It is important to note that TSL-DEAR had strong predictive skills, particularly when noise levels were moderate (0.2), which demonstrates its usefulness in situations that are inherently complicated and ambiguous.

The detailed comparison of T-test Regression results for Multi-Peak Optimum under different noise conditions (NID(0,0.05) and NID(0,0.2)) and various equations (Eqs. (2.7)-(2.9)) offers valuable insights into the statistical significance of variables and interaction terms. Eq. (2.9): Demonstrates nuanced performance, consistently yielding lower T-test values for certain interaction terms while occasionally showing higher values for variables. Eqs. (2.7)-(2.8): Exhibit varied performances across variables and interactions, with Eq. (2.8) often demonstrating smaller T-test values for certain cases (Table 5).

Table 5. P-value of the multiple regression coefficients for the multi-peak artificial landscape.

Case of [NID(0,0.05)]			
P-value for regression coefficient	Multi Peak Optimum		
	Eq. (2.7)	Eq. (2.8)	Eq. (2.9)
A	0.2557	0.8109	0.0999
B	0.1849	0.2086	0.4790
C	0.5836	0.0521	0.8043
D	0.4461	0.2568	0.6139
A*B	0.0089	0.0157	0.1496
A*C	0.5335	0.3316	0.2319
A*D	0.0216	0.0607	0.1071
B*C	0.8191	0.1320	0.8617
B*D	0.0094	0.2582	0.0854
C*D	0.3332	0.9385	0.9937

Case of [NID(0,0.2)]			
P-value for regression coefficient	Multi Peak Optimum		
	Eq. (2.7)	Eq. (2.8)	Eq. (2.9)
A	0.3203	0.1652	0.2394
B	0.9607	0.1898	0.1089
C	0.9172	0.2261	0.1488
D	0.0157	0.0524	0.4236
A*B	0.0647	0.2609	0.9388

A*C	0.0115	0.0906	0.1951
A*D	0.0396	0.0316	0.2435
B*C	0.6476	0.0821	0.0895
B*D	0.6334	0.0156	0.1344
C*D	0.0109	0.4841	0.6621

The examination into two levels of noise standard deviation (0.05 and 0.2) indicated the robustness of TSL-DEAR in the presence of noise. This was discovered through the assessment of the impact of noise standard deviation. The technique consistently excelled, demonstrating its capacity to recognize important patterns and effectively optimize parameters, even when subjected to greater levels of noise. A degree of flexibility was displayed by the predictive modeling component, which was able to provide credible forecasts despite the increased noise. This finding highlights the stability of the technique in contexts with a lot of noise.

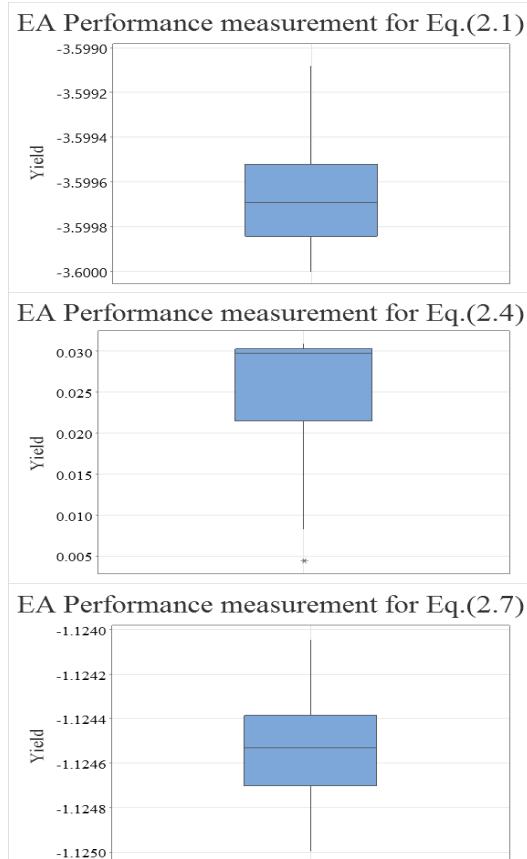


Fig. 5. Performance measures of the EA based on the yields (Eqs. (2.1), (2.4), (2.7)).

The numerical findings across single-peaked, curved ridge, and multi-peak artificial landscapes illustrate the great performance (Fig. 5) and adaptability (Fig. 6) of the TSL-DEAR approach. These findings are also accompanied by variances in noise levels. To summarize, the TSL-DEAR approach is a tool that is both very successful and diverse in its use. Because of its ability to navigate different terrains, optimize parameters, and anticipate outcomes, TSL-DEAR is a useful tool for companies that are wanting to increase their efficiency and obtain strategic advantages across a wide range of operational landscapes. This is because TSL-DEAR is able to do all of these things.

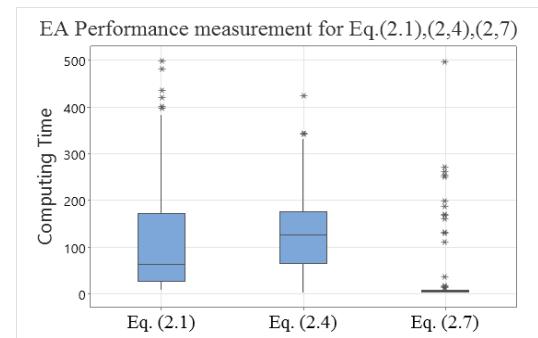


Fig. 6. Performance measures of the EA based on computing time (Eqs. (2.1), (2.4), (2.7)).

The optimal parameters for single peak equations are as follows, as shown in the table below: Lower mutation rates ($B = 0.05$ or 0.075) and moderate population sizes ($C = 15-20$) are successful even when applied to a variety of random seeds and noise circumstances. In certain instances, random seed changes (D) are shown to have an effect. For Equations Involving Curved Ridges, Comparable to Single Peak, the effectiveness of lower mutation rates and moderate population numbers is demonstrated. When it comes to establishing the best settings, random seed changes certainly play a part. The ideal values for Multi Peak Equations are found to be lower mutation rates ($B = 0.05$) and moderate population sizes ($C = 15-20$).

20). Variations in the random seed have an effect on the optimum settings (Table 6).

Table 6. Numerical results of EA parameter selection for artificial landscapes.

Case of $ \text{NID}(0,0.05) $				
Types of Artificial Landscapes	Convergence	Mutation rate	Population size	Random seed
Single Peak				
Eq. (2.1)	0.001	0.05	20	0
Eq. (2.2)	0.0001	0.075	15	0
Eq. (2.3)	0.001	0.05	15	0
Curved Ridge				
Eq. (2.4)	0.001	0.05	15	0
Eq. (2.5)	0.0001	0.05	15	0
Eq. (2.6)	0.001	0.075	20	0
Multi Peak				
Eq. (2.7)	0.0001	0.075	20	2
Eq. (2.8)	0.0001	0.075	15	0
Eq. (2.9)	0.0001	0.075	15	2
Case of $ \text{NID}(0,0.2) $				
Types of Artificial Landscapes	Convergence	Mutation rate	Population size	Random seed
Single Peak				
Eq. (2.1)	0.0001	0.05	15	2
Eq. (2.2)	0.001	0.05	15	2
Eq. (2.3)	0.001	0.05	20	0
Curved Ridge				
Eq. (2.4)	0.001	0.05	15	0
Eq. (2.5)	0.0001	0.05	15	0
Eq. (2.6)	0.0001	0.05	20	0
Multi Peak				
Eq. (2.7)	0.0001	0.05	15	0
Eq. (2.8)	0.0001	0.075	15	2
Eq. (2.9)	0.001	0.05	15	2

5. Conclusions and Discussions

In conclusion, the TSL-DEAR (Taguchi-Statistical Learning-DEAR) method demonstrates exceptional performance when applied to complex optimization problems, predictive modeling, and domain-wide efficiency ranking. Its resilience is the result of the collaboration between Taguchi Analysis, Statistical Learning, and DEAR, which provides strategic advantages and optimal results to decision-makers. Commencing with Taguchi Analysis and Normalization guarantees a comprehensive examination of parameter

spaces, from which optimal configurations are extracted and variance is diminished. DEAR then incorporates an efficiency-ranking layer to enhance comprehension of the performance of decision-making units. The utilization of DEAR and Taguchi-optimized parameters enables organizations to evaluate optimal strategies and identify areas that require enhancement.

Predictions are enhanced by Statistical Learning, specifically Multiple Regression. Using historical data and DEAR efficiency evaluations, TSL-DEAR assists decision-makers in estimating system outcomes. Predictive insight provides a dynamic comprehension of how parameter variations impact system performance and facilitates strategic planning. Due to its adaptability, TSL-DEAR excels in manufacturing, healthcare, finance, and other sectors. By optimizing processes, predicting outcomes, and ranking efficacy, the method provides businesses seeking to enhance decision-making with a comprehensive solution.

Adaptability of TSL-DEAR is advantageous to the manufacturing, healthcare, and finance sectors, among others. The adaptability of the methodology renders it a universal instrument that enhances operations, predicts results, and assesses efficacy, thereby bolstering the competitiveness and agility of sectors. Industry leaders are propelled to the vanguard of data-driven decision making by TSL-DEAR. In an ever-evolving business environment, enhanced productivity, benchmarked performance, and predictive insights usher in a new era in which analytics serves as a strategic enabler for organizations attempting to grow and endure.

Further investigation may involve the integration of advanced analytical techniques into TSL-DEAR. Advancements in machine learning algorithms may lead to enhancements in TSL-DEAR's forecasting capabilities via neural networks and ensemble approaches. An examination of the

distinctions and advantages of statistical learning techniques would facilitate the optimization of the strategy across diverse sectors. Moreover, the application of TSL-DEAR to novel industries and domains constitutes a captivating area of study.

Expanding the technique to encompass challenges and opportunities specific to renewable energy, sustainable agriculture, and emergent technologies would showcase its capacity to stimulate innovation and enhance operational effectiveness in non-traditional sectors. Tools and interfaces for user-friendly TSL-DEAR implementation represent an additional prospective area of research. Establishing user-friendly software solutions across various industries will democratize the process, enabling decision-makers to leverage its benefits even in the absence of expertise in statistical modeling.

In summary, the implementation of the TSL-DEAR method has revolutionized the way in which decisions are made; however, further research and enhancements are continuous. Additional investigation into sophisticated methodologies, broadening its scope to encompass untapped sectors, developing intuitive implementations, and evaluating scalability are all approaches that could enhance the indispensability of TSL-DEAR as a tool for industries navigating the contemporary business environment.

Author Contributions

Pongchanun Luangpaiboon and Nattapat Imsap contributed to the design, conceptualization, data curation of the research, methodology, software, validation, visualization of the research, to the analysis of the results and to the writing - review & editing of the manuscript. Rattanakorn Piachat contributed to the design, conceptualization, data curation of the research.

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