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Forecasting the Value of Indonesia's Exports using Model Hybrid Arimax-NN

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ABSTRACT

Time sequence data often exhibits both linear and nonlinear patterns, which can lead to inaccurate forecasts when using traditional methods that are limited to capturing only one type of pattern. To address this limitation, this study employs a hybrid method that combines the strengths of Autoregressive Integrated Moving Average with Exogenous Variables (ARIMAX) and neural networks (NN). The ARIMAX model effectively captures linear patterns, while the NN excels at modeling nonlinearities. The primary objective of this research is to optimize the ARIMAX-NN hybrid model for forecasting Indonesia's export values. Through rigorous model selection, the ARIMAX ([1,5,12],1,0)-NN 1 neuron model emerged as the best-performing configuration, achieving the lowest Mean Absolute Percentage Error (MAPE), Root Mean Squared Error (RMSE), and Mean Absolute Deviation (MAD) values. The forecasts for January-December 2024 reveal a pattern of decreasing export values during the month of Eid al-Fitr, a trend consistent with historical patterns and economic insights.

Keywords: ARIMAX; Calendar variation; Export value; MAPE; Neural network

1. Introduction

Autoregressive Integrated Moving Average (ARIMA) is a widely used method for time series forecasting. However, it has limitations in capturing recurring patterns across different periods within each year. These limitations arise due to the influence of specific events or other variables that can cause sharp spikes or declines in observation values. To address these shortcomings, alternative models are necessary, one of which is Autoregressive Integrated Moving Average with Exogenous Variables (ARIMAX) [1]. ARIMAX enhances ARIMA by incorporating the influence of observation values from specific periods and a set of ex-

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ogenous variables on the response variable [2]. Exogenous variables introduced into the model act as explanatory variables. Calendar effects are a common type of exogenous variable often employed in ARIMAX modeling [3].

While the ARIMAX model is effective in capturing linear relationships, it may struggle to accurately model nonlinear patterns often present in time series data. Meanwhile, a time series not only contains a linear relationship pattern but can also contain a nonlinear pattern. Therefore, there is a need for a method that can capture these two patterns, namely with a hybrid method which is expected to increase the accuracy of forecast results. One method that can capture nonlinear relationships is the neural network method [4]. One type of neural network method is the backpropagation neural network. The advantage of backpropagation neural networks is the ability to recognize patterns, trends, and complex relationships that may not be visible to other forecasting methods [5]. The combination of the ARIMAX model with the NN method will form a hybrid ARIMAX-NN model. In this study, the ARIMAX-NN hybrid model was chosen because of its superior ability to capture complex patterns in non-linear time series data. Compared to ARIMA-ANN, ARIMAX-NN offers greater flexibility in capturing more complex nonlinearity patterns. While ARIMA-LSTM (long short-term memory) has the advantage of handling long-term dependencies, its high computational complexity is a consideration. Thus, ARIMAX-NN is considered the most balanced choice between accuracy and complexity [6],

Previous research with the ARIMAX-NN model was conducted by Kusumaningrum, et al, with the results that the hybrid models of ARIMAX (0,1,2)-NN and ARIMAX (1,1,0)-NN with 1 and 2 neurons obtained MAPE values for each model of less than 5%. Research conducted by Eksiandayani, et al, found that the ARIMAX-NN model can be used to predict inflation with the amount of money supply as an exogenous variable. This hybrid model outperforms the ARIMA and ARIMAX models with a MAPE value of 0.6719% [7].

This study uses the ARIMAX-NN hybrid model to improve the accuracy of forecasting the value of Indonesia's exports. This hybrid model was chosen because of its ability to overcome the complexity of economic time series data, which often contain seasonal patterns, trends, and irregular fluctuations. Export is an activity of selling goods or services abroad and is one of the main factors of economic growth, especially in terms of gross domestic product (GDP) [8]. Based on data from the Central Statistics Agency (BPS), in April 2023 the value of Indonesia's exports decreased by 17.62% compared to March 2023 and 29.40% compared to April 2022. The export value in April 2023 reached USD19.29 billion. This decline is associated with the celebration of Eid al-Fitr in April 2023 [9]. Based on the description that has been explained previously, this study will forecast the value of Indonesia's exports using the ARIMAX-NN hybrid method. Accurate forecast results of export value prediction can help the government in formulating the right economic policies to encourage export growth.

2. Materials and Methods

2.1 Autoregressive integrated moving average exogenous

In forecasting the value of Indonesia's exports, a two-step approach is used. First, time series regression (TSR) is used to identify significant variables using dummy variables. The residuals obtained from the TSR model were then modeled using ARIMA, which is a statistical method for time series forecasting. The optimal ARIMA sequence is determined, and the ARIMA model is combined with the TSR model to produce the final ARIMAX model.

The Autoregressive Integrated Moving Average Exogenous (ARIMAX) model is an ARIMA model with the addition of exogenous variables as explanatory variables. The ARIMAX model of calendar variation is the ARIMAX model with the addition of variables such as a dummy for the effect of calendar variation. The ARIMAX model of calendar variations can be written in the following equation [4].

$$\begin{aligned} Z_t &= \beta_1 T_t + \beta_2 D_{1,t} + \beta_3 D_{2,t} + \cdots \\ &\quad + \beta_v D_{H,t} + \frac{\theta_q(B)}{\phi_p(B)(1-B)^d} e_t^*, \\ &\quad e_t^* \sim IIDN(0, \sigma^2), \end{aligned} \tag{2.1}$$

where $\beta_1, \beta_2, ..., \beta_v$ are parameter coefficient dummy variables, T_t is observation of trend variables at the time, $D_{H,t}$ is dummy variable for effect of calendar variation H, $\phi_p(B)$ is $1 - \phi_1 B - \cdots - \phi_p B^p$ (AR parameters of order p), $\theta_q(B)$ is $1 - \theta_1 B - \cdots - \theta_p B^q$ (MA parameters of order q), d is a differencing order, B is a backshift operator, and e_t^* is an error at the time t where $t = 1, 2, ..., n$.

2.2 Backpropagation neural network

The backpropagation neural network (NN) algorithm excels at handling complex pattern recognition problems [10]. This algorithm is one of the most efficient machine learning methods for multi-layer networks in NN because it is quite simple. This algorithm is called backpropagation because when given an input pattern as a training pattern, the pattern goes to the units on the hidden layer, and is passed to the units on the output layer to produce the network output. If the network output is not the same as the expected output, then the output will propagate backwards on the hidden layer, and be passed to the unit on the hidden layer, then passed to the unit on the input layer [11].

One of the activation functions used in the NN method is the binary sigmoid activation function. The binary sigmoid activation function is commonly used in NN models to introduce non-linearity, enabling the network to learn complex patterns. It maps input values to a range between 0 and 1. To ensure optimal performance, data normalization is essential, particularly for the binary sigmoid function. Min-max normalization is a suitable technique to scale data within a specific range, preventing negative values and improving the training process. After training, the normalized predictions can be denormalized to obtain the original scale [12].

2.3 Model hybrid ARIMAX-NN

The ARIMAX-NN model is a hybrid model that combines the linear capabilities of ARIMA with the nonlinear capabilities of neural networks [13]. The concept of hybrid work can be shown in the following equation:

$$Z_t = L_t + N_t + a_t^*, \tag{2.2}$$

where L_t represents the linear component, N_t represents the nonlinear component, and a_t^* is the error of the hybrid ARIMAX-NN model at time t [14].

2.4 Mean Absolute Percentage Error

Mean Absolute Percentage Error (MAPE), a common metric used to measure forecasting accuracy, is calculated by averaging the absolute percentage error for each forecasting period. A lower MAPE value indicates a more accurate forecast.

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Z_t - \hat{Z}_t}{Z_t}\right| \times 100\%, \quad (2.3)$$

where Z_t is the observation data at time t and $\hat{Z}_t$ is the prediction data at time t [15].

2.5 Root Mean Squared Error

Root Mean Squared Error (RMSE) is another commonly used metric to evaluate forecast accuracy. While RMSE is useful for comparing the forecast errors of different models for a specific variable, it may not be directly comparable across different variables. A lower RMSE value generally signifies a more accurate model [16].

$$RMSE = \frac{\sqrt{\sum_{i=1}^{n}(Z_t - \hat{Z}_t)^2}}{n}. \quad (2.4)$$

2.6 Mean Absolute Deviation

Mean Absolute Deviation (MAD) is a measure of forecast accuracy that calculates the average absolute error between predicted and actual values. A lower MAD value indicates a more accurate forecast) [16].

$$MAD = \frac{\sum_{i=1}^{n}|Z_t - \hat{Z}_t|}{n}. \quad (2.5)$$

3. Results and Discussion

3.1 Data description

This study utilizes monthly data on Indonesia's export value from January 2016 to December 2023, comprising a total of 96 months of data. This modeling is done using all historical data. Before forecasting Indonesia's export value data using the ARIMAX-NN hybrid model, we will begin with descriptive statistics in the form of a time sequence graph.

Based on Fig. 1, the value of Indonesia's exports exhibited an upward trend from January 2016 to October 2018, followed by a downward trend from November 2018 to May 2020. Subsequently, an upward trend was observed from June 2020 to August 2022, followed by another downward trend from September 2022 onwards. A notable pattern is that a decline in export values consistently occurs around the Eid al-Fitr holiday, as observed in July 2016, June 2017, June 2018, June 2019, May 2020, May 2021, May 2022, and April 2023. This indicates a significant calendar variation effect on Indonesia's export values.

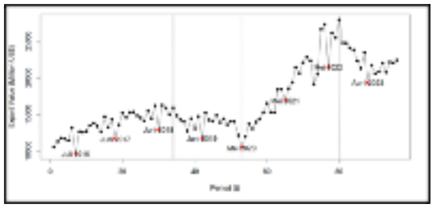


Fig. 1. Export value time sequence chart January-December 2024.

3.2 Time Series Regression (TSR)

Time Series Regression (TSR) aims to determine the influence of independent variables on a dependent variable within a time series model. In this study, the dependent variable is the value of Indonesia's exports (Z), which is influenced by two independent variables: a trend variable indicating an upward trend in the time series data and a dummy variable (T) capturing the effect of calendar variation. The dummy value of the calendar variation effect can be mathematically defined as follows:

$$D_{1,t} = \begin{cases} 1, & t = \text{satu bulan sebelum hari raya Idul Fitri} \\ 0, & t = \text{lainnya,} \end{cases}$$

$$D_{2,t} = \begin{cases} 1, & t = \text{bulan terjadinya hari raya Idul Fitri} \\ 0, & t = \text{lainnya,} \end{cases}$$

$$D_{3,t} = \begin{cases} 1, & t = \text{satu bulan setelah hari raya Idul Fitri} \\ 0, & t = \text{lainnya,} \end{cases}$$

The initial model of TSR can be written as follows:

$$Z_t = \beta_0 + \beta_1 T_t + \beta_2 D_{1,t} + \beta_3 D_{2,t} + \beta_4 D_{3,t} + \varepsilon_t \quad (3.1)$$

The data structure of the bound variable and the free variable can be presented in Table 1.

Table 1. Trend and Dummy Variables.

Period	Z_t	T_t	$D_{1,t}$	$D_{2,t}$	$D_{3,t}$
Jan 2016	10.581,9	1	0	0	0
Feb 2016	11.316,7	2	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮
Jun 2016	13.206,1	6	1	0	0
Jul 2016	9.649,5	7	0	1	0
Ags 2016	12.701,7	8	0	0	1
⋮	⋮	⋮	⋮	⋮	⋮
Des 2023	22.413,9	96	0	0	0

Parameter assessment used the Ordinary Least Squares (OLS) method. The estimated parameters of the TSR model are shown in Table 2.

Table 2. Results of Partial Testing of the Initial Model.

Parameter	Variable	*p-value*
β_0	*Intercept*	$< 2 \times 10^{-16}$
β_1	T	$< 2 \times 10^{-16}$
β_2	D_1	0.351
β_3	D_2	0.011
β_4	D_3	0.856

$$\hat{Z}_t = 10.533,580 + 130,408 T_t + 884,454 D_{1,t} - 2.458,742 D_{2,t} + 171,400 D_{3,t}. \quad (3.2)$$

This analysis employed a significance level (α) of 0.05 for both simultaneous and partial hypothesis testing. The null hypothesis for this test was that the parameters of both the trend variable and the dummy variable have no statistically significant impact on Indonesia's export value. The p-value obtained from the test was less than $< 2,2 \times 10^{-16}$. Since the p-value is less than the significance level ($\alpha = 0.05$), the null hypothesis is rejected. This leads to the conclusion that both the trend variable and the dummy variable have a statistically significant simultaneous impact on Indonesia's export value. A partial test was subsequently conducted on the model, and the results are presented in Table 2.

Based on Table 2, the p-values of parameters β_1 and β_3 are less than $\alpha = 0.05$. This suggests that β_1 and β_3 are statistically significant to the value of Indonesian exports. Conversely, the p-values of parameters β_2 and β_4 are greater than $\alpha = 0.05$, indicating that they are not significant. Parameters that do not significantly affect the value of Indonesian exports will be eliminated using backward elimination. This method removes the least significant parameter first, which in this case is β_4 (having the largest p-value). β_2 will then be removed as well. Consequently, the best TSR model equation is obtained as follows:

$$\hat{Z}_t = 10.639,402 + 130,206 T_t - 2.554,949 D_{2,t}. \quad (3.3)$$

The next step is to test the residual assumptions in Eq. (3.3) using the Ljung-Box test. The test results can be seen in Table 3.

Based on Table 3, the p-value is less than $\alpha = 0.05$ for all lags, indicating the rejection of the null hypothesis (H_0). This suggests the presence of autocorrelation in the residuals, meaning the residuals are not white noise. Therefore, the next step is to perform ARIMA modeling on the residuals of the TSR model.

Table 3. Pemeriksaan Residual White Noise Model TSR.

Was	p-*value*
12	$< 2,2 \times 10^{-16}$
24	$< 2,2 \times 10^{-16}$
36	$< 2,2 \times 10^{-16}$
48	$< 2,2 \times 10^{-16}$
60	$< 2,2 \times 10^{-16}$
72	$< 2,2 \times 10^{-16}$
84	$< 2,2 \times 10^{-16}$

3.3 Pemodelan Autoregressive Integrated Moving Average (ARIMA)

After the calculations are made, the residual TSR model displayed with a time sequence graph is presented in Fig. 2.

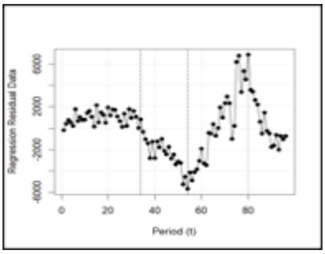


Fig. 2. Time sequence graph of TSR model residual data.

Based on Fig 2, it can be observed that the pattern of the TSR residual data tends to have a similar pattern to the actual data, namely experiencing an upward trend in the period $t = 1$ to $t = 34$, and experiencing a downward trend from period $t = 35$ to $t = 54$. After that, it experienced a significant increase from period $t = 55$ to $t = 80$, and a decrease from period $t = 81$ to $t = 86$. Therefore, visually, the TSR model residual data is still non-stationary.

The first step in building an ARIMA model is to ensure stationarity of the data, particularly in variance. This is often achieved through a Box-Cox transformation. The initial estimated value of λ at 0.7783, deviates significantly from 1, indicating non-stationarity in variance. Therefore, a transformation is necessary. The transformation involves raising the residual data of the TSR model in Eq. (3.2) to the power of λ, which in this case is 0.7783. Subsequently, the stationarity of the variance is re-evaluated using the transformed data (T^*). The analysis reveals that the estimated value of λ for the transformed residuals is 1, signifying stationarity in variance.

Results of the TSR model residual examination. The time series graph for the transformed data can be seen in Fig. 3.

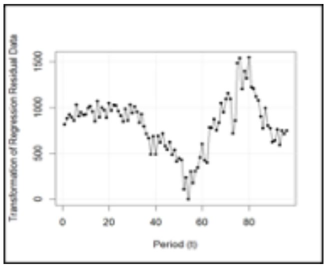


Fig. 3. Time streak chart T^*.

Based on Figs. 2-3, this implies that the data transformation process was not sufficient to achieve stationarity. Consequently, first-order differencing ($\triangle T^*$) is necessary, as shown in Eq. (3.4), to render the data stationary.

$$\triangle T^* = T^*_t - T^*_{t-1}. \tag{3.4}$$

The data appears to be stationary in terms of mean, as shown in Fig. 4.

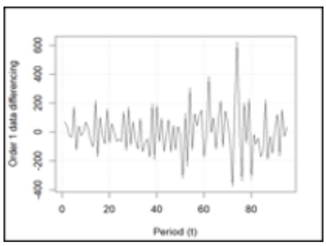
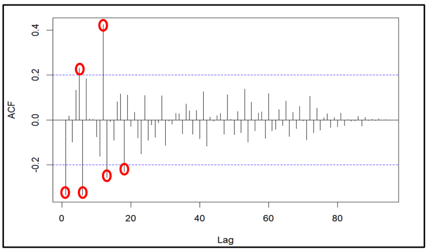


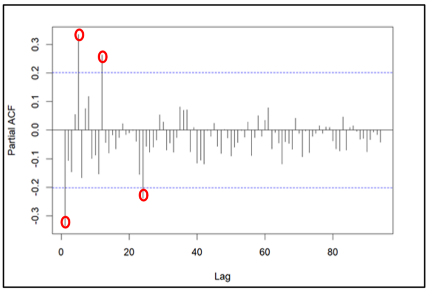
Fig. 4. Time streak chart $\triangle T^*$.

Identification of the provisional ARIMA model begins by analyzing the

ACF and PACF plots of the stationary data. The PACF plot for the stationary data is presented in Fig. 5.



(a)



(b)

Fig. 5. Graphs of (a) ACF and (b) PACF ΔT^*.

Based on the ACF and PACF plots in Fig. 5, significant autocorrelation was identified at lags 1, 5, 6, 12, 13, and 18 for the ACF, and at lags 1, 5, 12, and 24 for the PACF. To identify potential ARIMA models, the principle of parsimony was applied, resulting in the following candidate models: ARIMA(1,1,0), ARIMA(0,1,1), ARIMA (1,1,1), ARIMA([1,5,12],1,0), ARIMA (0,1,[1,5,6]), and ARIMA([1,5,12],1, [1,5,6]). After obtaining the provisional ARIMA model, the significance of the parameters of the provisional ARIMA model was then assessed and tested.

Based on Table 4, it can be seen that all parameters of the ARIMA(1,1,0), ARIMA(0,1,1), ARIMA([1,5,12],1,0), and ARIMA(0,1,5,6) models are significant. Meanwhile, the ARIMA(1,1,1) and ARIMA([1,5,12],1,[1,5,6]) models have insignificant parameters. Based on the

Table 4. Estimation and Significance Test of ARIMA.

Model	Parameter Estimation	p-value
ARIMA (1, 1, 0)	$\hat{\phi_1} = -0.333$	0.0008
ARIMA (0, 1, 1)	$\hat{\theta_1} = -0.370$	$6,772 \times 10^{-5}$
ARIMA (1, 1, 1)	$\hat{\phi_1} = -0.043$	0.847
	$\hat{\theta_1} = -0.337$	0.099
ARIMA ([1, 5, 12], 1, 1)	$\hat{\phi_1} = -0.294$	0.0008
	$\hat{\phi_5} = 0.198$	0.026
	$\hat{\phi_{12}} = 0.341$	0.0003
ARIMA (0, 1, [1, 5, 6])	$\hat{\theta_1} = 0.360$	0.0003
	$\hat{\theta_5} = -0.260$	0.012
	$\hat{\theta_6} = 0.221$	0.032
ARIMA ([1, 5, 12], 1, [1, 5, 6])	$\hat{\phi_1} = -0.294$	0.117
	$\hat{\phi_5} = 0.104$	0.617
	$\hat{\phi_{12}} = 0.346$	0.0004
	$\hat{\theta_1} = -0.014$	0.948
	$\hat{\theta_5} = 0.112$	0.631
	$\hat{\theta_{12}} = 0.114$	0.349

significance testing of the parameters of the ARIMA(1,1,0), ARIMA(0,1,1), ARIMA ([1,5,12],1,0), and ARIMA(0,1,5,6]) models, diagnostic examination can be continued.

The next step is to perform a white noise diagnostic check. Based on the results of the L-jung Box test, the results in Fig. 6 were obtained.

Based on Fig. 6, it can be observed that the p-value of the Ljung-Box test for the ARIMA([1,5,12],1,0) model lies above the significance line. Therefore, the null hypothesis (H_0) cannot be rejected, and it can be concluded that there is no autocorrelation among the residuals of the ARIMA([1,5,12],1,0) model, and the residuals are white noise. In contrast, for the ARIMA(1,1,0), ARIMA(1,1,1), and ARIMA(0,1,[1,5,6]) models, the p-value of the Ljung-Box test lies below the significance line. Hence, the null hypothesis (H_0) is rejected, and it can be concluded that there is autocorrelation among the residuals of the ARIMA(1,1,0), ARIMA(1,1,1), and ARIMA(0,1,[1,5,6]) models, and the resid-

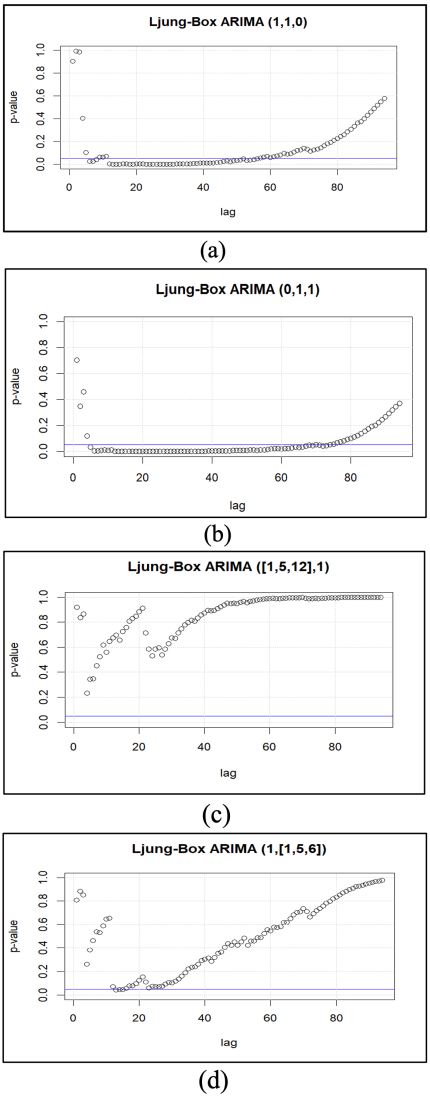


Fig. 6. Ljung-Box test graph of ARIMA model.

uals are not white noise.

The normality of the residuals was tested using the Kolmogorov-Smirnov test with a significance level (α) of 0.05. The null hypothesis in this test is that the residuals of the ARIMA([1,5,12],1,0) model are normally distributed. The test results showed a p-value of 0.062. Since the p-value $> \alpha = 0.05$, we fail to reject H0 and can conclude that the residuals of the ARIMA ([1,5,12],1,0) model are simultaneously normally distributed.

3.4 ARIMAX

The assessment and testing of the significance of the parameters of the provisional ARIMA model are obtained in Table 5.

Table 5. Estimation and Significance Test of ARIMAX Subset.

Parameter Estimation	p-value
$\hat{\beta}_1^* = 122{,}289$	0.440
$\hat{\beta}_3^* = -3.018{,}600$	< 0.001
$\hat{\phi}_1^* = -\ 0.248$	0.006
$\hat{\phi}_5^* = 0.205$	0.024
$\hat{\phi}_{12}^* = 0.354$	0.0003

Table 5, shows that the trend variable (T) is insignificant, necessitating its elimination using backward elimination. The ARIMAX subset is re-estimated and tested for significance, and the results are presented in Table 6.

Table 6. Post-elimination ARIMAX subset examination and significance testing.

Parameter Estimation	p-value
$\hat{\beta}_3^* = -3.018{,}600$	< 0.001
$\hat{\phi}_1^* = -\ 0.241$	0.007
$\hat{\phi}_5^* = 0.207$	0.020
$\hat{\phi}_{12}^* = 0.361$	0.0002

As shown in Table 6, all parameters of the ARIMAX([1,5,12],1,0) model are significant. Therefore, the best model is obtained as follows:

$$\begin{aligned}
\hat{Z}_t &= Z_{t-1} - 0.241Z_{t-1} \\
&\quad +0.241Z_{t-2} + 0.207Z_{t-5} \\
&\quad -0.207Z_{t-6} + 0.361Z_{t-12} \\
&\quad -0.361Z_{t-13} - 3.026, 6D_{2,t} \\
&\quad +3.026, 6D_{2,t-1} \\
&\quad +3.026, 5D_{2,t-1}(-0, 241) \\
&\quad -3.026, 6D_{2,t-2}(-0, 241)
\end{aligned}$$

$$+3.026,6D_{2,t-5}(0,207)$$
$$-3.026,6D_{2,t-6}(0,207)$$
$$+3.026,6D_{2,t-12}(0,361)$$
$$-3.026,6D_{2,t-13}(0,361) \quad (3.5)$$

To ensure model adequacy, a diagnostic check was conducted on the ARIMAX([1,5,12],1,0) model. As the model met all assumptions, it was deemed suitable for forecasting purposes. Forecast calculations were performed using Eq. (3.4) and the results for January-December 2024 are presented in Table 7.

Table 7. Forecasting results of indonesian export value using ARIMAX([1,5,12],1,0) model.

Period	Moon	Forecast Results
97	January 2024	22.022,697
98	February 2024	21.496,536
99	March 2024	22.668,559
100	April 2024	18.929,991
101	May 2024	21.996,615
102	June 2024	21.507,376
103	July 2024	21.610,543
104	August 2024	22.237,792
105	September 2024	21.487,876
106	October 2024	22.181,910
107	November 2024	21.859,697
108	December 2024	22.108,658

Based on the results of forecasting Indonesia's export values using the ARIMAX([1,5,12],1,0) model, the following accuracy metrics were obtained: MAPE of 0.0477%, RMSE of 116.1, and MAD of 810.7, using data from January 2014 to December 2023."

3.5 Hybrid ARIMAX-NN

The initial stage in hybrid ARIMAX-NN modeling involves standardizing the residual data of the ARIMAX([1, 5, 12], 1, 0) model. This ensures that all data used in the NN model have the same range of values. In this study, min-max standardization is employed.

The determination of this input variable is based on the amount of significant lag seen on the ACF and PACF graphs of the residual data, as shown in Fig. 7.

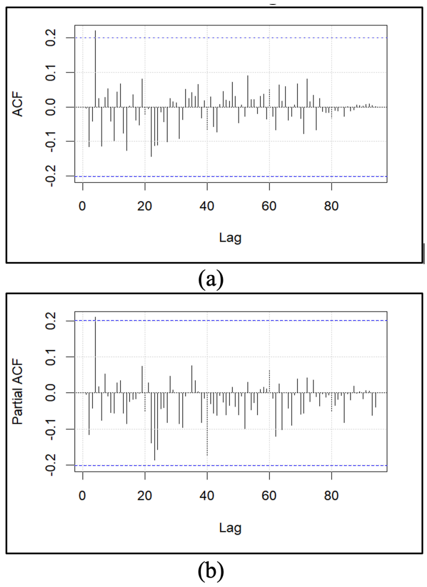
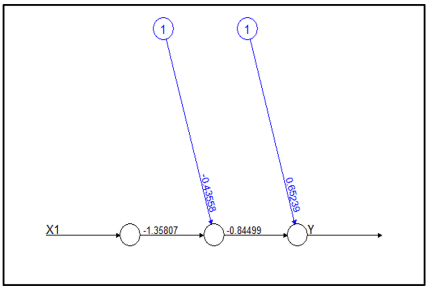


Fig. 7. Graphs of (a) ACF and (b) PACF of ARIMAX([1,5,12],1,0) residual data.

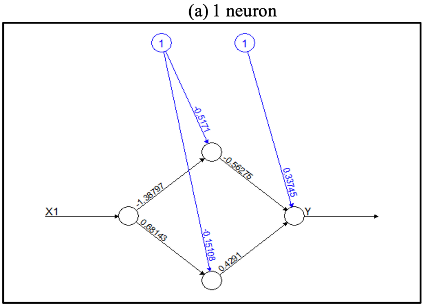
Based on Fig. 7, the ACF and PACF values are significant at lag 4. Consequently, the lagged error term, e^*_{t-4} is chosen as the input variable (x_1). The output data used is the standardized residual value of the ARIMAX([1, 5, 12], 1, 0) model.

Backpropagation training begins by determining the number of neurons in the hidden layer. This research evaluates 1 neuron, 2 neurons, 3 neurons, and 4 neurons with a learning rate of 0.01. The stopping conditions are a target error of 0.01 and a maximum of 5,000 iterations. The activation function used in this study is the binary sigmoid, which produces an output value between 0 and 1.

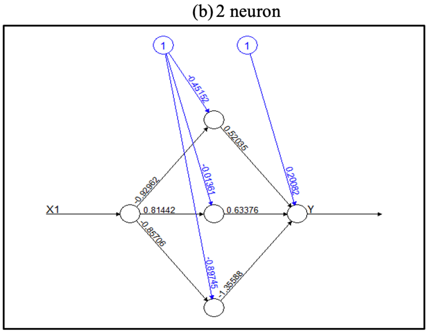
The backpropagation network architecture with 1,2,3, and 4 neurons using a binary sigmoid activation function based on the residual model ARIMAX([1,5,12],1,0) is shown in Fig. 8.



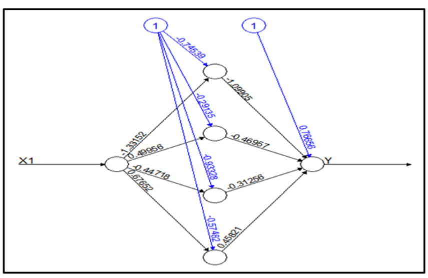
(a) 1 neuron



(b) 2 neuron



(c) 3 neuron



(d) 4 neuron

Fig. 8. Arsitektur backpropagation NN.

The artificial neural networks (NN) are trained using a backpropagation algorithm with binary sigmoid activation functions to optimize weights and minimize errors. The 5th to 95th data points are then predicted using the residual model of ARIMAX([1,5,12],1,0). These predicted values are subsequently re-normalized to match the original data range and then standardized. To determine the best ARIMAX-NN hybrid model, the number of neurons in the hidden layer is varied from 1 to 4. The performance of each model is evaluated using MAPE, RMSE, and MAD, as presented in Table 8.

Table 8. MAPE, RMSE, and MAD Values of ARIMAX-NN hybrid models.

Number of Neurons in the Hidden Layer	MAPE (%)	RMSE	MAD
1 Neuron	0,04870	1.113,0	840,0
2 Neuron	0,04869	1.114,7	840,5
3 Neuron	0,04878	1.116,0	842,1
4 Neuron	0,04937	1.113,1	848,1

Based on Table 9, it can be concluded that the ARIMAX-NN hybrid model with a variation in the number of neurons from 1 to 4 neurons provides very accurate results. Based on the MAPE value, it can be seen that in general the four configurations have very similar performance, with a very low MAPE value (around 0.0487% to 0.0493%). This shows that all configurations are capable of producing highly accurate predictions. However, if choosing between the two, the configuration with 2 neurons had the lowest MAPE value, although the difference was very small. Meanwhile, based on RMSE and MAD values, 1 neuron has the smallest RMSE and MAD values. Therefore, the ARIMAX([1,5,12],1,0)-NN 1 neuron model was chosen to predict the value of Indonesia's exports over the next 12 periods. The details of the prediction calculation with the ARIMAX([1,5,12],1,0)-NN 1 Neuron model will be explained next.

The calculation of the forecast value with the hybrid model of ARIMAX([1,5,12],1,0)-NN 1 Neuron is as follows:

$$
\begin{aligned}
\hat{Z}_t \;=\;& Z_{t-1} - 0.241Z_{t-1} \\
&+0.241Z_{t-2} + 0.207Z_{t-5} \\
&-0.207Z_{t-6} + 0.361Z_{t-12} \\
&-0.361Z_{t-13} - 3.026, 6D_{2,t} \\
&+3.026, 6D_{2,t-1} \\
&+3.026, 5D_{2,t-1}(-0, 241) \\
&-3.026, 6D_{2,t-2}(-0, 241) \\
&+3.026, 6D_{2,t-5}(0, 207) \\
&-3.026, 6D_{2,t-6}(0, 207) \\
&+3.026, 6D_{2,t-12}(0, 361) \\
&-3.026, 6D_{2,t-13}(0, 361) \\
&+\hat{N}_{1,t} \qquad (3.6)
\end{aligned}
$$

This hybrid ARIMAX-NN model can be used to calculate the forecasting value for the period January-December 2024. The forecasting results are shown in Table 9.

Table 9. Forecasting Indonesia's Export Value Using the ARIMAX([1,5,12],1,0) -NN 1 Neuron Hybrid Model.

t	ARIMAX ([1,5,12],1,0)	Forecasting NN 1 Neuron	Hybrid ARIMAX-NN
97	22.022,697	154,634	22.177,331
98	21.496,536	335,217	21.831,753
99	22.668,559	212,672	22.881,231
100	18.929,991	161,963	19.091,954
101	21.996,615	102,073	22.098,688
102	21.507,376	142,108	21.649,484
103	21.610,543	114,990	21.725,533
104	22.237,792	103,707	22.341,499
105	21.487,876	90,337	21.578,213
106	20.181,910	99,280	20.281,190
107	21.859,697	93,224	21.952,921
108	21.108,658	90,702	21.199,360

Based on the forecasting results, the time series graph comparing the actual data and the forecasting results can be seen in Fig. 9.

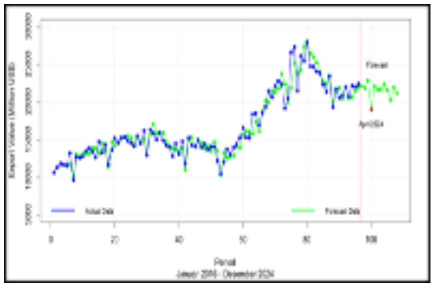


Fig. 9. Actual data, prediction, and forecasting results of the ARIMAX hybrid model ([1,5,12],1,0)-NN 1 neuron.

Based on Fig. 9, it is generally shown that the ARIMAX([1,5,12],1,0)-NN 1 neuron hybrid model has prediction results whose patterns tend to follow the actual data. The forecast of the ARIMAX([1,5,12],1,0)-NN 1 neuron model fluctuates every month, and it can be seen that there is a decrease in the value of Indonesia's exports in the month of Eid al-Fitr in April 2024.

Based on the accuracy metrics, the best model is the ARIMAX model ([1,5,12],1.0) with an accuracy value of MAPE 0.0468%, RMSE 116.1, and MAD 810.7. If desired, a model that can capture linear and nonlinear patterns is the ARIMAX([1,5,12],1,0)-1 neuron hybrid model with an accuracy value of MAPE 0.04870%, RMSE 1,113.0, and MAD 840.0. Therefore, it can be concluded that Indonesia's export value data tends to have a linear pattern.

While the ARIMAX-NN hybrid model offers the advantage of combining the strengths of ARIMA and NN models, it can still be affected by the limitations of the ARIMA component, particularly the assumption of normality in residuals. Violations of this assumption can arise from various factors, including the presence of outliers, non-constant variance (heteroscedasticity), and structural changes in the data, such as shifts in trends or seasonality.

4. Conclusion

Based on the accuracy metrics, the ARIMAX-NN 1-neuron hybrid model, with the lowest MAPE of 0.04870%, RMSE of 1,113.0, and MAD of 840.0, emerged as the best model for forecasting Indonesia's export values. Forecasts generated by this model for January-December 2024 indicate a fluctuating pattern, with a notable decline in export values during the month of Eid al-Fitr in April 2024.

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