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A Study of Right Truncated Lomax Rayleigh Distribution and Application for Group Acceptance Sampling Plan

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ABSTRACT

In this paper, the right-truncated Lomax-Rayleigh distribution (R-TLRD) is presented with some important statistical properties, such as the survival function, hazard function, moment, parameter estimation, and application for group acceptance sampling plans. The right-truncated Lomax-Rayleigh distribution provides the model corresponding to the three real datasets, while the Lomax-Rayleigh distribution fits only the two real datasets. Then, the group acceptance sampling plan when the product lifetime is determined as a right-truncated Lomax-Rayleigh distribution provides the optimum of the number of groups (g), the number of items for each group (r), the acceptance number (c), and the operating characteristic function (OCF) value. The group acceptance sampling plan under right-truncated Lomax-Rayleigh distribution gives a smaller sample size for testing than the group acceptance sampling plan under Weibull distribution.

Keywords: Group acceptance sampling plan; Lifetime distribution; Lomax-Rayleigh distribution; Truncated distribution

1. Introduction

Lifetime distribution, which is used for probabilistic explanations of the duration of the product lifetime, business failure, etc., is a science in which many researchers are interested in developing distributions to correspond to the different characteristics of the lifetime datasets. Distributions fitting for real datasets can be used for further studies in the industrial sector. There are many lifetime distributions that have evolved to become popular in the statistics literature, such as Weibull, exponential, and generalized half-normal distributions. However, these distributions may have limitations. To enable the lifetime distribution to be flexible and consistent with actual datasets, a lifetime distribution is usually developed by adding

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one or more shape parameters [1] and mixing distributions. The distribution that is a combination of the exponential distribution and the gamma distribution for example; Lindley [2] presents a single-parameter Lindley distribution (one-parameter Lindley distribution) and some statistical properties. Next, Shanker et al. [3] developed the one-parameter Lindley distribution, which is transformed by adding a shape parameter into a two-parameter distribution, along with presenting important statistical properties such as moments, parameter estimation, and applications to real datasets, Shanker et al. (2015) [4] presented the Akash distribution and its application to real data in engineering.

Venegas et al. [5] presented the Lomax-Rayleigh distribution, which is a compound between the Lomax and Rayleigh distributions. Moreover, some statistical properties and applications for the three real datasets are studied, and it was found to be more flexible and more consistent with real data than the Lomax and Rayleigh distributions. The Lomax-Rayleigh distribution is expected to be a good alternative model for explaining lifetime data. The cumulative distribution function (CDF) and the probability density function (PDF) of the Lomax-Rayleigh distribution, $X \sim LR(\alpha,\theta)$, are as Eqs. (1.1)-(1.2), respectively ([5]).

$$F(x;\alpha,\theta) = 1-\left(\frac{\theta}{\theta+x^2}\right)^{\alpha}, \qquad (1.1)$$

$$f(x;\alpha,\theta) = \frac{2\alpha\theta^{\alpha}x}{\left(\theta+x^2\right)^{\alpha+1}}, \qquad (1.2)$$

where θ, α are a scale parameter and a shape parameter, respectively. The sample plots of the PDF and CDF plots are shown in Fig. 1.

Fig. 1 reveals that a function *f(x)* is monotonically decreasing for $\theta<1$ and $\theta<\alpha$; for the other parameters, it is a unimodal shape. The Lomax-Rayleigh distribution has a simple model and is consistent with actual data.

Another option for developing a lifetime distribution is truncation of the parent distribution. A truncated distribution in statistics is a conditional distribution obtained by limiting the domain of another probability distribution. In real-world statistics, truncated distributions emerge when values that fall within a predetermined range or above a threshold are the only data that may be used to record or even learn about an event. Limiting the values of the distribution can be operated by the lie above (right side), below (left side), or both sides to give a criterion or within a specified range [6]. Additionally, data analysis is performed across multiple domains such as engineering, healthcare, finance, and demographics, where truncated data types commonly occur in applied statistics. It is utilized in situations where recording is possible, or when events are constrained to values that exceed or fall below a certain limit or reside within a defined range. Truncated distributions are very useful for data analysis [7]. The developed distribution is expected to be suitable for explaining product lifetime (the interval 0 to t). In this study, the Lomax-Rayleigh distribution is developed by truncation above to increase the fit for the lifetime data, and then it is applied for an acceptance sampling plan.

The single acceptance sampling plan is necessary to define the dependability of a single-item product with regard to its lifetime. However, in the case of inspecting a multiple number of items according to the number of testers available for the experimenter to test, a group acceptance sampling plan (GASP) is a good choice for the situation because the GASP reduces testing time and cost [8, 9].

Because the lifespan of a product ranges from 0 to t, in this study the right truncated Lomax-Rayleigh distribution with application for group acceptance sampling plans is proposed to increase the potential for industrial work. The manuscript is structured as follows: the presentation of R-TLRD along with some statistical properties, application to real datasets and testing goodness of fit, and application to GASP, respectively.

2. Right truncated Lomax-Rayleigh distribution

This part reveals the details about the right-truncated Lomax-Rayleigh distribution with some statistical properties.

2.1 Right truncated distribution

Suppose $X \in (-\infty, \infty)$, a continuous random variable, has a baseline distribution with the parameter Θ. The PDF of X is defined on the interval $[a,b]$, where $-\infty < a \le x \le b < \infty$. If $a = 0$, the distribution of X is called the right-truncated distribution, it is written as x~RTD(Θ,b) and the PDF of the RTD is shown as Eq. (2.3) [6, 10].

$$t(x;\Theta,b) = \frac{f(x;\Theta)}{F(b;\Theta)}, \quad 0 \le x \le b < \infty, \tag{2.3}$$

where $f(x) = \begin{cases} t(x), & 0 \le x \le b \\ 0, & \text{otherwise} \end{cases}, \quad t(x) \ge 0, \ \forall x.$

2.2 R-TLRD

Let X be a random variable of the right truncated Lomax-Rayleigh distribution (R-TLRD) on $0 \le x \le b$. The PDF and CDF of the R-TLRD, are presented in Eqs. (2.4)-(2.5), respectively . The properties of R-TLRD are verified as detailed in Appendices A and B.

$$t(x \mid x \le b) = \frac{2\alpha x\theta^{\alpha}}{\left[1-\theta^{\alpha}\left(\theta+b^2\right)^{-\alpha}\right]\left(\theta+x^2\right)^{\alpha+1}}, \tag{2.4}$$

$$T(x \mid x \le b) = \frac{1-\theta^{\alpha}\left(\theta+x^2\right)^{-\alpha}}{1-\theta^{\alpha}\left(\theta+b^2\right)^{-\alpha}}. \tag{2.5}$$

The PDF and CDF plots of R-TLRD are presented in Fig. 2. The shapes of the PDF and CDF plots of R-TLRD are similar to LRD. However, the R-TLRD lines tend to have a higher PDF than LRD when the parameter values are close together.

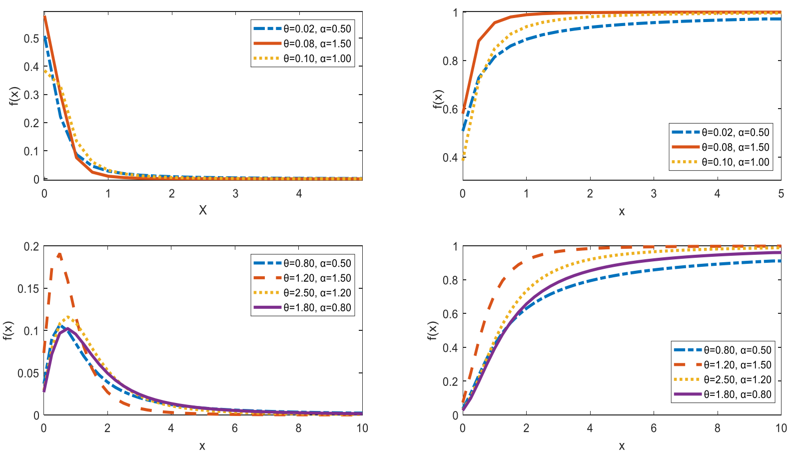

Fig. 1. The PDF and CDF plots of LRD for difference parameters.

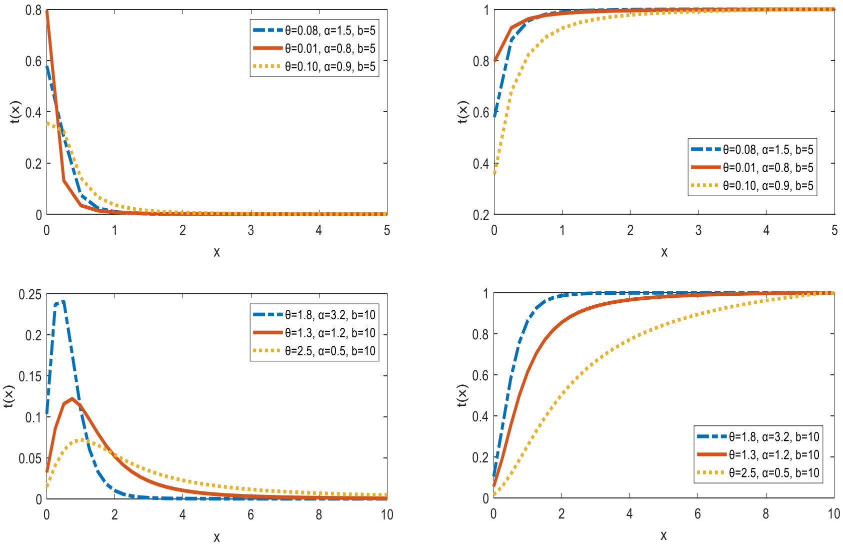

Fig. 2. The PDF and CDF of R-TLRD for difference parameters.

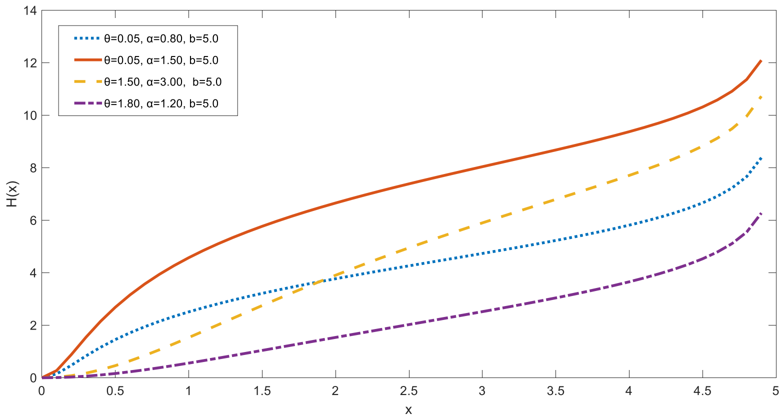

Fig. 3. The H(x) plots of R-TLRD for difference parameters.

2.2.1 Statistical properties

The proposed distribution is represented in terms of survival function, hazard function, moments, and parameter estimation, respectively.

2.2.1.1 Survival function and hazard function

S(x), a survival function, is the function that provides the probability that an object or a product of interest will survive longer than time x, and *H(x)*, a hazard function, is the ratio of $t(x)$ to *S(x)*. *S(x)* and *H(x)* are as Eqs. (2.6)-(2.7).

$$S(x;\alpha,\theta,b)=\frac{\theta^{\alpha}\left[\left(\theta+x^{2}\right)^{-\alpha}-\left(\theta+b^{2}\right)^{-\alpha}\right]}{1-\theta^{\alpha}\left(\theta+b^{2}\right)^{-\alpha}}, \quad (2.6)$$

$$H(x;\alpha,\theta,b)=\frac{2\alpha x}{\left[\left(\theta+x^{2}\right)^{-\alpha}-\left(\theta+b^{2}\right)^{-\alpha}\right]\left(\theta+x^{2}\right)^{\alpha+1}}. \quad (2.7)$$

Hazard curve plots show monotonically increasing shapes (see in Fig. 3).

2.2.1.2 Moments of R-TLRD

Moments of the new distribution provide some characteristics of R-TLRD, such as mean, variance, skewness, kurtosis, etc. The moment for $r=1,2,...$ and $\alpha>\frac{r}{2}$ is shown through a moment-generating function that is [5, 11] :

$$\mu_r = E\left(X^r\right)$$
$$=\frac{\alpha\theta^{\frac{r}{2}}}{\left[\theta^{\alpha}\left(\theta+b^2\right)^{-\alpha}-1\right]}\left[\frac{\Gamma\left(\alpha-\frac{r}{2}\right)\Gamma\left(\frac{r}{2}+1\right)}{\Gamma\left(\alpha+1\right)}\right], \quad (2.8)$$

where $\Gamma\left(\alpha\right)=\int_0^1 u^{\alpha-1}\left(1-u\right)^{\beta-1}du$ is the gamma function, $\beta\left(\alpha,\beta\right)=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma\left(\alpha+\beta\right)}$ is the beta function.

2.2.1.3 Parameter estimation

The parameter estimation of R-TLRD is solved using maximum likelihood estimation (MLE). Let $\tilde{x}=\left(x_1,x_2,...,x_n\right)$ be a random sample with a size n. X ~ R-TLRD $\left(\alpha,\theta,b\right)$ is independent and identically distributed. The likelihood function of R-TLRD($L\left(x_i;\alpha,\theta,b\right)$) is described in Eq. (2.9).

$$L\left(x_i;\alpha,\theta,b\right)$$
$$=\prod_{i=1}^{n}\left(\frac{2\alpha x_i\theta^{\alpha}}{\left[1-\theta^{\alpha}\left(\theta+b^2\right)^{-\alpha}\right]\left(\theta+x_i^2\right)^{\alpha+1}}\right). \quad (2.9)$$

The log likelihood function $\left(\ell\left(x_i;\alpha,\theta,b\right)\right)$ of R-TLRD is as in Eq. (2.10).

$$\ell\left(x_i;\alpha,\theta,b\right)=n\log\left(2\alpha\theta^{\alpha}\right)+\sum_{i=1}^{n}\log\left(x_i\right)$$
$$-n\log\left(1-\theta^{\alpha}\left(\theta+b^2\right)^{-\alpha}\right) \quad (2.10)$$
$$-\left(\alpha+1\right)\sum_{i=1}^{n}\log\left(\theta+x_i^2\right).$$

For $\hat{b}=\max(x_i)$, the MLE value of parameter $\hat{\alpha}$ and $\hat{\theta}$ are obtained by

$$\frac{\partial\left(\ell\left(x_i;\alpha,\theta,b\right)\right)}{\partial\alpha}=\frac{n\left(2\theta^{\alpha}+2\alpha\theta^{\alpha}\log(\theta)\right)}{2\alpha\theta^{\alpha}}$$
$$-n\left(\frac{\theta^{\alpha}\log\left(\theta\right)-\theta^{\alpha}\log\left(b^2+\theta\right)}{\left(b^2+\theta\right)^{\alpha}}\right)$$
$$\times\left(\frac{1}{\theta^{\alpha}\left(b^2+\theta\right)^{-\alpha}-1}\right)-\log(x_i^2+\theta),$$

and

$$\frac{\partial\left(\ell\left(x_i;\alpha,\theta,b\right)\right)}{\partial\theta}=n\left(\frac{\alpha\theta^{\alpha}}{\left(b^2+\theta\right)^{\alpha+1}}-\frac{\alpha\theta^{\alpha-1}}{\left(b^2+\theta\right)^{\alpha}}\right)$$
$$\times\left(\frac{1}{\theta^{\alpha}\left(b^2+\theta\right)^{-\alpha}-1}\right)-\frac{\alpha+1}{x_i^2+\theta}+\frac{n\alpha\theta^{\alpha-1}}{\theta^{\alpha}},$$

In which the expression is not in closed form. The solution of parameter estimates $\hat{\alpha}$ and $\hat{\theta}$ are solved by using the Newton- Raphson method [12] with stopping criteria $\varepsilon\le 0.0005$.

2.3 Application to real data

Three real datasets are used for numerical experiments as follows:

The first dataset represents the failure times (unit: hours) for a sample of 50 electronic devices, as shown in [13]. A total of 50 observations are discussed as: 26.3, 78.5, 29.8, 22.6, 113.1, 157.4, 2.4, 51.9, 29.3, 40.3, 216.6, 30.5, 31.6, 57.5, 38.1, 113.7, 1.0, 96.8, 63.3, 72.1, 107.4, 39.6, 29.0, 11.0, 105.2, 36.7, 7.1, 85.5, 24.6, 28.0, 23.6, 14.7, 24.3, 46.9, 56.9, 293.4, 33.0, 47.0, 51.9, 20.0, 20.3, 158.9, 54.0, 14.8, 81.2, 46.0, 42.8, 8.9, 35.7, 32.3.

The second dataset presents the 63 service times (unit: a thousand hours) of aircraft windshields. The data is as follows [14]: 0.046, 1.436, 2.592, 0.140, 1.492, 2.600, 0.150, 1.580, 2.670, 0.248, 1.719, 2.717, 0.280, 1.794, 2.819, 0.313, 1.915, 2.820, 0.389, 1.920, 2.878, 0.487, 1.963, 2.950, 0.622, 1.978, 3.003, 0.900, 2.053, 3.102, 0.952, 2.065, 3.304, 0.996, 2.117, 3.483, 1.003, 2.137, 3.500, 1.010, 2.141, 3.622, 1.085, 2.163, 3.665, 1.092, 2.183, 3.695, 1.152, 2.240, 4.015, 1.183, 2.341, 4.628, 1.244, 2.435, 4.806, 1.249, 2.464, 4.881, 1.262, 2.543, 5.140.

The third dataset contains the length of the fault in each case (from the time of detection to the time of failure). The collapse happens in a couple of minutes, demonstrating its instantaneous nature: 333, 119, 280, 245, 125, 242, 114, 233, 494, 131, 246, 73, 254 [15].

The efficiency of the model is compared using the log-likelihood function of the model (-LL), the Akaike Information Criterion (AIC), and the Kolmogorov-Smirnov (K-S tests), when *n* and *p* are defined as the number of samples and the number of parameters, as shown in Eqs. (2.11)-(2.12) [16], respectively.

$$AIC = -2\log\left(LL\right) + 2p, \tag{2.11}$$

$$K - S = \sup_{x}\left|F_n\left(x\right) - F_0\left(x\right)\right|, \tag{2.12}$$

The distribution that has the lowest AIC and K-S values and p-values > 0.05 is selected as the optimal one for fitting the data. The parameter estimations and goodness of fit tests are shown in Table 1. The results reveal that both distributions fit datasets 1 and 2, while dataset 3 fits R-TLRD but not LRD. For datasets 2 and 3, R-TLRD provides a lower -LL, AIC, and K-S test (p-values > 0.05), which means R-TLRD is more corresponding to the two datasets than LRD. Dataset 1 shows that both LRD and R-TLRD fit the data well, especially with similar values for AIC, K-S, and p-value.

3. Group Acceptance Sampling Plan under R-TLRD

To increase efficiency for industrial applications, the R-TLRD is practical for group acceptance sampling plan. Symbols and variables are defined for ease of study as [9;17]:

μ : the true average product life,

μ_0 : the specified life of an item,

g : the number of groups,

r : the number of items for each group,

c : the acceptance number,

t_0 : the experimental time $\left(t_0 = a\mu_0\right)$, where a is a specified constant (or a termination ratio),

β : the consumer's risk (or the probability of accepting a bad lot),

γ : the producer's risk (or the probability of rejecting a good lot).

3.1 GASP under R-TLRD

By using the GASP, the sample size of the speculated lot is $n = r \times g$ .The sampling plan $r = 1$ is called a single acceptance sampling plan. If the sampling information supports the hypothesis $H_0 = \mu \geq \mu_0$, a product lot is speculated to be a good product, and the product lot is accepted. The probability of lot acceptance is given in Eq. (2.13),

$$L\left(p\right) = \left[\sum_{i=0}^{c}\binom{r}{i}p^i\left(1-p\right)^{r-i}\right]^g. \tag{2.13}$$

In this study, the GASP is developed under the product lifetime R-TLRD as:

$$p = F(x) = \frac{1-\theta^{\alpha}\left(\theta + M^{2}\right)^{-\alpha}}{1-\theta^{\alpha}\left(\theta + b^{2}\right)^{-\alpha}},$$

where

$$M = \frac{\alpha\theta^{\frac{1}{2}}}{\left(\theta^{\alpha}\left(\theta+b^{2}\right)^{-\alpha}-1\right)} \cdot \left[\frac{\Gamma(\alpha-0.5)\Gamma(1.5)}{\Gamma(\alpha+1)}\right] \cdot \frac{\alpha}{\mu/\mu_0}$$

and $\mu/\mu_0 = 4{,}6$ and 8. The number of groups is determined under the following conditions:

$$P_a\left(p_1 \middle| \frac{\mu}{\mu_0} = 1\right) = \left[\sum_{i=0}^{c}\binom{r}{i} p_1^{i}\left(1-p_1\right)^{r-i}\right]^{g} \le \beta,$$

$$P_a\left(p_2 \middle| \frac{\mu}{\mu_0} = d\right) = \left[\sum_{i=0}^{c}\binom{r}{i} p_2^{i}\left(1-p_2\right)^{r-i}\right]^{g} \ge 1-\gamma,$$

where d = 4, 6 and 8,

$$p_1 = \frac{1-\theta^{\alpha}\left(\theta+M_1^{2}\right)^{-\alpha}}{1-\theta^{\alpha}\left(\theta+b^{2}\right)^{-\alpha}},\ p_2 = \frac{1-\theta^{\alpha}\left(\theta+M_2^{2}\right)^{-\alpha}}{1-\theta^{\alpha}\left(\theta+b^{2}\right)^{-\alpha}},$$

$$M_1 = \frac{\alpha\theta^{\frac{1}{2}}}{\left(\theta^{\alpha}\left(\theta+b^{2}\right)^{-\alpha}-1\right)}\left[\frac{\Gamma(\alpha-0.5)\Gamma(1.5)}{\Gamma(\alpha+1)}\right] \times \alpha$$

and

$$M_2 = \frac{\alpha\theta^{\frac{1}{2}}}{\left(\theta^{\alpha}\left(\theta+b^{2}\right)^{-\alpha}-1\right)}\left[\frac{\Gamma(\alpha-0.5)\Gamma(1.5)}{\Gamma(\alpha+1)}\right] \times \frac{\alpha}{\mu/\mu_0}.$$

The minimization of both producer's risk and consumer's risk is one approach to optimizing acceptance sampling plans using nonlinear optimization.

Table 1. MLEs , -LL, AIC, and K-S tests for the three real datasets.

Dataset	Distribution	P.E. α	θ	b	-LL	AIC	K-S (p-value)
1	LRD	1.1039	1867.01	-	250.5932	505.1865	0.0543 (0.9967)
	R-TLRD	0.7998	1269.3284	293.40	249.5463	505.0926	0.05674 (0.9763)
2	LRD	15.9972	88.1643	-	102.4106	208.8212	0.0849 (0.7217)
	R-TLRD	1.0288	5.0916	5.14	100.2586	206.5172	0.0766 (0.7324)
3	LRD	0.02549	3×10^{-13}	-	120.4163	244.8327	1.5185 (<0.0001)
	R-TLRD	3.5709	224,458.6282	494	78.2430	160.4861	0.1959 (0.6319)

3.2 Numerical method for GASP under R-TLRD

GASP under R-TLRD when $\mu/\mu_0 = 4{,}6$ and 8, $\gamma = 0.05, \beta = 0.10, 0.05$ and 0.01 are brought into the numerical process. The industry typically selects a value of 0.05 to reduce product rejection and control inspection expenses. Values are varied based on the severity of the defect; non-critical products may have higher values, whereas high-risk products necessitate lower values. The optimal parameters (g, r, c) of the developed GASP are set and chosen to simultaneously satisfy both the producer's risk (γ) and consumer's risk (β) to the maximum value of $L(p)$. Two parameter values of R-TLRD are used in the process, viz. (α, θ, b) = (0.8,0.3,10) and (α, θ, b) = (3.5709, 224,458.6282, 494) or real data sets that estimate parameters from the 3rd dataset in Table 1. The optimal values (g,r,c) of GASP under R-TLRD to maximize $L(p)$ can be solved using the nonlinear optimization problem as follows [18]:

Objective function :

Maximize $L(p)$

Subject to :

$$P_a(p_1) \le \beta, \quad P_a(p_2) \ge 1-\gamma,$$
$$g, r > 0,\ c \ge 0 \ \text{ and } \ r \ge c.$$

Table 2. Optimal g,r, and c values for GASP under R-TLRD when $\alpha = 0.8$, $\theta = 0.3$, $b = 10$.

β	μ/μ_0	$a = 0.5$ g,r,c	$a = 0.5$ OC	$a = 0.8$ g,r,c	$a = 0.8$ OC	$a = 1.0$ g,r,c	$a = 1.0$ OC
0.1	4	3,3,1	0.9671	2,4,2	0.9790	2,3,2	0.9826
	6	3,3,1	0.9927	2,4,2	0.9974	1,5,2	0.9902
	8	3,3,1	0.9976	3,2,1	0.9950	1,5,2	0.9978
0.05	4	4,5,2	0.9913	5,3,2	0.9853	5,3,2	0.9570
	6	4,5,2	0.9991	3,4,2	0.9961	4,2,1	0.9568
	8	4,5,2	0.9998	3,4,2	0.9992	4,2,1	0.9847
0.01	4	12,6,2	0.9513	10,3,2	0.9708	6,5,3	0.9556
	6	10,4,1	0.9534	5,7,2	0.9525	5,5,2	0.9520
	8	7,8,1	0.9518	5,4,1	0.9537	4,3,1	0.9564

Table 3. Optimal g,r, and c values for GASP under R-TLRD when α =3.5709, θ=224,458.6282, b = 494 (See in the third data from Table1).

β	μ/μ_0	$a = 0.5$ g,r,c	$a = 0.5$ OC	$a = 0.8$ g,r,c	$a = 0.8$ OC	$a = 1.0$ g,r,c	$a = 1.0$ OC
0.1	4	2,8,1	0.9811	2,5,2	0.9979	3,3,2	0.9988
	6	3,5,1	0.9978	2,4,1	0.9945	1,8,3	0.9999
	8	3,4,1	0.9996	2,4,1	0.9982	1,8,3	1
0.05	4	3,9,1	0.9642	3,6,3	0.9998	4,5,3	0.9994
	6	3,9,1	0.9924	3,6,2	0.9994	**3,3,1**	**0.99**
	8	3,9,1	0.9975	3,6,2	0.9999	3,3,1	0.9967
0.01	4	58,10,2	0.9567	46,5,2	0.9533	14,5,2	0.9504
	6	34,7,1	0.9503	4,8,1	0.9523	3,6,1	0.9539
	8	21,10,1	0.9787	4,8,1	0.9839	3,6,1	0.9844

Tables 2-3 show the results of GASP under R-TLRD at various values of $(\alpha, \theta, b) = (0.8, 0.3, 10)$ and $(\alpha, \theta, b) = (3.5709, 224, 458.6282, 494)$ or real dataset. They propose the optimal values of g,r,c, and OC for fixing the values of a = 0.5,0.8 and 1.0, $\gamma = 0.05$, β =0.10, 0.05 and 0.01, and μ/μ_0= 4,6,and 8, respectively. For the fixed values of a and β, the results reveal that the sample sizes $(n = g \times r)$ and c tend to decrease when μ/μ_0 increases. On the other hand, if μ/μ_0 and a are fixed, the sample size $(n = g \times r)$ increases when β decreases. Table 2 gives an example to explain that if the consumer's risk(β) is set to 0.05, μ = 4,000 min, μ_0 =1,000 min and t_0 = 500 min, respectively, the optimal sample size should be required to be 20 ($g \times r = 4 \times 5 = 20$), where the acceptance number c does not exceed 2. Table 4, shows the application of GASP under R-TLRD for the real dataset. The trend of the data analysis results is the same as in Table 3. The results are compared with the GASP under the Weibull distribution, a basic distribution for explanation lifetime, in Table 5. The Weibull distribution corresponds to this dataset, with the shape and scale parameters being 2.1754 and 251.8302, respectively. The goodness-of-fit test reveals a K-S-test p-value = 0.6957. Histograms for data and the theoretical probability density functions of distributions are illustrated in Fig. 4. The optimal g, r ,and c values of GASP under Weibull distribution are calculated when $\beta = 0.05$, $\mu / \mu_0 = 4,6,8$ and $a = t_0 / \mu_0 = 4,6,8$, respectively. The GASP under R-TLRD provides smaller g and r values than the GASP under WD when a = 0.5 and 1.0.

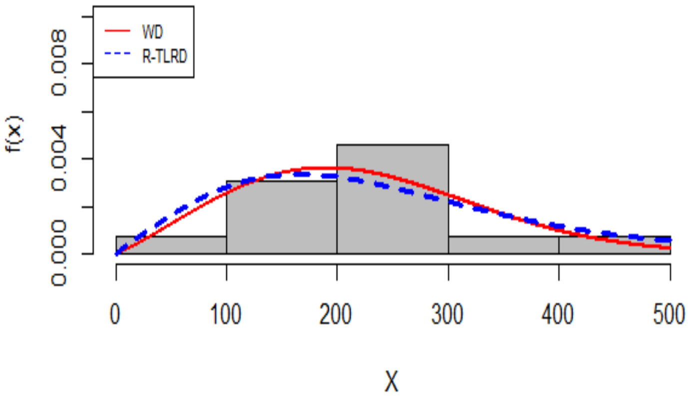

Fig. 4. Plots the histogram of the third dataset and the theoretical pdf of distributions.

Table 4. A comparison of optimal g,r, c, and OC values between the GASP under WD and the GASP under R-TLRD of the third dataset from Table 3 when $\beta = 0.05$.

β	μ / μ_0	a = 0.5		a = 0.8		a = 1.0	
		g,r,c	OC	g,r,c	OC	g,r,c	OC
0.05	4	4,9,1	0.9905	3,6,1	0.9780	3,7,3	0.9998
(WD)	6	5,7,1	0.9988	3,6,1	0.9960	3,4,1	0.9958
	8	5,7,1	0.9996	3,6,1	0.9988	3,4,1	0.9988
0.05	4	3,9,1	0.9642	3,6,3	0.9998	4,5,3	0.9994
(R-TLRD)	6	3,9,1	0.9924	3,6,2	0.9994	3,3,1	0.9900
	8	3,9,1	0.9975	3,6,2	0.9999	3,3,1	0.9967

4. Conclusions

The right truncated Lomax-Rayleigh distribution (R-TLRD) is presented with some statistical properties, such as the survival function, hazard function, moment, and parameter estimation. The developed distribution is applied to three real datasets, with a performance comparison between the developed distribution and the baseline distribution using the goodness-of-fit tests. The results show that the right-truncated Lomax-Rayleigh distribution is consistent with all datasets of real data. Moreover, the proposed distribution is applied to the group acceptance sampling plan. The operational characteristic function (OCF) value is shown together with the optimal value for the number of groups (*g*), items for each group (*r*), and acceptance number (*c*). In terms of a minimum *g, r,* and *c* demand to reach a choice, the designed group acceptance sampling plan under the appropriate truncated Lomax-Rayleigh distribution offers. Decision-makers using GASP will determine and achieve a minimum sample size. The value would be set based on the desired level of quality control, balancing the risk of rejecting good lots to detect defects. By optimizing these parameters, decision-makers can minimize sample size while maintaining effective quality assurance. Since the GASP under R-TLRD provides a smaller number of groups and a smaller number of items for each group, it would be interesting to develop other acceptance sampling plans under R-TLRD in future work, such as a double acceptance sampling plan.

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Appendix

Appendix A

The pdf property verification of R-TLRD

$$t\left(x \middle| x \le b\right) = \frac{2\alpha x\theta^{\alpha}}{\left[1-\theta^{\alpha}\left(\theta+b^2\right)^{-\alpha}\right]\left(\theta+x^2\right)^{\alpha+1}}.$$

Proof.

$$\int_0^b \left(\frac{2\alpha x\theta^{\alpha}}{\left[1-\theta^{\alpha}\left(\theta+b^2\right)^{-\alpha}\right]\left(\theta+x^2\right)^{\alpha+1}} \right) dx$$

$$= \frac{\alpha\theta^{\alpha}}{\left[1-\theta^{\alpha}\left(\theta+b^2\right)^{-\alpha}\right]} \int_0^b \left(\frac{1}{\left(\theta+x^2\right)^{\alpha+1}} \right) d\left(\theta+x^2\right),$$

$$= \frac{\alpha\theta^{\alpha}}{\left[1-\theta^{\alpha}\left(\theta+b^2\right)^{-\alpha}\right]} \left[\frac{\left(\theta+x^2\right)^{-\alpha}}{-\alpha} \right]_{x=0}^{x=b} = 1$$

Appendix B

The calculation of the cdf of R-TLRD

$$T\left(x \middle| x \le b\right) = \frac{\alpha\theta^{\alpha}}{\left[1-\theta^{\alpha}\left(\theta+b^2\right)^{-\alpha}\right]} \times \int_0^x \left(\frac{1}{\left(\theta+x^2\right)^{\alpha+1}} \right) d\left(\theta+x^2\right),$$

$$= \frac{\alpha\theta^{\alpha}}{\left[1-\theta^{\alpha}\left(\theta+b^2\right)^{-\alpha}\right]} \times \left[\frac{\left(\theta+x^2\right)^{-\alpha}}{-\alpha} \right]_{x=0}^{x=x}$$

$$= \frac{1-\theta^{\alpha}\left(\theta+x^2\right)^{-\alpha}}{1-\theta^{\alpha}\left(\theta+b^2\right)^{-\alpha}}.$$