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On Designing of Modified Exponentially Weighted Moving Average Control Chart based on Sign Rank for Zero-Inflated Data

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ABSTRACT

The performance of parametric and nonparametric control charts that are carefully created for zero-inflated count data—which are commonly modeled using zero-inflated Poisson (ZIP) and zero-inflated binomial (ZIB) distributions—is thoroughly assessed in this study. The evaluation of the three primary control chart performance metrics—average run length (ARL), median run length (MRL), and standard deviation run length (SRL)—forms the basis of the comparative analysis. The performance evaluation's new results clearly show that nonparametric control charts are more effective at identifying shifts in the probability of small-scale inflation. On the other hand, changes in large-scale inflation are better detected by parametric control charts. These empirical observations are further substantiated through the applications of real-world data, which serves as a robust case study for assessing the effectiveness of both types of control charts in practical settings.

Keywords: Average run length; Median run length; Standard deviation run length; Zero-inflation binomial; Zero-inflation poisson

1. Introduction

In quality control, data often shows more zeros than expected, especially when counting occurrences due to factors beyond chance. To address this issue, zero inflation models in count data both ZIB and ZIP models were created. These models help identify zeros caused by specific factors rather than random chance. For example, they can distinguish between products with no defects due to high quality stan-

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dards and those with no defects due to random variation.

Zero-inflated Poisson (ZIP) and zero-inflated binomial (ZIB) control charts are significance designed to handle data with excessive zeros, a common issue that traditional parametric control charts struggle to address. These specialized charts offer a better solution for managing data with zero inflation characteristics (see more detail in [1, 2]).

Parametric and nonparametric control charts are two main types used in quality control. Parametric control chart, pioneered by Shewhart [3], rely on known statistical distributions (like Normal or Poisson) to monitor quality characteristics in manufacturing and service processes [4]. Nonparametric control charts, developed for situations where the population distribution is unknown or process parameters cannot be estimated, use methods like rank-based analysis [5, 6].

Parametric control charts are used when the data are normally distributed or when the mean and standard deviation parameters can be estimated. Otherwise, these charts may not be suitable for monitoring process changes. Control charts, first introduced in [3], are valuable tools for detecting significant shifts in process outputs. Modern process monitoring systems often used Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA) control charts, respectively [7, 8]. These charts, known as time-varying control charts, consider both past and current data. The Modified Exponentially Weighted Moving Average (Modified EWMA) chart [9], is an improvement on the EWMA chart by increasing the term of the difference value of adjacent data. If the mean process increases, the term that makes the statistic Modified EWMA also increase, which makes the detection faster.

An appropriate substitute for a parametric control chart in circumstances where assumptions are not addressed is a nonparametric control chart (NP). NP charts offer several benefits, including ease of use, flexibility, robustness to outliers, and the elimination of variance estimation. Graham [10] suggested an EWMA-SR chart in 2011 to track slight but consistent changes in the process mean. Additionally, to detect changes in the average of a continuous distribution, the Wilcoxon sign-rank-based MEWMA chart (MEWMA-SR) was developed. The MEWMA-SR chart performs well, especially for small subgroup sizes and small changes, as shown by the evaluation and numerical results (see [11–13]).

While many production processes follow a normal distribution, quality characteristics can sometimes have non-normal or discrete distributions. For instance, a zero-inflated binomial (ZIB) distribution is frequently used to describe the percentage of defective products or the total number of defects. The zero-inflated Poisson (ZIP) distribution is appropriate for counting procedures, and a binomial distribution is appropriate when gauging quality by the quantity of nonconformities.

Numerous studies, spanning from earlier periods to the present, have focused on quality control methods specifically designed for ZIB and ZIP data that have been addressed. Xie et al. [14] compared various control chart methods for defect data, which is directly relevant to applying control charts for zero- inflated models. Li et al. [15] presented a control chart specifically for ZIP distributed data, making it directly relevant to understanding how zero-inflation affects control chart designs and interpretations in quality control. A cumulative sum (CUSUM) control chart tailored for ZIP data was proposed by Zhu et al. [16]. It is a useful resource for comprehending sophisticated control chart methods that take manufacturing or defect data with zero inflation into account. Exponentially weighted moving average (EWMA) control charts for zero-inflated Poisson processes were covered by Zhou et al. [17]. It presents statistical methods designed specifically for quality control in situations with zero inflation. Although not unique to models with zero inflation. Phantu et al. [18] investigated the mixed DMA-EWMA control chart for both symmetric and asymmetric distributions which the performance of the mixed control chart is superior to single control chart. A thorough analysis of control charts for count data was given by Castagliola et al. [19], which is crucial for comprehending how ZIB and ZIP models fit into the larger framework of control chart methodologies. Although not directly about control charts, Beyene et al. [20] discussed applications of zero-inflated models in data sets with excess zeros, offering context for the kinds of data where these models would be useful in quality control. Bakar et al. [21] proposed a control chart for ZIB data, extending traditional binomial control charts to handle situations where excessive zeros occur, such as in defect monitoring.

Investigating parametric and nonparametric control charts made especially for integer data characteristics that adhere to ZIP or ZIB distributions is the aim of this study. Of particular interest is their capacity to identify changes in data that contains a large number of zeros.

The following is the structure of the article's following sections: Inflation distributions, nonparametric properties, and control chart features are briefly reviewed in Section 2. Section 3 assesses how well the control chart detects changes in the probability of the extra zero parameter. In Section 4, the control chart's use with simulated and real-world data is demonstrated and contrasted with alternative approaches. Section 5 concludes with a summary of the main conclusions, a discussion of the limitations, and possible future research directions.

2. Materials and Methods

2.1 Zero-inflated distributions

This research focuses on the study of data from a counting process in which a large number of zeros occur. This is sometimes called a zero-inflation distribution, which in industry tends to occur with high yield process.

2.1.1 Zero-inflation binomial

A generalization of the regular binomial distribution, the zero-inflated binomial (ZIB) distribution can be used to simulate count processes with an excessive number of zeros. As with the conventional binomial distribution, if X is a ZIB random variable, it is defined on $0, 1, ..., n$ by definition, and its probability mass function (pmf.) equals when $x = 0$ as following:

$$f_{ZIB}(0|\phi, n, p) = \phi + (1-\phi) f_B(0|n, p),$$

while for $xin\{1, 2, ..., n\}$

$$f_{ZIB}(x|\phi, n, p) = \phi + (1-\phi) f_B(x|n, p),$$

where

$$f_B(x|n, p) = \binom{n}{x} p^x (1-p)^{n-x},$$

is the pmf. of the binomial distribution, where $p \in [0, 1]$ is the probability of a predefined event and $\phi \in [0, 1]$ is the zero-inflation parameter. The ZIB distribution and the binomial distribution with parameters n and p coincide if $\phi = 0$. Otherwise, if $\phi = 1$, the ZIB distribution reduces to the Dirac distribution on $x = 0$. Furthermore, the cumulative distribution function (cdf.) of X is given by

$$f_{ZIB}(x|\phi, n, p) = \phi + (1 - \phi)F_B(x|n, p),$$

where

$$F_B(x|n, p) = \sum_{z=0}^{x} \binom{n}{z} p^z (1 - p)^{n-z},$$

is the cdf. of the standard binomial distribution with parameters n and p.

The following two expressions provide the mean and variance of the ZIB distribution with parameters (ϕ, n, p), respectively.

$$\mu_{ZIB} = np(1 - \phi),$$

and

$$\delta^2_{ZIB} = np(1 - p + np\phi)(1 - \phi).$$

2.1.2 Zero-inflation poisson

In order to fit count data with excessive zero values—which can be thought of as a combination of a degenerate distribution at zero with probability $1 - \rho$ and a Poisson distribution with probability ρ, zero-inflated Poisson regression models have been introduced widely. Letting $Y_1, , Y_n$ be a sequence of independently and identically random variables, in which $Y_i \sim ZIP(\rho, \theta_i)$ then the pmf. of the ZIP model is

$$P(Y_i = y_i) = \begin{cases} \rho + (1 - \rho)e^{-\theta} & ; y_i = 0, \\ (1 - \rho)\frac{\theta_i^{y_i} e^{\theta_i}}{y_i} y_i & ; y_i = 1, 2, ..., \end{cases}$$

where $0 < \rho < 1$, assuming a fixed probability, is the likelihood of excessive zeros. The following are the ZIP random variable's mean and variance: $\mu_{ZIP} = (1-\rho)\theta$, and $\sigma^2_{ZIP} = (1 - \rho)(\theta + \rho\theta^2)$.

This section, which is divided into three sections, presents relevant theory. The control chart's nonparametric characteristics are covered in detail in Section 2.1. The conceptual design of the traditional control chart as an exponentially weighted moving average (EWMA) control chart and its conversion to an exponentially weighted moving average (MEWMA) control chart are explained in Section 2.2. It also proposes the modified exponentially weighted moving average based on sign rank (MEWMA-SR) control chart and discusses the current nonparametric control chart that has been converted into an exponentially weighted moving average based on sign rank (EWMA-SR) control chart. Lastly, Section 2.3 describes the process for assessing achievement.

2.2 Sign rank

Take into consideration $A_t = A_{t1}, A_{t2}, ..., A_{tk}$ a sample of size n from a procedure that has an arbitrary distribution with a process target (α). Eq. (2.1) represents the distribution of the observed quantity and the desired quantity, indicated as within groups,

$$\beta_{tk} = A_{tk} - \alpha, \ t = 1, 2, 3, ...; k = 1, 2, 3, ... \tag{2.1}$$

The S_t statistic is calculated as:

$$S_t = \sum_{k=1}^{n} I_{tk} \text{where } I_{tk} = \begin{cases} 1 & , \beta_{tk} > 0, \\ 0 & , \text{otherwise.} \end{cases}$$

The sign statistic for the case of the control state is the total number of observations that fit a binomial distribution with the in control parameter ($n, p = 0.5$). The procedure, which is $p = P(\beta \leq 0) = P(\beta > 0) = 0.5$ for an in control process, is represented by the fraction. On the other hand, when the process becomes uncontrollable, $q \neq 0.5$.

Finding J_{tk} the absolute rank difference $|A_{tk} - \alpha|$ within the subset t^{th} is important. The following is the expression of the sign rank statistic:

$$SR_t = \sum_{k=1}^{n} I_{tk} J_{tk},$$

where

$$I_{tk} = \begin{cases} 1 & ; (A_{tk} - \alpha) > 0 \\ 0 & ; (A_{tk} - \alpha) = 0 \\ -1 & ; (A_{tk} - \alpha) < 0 \end{cases}.$$

2.3 The characteristics of control charts

The control chart under examination can be shown as follows:

2.3.1 Exponentially Weighted Moving Average (EWMA) control chart

This control chart is a time-weighted technique that incorporates historical data and was first presented by [7] in 1959. Particularly when handling small changes, it exhibits remarkable sensitivity in identifying variation in the process. Eq. (2.2), which represents the EWMA statistic, can be explained as follows:

$$EWMA_t = \lambda Y_t + (1-\lambda) EWMA_{t-1}, t = 1, 2, ..., \tag{2.2}$$

where the parameter λ, which ranges from 0 to 1 applied to weight historical data. With independent and regularly spaced observations Y_t, the initial value $EWMA_0$ is typically assumed to be expectation of process observation μ_0. Consequently, the following equation can be used to characterize the mean and variance:

$$E(EWMA) = \mu_0,$$

and

$$V(EWMA_t) = \sigma^2 \left(\frac{\lambda}{2-\lambda} (1 - (1-\lambda)^{2t}) \right), \tag{2.3}$$

where μ_0 and σ^2 are the mean and variance. As t approaches infinity from Eq. (2.3), the variance asymptotically is

$$V(EWMA) = \sigma^2 \frac{\lambda}{2-\lambda}. \tag{2.4}$$

Thus, the control limit of the EWMA chart follows Eq. (2.4)

$$UCL_{EWMA}/LCL_{EWMA} = \mu_0 \pm L_1 \sigma \sqrt{\frac{\lambda}{2-\lambda}}. \tag{2.5}$$

The desired ARL_0 is represented by L_1, the width of control limit for the EWMA chart. The Monte Carlo simulation method can be used to determine this value in order to reach the desired ARL_0.

2.3.2 Modified Exponentially Weighted Moving Average (MEWMA) control chart

The MEWMA control chart was initiated by appending the additional term $k(Y_t - Y_{t-1})$ to the EWMA statistic in order to improve detection performance. MEWMA statistics for the same dataset were higher than the EWMA chart after this adjustment [9]. The MEWMA chart's statistics are as follows:

$$\begin{aligned} MEWMA_t = \lambda Y_t + (1-\lambda) MEWMA_{t-1} \\ + k(Y_t - Y_{t-1}), t = 1, 2, ... \end{aligned} \tag{2.6}$$

The constant k is represented in the equation. When k is set to 1, the MEWMA chart statistic takes on a similar form with $k = 1$ (see details [9]). The following are the MEWMAt mean and deviation for an in-control process:

$$E(MEWMA) = \mu_0 \tag{2.7}$$

and

$$V(MEWMA) = \sigma^2 \frac{\lambda + 2\lambda k + 2k^2}{2-\lambda}. \tag{2.8}$$

The MEWMA chart's upper and lower control limits are described as follows:

$$UCL_{MEWMA}/LCL_{MEWMA} = \mu_0 \pm L_2\sigma\sqrt{\frac{\lambda + 2\lambda k + 2k^2}{2-\lambda}}. \tag{2.9}$$

The MEWMA chart's control limit width is determined by the control limit coefficient L_2, whose value is chosen to achieve an in control average run length (ARL_0).

2.3.3 Exponentially Weighted Moving Average-Sign Rank (EWMA-SR) control chart

[8] states that a normal distribution is assumed by this commonly used control chart for production processes. However, it becomes clear that non-normal distributions can be seen in production processes. To address this, a nonparametric method called the EWMA-SR control chart is presented, which is meant to track variations in the process mean. The mathematical definition of the EWMA-SR statistic is Eq. (2.10).

$$EWMA - SR_t = \lambda SR_t + (1-\lambda)EWMA - SR_{t-1},\ t = 1, 2, ... \tag{2.10}$$

where λ is the weight with a range of $0 < \lambda < 1$. The following describes the EWMA-SRt mean and asymptotic variance for the controlled process:

$$E(EWMA - SR) = 0 \tag{2.11}$$

and

$$V(EWMA{-}SR) = \frac{\lambda}{2-\lambda}\left(\frac{n(n-1)(2n+1))}{6}\right). \tag{2.12}$$

Consequently, the EWMA-SR chart's upper and lower control limits match the definitions listed below:

$$UCL_{EWMA-SR}/LCL_{EWMA-SR} = \pm L_3\sqrt{\frac{\lambda}{2-\lambda}\left(\frac{n(n+1)(2n+1)}{6}\right)}, \tag{2.13}$$

where the desired ARL0 selects the L3 coefficient, which is used to determine the control limits of the EWMA-SR chart, and the mean number of samples is anticipated prior to a false alarm.

2.3.4 Modified Exponentially Weighted Moving Average-Sign Rank (MEWMA-SR) control chart

The MEWMA-SR chart was created by the MEWMA sign by adding a supplementary rank phase, which works especially well when the process changes slightly [10]. The process median can be monitored with the help of SR statistics. The following statistical value is present in the MEWMA-SR chart:

$$MEWMA - SR_t = \lambda SR_t + (1-\lambda)MEWMA - SR_{t-1} + k(SR_t - SR_{t-1}),\quad t = 1, 2, ... \tag{2.14}$$

The mean and asymptotic variance of the MEWMA-SR control chart are as follows:

$$E(MEWMA - SR) = 0 \tag{2.15}$$

and

$$V(EWMA - SR) = \frac{\lambda + 2\lambda k + 2k^2}{2-\lambda}\,\frac{n(n-1)(2n+1)}{6}. \tag{2.16}$$

Thus, the asymptotic control limit of the MEWMA-SR control chart is as follows:

$$UCL_{MEWMA-SR}/LCL_{MEWMA-SR}$$

$$= \pm L_4 \sqrt{\frac{\lambda + 2\lambda k + 2k^2}{2-\lambda} \frac{n(n-1)(2n+1)}{6}}, \quad (2.17)$$

where L_4 is the factor of control limit for the MEWMA-SR chart, corresponding with the designated ARL_0, The MEWMA-SR will determine whether samples are out of control for $MEWMA - SR_t > UCLMEWMA - SR$ or $MEWMA - SR_t < LCLMEWMA - SR$.

2.4 Comparison of control chart and calculation step

In the literature on statistical process control, run-length metrics like average, standard deviation, and median—referred to as ARL, SRL, and MRL, respectively—are used to evaluate the control chart's performance. Prior to an out-of-control sample, the samples' average, standard deviation, and median are known as the ARL, SRL, and MRL. Additionally, in-control ARL, or ARL_0, is measured when the process is in a stable state, whereas out-of-control, or ARL_1, is calculated when the process is unstable or shifted.

Eq. (2.18) provides a mathematical expression for the average run length (ARL), where RL is the number of samples needed before the system becomes uncomfortable for the first time. It can be calculated in this way:

$$ARL = \frac{\sum_{t=1}^{t} RL_t}{N}. \quad (2.18)$$

The standard deviation run length (SRL) is a statistical measure used in quality control and process monitoring to analyze the variability of run lengths before a signal is detected. It provides insights into how consistent or variable the performance of a process is over time. It can be calculated as follows:

$$SRL = \sqrt{E(RL_i)^2 - ARL^2}. \quad (2.19)$$

The median run length (MRL) is a statistical measure often used in quality control and process monitoring. It provides insights into the average performance of a process by measuring the length of time (or number of units) until a change or an outlier is detected. Thus, the MRL is calculated as follows:

$$MRL = Median(RL_i). \quad (2.20)$$

Using a Monte Carlo simulation with 100,000 iterations (N) and $ARL_0 = 370$ in-control case parameters, the run length properties of the control chart can be investigated. In order to align the final value with roughly the ARL_0 equivalent, selecting the control limit coefficient is crucial.

The following are the steps that describe the process of finding a solution:

Step 1: Create N arbitrary samples, such as the ZIP and ZIB distributions, from the zero-inflation distribution.

Step 2: Fit the observed count data to the zero-inflation distribution. Section 2 explains the process for fitting two models.

Step 3: For every control chart, estimate the expected proportion of zero with regard to UCL and LCL.

Step 4: Determine the L at ARL_0 values of 370 by computing the suggested tracking numerical data.

Step 5: Determine which weighting values (λ) are present in the control chart and the percentage of zero for ZIP (ϕ) and ZIB (θ when the process is out of control.

Step 6: Determine the control chart's statistics and control limits.

Step 7: Write down the control chart's run length (RL) until the data surpasses the control limit.

Step 8: To calculate the ARL1, SRL1, and MRL1 and assess the control chart's efficacy, repeat iteration 20,000 times (N).

For each control chart, the average run length value was finally calculated. The EWMA, MEWMA, EWMA-SR, and MEWMA-SR charts were then used to compare the performance of the suggested chart. Additionally, assuming a fixed ARL_0 for all charts, a chart is deemed the best for performance comparison if its ARL_1 is smaller than that of the others.

3. Performance Evaluation of ZIB and ZIP Model

In this study, the performance control chart is analyzed. In Section 3.1 (in control) and Section 3.2 (out of control), the results are carried on from Monte Carlo simulation and compared the performance of the proposed chart with the four charts currently in use are presented. In Section 3.3 a comparison of a case study control chart is shown.

The in control and out of control performance of the ZIB and ZIP models for the control chart when the ZIB and ZIP parameters are known are examined in this section.

3.1 In-control performance

Calculate the run-length distribution of the ZIB and ZIP models for the EWMA, MEWMA, EWMA-SR, and MEWMA-SR charts. We set the smoothing constant for the control chart $w = 0.05$.

In the simulation study of the ZIB model, given $ARL_0 = 370$, the IC values of ZIB parameters $\phi_0 = 0.1, 0.5, 0.9$, and $p_0 = 0.1, 0.5$. Next, in the simulation study of the ZIP model, IC values of ZIP parameter $\theta_0 = 0.1, 0.2$ and $\rho_0 = 0.1, 0.5$.

These values are used for the design of all charts. Furthermore, the IC standard deviation of the run length (SRL_0) is reported to obtain more information about the run-length distribution. In addition, The IC median of the run length distribution (referred as MRL_0) is reported about data trend.

3.2 Out-control performance

Monitoring increases in the ZIP and ZIP parameter dispersion parameters is of interest to us in this study. All of the ARL_1, SRL_1, and MRL_1 charts' performance comparison results can be displayed as follows.

3.2.1 Comparison control chart of ZIB model

We study the OOC performance of the ZIB for EMWA, MEWMA, EWMA-SR and MEWMA-SR charts when $ARL_0 = 370$ and either one ZIB parameters ϕ shift from ϕ_0 to $\phi_1 = (1 +)\phi_0$. The results are shown in the Tables 1 to 6. From these tables, we observe the following:

Table 1 shows that, for a fixed value of p and a specific sample size n, as ϕ_0 increase, all charts become more effective, as indicated by a decrease in ARL_1. For instance, when $(p_0, n, \phi_0) = (0.1, 200, 0.1)$ and $\eta = 0.01$. The MEWMA-SR chart demonstrates the best performance. Next, when $0.05 \leq \eta \leq 0.10$, the EWMA-SR chart is the most effective performance. Finally, for $\eta > 0.10$, the EWMA chart is one of the most suitable control chart for detecting shift in dispersion parameter of ZIB model. The comparison of ARL_1 performance with increasing dispersion parameter of ZIB model yield the same numerical results as SRL_1 and MRL_1.

Next, Table 2 presents the ZIB model with parameters $(0.1, 200, 0.5)$. The numerical results indicated that the MEWMA-SR chart performs best when $\eta = 0.01$. Following this, the EWMA-SR chart shows the most effective performance when $0.05 \leq \eta \leq 0.10$. Finally, the EWMA chart performs best when $\eta > 0.10$. The numerical

Table 1. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIB distribution when $ARL_0 = 370$, proportion of zero $\phi = 0.1$, and $p_0 = 0.1$.

Shift	EWMA			MEWMA			EWMA-SR			MEWMA-SR		
(η)	$L_1 = 5.746$			$L_2 = 4.288$			$L_3 = 5.127$			$L_4 = 4.169$		
	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL
0.01	361.03	357.7	338	358.32	354.56	331	353.97	350.89	328	**352.12**	**347.51**	**323**
0.05	346.13	342.05	312	342.56	339.16	307	**340.41**	**336.62**	**304**	345.31	340.12	305
0.1	328.03	324.18	284	325.17	321.52	275	**325.46**	**321.74**	**272**	330.48	326.55	273
0.15	**297.43**	**294.57**	**259**	298.18	294.57	260	299.51	295.78	261	308.41	304.91	261
0.2	**264.18**	**261.5**	**227**	265.37	261.97	229	267.36	262.33	230	269.45	264.31	231
0.25	**240.18**	**236.74**	**197**	241.72	237.01	200	243.59	240.87	203	244.82	240.16	205
0.3	**152.34**	**148.69**	**133**	155.31	151.63	137	155.25	152.85	139	154.26	151.2	142
0.35	**106.35**	**102.37**	**85**	109.48	105.33	88	108.79	105.5	90	110.72	107.64	87
0.4	**92.34**	**89.42**	**74**	92.53	89.57	77	93.32	89.63	79	95.92	91.61	76
0.5	**42.29**	**38.74**	**31**	43.4	40.88	35	43.95	40.25	38	50.77	46.27	26
1	**34.21**	**30.12**	**22**	35.72	31.75	24	36.69	32.95	25	39.15	33.4	25

The median run length is MRL, the standard deviation of RL is SRL, and the minimum of ARL1 is bolded.

Table 2. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIB distribution when $ARL_0 = 370$, proportion of zero $\phi = 0.5$, and $p_0 = 0.1$.

Shift	EWMA			MEWMA			EWMA-SR			MEWMA-SR		
(η)	$L_1 = 5.638$			$L_2 = 4.274$			$L_3 = 5.116$			$L_4 = 4.133$		
	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL
0.01	358.57	355.11	335	356.21	352.67	332	351.28	347.06	329	**349.55**	**345.2**	**325**
0.05	343.18	339.17	310	340.22	336.96	308	**338.65**	**334.58**	**306**	342.29	338.36	310
0.1	326.96	324.69	282	324.41	320.12	281	**323.53**	**319.65**	**280**	328.12	324.52	284
0.15	**296.13**	**292.31**	**256**	298.18	294.6	258	299.51	295.85	259	308.41	304.86	265
0.2	**262.53**	**258.41**	**224**	263.71	259.13	227	265.94	261.4	228	266.03	262.11	230
0.25	**238.62**	**234.61**	**194**	239.05	234.52	195	241.27	237.85	196	242.31	237.41	197
0.3	**150.85**	**146.56**	**130**	153.65	149.64	132	153.33	150.36	134	154.28	150.37	132
0.35	**104.52**	**100.64**	**82**	107.37	103.23	83	106.48	102.75	85	108.72	104.97	84
0.4	**89.21**	**85.46**	**71**	90.57	86	72	91.88	87.64	73	93.68	89.66	72
0.5	**39.96**	**35.22**	**28**	41.14	36.45	29	42.01	38.51	30	47.58	43.53	29
1	**30.23**	**26.57**	**19**	35.63	31.58	21	34.29	30.56	22	36.75	32.55	22

The median run length is MRL, the standard deviation of RL is SRL, and the minimum of ARL_1 is bolded.

results are reflected in the ARL_1, SRL_1, and MRL_1 minimum values.

Table 3 illustrates the ZIB model with parameters $(0.1, 200, 0.9)$. The results show that the MEWMA-SR chart performs best when $\eta = 0.01$. Subsequently, the EWMA-SR chart demonstrates optimal performance for $0.05 \leq \eta \leq 0.10$, while the EWMA chart is most effective when $\eta > 0.10$. These findings are evident from the minimum values of ARL_1, SRL_1, and MRL_1. The results for the average run length, median run length, and standard deviation of run length from Tables 1-3 are shown in Fig. 1.

Furthermore, Tables 4-6 illustrates the performance of all control charts with a fixed $p_0 = 0.5$ and varied $\phi_0 = 0.1, 0.5$ and 0.9, respectively. The numerical results for $p_0 = 0.1$ are identical to those for $p_0 = 0.5$. The results for the average run length (ARL), median run length (MRL), and standard deviation of run length (SRL), as summarized in Tables 4–6, are presented in detail in Fig. 2 for further illustration and analysis.

Table 3. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIB distribution when $ARL_0 = 370$, proportion of zero $\phi = 0.9$, and $p_0 = 0.1$.

Shift	EWMA			MEWMA			EWMA-SR			MEWMA-SR		
(η)	$L_1 = 5.597$			$L_2 = 4.245$			$L_3 = 5.069$			$L_4 = 4.112$		
	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL
0.01	356.92	352.97	334	353.47	349.15	331	349.83	345.02	331	**346.53**	**342.62**	**329**
0.05	340.54	336.52	310	338.2	334.61	309	**335.17**	**331.25**	**308**	340.47	336.5	310
0.1	324.52	320.15	280	322.34	319.64	281	**320.98**	**316.38**	**279**	326.53	322.47	280
0.15	**294.35**	**290.18**	**154**	296.07	295.47	155	298.25	295.64	157	305.34	301.75	156
0.2	**260.28**	**256.38**	**222**	261.45	258.33	224	263.21	259.86	225	263.75	259.64	224
0.25	**236.27**	**232.65**	**192**	236.49	232.75	193	237.53	234.52	194	240.13	236.75	193
0.3	**148.37**	**145.07**	**128**	151.59	148.92	129	151.83	147.65	130	152.6	148.63	130
0.35	**102.45**	**98.66**	**80**	104.63	100.32	82	104.58	99.64	84	105.92	92.86	83
0.4	**86.1**	**82.56**	**68**	88.61	84.97	70	88.1	84.3	71	90.65	86.77	70
0.5	**36.25**	**33.2**	**25**	38.73	34.87	26	40.18	36.5	27	44.66	41.21	27
1	**28.6**	**24.82**	**16**	33.28	30.65	17	33.52	30.87	18	34.43	31.66	17

The median run length is MRL, the standard deviation of RL is SRL, and the minimum of ARL_1 is bolded.

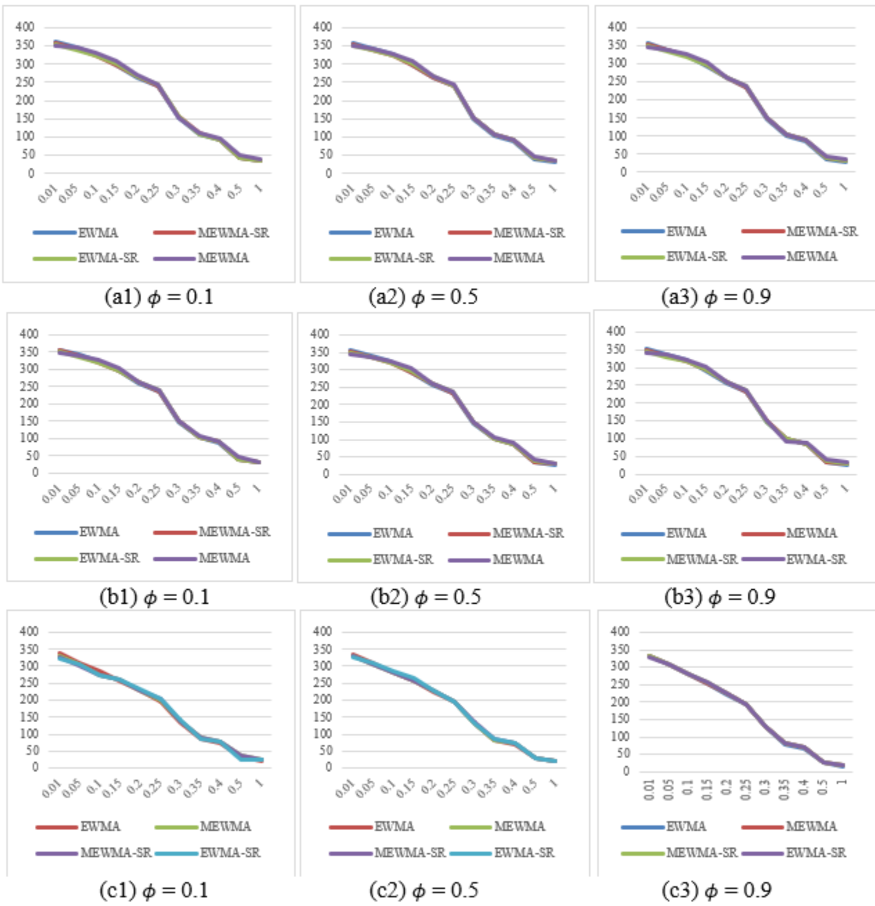


Fig. 1. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIB distribution with $p = 0.1$: (a1)-(a3) is ARL, (b1)-(b3) is SRL, and (c1)-(c3) is MRL.

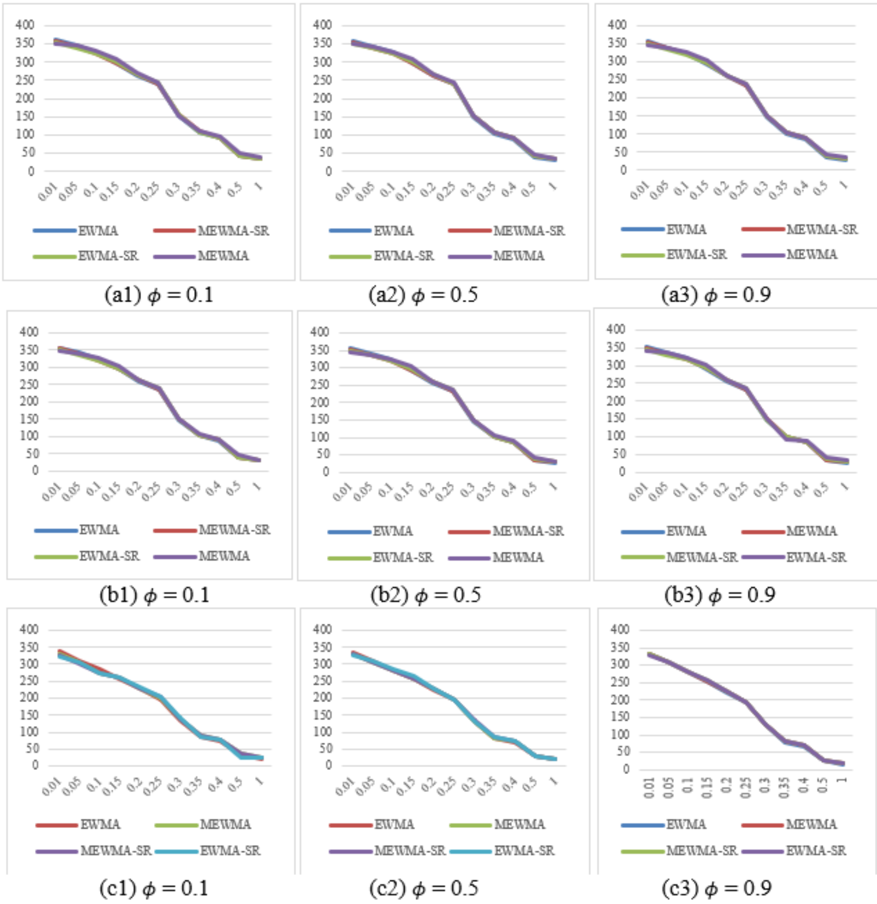


Fig. 2. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIB distribution with $p = 0.5$: (a1) - (a3) is ARL, (b1) - (b3) is SRL, and (c1) - (c3) is MRL.

Table 4. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIB distribution when $ARL_0 = 370$, proportion zero$\phi = 0.1$, and $p_0 = 0.5$.

Shift	EWMA			MEWMA			EWMA-SR			MEWMA-SR		
(η)	$L_1 = 5.897$			$L_2 = 4.325$			$L_3 = 5.261$			$L_4 = 4.287$		
	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL
0.01	352.53	348.26	340	349.85	345.57	338	346.25	343.01	337	**343.86**	**339.52**	**336**
0.05	346.13	342.75	319	342.56	338.53	318	**335.29**	**331.64**	**316**	338.53	334.2	317
0.1	325.63	321.66	294	324.12	320.14	293	**323.53**	**319.57**	**292**	327.91	322.67	294
0.15	**295.73**	**291.58**	**267**	297.66	295.41	268	297.41	293.65	270	305.01	301.49	296
0.2	**261.78**	**257.65**	**227**	263.21	259.64	229	265.92	261.06	230	266.47	261.37	228
0.25	**237.8**	**233.66**	**185**	238.27	234.67	186	240.09	236.46	190	241.63	237.57	188
0.3	**152.34**	**147**	**137**	153.75	149.5	139	153.95	149.7	140	153.37	150.35	139
0.35	**104.21**	**101.7**	**86**	107.72	103.68	88	106.33	102.17	89	108.37	104.69	87
0.4	**88.22**	**84.32**	**45**	89.66	85.69	46	91.64	87.37	48	93.27	89.77	48
0.5	**39.56**	**35.62**	**24**	41.37	37.16	25	42.47	38.6	25	45.82	41.31	25
1	**30.63**	**26.55**	**16**	32.58	28.64	17	34.65	30.47	17	36.28	32.55	17

The median run length is MRL, the standard deviation of RL is SRL, and the minimum of ARL_1 is bolded.

Table 5. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIB distribution when $ARL_0 = 370$, proportion of zero $\phi = 0.5$, and $p_0 = 0.5$.

Shift	EWMA			MEWMA			EWMA-SR			MEWMA-SR		
(η)	$L_1 = 5.766$			$L_2 = 4.297$			$L_3 = 5.137$			$L_4 = 4.115$		
	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL
0.01	348.62	344.06	338	345.23	341.66	337	343.64	339.43	337	**338.57**	**334.86**	**335**
0.05	344.6	340.2	317	340.7	336.05	316	**332.48**	**329.52**	**314**	336.3	332.61	315
0.1	323.9	319.04	292	321.78	316.75	290	**320.97**	**316.53**	**289**	324.66	320.9	290
0.15	**292.86**	**289.65**	**264**	295.27	291.64	268	295.34	291.31	269	297.64	295.46	267
0.2	**258.49**	**254.2**	**224**	260.63	256.45	225	263.56	259.82	226	264.69	260.33	225
0.25	**233.28**	**230.42**	**183**	235.42	231.06	186	237.12	234.68	187	238.13	234.85	186
0.3	**148.76**	**144.64**	**135**	150.32	146.54	137	151.66	147.31	138	152	148.61	138
0.35	**101.25**	**87.3**	**83**	103.44	89.34	86	104.65	100.53	89	105.49	101.64	87
0.4	**85.42**	**81.65**	**41**	87.47	83.58	42	89.25	86	43	90.29	86.23	42
0.5	**36.65**	**32.55**	**20**	39.53	35.68	21	40.51	36.1	22	41.26	27.16	22
1	**27.06**	**23.5**	**14**	30.63	26.56	16	31.78	28.46	15	33.64	29.07	15

The median run length is MRL, the standard deviation of RL is SRL, and the minimum of ARL_1 is bolded.

Table 6. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIB distribution when $ARL_0 = 370$, proportion of zero $\phi = 0.9$, and $p_0 = 0.5$.

Shift	EWMA			MEWMA			EWMA-SR			MEWMA-SR		
(η)	$L_1 = 5.766$			$L_2 = 4.297$			$L_3 = 5.137$			$L_4 = 4.115$		
	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL
0.01	345.26	341.99	335	341.69	337.26	333	340.52	336.26	331	**334.6**	**330.78**	**328**
0.05	340.61	336.04	316	337.28	334.16	315	**328.27**	**324.62**	**314**	333.32	329.6	315
0.1	321.53	317.48	290	317.68	314.65	289	**315.86**	**311.69**	**287**	320.35	316.21	290
0.15	**289.53**	**285.65**	**262**	292.53	289.64	263	293.14	289.65	265	295.86	291.65	264
0.2	**255.3**	**251.7**	**221**	257.07	254.33	224	260.39	256.47	226	261.11	257.82	225
0.25	**230.17**	**226.55**	**180**	233.55	230.42	182	234.09	230.19	185	235.62	231.65	184
0.3	**145.62**	**141.02**	**132**	147.63	144.64	134	148.25	144.63	135	147.33	144.61	135
0.35	**98.34**	**94.81**	**81**	100.25	86.46	83	101.34	86.75	84	103.54	99.65	83
0.4	**82.46**	**78.63**	**38**	85.69	84.62	40	86.94	82.02	41	87.22	84.31	40
0.5	**33.56**	**29.59**	**19**	36.25	32.16	21	38.21	34.87	22	38.74	34.53	21
1	**25.41**	**21.46**	**12**	28.64	24.16	13	28.64	24.16	15	30.66	26.37	14

The median run length is MRL, the standard deviation of RL is SRL, and the minimum of ARL_1 is bolded.

3.2.2 Comparison control chart of ZIP model

The OOC. performance of the ZIP model for EMWA, MEWMA, EWMA-SR, and MEWMA-SR charts is evaluated when dispersion ZIP parameters θ shift from their IC values of $\theta_0 = 0.1, 0.2$, and $\rho_0 = 0.1, 0.5$, respectively. The results are presented in Tables 7-10, and the following observations can be made:

Table 7 indicates that for a set value of ρ and a particular sample size n, as θ_0 increase, all charts become more effective, resulting in a decrease in ARL_1. For example, when $(\rho_0, n, \theta_0) = (0.1, 200, 0.1)$ and $\eta = 0.01$, the MEWMA-SR chart performs optimally. Next, the EWMA-SR chart performs best when $0.05 \leq \eta \leq 0.10$. When $\eta = 0.15$, the MEWMA chart works best. Finally, for $\eta > 0.15$, the EWMA chart is one of the best at spotting shifts in the dispersion parameter of the ZIP model. When ARL_1 performance is compared with increasing dispersion parameters in the ZIP model, the numerical findings are identical to SRL_1 and MRL_1.

Table 7. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIP distribution when $ARL_0 = 370$, proportion of zero $\theta = 0.1$, and $\rho_0 = 0.1$.

Shift	EWMA			MEWMA			EWMA-SR			MEWMA-SR		
(η)	$L_1 = 5.746$			$L_2 = 4.288$			$L_3 = 5.127$			$L_4 = 4.169$		
	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL
0.01	372.42	368.12	357	370.61	365.5	352	365.21	360.12	348	**364.18**	**359.64**	**345**
0.05	367.59	362.53	336	365.99	361.53	334	**359.16**	**356.25**	**332**	361.03	356.48	335
0.1	342.11	339.65	320	338.67	335.2	318	**335.96**	**331.63**	**316**	347.06	343.08	319
0.15	**318.56**	**315.46**	**297**	**314.56**	**310.62**	**294**	319.5	314.96	296	320.17	316.5	297
0.2	**289.42**	**285.47**	**264**	292.32	288.64	265	297.42	294.67	266	298.31	294.58	265
0.25	**261.01**	**257.89**	**238**	270.48	266.75	240	273.96	269.42	241	285	281.46	240
0.3	**224.11**	**220.31**	**203**	231.63	228.65	205	234.14	230.64	198	237.94	234.88	197
0.35	**186.26**	**183.46**	**173**	191.47	187.65	175	207.56	204.71	176	210.32	207.46	175
0.4	**143.86**	**140.98**	**114**	152.35	148.93	116	163.41	160.75	118	167.2	163.59	117
0.5	**89.68**	**85.13**	**65**	93.43	90.74	67	112.05	107.56	68	118.64	114.02	67
1	**24.82**	**20.32**	**16**	34.06	30.57	17	54.93	50.78	17	57.1	49.56	17

Bold is the minimum of ARL_1, SRL is the standard deviation of RL, and MRL is the median run length.

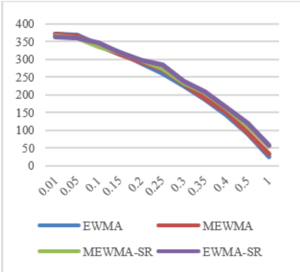
Table 8. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIP distribution when $ARL_0 = 370$, proportion of zero $\theta = 0.2$, and $\rho_0 = 0.1$.

Shift	EWMA			MEWMA			EWMA-SR			MEWMA-SR		
(η)	$L_1 = 5.746$			$L_2 = 4.288$			$L_3 = 5.127$			$L_4 = 4.169$		
	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL
0.01	372.42	368.12	357	370.61	365.5	352	365.21	360.12	348	**364.18**	**359.64**	**345**
0.05	367.59	362.53	336	365.99	361.53	334	**359.16**	**356.25**	**332**	361.03	356.48	335
0.1	342.11	339.65	320	338.67	335.2	318	**335.96**	**331.63**	**316**	347.06	343.08	319
0.15	**318.56**	**315.46**	**297**	**314.56**	**310.62**	**294**	319.5	314.96	296	320.17	316.5	297
0.2	**289.42**	**285.47**	**264**	292.32	288.64	265	297.42	294.67	266	298.31	294.58	265
0.25	**261.01**	**257.89**	**238**	270.48	266.75	240	273.96	269.42	241	285	281.46	240
0.3	**224.11**	**220.31**	**203**	231.63	228.65	205	234.14	230.64	198	237.94	234.88	197
0.35	**186.26**	**183.46**	**173**	191.47	187.65	175	207.56	204.71	176	210.32	207.46	175
0.4	**143.86**	**140.98**	**114**	152.35	148.93	116	163.41	160.75	118	167.2	163.59	117
0.5	**89.68**	**85.13**	**65**	93.43	90.74	67	112.05	107.56	68	118.64	114.02	67
1	**24.82**	**20.32**	**16**	34.06	30.57	17	54.93	50.78	17	57.1	49.56	17

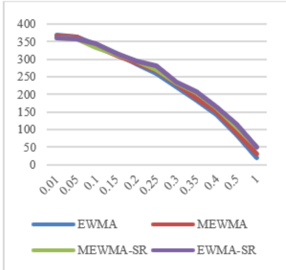
Bold is the minimum of ARL_1, SRL is the standard deviation of RL, and MRL is the median run length..

Table 8 shows the ZIP model's parameter $(0.1, 200, 0.2)$. The numerical finding showed that the MEWMA-SR chart operates best at $\eta = 0.01$. The EWMA-SR chart indicates optimal performance for $0.05 \leq \eta \leq 0.10$. Next, The MEWMA chart is also the best at detecting parameter shift when $\eta = 0.15$. Finally, the EWMA chart performs best when η is greater than 0.10. The numerical results are reflected in the minimum values for ARL_1, SRL_1, and MRL_1. Fig. 3 provides a detailed illustration of the results for the average run length (ARL), median run length (MRL), and standard deviation of run length (SRL) as summarized in Tables 7–8.

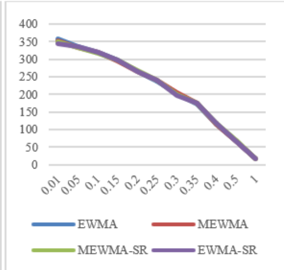
Furthermore, Tables 9-10 presents the performance of all control charts under a fixed probability parameter of $\rho_0 = 0.5$ and varying dispersion parameter of $\theta_0 = 0.1$, and 0.2, respectively. The numerical outcomes for the probability parameter of $\rho_0 = 0.1$ are indistinguishable from those obtained for $p_0 = 0.5$. The results for the average run length (ARL), median run length (MRL), and standard deviation of



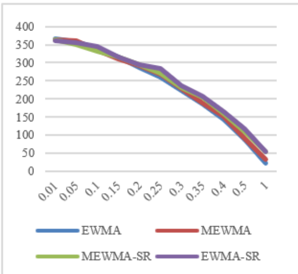
(a1) ARL for $\theta = 0.1$



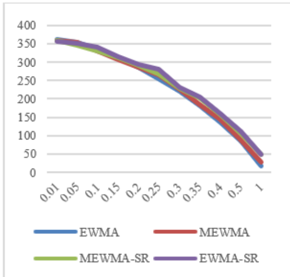
(b1) SRL for $\theta = 0.1$



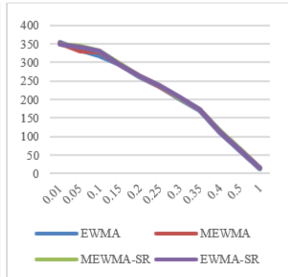
(c1) MRL for $\theta = 0.1$



(a2) ARL for $\theta = 0.2$



(b2) SRL for $\theta = 0.2$



(c2) MRL for $\theta = 0.2$

Fig. 3. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIP distribution with (ρ) = 0.1: (a1)-(a2) is ARL, (b1)-(b2) is SRL, and (c1)-(c2) is MRL.

Table 9. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIP distribution when $ARL_0 = 370$, proportion of zero $\theta = 0.1$, and $\rho = 0.5$.

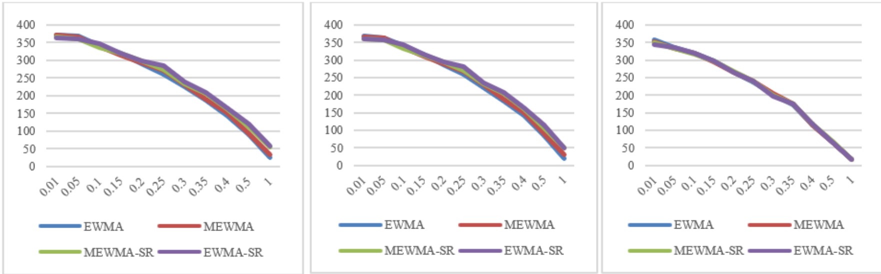
Shift	EWMA			MEWMA			EWMA-SR			MEWMA-SR		
(η)	$L_1 = 5.746$			$L_2 = 4.288$			$L_3 = 5.127$			$L_4 = 4.169$		
	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL
0.01	375.86	370.16	358	364.29	361.84	355	371.5	366.53	354	**361.33**	**357.23**	**352**
0.05	352.65	348.19	339	349.38	345.63	337	**346.66**	**342.13**	**332**	349.03	345.61	334
0.1	328.16	324.63	302	325.9	320.14	299	**321.01**	**318.06**	**296**	326.45	322.53	298
0.15	**284.17**	**280.64**	**267**	**283.48**	**279.64**	**265**	285.35	284.66	268	291.43	287.5	269
0.2	**241.1**	**237.19**	**223**	**240.06**	**236.82**	**220**	254.43	250.13	222	259.36	255.64	225
0.25	**219.35**	**215.64**	**194**	223.42	220.34	195	225.63	221.51	196	226.42	223.74	197
0.3	**175.43**	**171.31**	**153**	178.64	174.26	156	179.72	175.42	157	181.02	178.01	157
0.35	**124.05**	**121.78**	**102**	129.34	125.34	104	131.29	127.99	105	133.64	130.42	104
0.4	**94.36**	**90.75**	**74**	96.35	95.65	76	98.2	94.21	78	99.57	95.47	76
0.5	**53.24**	**48.67**	**26**	55.33	51.33	28	57.5	53.46	29	58.64	54.62	28
1	**12.49**	**9.22**	**8**	13.42	10.67	10	14.26	11.02	11	15.34	12.69	10

Bold is the minimum of ARL_1, SRL is the standard deviation of RL, and MRL is the median run length..

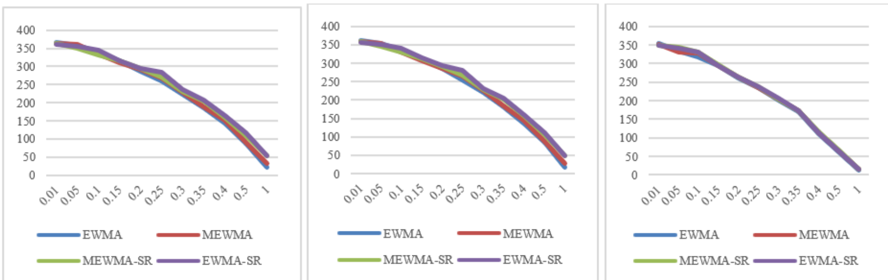
run length (SRL) from Tables 9–10 are visually represented in Fig. 4 for clarity and analysis.

3.3 Illustrative example

This section presents a practical example of the Zero-Inflated Binomial (ZIB) distribution, where the binomial probability of observing defects is 0.0197 and the proportion of zero-defect batches is 0.762. The dataset, which contains 150 observations from a manufacturing process producing hand brake cables, highlights the number of defects per batch. Each batch con-



(a1) ARL for $\theta = 0.1$ (b1) SRL for $\theta = 0.1$ (c1) MRL for $\theta = 0.1$



(a2) ARL for $\theta = 0.2$ (b2) SRL for $\theta = 0.2$ (c2) MRL for $\theta = 0.2$

Fig. 4. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIP distribution with $(\rho) = 0.1$: (a1)-(a2) is ARL, (b1)-(b2) is SRL, and (c1)-(c2) is MRL.

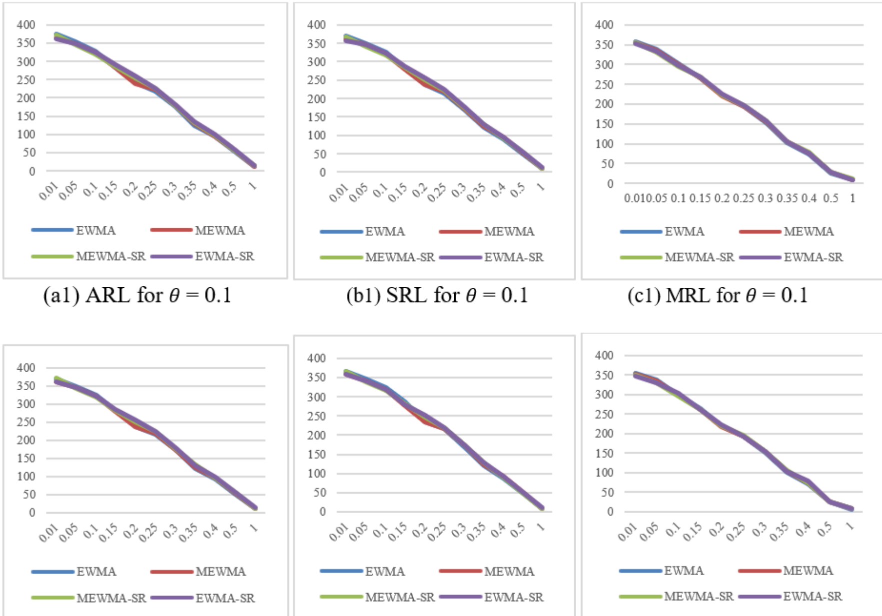
Table 10. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIP distribution when $ARL_0 = 370$, proportion of zero $\theta = 0.2$, and $\rho = 0.5$.

Shift	EWMA			MEWMA			EWMA-SR			MEWMA-SR		
(η)	$L_1 = 5.746$			$L_2 = 4.288$			$L_3 = 5.127$			$L_4 = 4.169$		
	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL	ARL	SRL	MRL
0.01	370.01	367.14	354	361.64	357.42	351	371.5	365.94	350	**361.33**	**358.99**	**348**
0.05	348.65	345.23	336	345.66	340.7	334	**342.77**	**338.17**	**329**	345.65	341.78	330
0.1	325.23	322.62	298	321.45	317.65	297	**318.69**	**315.26**	**296**	322.19	318.7	302
0.15	**282.75**	**285.5**	**263**	**280.75**	**276.46**	**261**	283.02	280.42	262	285.05	280	261
0.2	**238.29**	**235.64**	**220**	**238.26**	**235.2**	**218**	251.47	247.3	220	256.65	252.46	222
0.25	**217.21**	**215.8**	**192**	220.49	216.85	194	223.67	219.64	195	224.33	220.17	164
0.3	**173.46**	**170.63**	**151**	175.65	173.64	153	177.69	174.57	154	179.62	175.25	153
0.35	**122.1**	**120.5**	**100**	125.12	123.63	103	131.29	128.46	104	130.14	127.91	102
0.4	**91.66**	**88.7**	**72**	94.83	91.85	73	95.46	91.53	74	97.99	92.67	78
0.5	**51.76**	**48.64**	**24**	52.64	49.67	25	55.01	49.37	24	56.2	50.31	24
1	**10.7**	**8.02**	**6**	11.6	8.33	**7**	12.65	9.66	7	13.16	10.96	7

Bold is the minimum of ARL_1, SRL is the standard deviation of RL, and MRL is the median run length..

sists of 20 cables inspected and categorized as conforming or non-conforming [22]. The data presented in Fig. 5 show that 76.2% of the batches are defect-free, necessitating a ZIB model to account for this excess of zero-defect observations. The model allows us to understand the distribution of non-conforming cables better and offers insights into improving the manufacturing process.

In this analysis, EWMA, MEWMA, EWMA-SR, and MEWMA-SR were evaluated for monitoring changes in a manufacturing process with data following a Zero-Inflated Binomial (ZIB) distribution. The



(a1) ARL for $\theta = 0.1$ (b1) SRL for $\theta = 0.1$ (c1) MRL for $\theta = 0.1$

(a2) ARL for $\theta = 0.2$ (b2) SRL for $\theta = 0.2$ (c2) MRL for $\theta = 0.2$

Fig. 5. Performance of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for ZIP distribution with $(\rho) = 0.5$: (a1) - (a2) is ARL, (b1) - (b2) is SRL, and (c1) - (c2) is MRL.

Average Run Length (ARL_0) was set at 268.74, meaning false alarms would occur roughly every 268 observations when the process is under control.

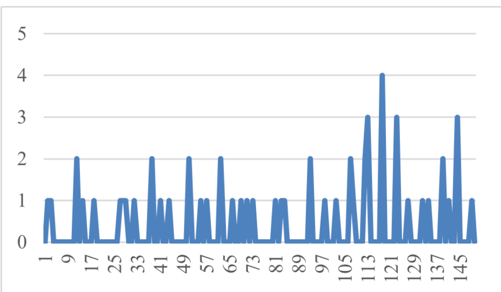


Fig. 6. Dataset.

The results showed that the EWMA chart detected a process change at the 119th observation, present in Fig. 6, while the MEWMA, EWMA-SR, and MEWMA-SR charts detected changes earlier, at the 115th observation, displayed in Figs. 7-9, respectively. The results indicate that the latter three charts are more sensitive and effective in detecting process shifts, mainly when dealing with ZIB distributions. Faster detection allows for quicker corrective action, enhancing quality control in manufacturing.

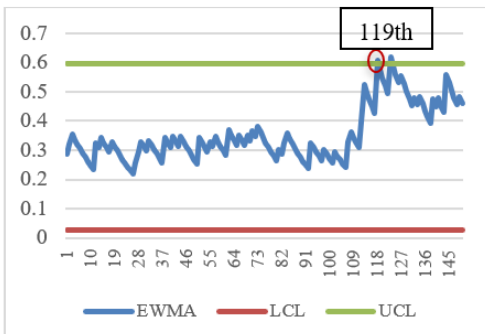


Fig. 7. EWMA chart.

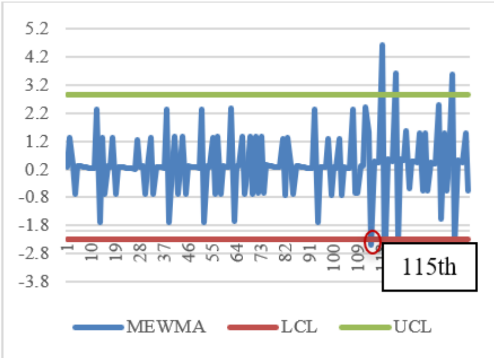


Fig. 8. MEWMA chart.

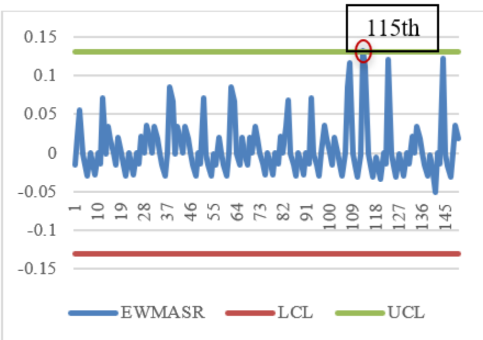


Fig. 9. EWMA-SR chart.

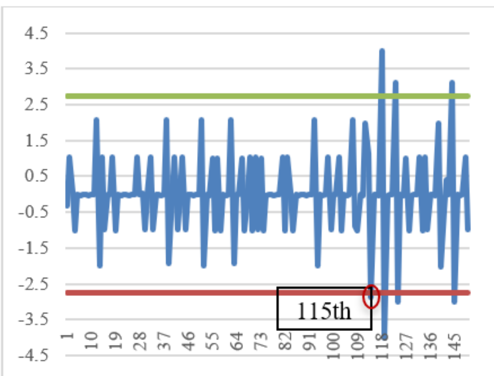


Fig. 10. MEWMA-SR chart.

4. Conclusion, Limitation and Future Research Idea

The study examines the effectiveness of parametric and nonparametric control charts, such as modified EWMA (MEWMA) and exponentially weighted moving average (EWMA) charts, as well as their nonparametric equivalents, MEWMA-sign rank (MEWMA-SR) and EWMA-sign rank (EWMA-SR). In addition, the numerical results from the simulation mothod are shown that the proposed control chart is superior to other control charts for tiny magnitude of change where $\eta 1$. The data employed for this analysis is categorized into zero-inflated binomial (ZIB) and zero-inflated Poisson (ZIP) models. The parameters were set to increase the zero inflation component of the distribution while maintaining the same distribution parameters. Based on the simulation results, nonparametric control charts are more sensitive to changes in the zero-inflation component of ZIB and ZIP models than parametric charts. In addition, for moderate to significant increases in the dispersion parameters of ZIB and ZIP models, parametric control charts demonstrate superior performance in detecting changes compared to nonparametric control charts. This study, however, departs from the assumption of constant parameters by analyzing the control chart performance in situations where the zero parameters of the ZIP and ZIB models are changed. Therefore, under these conditions, the effectiveness of the chart should be reassessed. The effectiveness of parametric and nonparametric control charts will be investigated further in the context of data counting with other zero inflation models that optimize with process monitoring under inflation and deflation data and robustness to contaminated outlier case study.

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References

[1] Lambert D. Zero-Inflated Poisson regression, with an application to defects in manufacturing. Technometrics. 1992; 34(1): 1-14.

[2] Hall DB. Zero-inflated Poisson and binomial regression with random effects: A Case Study. Biometrics. 2000; 56(4): 1030-1039.

[3] Shewhart WA. Economic control of quality of manufactured product. New York; D. Van Nostrand Company; 1931.

[4] Reynolds MR. Stoumbos ZG. Control charts for monitoring the mean and variance of autocorrelated processes. J. Qual. Technol. 2000; 32(1), 1-14.

[5] Montgomery DC. Introduction to Statistical Quality Control (8th ed.); 2019.

[6] Li Z. Wang Z. Tsung F. Quantile control chart for monitoring process location. J. Qual Technol. 2014; 46(1): 74-90.

[7] Page ES. Continuous inspection schemes, Biometrika, 1954; 41: 100-44.

[8] Roberts SW. Control chart tests based on geometric moving average, Technometrics. 1958; 1: 239-50.

[9] Patel AK. Divecha J. Modified exponentially weighted moving average (EWMA) control chart for an analytical process data. J. Chem. Eng. Mater Sci. 2011; 2(1): 12–20.

[10] Graham MA. Chakraborti S. Human SW. A nonparametric exponentially weighted moving average signed-rank chart for monitoring location. Comput. Stat. Data Anal. 2011; 55: 2490–503.

[11] Petcharat K. Sukparungsee S. Development of a new MEWMA-Wilcoxon sign rank chart for detection of change in mean parameters. Appl. Sci. Eng. Prog. 2023; 16(2): 5892.

[12] Talordphop K. Sukparungsee S. Areepong Y. Monitoring of mean processes with mixed moving average – modified exponentially weighted moving average control charts. Appl. Sci. Eng. Prog. 2023; 16(4): 6498.

[13] Talordphop K. Sukparungsee S. Areepong Y. On designing new mixed modified exponentially weighted moving average - exponentially weighted moving average control chart. Results in Engineering. 2023; 18: 101152.

[14] Xie M. Goh TN. Lu XS. A comparative study of control charts for defects. Int. J. Qual. Reliab. Manag. 1997; 14(1): 64-73.

[15] Li H. Xie M. Goh TN. A control chart for the zero-inflated Poisson distribution. J. Qual. Technol. 2009; 41(4): 389-400.

[16] Zhu X. Wang K. Wang Y. Zero-inflated Poisson cumulative sum control chart. Qual. Reliab. Eng. Int. 2012; 28(3): 367-79.

[17] Zhou H. Liu S. Shu L. EWMA control charts for monitoring zero-inflated Poisson processes. Commun. Stat. - Theor M. 2017; 46(9): 4323-34.

[18] Phantu P. Areepong Y. Sukparungsee S. Average run length formulas for mixed double moving average - exponentially weighted moving average control chart. Sci. Technol. Asia. 2023; 28(1): 77–89.

[19] Castagliola P. Celano G. Psarakis S. A review of CUSUM and EWMA control charts for count data. Qual. Reliab. Eng. Int. 2009; 25(5): 543-70.

[20] Beyene J. Moineddin R. Application of zero-inflated and hurdle models to survey data with extra zeros. Int. J. Stat. Sci. 2005; 4: 53-64.

[21] Bakar, MRA. Riaz M. A control chart for monitoring zero-inflated binomial processes. J. Appl. Stat. 2018; 45(7): 1270-89.

[22] Alevizakos, V. Koukouvinos, C. Monitoring of zero-inflated binomial processes with a DEWMA control chart. J. Appl. Stat. 2020.