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Bezier Curve in Modeling Bonds Yield Curve

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ABSTRACT

Before buying bonds, investors must ensure that the bonds have good prospects and can provide profits as expected, so investors must analyze the yield curve. The results of yield curve analysis will help investors to model the yield curve and help make decisions about investing in certain bond products. Bond yield curves can be modeled using Bezier curves. The Bezier curve was chosen because the Bezier curve has a control value which functions to direct the points on the curve to move from the starting point to the ending point. The more control values, the more complex the Bezier curve will be and the Bezier curve can be adjusted to real data. This study is an applied study that compares high order Bezier curves (6th order) with low order Bezier curves (5th order). The data used in this study are bond data obtained from the Indonesia Stock Exchange from July 27 and 28, 2023. The 5th order Bezier curve produces Mean Square Error (MSE) for July 27 and 28, 2023 of 0.1639 and 0.0688, respectively. Meanwhile, the MSE produced by the 6th order Bezier curve for the data for July 27 and 28, 2023 is 0.1607 and 0.0668, respectively. The results of the research show that the Bezier order 6 curve has a smaller error compared to the Bezier order 5 curve. More control values on the order 6 Bezier curve make the curve move more complexly and adapt to real bond yield data.

Keywords: Bezier curve; Bonds; Yield curve

1. Introduction

The investment industry is constantly evolving from year to year. This is proven by the rising number of investors who invest their money in the capital market. Investing in stocks, bonds, options, or other financial assets is an example of a form of investment. Investors invest in order to raise capital and gain profits. Investors must conduct due diligence on investment products prior

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to investing in order to optimize returns.

Bonds are a common type of financial instrument traded on the capital market. Bonds are long-term debt securities in which the bond issuer borrows funds (loans) from the bond buyer and agrees to repay the borrowed money plus interest within a specified time. Bonds are classified into two types: government bonds and corporate bonds. The government issues government bonds and the funds raised are used to fund the government's operations. Meanwhile, corporate bonds are issued by companies.

Bond investing has several advantages, mainly when investors invest in government bonds. Government bonds are considered risk-free investments since the government always pays interest and principal on time. The government issues various bonds, including zero-coupon bonds, Indonesian retail bonds, and state securities.

Before buying bonds, investors must consider several factors, including the bond issuer, the maturity date, the face value of the bonds, the bond price, and the bond yield. Investors must study bond yields in order to determine the benefits of holding bonds until maturity. The yield on a bond is the annual profit that investors will earn. Investors cannot determine bond yields by comparing bond prices to one another, as bond prices are influenced by various factors, including the bond's value, maturity, interest rate, and coupons obtained. Investors cannot immediately decide to buy bonds based on the yield. Investors should perform an in-depth analysis of the yield-to-maturity correlation. The link between yield to maturity and time to maturity is referred to as the interest rate's term structure. The yield curve can be used to describe the structure of the interest rate period. To verify that the bonds chosen by investors have favorable prospects and can provide the required profits, investors must conduct a yield curve analysis. The yield curve research results will assist investors in modeling yield curves and making investment decisions in specific bond products.

Numerous studies have been conducted to model yield curves, including the Nelson Siegel model [1] and the Nelson Siegel Svensson model [2], which model the yield curve by adjusting its shape using data-derived parameters. Along with the Nelson Siegel and Nelson Siegel Svensson models are Cubic Spline curves [3] and Cubic B Spline models [4]. Spline models alter the shape of the curve by applying the spline function to many nodes. All of these models apply an empirical approach to yield curve estimation.

Research by Permatasari and Iriawan [5], shows that the Nelson Siegel Svensson model and the Cubic Spline Smoothing model each have advantages and disadvantages. The Nelson Siegel Svensson model is a parsimony model but has a larger error value. Parsimony means that the model formed is simple but informative in explaining the model and easy to interpret. While the Cubic Spline Smoothing model has a smaller error value but is not parsimony and has a fluctuating return value. Based on these results, it is necessary to use a new method that is stable and accurate to model the yield curve.

In 1962, Pierre Bezier developed the Bezier curve for vehicle graphic design. Bezier curves are capable of describing polynomial forms with complex coefficients. Bezier curves are used to model curves smoothly so that they can be represented graphically as geometric planes. They are frequently used in graphic design for various purposes, including engineering, automotive, crafts, and industry. The Bezier curve is the most numerically sta-

ble polynomial-based curve [6]. Due to the curve's stability, it is capable of accurately estimating geometric shapes. For example, the Bezier curve has been used to design rattan tables and chairs [7], to design ceramics [8], to reconstruct magnetic resonance imaging (MRI) image results [9], to illustrate the movement path of a mobile robot [10], to illustrate a collision-free trajectory for multiple drones [11], and to simulate collision-free paths for autonomous cars [12].

The Bezier curve has many uses in a variety of sectors of science. The primary rationale for utilizing the Bezier curve is that it is highly accurate at calculating the curve or geometric plane from empirical data. As a result of the advantages inherent in the Bezier curve, this study uses it to represent the yield curve. Bezier curves are classified into two types, low-order and high-order. The purpose of this study is to compare a low-order and a high-order Bezier curve. The 6th order Bezier curve has seven control values and is classified as a high order Bezier curve [10). As a result, bond yield curve modeling in this work uses a 5th order Bezier curve as the low-order Bezier curve and a 6th order Bezier curve as the high-order Bezier curve.

Previous studies have not compared low-order and high-order Bezier curves, especially those applied to the bond yield curve. This study examines low-order and high-order Bezier curves in modeling the yield curve and discusses the resulting errors so that the right Bezier curve is obtained in modeling the bond yield curve.

2. Method

This research involves a review of the literature that is then applied to a case study. The process in this research begins with reviewing the literature on bonds, yield curves, and Bezier curves. After conducting a literature study, it is continued by analyzing the literature to compile the equations of the 5th and 6th order Bezier curves. The Bezier curve equations formed are then used to model the yield curve on government bonds of the type of Government Securities with a fixed rate interest rate. The data used to model the bond yield curve is bond data obtained from the Indonesia Stock Exchange from July 27 and 28, 2023. Bond data on July 27, 2023 amounted to 39 data and on July 28, 2023 amounted to 38 data.

The following are the steps in modeling the yield curve with the Bezier curve approach:

a. Sorting the yield values based on time to maturity from the smallest to the largest,

b. Determining the control values $P_0, P_1, P_2, ..., P_n$ using the least squares method,

c. Substituting the values $P_0, P_1, P_2, ..., P_n$ in the general form of the Bezier curve,

$$B(t) = (1-t)^n P_0 + \binom{n}{1}(1-t)^{n-1} t P_1 + \cdots + t^n P_n, \quad t \in [0,1].$$

d. Determining the yield with the Bezier curve approach,

e. Forming a bond yield curve with the Bezier curve approach,

f. Comparing the actual yield data in the bond market with the yield from the Bezier curve approach to determine the resulting error.



Fig. 1. Research flow.

The final stage is to draw conclusions based on the previous stage's analysis and discussion findings. The research method is displayed in Fig. 1.

3. Results and Discussion

A Bezier curve has $n + 1$ defining points, namely $P_0, P_1 ..., P_n$. The point P_0 is the curve's origin that curves in the P_n curve's endpoint direction. The Bezier curve plots the curve's points as a function of time t, with t ranging from 0 to 1. Apart from the control points $P_0, P_1, \ldots, P_n$, a partition of t is also required to characterize the Bezier curve, that is Δt. The value of Δt is variable; it can be 0.1; 0.2; 0.01; or 0.02, for example. According to Sederberg [13] a Bezier curve of n order with the parameters $P_0, P_1, P_2, ..., P_n$ can be generalized as follows:

$$\begin{aligned} B(t) &= \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^i P_i \\ &= (1-t)^n P_0 + \binom{n}{i} (1-t)^{(}n-1)tP_1 \\ &+ \cdots + t^n P_n, \quad t \in [0,1]. \end{aligned} \tag{3.1}$$

The t variable in the equation indicates the distance moved by the Bezier curve from P_0 to P_n. If $t = 0.2$, the function $B(t)$ is one-fifth of the way between P_0 and P_n. When $t = 0$, the curve is at P_0; when $t = 1$, the curve is at P_n [14].

3.1 5^{th} order Bezier curve

According to Eq. (3.1), the general form of the 5^{th} order Bezier curve is

$$\begin{aligned} B(t_i) &= (1-t_i)^5 P_0 + 5t_i(1-t_i)^4 P_1 \\ &+ 10t_i^2(1-t_i)^3 P_2 \\ &+ 10t_i^3(1-t_i)^2 P_3 \\ &+ 5t_i^4(1-t_i)P_4 + t_i^5 P_5, \quad t \in [0,1]. \end{aligned} \tag{3.2}$$

Bond yield data, when represented using a Bezier curve of 5^{th} order, requires six yield control values, namely P_0, P_1, P_2, P_3, P_4 and P_5. The initial and final control values (P_0, P_5), are the values of the first and last sorted bond yield data, respectively, so no search is required. The center control value (P_1, P_2, P_3, P_4) on the other hand, must be calculated first. The least-squares error approach was used to find the control values P_1, P_2, P_3, and P_4. If there are n data, where y_i denotes the observation yield data and $B(t_i)$ denotes the estimation, the sum of squares is the minimum result as follows:

$$S = \sum_{i=1}^{n} [y_i - B(t_i)]^2. \tag{3.3}$$

The results of the minimum squares error method application to Eq. (3.3) yielded the formulas P_1, P_2, P_3, and P_4 as Eq. (3.4).

$$P_1 = \frac{\begin{vmatrix} A_1 & C_1 & D_1 & E_1 \\ A_2 & C_2 & D_2 & E_2 \\ A_3 & C_3 & D_3 & E_3 \\ A_4 & C_4 & D_4 & E_4 \end{vmatrix}}{\begin{vmatrix} B_1 & C_1 & D_1 & E_1 \\ B_2 & C_2 & D_2 & E_2 \\ B_3 & C_3 & D_3 & E_3 \\ B_4 & C_4 & D_4 & E_4 \end{vmatrix}},$$

$$P_2 = \frac{\begin{vmatrix} B_1 & A_1 & D_1 & E_1 \\ B_2 & A_2 & D_2 & E_2 \\ B_3 & A_3 & D_3 & E_3 \\ B_4 & A_4 & D_4 & E_4 \end{vmatrix}}{\begin{vmatrix} B_1 & C_1 & D_1 & E_1 \\ B_2 & C_2 & D_2 & E_2 \\ B_3 & C_3 & D_3 & E_3 \\ B_4 & C_4 & D_4 & E_4 \end{vmatrix}},$$

$$P_3 = \frac{\begin{vmatrix} B_1 & C_1 & A_1 & E_1 \\ B_2 & C_2 & A_2 & E_2 \\ B_3 & C_3 & A_3 & E_3 \\ B_4 & C_4 & A_4 & E_4 \end{vmatrix}}{\begin{vmatrix} B_1 & C_1 & D_1 & E_1 \\ B_2 & C_2 & D_2 & E_2 \\ B_3 & C_3 & D_3 & E_3 \\ B_4 & C_4 & D_4 & E_4 \end{vmatrix}},$$

$$P_4 = \frac{\begin{vmatrix} B_1 & C_1 & D_1 & A_1 \\ B_2 & C_2 & D_2 & A_2 \\ B_3 & C_3 & D_3 & A_3 \\ B_4 & C_4 & D_4 & A_4 \end{vmatrix}}{\begin{vmatrix} B_1 & C_1 & D_1 & E_1 \\ B_2 & C_2 & D_2 & E_2 \\ B_3 & C_3 & D_3 & E_3 \\ B_4 & C_4 & D_4 & E_4 \end{vmatrix}}, \qquad (3.4)$$

with

$$A_1 = \sum_{i=1}^{n} 5t_i(1-t_i)^4[y_i - (1-t_i)^5 P_0 - t_i^5 P_5],$$

$$A_2 = \sum_{i=1}^{n} 10t_i^2(1-t_i)^3[y_i - (1-t_i)^5 P_0 - t_i^5 P_5],$$

$$A_3 = \sum_{i=1}^{n} 10t_i^3(1-t_i)^2[y_i - (1-t_i)^5 P_0 - t_i^5 P_5],$$

$$A_4 = \sum_{i=1}^{n} 10t_i^4(1-t_i)[y_i - (1-t_i)^5 P_0 - t_i^5 P_5],$$

$$B_1 = 25\sum_{i=1}^{n} t_i^2(1-t_i)^8,$$

$$B_2 = 50\sum_{(} i=1)^{n} t_i^3(1-t_i)^7,$$

$$B_3 = 50\sum_{i=1}^{n} t_i^4(1-t_i)^6,$$

$$B_4 = 50\sum_{i=1}^{n} t_i^5(1-t_i)^5,$$

$$C_1 = 50\sum_{i=1}^{n} t_i^3(1-t_i)^7,$$

$$C_2 = 100\sum_{i=1}^{n} t_i^4(1-t_i)^6,$$

$$C_3 = 100\sum_{i=1}^{n} t_i^5(1-t_i)^5,$$

$$C_4 = 50\sum_{i=1}^{n} t_i^6(1-t_i)^4,$$

$$D_1 = 50\sum_{i=1}^{n} t_i^4(1-t_i)^6,$$

$$D_2 = 100\sum_{i=1}^{n} t_i^5(1-t_i)^5,$$

$$D_3 = 100\sum_{i=1}^{n} t_i^6(1-t_i)^4,$$

$$D_4 = 50\sum_{i=1}^{n} t_i^7(1-t_i)^3,$$

$$E_1 = 25\sum_{i=1}^{n} t_i^5(1-t_i)^5,$$

$$E_2 = 50\sum_{i=1}^{n} t_i^6(1-t_i)^4,$$

$$E_3 = 50\sum_{i=1}^{n} t_i^7(1-t_i)^3,$$

$$E_4 = 25\sum_{i=1}^{n} t_i^8(1-t_i)^2,$$

3.2 6^{th} order Bezier curve

The 6^{th} order Bezier curve is constructed by starting with the value of P_0 moving towards the final value of P_6 with the direction of P_1, P_2, P_3, P_4, and P_5. The parametric form of a Bezier cubic curve of 6^{th} order is as follows:

$$\begin{aligned} B(t_i) = {} & (1-t_i)^6 P_0 + 6t_i(1-t_i)^5 P_1 \\ & + 15t_i^2(1-t_i)^4 P_2 \\ & + 20t_i^3(1-t_i)^3 P_3 \\ & + 15t_i^4(1-t_i)^2 P_4 + t_i^5 P_5 \\ & + 6t_i^5(1-t_i)P_5 + t_i^6 P_6, \quad t \in [0,1]. \end{aligned} \qquad (3.5)$$

According to Eq. (3.5), to model bond yield data with a Bezier curve of 6th order, seven yield control values are required, namely $P_0, P_1, P_2, P_3, P_4, P_5$ and P_6. The control values P_0 and P_6 are de-

rived from the sorted bond yield data's first and last yields. P_1, P_2, P_3, P_4, and P_5 must be determined first using the least-squares error approach. The formula gives the result of the least squares error method's elaboration P_1, P_2, P_3, P_4, and P_5 as Eq. (3.6).

$$P_1 = \frac{\begin{vmatrix} A_1 & C_1 & D_1 & E_1 & F_1 \\ A_2 & C_2 & D_2 & E_2 & F_2 \\ A_3 & C_3 & D_3 & E_3 & F_3 \\ A_4 & C_4 & D_4 & E_4 & F_4 \\ A_5 & C_5 & D_5 & E_5 & F_5 \end{vmatrix}}{\begin{vmatrix} B_1 & C_1 & D_1 & E_1 & F_1 \\ B_2 & C_2 & D_2 & E_2 & F_2 \\ B_3 & C_3 & D_3 & E_3 & F_3 \\ B_4 & C_4 & D_4 & E_4 & F_4 \\ B_5 & C_5 & D_5 & E_5 & F_5 \end{vmatrix}},$$

$$P_2 = \frac{\begin{vmatrix} B_1 & A_1 & D_1 & E_1 & F_1 \\ B_2 & A_2 & D_2 & E_2 & F_2 \\ B_3 & A_3 & D_3 & E_3 & F_3 \\ B_4 & A_4 & D_4 & E_4 & F_4 \\ B_5 & A_5 & D_5 & E_5 & F_5 \end{vmatrix}}{\begin{vmatrix} B_1 & C_1 & D_1 & E_1 & F_1 \\ B_2 & C_2 & D_2 & E_2 & F_2 \\ B_3 & C_3 & D_3 & E_3 & F_3 \\ B_4 & C_4 & D_4 & E_4 & F_4 \\ B_5 & C_5 & D_5 & E_5 & F_5 \end{vmatrix}},$$

$$P_3 = \frac{\begin{vmatrix} B_1 & C_1 & A_1 & E_1 & F_1 \\ B_2 & C_2 & A_2 & E_2 & F_2 \\ B_3 & C_3 & A_3 & E_3 & F_3 \\ B_4 & C_4 & A_4 & E_4 & F_4 \\ B_5 & C_5 & A_5 & E_5 & F_5 \end{vmatrix}}{\begin{vmatrix} B_1 & C_1 & D_1 & E_1 & F_1 \\ B_2 & C_2 & D_2 & E_2 & F_2 \\ B_3 & C_3 & D_3 & E_3 & F_3 \\ B_4 & C_4 & D_4 & E_4 & F_4 \\ B_5 & C_5 & D_5 & E_5 & F_5 \end{vmatrix}},$$

$$P_4 = \frac{\begin{vmatrix} B_1 & C_1 & D_1 & A_1 & F_1 \\ B_2 & C_2 & D_2 & A_2 & F_2 \\ B_3 & C_3 & D_3 & A_3 & F_3 \\ B_4 & C_4 & D_4 & A_4 & F_4 \\ B_5 & C_5 & D_5 & A_5 & F_5 \end{vmatrix}}{\begin{vmatrix} B_1 & C_1 & D_1 & E_1 & F_1 \\ B_2 & C_2 & D_2 & E_2 & F_2 \\ B_3 & C_3 & D_3 & E_3 & F_3 \\ B_4 & C_4 & D_4 & E_4 & F_4 \\ B_5 & C_5 & D_5 & E_5 & F_5 \end{vmatrix}},$$

$$P_5 = \frac{\begin{vmatrix} B_1 & C_1 & D_1 & E_1 & A_1 \\ B_2 & C_2 & D_2 & E_2 & A_2 \\ B_3 & C_3 & D_3 & E_3 & A_3 \\ B_4 & C_4 & D_4 & E_4 & A_4 \\ B_5 & C_5 & D_5 & E_5 & A_5 \end{vmatrix}}{\begin{vmatrix} B_1 & C_1 & D_1 & E_1 & F_1 \\ B_2 & C_2 & D_2 & E_2 & F_2 \\ B_3 & C_3 & D_3 & E_3 & F_3 \\ B_4 & C_4 & D_4 & E_4 & F_4 \\ B_5 & C_5 & D_5 & E_5 & F_5 \end{vmatrix}}, \quad (3.6)$$

with

$$A_1 = \sum_{i=1}^{n} 6t_i(1-t_i)^5[y_i - (1-t_i)^6 P_0 - t_i^6 P_6],$$

$$A_2 = \sum_{i=1}^{n} 15t_i^2(1-t_i)^4[y_i - (1-t_i)^6 P_0 - t_i^6 P_6],$$

$$A_3 = \sum_{i=1}^{n} 20t_i^3(1-t_i)^3[y_i - (1-t_i)^6 P_0 - t_i^6 P_6],$$

$$A_4 = \sum_{i=1}^{n} 15t_i^4(1-t_i)^2[y_i - (1-t_i)^6 P_0 - t_i^6 P_6],$$

$$A_5 = \sum_{i=1}^{n} 6t_i^5(1-t_i)[y_i - (1-t_i)^6 P_0 - t_i^6 P_6],$$

$$B_1 = 36 \sum_{i=1}^{n} t_i^2(1-t_i)^1 0,$$

$$B_2 = 90 \sum_{(} i = 1)^n t_i^3(1-t_i)^9,$$

$$B_3 = 120 \sum_{i=1}^{n} t_i^4(1-t_i)^8,$$

$$B_4 = 90 \sum_{i=1}^{n} t_i^5(1-t_i)^7,$$

$$B_5 = 36\sum_{i=1}^{n} t_i^6(1-t_i)^6,$$

$$C_1 = 90\sum_{i=1}^{n} t_i^3(1-t_i)^9,$$

$$C_2 = 225\sum_{i=1}^{n} t_i^4(1-t_i)^8,$$

$$C_3 = 300\sum_{i=1}^{n} t_i^5(1-t_i)^7,$$

$$C_4 = 225\sum_{i=1}^{n} t_i^6(1-t_i)^6,$$

$$C_4 = 90\sum_{i=1}^{n} t_i^7(1-t_i)^5,$$

$$D_1 = 120\sum_{i=1}^{n} t_i^4(1-t_i)^8,$$

$$D_2 = 300\sum_{i=1}^{n} t_i^5(1-t_i)^7,$$

$$D_3 = 400\sum_{i=1}^{n} t_i^6(1-t_i)^6,$$

$$D_4 = 300\sum_{i=1}^{n} t_i^7(1-t_i)^5,$$

$$D_5 = 300\sum_{i=1}^{n} t_i^8(1-t_i)^4,$$

$$E_1 = 90\sum_{i=1}^{n} t_i^5(1-t_i)^7,$$

$$E_2 = 225\sum_{i=1}^{n} t_i^6(1-t_i)^6,$$

$$E_3 = 300\sum_{i=1}^{n} t_i^7(1-t_i)^5,$$

$$E_4 = 225\sum_{i=1}^{n} t_i^8(1-t_i)^4,$$

$$E_5 = 90\sum_{i=1}^{n} t_i^9(1-t_i)^3,$$

$$F_1 = 36\sum_{i=1}^{n} t_i^6(1-t_i)^6,$$

$$F_2 = 90\sum_{i=1}^{n} t_i^7(1-t_i)^5,$$

$$F_3 = 120\sum_{i=1}^{n} t_i^8(1-t_i)^4,$$

$$F_4 = 90\sum_{i=1}^{n} t_i^9(1-t_i)^3,$$

$$F_5 = 36\sum_{i=1}^{n} t_i^{10}(1-t_i)^2.$$

The data used to model the bond yield curve are from Fix Rate Bonds issued on July 27 and 28, 2023. On 27 July 2023, there are 39 bond data points, and on 28 July, there are 38 bond data points. The data is then pre-sorted from shortest to longest time to maturity.

The results of data analysis using a 5th order Bezier curve for bond data on 27 July 2023 obtained the values of $B_1 = 1.9192, B_2 = 1.4394, B_3 = 0.8225, B_4 = 0.3427, C_1 = 1.4394, C_2 = 1.6450, C_3 = 1.3709, C_4 0.8225, D_1 = 0.8225, D_2 = 1.3709, D_3 = 1.6450, D_4 = 1.4394, E_1 = 0.3427, E_2 = 0.8225, E_3 = 1.4394, E_4 = 1.9192, A_1 = 28.3536, A_2 = 34.2531, A_3 = 35.4147, A_4 = 31.4989.$ The control values P_0 and P_5 obtained from the bond yield data are $P_0 = 3.122$ and $P_5 = 6.7$. Control values P_1, P_2, P_3, and P_4 determined using Eq. (3.4) are $P_1 = 5.3114, P_2 = 7.5638, P_3 = 5.4539$ and $P_4 = 8.1322$.

The bond yield curve equation formed using the 5th order Bezier curve for data on 27 July 2023 is as Eq. (3.7).

$$\begin{aligned} B(t_i) = {} & (1-t_i)^5 3.122 + 5t_i(1-t_i)^4 5.3114 \\ & + 10t_i^2(1-t_i)^3 7.5638 \\ & + 5t_i^4(1-t_i)8.1322 \\ & + 10t_i^3(1-t_i)^2 5.4539 + t_i^5 6.7 \quad (3.7) \end{aligned}$$

The yield curve generated using the 5^{th} order Bezier curve approach for bond data on 27 July 2023 is as shown in Fig. 2.

Simulation of Eq. (3.7) can be done to estimate the yield bonds, $B(t_i)$ and com-

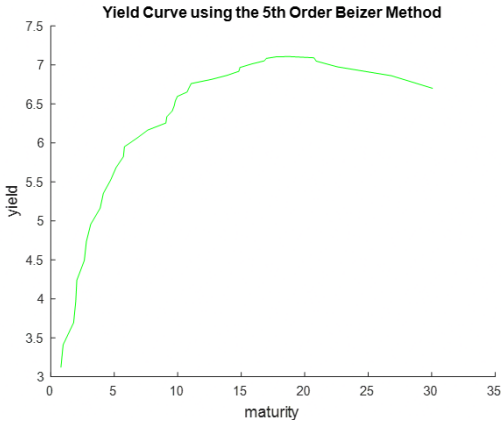


Fig. 2. Bond yield curve using the 5th order bezier method for bond data on 27 July 2023.

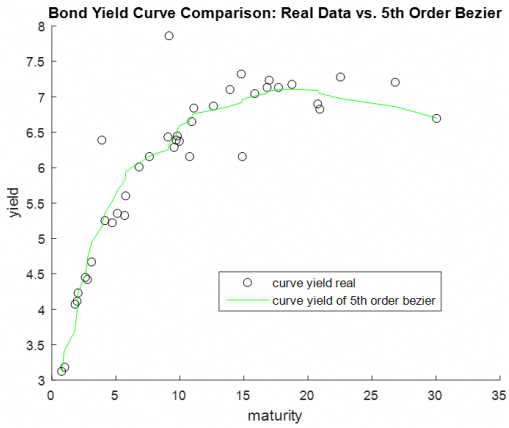


Fig. 3. Comparison of bond yield curves: real data vs. 5th order bezier for bonds data on 27 July 2023.

pare them with bond yields in the market, y_i. In the data dated July 27, 2023, the value of $B(t_i) = y_i$ for the smallest time to maturity is 0.8 years and the largest is 30.07 years. The value of $B(t_i)$ for time to maturity 2.05 years is 4.2376 while for the yield value in the market is 4.227; there is a square of error 0.0001. When time to maturity 6.81 years, value $B(t_i) = 6.0636$ and $y_i = 6.014$. The ratio results in the square of the error of 0.0025. Simulating the value of $B(t_i)$ for the entire time to maturity in the data on 27 July 2023 and comparing the results with y_i resulted in a Mean Square Error. On 27 July 2023 data, the Mean Square Error (MSE) of the 5th order Bezier curve used to model bond yield curves is 0.1639. In Fig. 3, the real yield data from the bond market is compared to the yield calculated using the 5th order Bezier approach.

The results of the analysis of the 5th order Bezier curve for bond data on July 28, 2023, obtained the value of $B_1 = 1.8687, B_2 = 1.4015, B_3 = 0.8009, B_4 = 0.3337, C_1 = 1.4015, C_2 = 1.6017, C_3 = 1.3348, C_4 = 0.8009, D_1 = 0.8009, D_2 = 1.3348, D_3 = 1.6017, D_4 = 1.4015, E_1 = 0.3337, E_2 = 0.8009, E_3 = 1.4015, E_4 = 1.8687, A_1 = 28.3260, A_2 = 33.3546, A_3 = 33.9046, A_4 = 29.7567$. Control values P_1, P_2, P_3, and P_4 determined based on Eq. (3.4) are $P_11 = 6.4105, P_2 = 6.4034, P_3 = 6.0959, P_4 = 7.4630$ The initial and final control values are based on bond yield data, namely $P_0 = 3.0960$ and $P_5 = 6.855$. According to Eq. (3.2), the general form of a 5th order Bezier curve using the control values P_0, P_1, P_2, P_3, P_4 and P_5 for data on 28 July 2023 is

$$\begin{aligned} B(t_i) &= (1-t_i)^5 3.0960 \\ &\quad + 5t_i(1-t_i)^4 6.4105 \\ &\quad + 10t_i^3(1-t_i)^2 6.0959 \\ &\quad + 5t_i^4(1-t_i)7.4630 + t_i^5 6.855. \end{aligned} \tag{3.8}$$

The yield curve resulting from Eq. (3.8) is as shown in Fig. 4.

On 28 July 2023, when the time to maturity is 4.14 years, the value of $B(t_i)$ obtained using Eq. (3.8) is 5.0347 while the value of y_i is 5.187. The resulting squared error value is 0.0232. At the time to maturity the other is 9.72 years, the value $B(t_i) =$

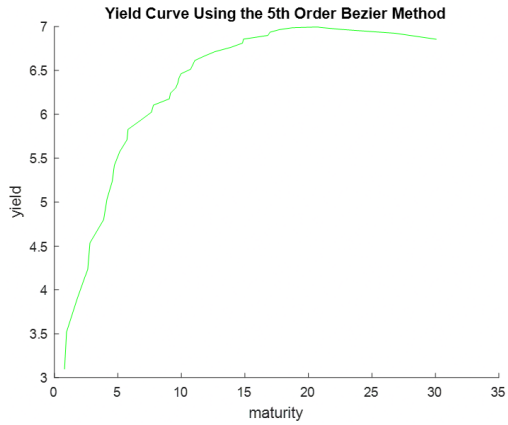


Fig. 4. Bond yield curve using the 5th order bezier method for bond data on 28 July 2023.

6.3578 and $y_i = 6.404$, where the square of the error is obtained 0.0021. The MSE value of 0.0688 is obtained by comparing the real yield data from the bond market to the yield calculated using the 5th order Bezier approach in equation modeling (3.8) for all time to maturity data on 28 July 2023. Fig. 5 shows the yield comparison curves for real and approximate yield data.

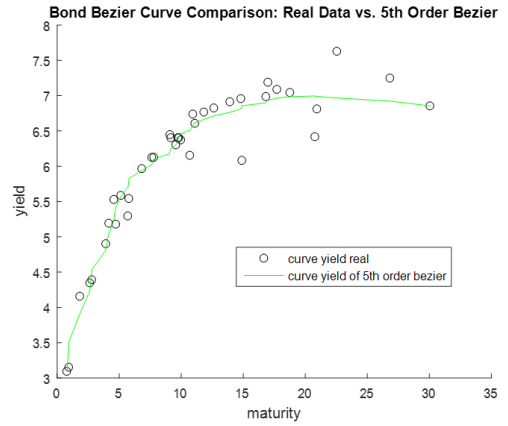


Fig. 5. Comparison of bond yield curves: real data vs. 5th order bezier for bonds data on 28 July 2023.

Bond yield curve modeled through a 6th order Bezier curve requires 7 control values. The initial control value P_0 and the final control value P_6 obtained from the sorted yield data are $P_0 = 3.122$ and $P_6 = 6.7$. Based on Eq. (3.6), the intermediate control values obtained for the bond data dated July 27, 2023 are $P_1 = 5.6574, P_2 = 4.5383, P_3 = 9.7075, P_4 = 4.0720$, and $P_5 = 8.6044$. The control values $P_1, P_2, P_3, P_4$and P_5 are determined after obtaining the following values of A_i, B_i, C_i, D_i, E_i, and F_i, namely $B_1 = 1.5944, B_2 = 1.1958, B_3 = 0.7086, B_4 = 0.3322, B_5 = 0.1139, C_1 = 1.1958, C_2 = 1.3287, C_3 = 1.1072, C_4 = 0.7118, C_5 = 0.3322, D_1 = 0.7086, D_2 = 1.1072, D_3 = 1.2654, D_4 = 1.1072, D_5 = 0.7086, E_1 = 0.3322, E_2 = 0.7118, E_3 = 1.1072, E_4 = 1.3287, E_5 = 1.1958, F_1 = 0.1139, F_2 = 0.3322, F_3 = 0.7086, F_4 = 1.1958, F_5 = 1.5944, A_1 = 23.6584, A_2 = 29.3000, A_3 = 31.9237, A_4 = 31.5575$, and $A_5 = 27.6185$.

The general form of the 6th order Bezier curve for data on 27 July 2023 is as follows Eq. (3.9) based on Eq. (3.5) and the derived control value.

$$\begin{aligned} B(t_i) &= (1-t_i)^6 3.122 \\ &+ 6t_i(1-t_i)^5 5.6574 \\ &+ 15t_i^2(1-t_i)^4 4.5383 \\ &+ 20t_i^3(1-t_i)^3 9.7075 \\ &+ 15t_i^4(1-t_i)^2 4.0720 \\ &+ 6t_i^5(1-t_i)8.6044 + t_i^6 6.7. \end{aligned} \quad (3.9)$$

The yield curve in Fig. 6 was generated using a 6th order Bezier curve approach based on Eq. (3.9).

Simulation of the 6th order Bezier curve equation for data on 27 July 2023 produces a value of $B(t_i) = y_i$ for the smallest time to maturity of 0.8 years and the largest is 30.07 years. For time to maturity 7.64 years, the value $B(t_i) = 6.1814$ and the value$y_i = 6.152$. There is an error square

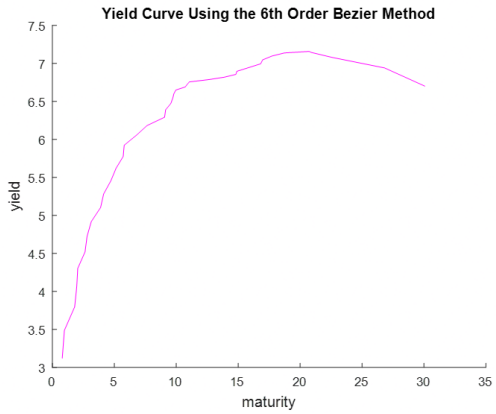


Fig. 6. Bond yield curve using the 6th order bezier method for bond data on 27 July 2023.

for time to maturity of 7.64 years which is 0.0009. The value $B(t_i) = 7.0963$ while the value $y_i = 7.128$ at the time to maturity 17.73 years so that the error square obtained is 0.001. The bond yield estimation from Eq. (3.9) compared with bond data in the market for all time to maturity data on 27 July 2023 produces an MSE value of 0.1607. Fig. 7 illustrates the comparison between real yield data from the bond market and the yield calculated using the 6th order Bezier approach.

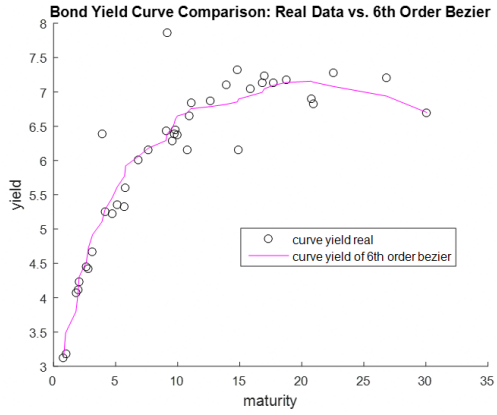


Fig. 7. Comparison of bond yield curves: real data vs. 6th order bezier for bonds data on 27 July 2023.

Analysis of bond yield data using a 6th order Bezier curve for data on July 28, 2023 yields a value of$B_1 = 1.5524, B_2 = 1.1644, B_3 = 0.6900, B_4 = 0.3234, B_5 = 0.1109, C_1 = 1.1644, C_2 = 1.2937, C_3 = 1.0781, C_4 = 0.6931, C_5 = 0.3234, D_1 = 0.6900, D_2 = 1.0781, D_3 = 1.2321, D_4 = 1.0781, D_5 = 0.6900, E_1 = 0.3234, E_2 = 0.6931, E_3 = 1.0781, E_4 = 1.2937, E_5 = 1.1644, F_1 = 0.1109, F_2 = 0.3234, F_3 = 0.6900, F_4 = 1.1644, F_5 = 1.5524, A_1 = 23.8042, A_2 = 28.7570, A_3 = 30.8045, A_4 = 30.1090, and A_5 = 26.0942.$ The obtained values of $A_i, B_i, C_i, D_i, E_i,$ and F_i were used to determine the control values $P_1, P_2, P_3, P_4,$ and P_5. The control values $P_1, P_2, P_3, P_4,$ and P_5 obtained using Eq. (3.6) are $P_1 = 6.4190, P_2 = 4.6112, P_3 = 8.7731, P_4 = 4.7570,$ and $P_5 = 7.9225$. Meanwhile, the initial and final control values of the bond yield data are $P_0 = 3.0960$ and $P_6 = 6.8550$.

Eq. (3.10) is the general form of the 6th order Bezier curve for data obtained on 28 July 2023, which is generated using the control value derived in Eq. (3.5).

$$\begin{aligned} B(t_i) = {} & (1-t_i)^6 3.0960 \\ & + 6t_i(1-t_i)^5 6.4190 \\ & + 15t_i^2(1-t_i)^4 4.6112 \\ & + 20t_i^3(1-t_i)^3 8.7731 \\ & + 15t_i^4(1-t_i)^2 4.7570 \\ & + 6t_i^5(1-t_i)7.9225 + t_i^6 6.8550. \end{aligned} \tag{3.10}$$

The yield curve for bond data on 28 July 2023 created using the 6th order Bezier curve approach is represented in Fig. 8.

On 28 July 2023, the simulation result of the 6th order Bezier curve equation at the time to maturity 7.6 years is $B(t_i) = 6.1076$. The bond yield value obtained from the market is $y_i = 6.132$ which causes the square of error 0.0006. At the time to ma-

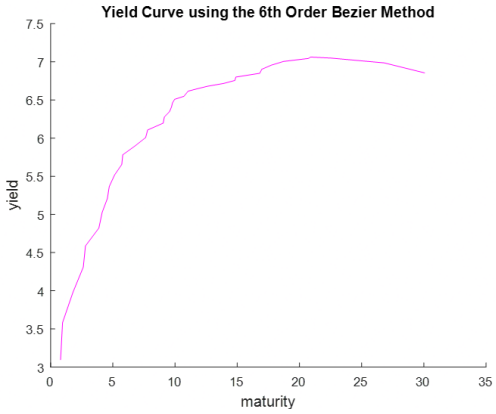


Fig. 8. Bond yield curve using the 6th order bezier method for bond data on 28 July 2023.

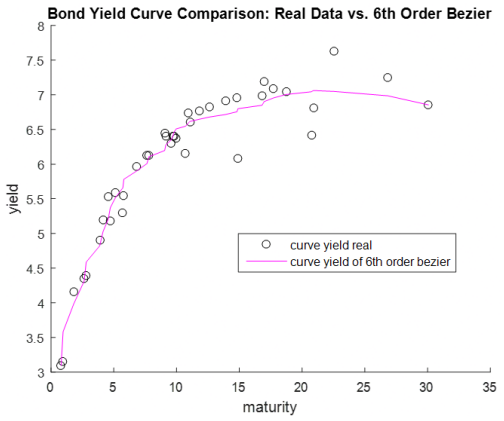


Fig. 9. Comparison of bond yield curves: real data vs. 6th order bezier for bonds data on 28 July 2023.

turity 18.73 years, the value $B(t_i) = 7.0050$ while $y_i = 7.04$. The difference in value between $B(t_i)$ and y_i gives the results of a square error of 0.0012. MSE of the 6th order Bezier curve for all time to maturity data on 28 July 2023 by comparing the bond yield from Eq. (3.10) approach and the real bond yield is 0.0668. In Fig. 9, the real yield data from the bond market is compared to the yield calculated using the 6th Order Bezier approach.

The Bezier curve starts at P_0 to P_1, P_2, and so on until it reaches P_n. The initial data for bond yields is P_0, and the final data for bond yields is P_n. The intermediate control value must be determined while generating the Bezier curve, namely the points $P_1, P_2, ..., P_(n-1)$. The intermediate control variables $P_1, P_2, ..., P_(n-1)$ do not lie on the Bezier curve's curve. The control value is used to direct the points on the curve in the direction of the starting point. The more control values added to the Bezier curve, the more complex it becomes and the more adaptive it becomes to real data. The more control values there are, the smoother the curve.

Bezier curves are classified into two types, low-order and high-order. According to research by Song et al. [10], the high-order Bezier curve has 7 control values. This indicates that the high-order Bezier curve starts with the 6th order Bezier curve, which contains 7 control variables. Meanwhile, a Bezier curve with a control value less than 7 is referred to as a low-order Bezier curve. In this study, a high-order Bezier curve of 6th order is compared against a low-order Bezier curve of 5th order to determine which yield curve model better fits the market's bond yield data.

Bond yield curve modeling using 5^{th} and 6^{th} order Bezier curves indicates that the 6^{th} order Bezier curve is more closely related to the bond yield curve for all bond data. This is proves that the MSE of the 6^{th} order Bezier curve is less than that of the 5th order Bezier curve.

The 6th order Bezier curve is closer to the bond yield curve since it contains more control values, particularly seven control values. The 5th order Bezier curve, on the other hand, has only six control values. Additional control parameters on this 6th order Bezier curve complicate the curve's movement and adjustment to actual bond yield data. This is consistent with Song et

al. [10], which indicates that a high-order Bezier curve creates a smoother than a low-order Bezier curve. Song et al. defined a high-order Bezier curve using seven control parameters [10]. The 6th order Bezier curve contains seven control parameters. Furthermore, Triadi et al. suggest that the 6th order Bezier curve results in a more diversified curve [15].

The MSE of the 6th order Bezier curve is lower than that of the 5th order. However, the 5th and 6th order Bezier curves are very similar. Thus, Bezier curves of 5th and 6th order are capable of accurately modeling the bond yield curve.

4. Conclusion

The 5^{th} and 6^{th} order Bezier curves are applied to model the bond yield curve for July 27 and 28, 2021 data. Based on the calculation results, it is obtained that the 6th order Bezier curve is closer to the bond yield curve for all data because the MSE value of the 6th order Bezier curve is smaller than the 5^{th} order Bezier curve. So, the 6^{th} order Bezier curve can model the bond yield curve with a smaller MSE. The 6^{th} order Bezier curve is closer to the bond yield curve because the 6th order Bezier curve has more control values, namely seven control values. The more control values on the 6^{th} order Bezier curve make the curve more flexible to move and adjust to the actual bond yield data. However, the MSE of the 5^{th} and 6^{th} order Bezier curves are not much different. So, the 5th and 6^{th} order Bezier curves can model the bond yield curve well, but the 6th order Bezier curve produces a smaller MSE value.

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