



The Effectiveness of Mixed Non – Parametric Process Monitoring Charts based on Tukey for ZIB and ZIP Processes

Saowanit Sukparungsee¹, Suganya Phantu^{2,*}

¹*Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok 10800, Thailand*

²*Faculty of Science, Energy, and Environment, King Mongkut's University of Technology North Bangkok, Rayong 21120, Thailand*

Received 27 July 2025; Received in revised form 27 June 2026

Accepted 29 June 2026; Available online 30 June 2026

ABSTRACT

This study examines the application of non-parametric Tukey-based control charts to address the limitations of standard control charts in monitoring high-yield manufacturing processes that frequently exhibit zero counts. Specifically, it compares the Tukey control chart (TCC), Tukey-based mixed exponentially weighted moving average - moving average control chart (TEMCC), and Tukey-based mixed moving average - exponentially weighted moving average control chart (TMECC) in their ability to detect parameter shifts in zero-inflated binomial and Poisson (ZIB/ZIP) processes, better suited for this data type. Through a Monte Carlo simulation, TMECC demonstrated superior performance in detecting process changes, especially under high zero inflation. A case study highlights the practical advantages of utilizing ZIB and ZIP models for enhanced quality control in manufacturing environments.

Keywords: Mixed non-parametric control chart; Tukey control chart; Zero-inflated process

1. Introduction

Manufacturers strive for high-quality production, where minimizing nonconformities is essential for process efficiency. In statistical process monitoring (SPM), count data is typically modeled using Poisson or binomial distributions, assuming defects occur randomly and independently over time. These models help detect deviations, assess defect rates, and maintain

process stability. However, when advanced technology is integrated into manufacturing processes to enhance quality control, it often significantly reduces defects, resulting in a substantial reduction in data errors.

Numerous studies have highlighted that applying standard Poisson or binomial distributions to datasets with excessive zeros can lead to underestimating the process mean and variance. This misrepresentation can have

profound implications for quality control, as it causes process monitoring charts to signal false alarms more frequently due to overly restrictive control boundaries. Such false signals can prompt unnecessary process adjustments, leading to increased operational costs and reduced production efficiency.

For instance, it has been demonstrated that traditional Poisson and binomial models fail to adequately accommodate zero inflation, leading to misleading process monitoring outcomes. Zero-inflated models have been proposed as a more suitable alternative for analyzing attribute counts of occurrences with a high prevalence of zeros. To explain the surplus of zeros in our counts, the zero-inflated binomial (ZIB) and Poisson (ZIP) distributions use a two-part model. One part covers the standard counts, and the other accounts for zero inflation [1-3].

This study focuses on implementing and monitoring manufacturing processes that exhibit zero-inflated count characteristics. By leveraging ZIB and ZIP models, more accurate assessments of process stability can be achieved, reducing false alarms and enhancing the reliability of quality control mechanisms in modern manufacturing environments.

Process monitoring charts are essential statistical process control (SPC) tools for monitoring process quality. Initially introduced, the Shewhart chart effectively detects large process shifts but is less efficient for small shifts [4]. To overcome this limitation, process monitoring charts with built-in memory, such as control charts using averages that give more weight to recent data [5] and control charts using averages of recent data points were developed [6]. These charts incorporate both past and present data for improved sensitivity. Traditional SPC charts often assume a specific probability distribution, typically the expected one, but this assumption is not always valid. As a result, non-parametric or distribution-free control charts have gained popularity for their flexibility in handling unknown or non-normal distributions. Comprehensive reviews of these

methods have been conducted by various researchers, further advancing the field of non-parametric statistical process control (SPC).

Parametric control charts commonly detect process changes in the mean or standard deviation, but they rely on assumptions of normality, variance homogeneity, and data independence. Real-world data often have unknown or uncertain distributions and parameters, which hinders the usefulness of standard statistical approaches. To overcome these challenges, non-parametric control charts have been developed, offering greater flexibility by eliminating the need for strict distributional assumptions.

Over the years, various researchers have developed advanced control charts to improve process monitoring. The arcsine control chart for mean shift detection [7] was followed by the development of Tukey's control chart (TCC) [8], which was specifically designed for single or small-group data. Including the exponentially weighted moving average (EWMA) sign and EWMA arcsine charts, makes it possible to better identify small shifts without assuming a specific data distribution [9].

Subsequent improvements included the evaluation of TCC, which confirmed its superior performance over Shewhart and exponentially weighted moving average (EWMA) control charts [10]. A mixed TCC and cumulative sum (CUSUM) control chart, which applies to both symmetric and asymmetric distributions, provides it with an edge over TCC and CUSUM [11]. A proposal was made to combine EWMA and TCC, demonstrating higher efficiency [12]. The Tukey-based EWMA-CUSUM, proving its superiority over traditional methods [13]. An EWMA-TCC for monitoring process spread which outperformed both EWMA and TCC [14]. Additionally, numerous researchers have contributed to advancements in non-parametric control chart techniques, further enhancing statistical process monitoring [15-17].

This study was inspired by the examination of non-parametric Tukey-based process monitoring charts for monitoring zero-inflated data, particularly in processes that follow binomial and Poisson distributions, which account for extra zeros [18]. The remainder of the paper is organized as follows: Section 2 discusses the characteristics of zero-inflated data, the non-parametric Tukey-based control chart, and its performance measures. Section 3 evaluates the performance of process monitoring charts within acceptable limits (WAL) and outside acceptable limits (OAL) conditions. Finally, Section 4 concludes with a summary and discussion of the findings.

2. Materials and Methods

The theoretical framework of this research is divided into three key parts. First, it examines zero-inflated data distributions, focusing on their characteristics and the relevance of models such as the binomial and Poisson models, which are used in statistical process monitoring to address extra zeros. Second, it explores non-parametric Tukey-based control charts, highlighting their principles, advantages, and applications compared to traditional parametric methods. Finally, it discusses performance metrics of the process monitoring chart, detailing key evaluation measures such as average time until an alarm (ATA), variability of run length (VRL), and middle run length (MRL) to assess the effectiveness of different process monitoring chart approaches.

2.1 The Characteristics of Zero-Inflated Data

The zero-inflated data in this study is categorized into two parts as follows:

2.1.1 A binomial distribution with zero-inflation

The binomial distribution with zero inflation (ZIB) extends to a binomial model designed to address excess zero counts. It assumes that independent random events occur, each with a probability of θ , leading to

non-conforming items that follow a binomial distribution with a proportion of p . Assuming no shocks occur, the count remains zero. The probability mass function (PMF) of a ZIB variable is defined accordingly.

$$f(x; \theta, n, p) = \begin{cases} 1 - \theta + \theta(1-p)^n & ; x = 0 \\ \theta \binom{n}{x} p^x (1-p)^{n-x} & ; x > 0 \end{cases} \quad (2.1)$$

A random sample of N units can be used to estimate the ZIB parameters θ and p . Their maximum likelihood estimators (MLEs) are computed numerically using Eqs. (2.2)-(2.3).

$$\hat{\theta} = \frac{n - n_0}{n} \frac{1}{1 - (1-p)^n}, \quad (2.2)$$

$$\text{and} \quad \hat{p} = \frac{\bar{X}}{n\hat{\theta}}. \quad (2.3)$$

The MLEs of $\hat{\theta}$ and $\hat{p}$, along with the arithmetic average of the sample $\bar{X}$ and the count of data points that are zero n_0 , are used to determine the expected value and dispersion of X .

$$E(X) = np\theta, \quad (2.4)$$

and

$$V(X) = n(n-1)p^2\theta + np\theta(1-np\theta). \quad (2.5)$$

2.1.2 A Poisson distribution with zero-inflation

The Poisson distribution with zero inflation (ZIP) extends the Poisson distribution to handle excess zeros. Defects are modeled as random events (Poisson distribution, rate λ) with a probability π , resulting in a specific probability density function (PDF).

$$f(y; \pi, \lambda) = \begin{cases} 1 - \pi + \pi e^{-\lambda} & ; y = 0 \\ \pi \frac{\lambda^y e^{-\lambda}}{y!} & ; y > 0 \end{cases} \quad (2.6)$$

The estimation of parameters $\hat{\pi}$, and $\hat{\lambda}$, which are generally unknown, is performed using maximum likelihood estimation, informed by observed non-conformity data.

$$\hat{\lambda} = \frac{\bar{Y}}{\hat{\pi}} \frac{n - n_0}{n}, \quad (2.7)$$

and

$$\hat{\pi} = \frac{1}{1 - e^{-\hat{\lambda}}} \frac{n - n_0}{n}. \quad (2.8)$$

The estimators $\hat{\pi}$ and $\hat{\lambda}$ are derived using the mean of positive observations, total observations n , and the count of zeros n_0 . The mean and variance are then determined accordingly.

$$E(Y) = (1 - \pi)\lambda, \quad (2.9)$$

and

$$V(Y) = (1 - \pi)\lambda + \left(\frac{\pi}{1 - \pi}\right)((1 - \pi)\lambda)^2. \quad (2.10)$$

2.2 Characteristics of zero-inflated data

This section describes the characteristics of non-parametric Tukey-based process monitoring charts, which can be categorized as follows:

2.2.1 Tukey's control chart

The Tukey process monitoring chart is a non-parametric process monitoring chart [8]; it is an observed-value-based process monitoring chart that utilizes the principles of the Boxplot to establish its boundary of control. Let O_i represent an observation derived from the zero-inflated distribution of interest. The subsequent definitions delineate the boundary of controls applicable to the Tukey process monitoring chart:

$$\begin{aligned} UBC &= Q_3 + c_1(IQR) \\ LBC &= Q_1 - c_1(IQR) \end{aligned} \quad (2.11)$$

UBC (Upper Bound of Control) and LBC (Lower Bound of Control) represent the

boundaries for monitoring process variation. The terms Q_1 and Q_3 refer to the data points at the 25th and 75th percentiles, respectively, which are the 25th and 75th percentiles of the data distribution. The range between the 25th and 75th percentiles (IQR) is calculated as $Q_3 - Q_1$ and represents the spread of the middle 50% of the data. The parameter c_1 is the boundary of the control coefficient used in Tukey's control chart to determine the threshold for detecting outliers or process shifts.

2.2.2 Tukey-based exponentially weighted moving average – moving average control chart

The TEMCC (Tukey-based exponentially weighted moving average – moving average control chart) is constructed by integrating the mixed exponentially weighted moving average – moving average control chart (mixed EWMA-MA) with the TCC. The proposed hybrid approach leverages the statistical characteristics of the mixed EWMA-MA process monitoring chart, which is derived from the exponentially weighted moving average (EWMA) and moving average (MA) process monitoring charts [18]. By synergistically combining the smoothing capability of EWMA with the averaging function of MA, the mixed EWMA-MA process monitoring chart demonstrates improved detection of both subtle and moderate process deviations. The associated statistical representation of the mixed EWMA-MA process monitoring chart is provided in Eq. (2.12).

$$MA_i = \begin{cases} \frac{Z_i + Z_{i-1} + Z_{i-2} + \dots}{i} & ; i < w \\ \frac{Z_i + Z_{i-1} + \dots + Z_{i-w+1}}{w} & ; i \geq w \end{cases} \quad (2.12)$$

where w represents the process monitoring chart width, the range within which the process variation is monitored is defined. The term Z_i refers to the calculated EWMA value at time i ,

which incorporates past observations to smooth fluctuations in process data. The parameter α is the weighting factor for previous observations, constrained within the range $0 \leq \alpha \leq 1$, where smaller values α emphasize historical data and larger values give more weight to recent observations. The variable O_i represents the observed value at time i . Given these definitions, the statistical formulation of the chart using exponentially smoothed averages is expressed in Eq. (2.13).

$$Z_i = \alpha O_i + (1 - \alpha) Z_{i-1}; i = 1, 2, \dots \quad (2.13)$$

The UBC and LBC of the TEMCC are established on the assumption that the underlying data follow the same statistical distribution as the TCC. However, the variance used in TEMCC is adjusted by incorporating elements from the mixed EWMA-MA and TCC, allowing for more robust detection of process shifts in zero-inflated data. This combination enhances sensitivity to variations while maintaining control over false alarms. Consequently, the boundary of controls for TEMCC, when the index i exceeds the process monitoring chart width w , is mathematically defined by Eq. (2.14).

$$\begin{aligned} UBC &= Q_3 + c_2(IQR) \sqrt{\left(\frac{1}{w}\right) \left(\frac{\alpha}{2-\alpha}\right)} \\ LBC &= Q_1 - c_2(IQR) \sqrt{\left(\frac{1}{w}\right) \left(\frac{\alpha}{2-\alpha}\right)} \end{aligned}, \quad (2.14)$$

when the index i is less than the process monitoring chart width w ($i < w$), the control boundary is modified for the reduced number of measured values, ensuring stability in the monitoring process. In this case, the boundary of control is defined as presented in Eq. (2.15).

$$\begin{aligned} UBC &= Q_3 + c_2(IQR) \sqrt{\left(\frac{1}{i}\right) \left(\frac{\alpha}{2-\alpha}\right)} \\ LBC &= Q_1 - c_2(IQR) \sqrt{\left(\frac{1}{i}\right) \left(\frac{\alpha}{2-\alpha}\right)} \end{aligned}, \quad (2.15)$$

where c_2 represents the coefficient used to determine the boundary of control for the TEMCC, influencing the threshold for detecting process shifts.

2.2.3 Tukey-based moving average – exponentially weighted moving average control chart

Similarly, the TMECC (Tukey-based mixed moving average - exponentially weighted moving average control chart) is constructed by integrating the mixed moving average – exponentially weighted moving average (mixed MA-EWMA) process monitoring chart with the TCC. This integration leverages the statistical characteristics of the mixed MA-EWMA process monitoring chart, enhancing its ability to observe process variations in zero-inflated data. For a more in-depth discussion of its development and applications [18].

TMECC combines the strengths of both the MA and the EWMA process monitoring chart. The MA process monitoring chart provides a smoothing effect by averaging past observations, thereby reducing random noise. In contrast, the EWMA process monitoring chart assigns exponentially decreasing weights to historical data, allowing for the early detection of gradual process shifts. By integrating these methods with Tukey's approach, the TMECC enhances sensitivity to variations while maintaining robustness against false alarms.

The mathematical model intended for the mixed MA-EWMA process monitoring chart and the statistical values obtained from the MA process monitoring chart serve as inputs for the EWMA process monitoring chart, ensuring a progressive smoothing mechanism that effectively captures sudden and gradual process changes. The formulation of this approach is based on Eq. (2.13). Consequently, the statistical representation of the mixed MA-EWMA process monitoring chart is expressed as follows:

$$Z_i = \alpha MA_i + (1 - \alpha)Z_{i-1} ; i=1,2,\dots \quad (2.16)$$

where α is the weighting parameter that determines the influence of past data on the current estimate, taking values within the range $0 \leq \alpha \leq 1$. A lower value α gives more weight to historical observations, making the process monitoring chart more stable, while a higher value prioritizes recent data, increasing responsiveness to process changes.

The term Z_0 represents the initial value of the process monitoring statistic and is set equal to the target mean, Z_0 , to ensure a proper starting point for tracking deviations. This initialization helps maintain consistency and accuracy when applying the process monitoring chart.

In the MA process monitoring chart, the process monitoring chart width w defines the number of past observations included in calculating the moving average at each step. Measurement at time i is computed by averaging data within this window, allowing for a smoother representation of process trends while reducing the impact of short-term fluctuations.

MA statistic calculations are classified into two cases, detailed below:

$$MA_i = \begin{cases} \frac{O_i + O_{i-1} + O_{i-2} + \dots}{i} & ; i < w \\ \frac{O_i + O_{i-1} + \dots + O_{i-w+1}}{w} & ; i \geq w \end{cases} \quad (2.17)$$

Therefore, the boundary of control for the TMECC is mathematically derived based on its statistical properties and is explicitly formulated in Eq. (2.18). These limits effectively detect shifts in zero-inflated data while striking a balance between sensitivity and robustness.

$$\begin{aligned} UBC &= \bar{Q}_3 + c_3(IQR) \sqrt{\left(\frac{1}{w}\right) \left(\frac{\alpha}{2-\alpha}\right)} \\ LBC &= \bar{Q}_1 - c_3(IQR) \sqrt{\left(\frac{1}{w}\right) \left(\frac{\alpha}{2-\alpha}\right)} \end{aligned} \quad (2.18)$$

where c_3 represents the coefficient used to determine the boundary of control for the TMECC, influencing the threshold for detecting process shifts and anomalies.

2.3 The design of performance for process monitoring chart

This section analyzes the run-length distribution of the process monitoring chart, assuming that the within-acceptable-limits (WAL) parameters, θ_0 for θ and π_0 for π , are known beforehand. It examines the WAL design, outside acceptable limits (OAL) performance, and steady-state behavior to assess the effectiveness of the three process monitoring charts in detecting process shifts. These evaluations provide insights into their sensitivity, detection speed, and long-term stability in monitoring zero-inflated data.

The average time until an alarm (ATA) is a key performance metric for the process monitoring chart, representing the expected number of plotted observations before a process shift is signaled. It is used to evaluate the effectiveness of a process monitoring chart in distinguishing between within-acceptable-limits (WAL) and outside-acceptable-limits (OAL) conditions. A well-designed chart should have a high ATA_0 in the WAL state to minimize false alarms and a low ATA_1 in the OAL state to detect process deviations quickly. Balancing these values ensures efficient monitoring, reducing unnecessary disruptions while enabling timely detection of shifts. The mathematical expressions for ATA_0 and ATA_1 are provided for further analysis.

$$ATA_0 = 1/\phi, \quad (2.19)$$

where ϕ represents the probability of a Type I error, which refers to the likelihood that a plotted point exceeds the control boundary when the process is in control.

$$ATA_1 = 1/1-\beta, \quad (2.20)$$

where β represents the probability of a Type II error, indicating the likelihood that a process shift occurs but remains undetected, meaning the plotted points stay within the control boundary despite an actual change in the process.

In this study, detection accuracy and detection efficiency are treated as different evaluation aspects. Detection accuracy reflects the correctness of the monitoring decision. Under the WAL condition, this is represented by the ability to avoid false alarms, where the false-alarm probability is denoted by ϕ and $ATA_0 = 1/\phi$. Under the OAL condition, detection accuracy is related to the ability to detect an actual shift, where β denotes the Type II error or missed-detection probability and $ATA_1 = 1/(1 - \beta)$. In contrast, detection efficiency focuses on the speed and consistency of detection after a shift occurs. Therefore, ATA_1 , VRL_1 , and MRL_1 are used to compare how quickly and stably the charts detect OAL conditions.

The variability of run length (VRL) and middle run length (MRL) are key performance metrics for evaluating process monitoring charts. While VRL measures variability in detection time, MRL is preferred in cases where the run-length distribution is asymmetric, as it is less influenced by skewness. Additional criteria, such as chattering, false detection rate, missed detection rate, and average detection delay, are also used to evaluate the efficiency of process monitoring charts. These metrics help determine the chart's ability to balance sensitivity and stability in detecting process shifts. The ATA, VRL, and MRL formulas are provided for further analysis.

$$ATA = \frac{\sum_{i=1}^N RL_i}{N} ; i = 1, 2, \dots, N, \quad (2.21)$$

$$VRL = \sqrt{E(RL)^2 - ARL^2}, \quad (2.22)$$

$$MRL = \text{Median}(RL), \quad (2.23)$$

RL_i represents the number of samples before the process exceeds the control boundary, while N is the total number of simulation repetitions set to 200,000.

A common ATA_0 , VRL_0 , and MRL_0 should be established to compare the performance of process monitoring charts. The chart with lower ATA_1 , VRL_1 , and MRL_1 is more effective in detecting process shifts. For computing the run-length distribution of the ZIB and ZIP process monitoring charts, we employ the Monte Carlo simulation method. The simulation algorithm consists of the following five steps:

Step 1: Generate 500 random variables O_i ($i = 1, 2, \dots, 500$) from the ZIB distribution with parameters (θ, n, p) and the ZIP distribution with parameters (π, λ) . These random samples are generated using R statistical software, ensuring accurate modeling of zero-inflated data characteristics for subsequent analysis in the Monte Carlo simulation.

Step 2: Given appropriate values for w (process monitoring chart width), α (significance level), and c (boundary of control coefficient), the boundary of control for the process monitoring charts are computed using Eqs. (2.11), (2.14)-(2.15), (2.17)-(2.18), respectively. The charting statistics, derived from Eqs. (2.12) and (2.16), are then compared against the corresponding boundary of control for $N = 1, 2, \dots, 200,000$ to determine the run length, which represents the number of observations before the first outside-acceptable-limits signal occurs.

Step 3: After 200,000 iterations, the ATA, VRL, and MRL are calculated based on specific zero-inflated parameters and the process monitoring chart combination coefficient to assess detection performance, false alarms, and stability.

Step 4: The boundary of control width (c), which corresponds to the target within acceptable limits ($ATA_0 \approx 370$), is obtained through an iterative process by repeatedly executing Steps (1) to (3). During this

procedure, the parameters are set as $\theta = \theta_0$ for the ZIB distribution and $\pi = \pi_0$ for the ZIP distribution. This iterative approach ensures that the process monitoring charts are correctly tuned to achieve the desired performance in detecting deviations while minimizing false alarms.

Step 5: Using the optimized process monitoring chart coefficient for the desired ATA_0 , VRL_0 , and MRL_0 , the ATA_1 , VRL_1 , and MRL_1 are calculated for specific shifts in θ_0 and π_0 by executing Steps (1) to (3), evaluating the process monitoring chart's effectiveness in detecting process deviations.

3. Performance Evaluation

This section evaluates the performance of the proposed process monitoring chart by considering two aspects: detection accuracy and detection efficiency. In this study, detection accuracy refers to the ability of a process monitoring chart to correctly distinguish between within-acceptable-limits (WAL) and outside-acceptable-limits (OAL) conditions. Under the WAL condition, accuracy is associated with the false-alarm probability, or Type I error. A chart with a lower false-alarm probability is considered more accurate in maintaining the process as in control when no actual shift occurs. Under the OAL condition, accuracy is associated with the probability of detecting a true process shift, which is related to the Type II error or missed-detection probability.

Detection efficiency, on the other hand, refers to how rapidly and consistently a chart signals after a process shift occurs. It is measured using three run-length-based criteria: the average time until an alarm under the OAL condition (ATA_1), the variability of run length under the OAL condition (VRL_1), and the median run length under the OAL condition (MRL_1). A smaller ATA_1 indicates faster detection, while smaller VRL_1 and MRL_1 indicate more stable and consistent detection performance. To ensure a fair comparison, all competing charts are

calibrated to have the same WAL performance, with ATA_0 approximately equal to 370. Therefore, after controlling the false-alarm rate, the chart with the lowest ATA_1 , VRL_1 , and MRL_1 is considered the most efficient in detecting process shifts.

The performance evaluation is divided into three parts: (1) determining appropriate chart parameter settings under the WAL condition, (2) evaluating the chart performance under OAL conditions and comparing the proposed chart with existing methods, and (3) applying the charts to real-world zero-inflated datasets to demonstrate their practical effectiveness.

3.1 The within acceptable limits design

Since the ZIB distribution lacks a standardized form, this simulation study defines specific parameter configurations to evaluate the performance of process monitoring charts. The within acceptable limits (WAL) shock probability is set at $\theta_0 = 0.2$, with a sample size of $n = 500$ and a WAL proportion of non-conforming units at $p_0 \in \{0.01, 0.05\}$. Process shifts are introduced to analyze detection efficiency with magnitudes δ ranging from 0.01 to 1.00.

Additionally, case 2: parameter configuration for ZIP-distributed data focuses on optimizing parameter settings for datasets that follow a Poisson distribution with a high proportion of zeros. We examine values for the within-acceptable-limits (WAL) shock probability, setting $\pi_0 = 0.2$, and the WAL proportion of non-conforming units, defined as $\lambda_0 \in \{1, 5\}$. Additionally, we introduce different shift magnitudes δ , considering values from $\delta \in \{0.01, 0.05, 0.10, 0.25, 0.50, 0.75, 1.00\}$ to evaluate the process monitoring chart's ability to detect process variations effectively.

Moreover, the process monitoring chart parameters are set as follows: the moving average window width is fixed at 3, and the weighting parameter α is set to 0.05.

3.2 The outside acceptable limits design

This study aims to evaluate the efficiency of a non-parametric Tukey-based process monitoring chart, specifically TCC, TEMCC, and TMECC. The focus is on monitoring shifts in the zero-inflated parameters, namely θ_1 for the ZIB process and π_1 for the ZIP process. The parameter of out-of-control is defined as follows:

In Case 1, the ZIB process was configured with $\theta = \theta_0$ in the within acceptable limits (WAL) state and $\theta = \theta_1$ in the outside acceptable limits (OAL) state, where $\theta_1 = \theta_0 + \delta\sigma$. Here, δ represents the shift magnitude and σ is the typical spread we see in the process when it's stable; the experimental design enables the determination of the process monitoring chart's sensitivity to variations in the zero-inflation parameter.

Next, in Case 2, the ZIP process was configured with $\pi = \pi_0$ in the WAL state $\pi = \pi_1$ and OAL state $\pi_1 = \pi_0 + \delta\sigma$. Here, δ it represents the shift magnitude and the parameter σ indicates the standard deviation of the process in its controlled state. This setup

enables a systematic evaluation of the process monitoring chart's ability to detect changes in the zero-inflated probability parameter (π), ensuring accurate monitoring of process deviations.

The efficiency of the process monitoring chart was evaluated using three key performance metrics: ATA, VRL, and MRL. A process monitoring chart was considered the most advantageous if it had the lowest ATA_1 , VRL_1 , and MRL_1 values, indicating quicker and more consistent detection of process shifts. These metrics comprehensively assessed the chart's ability to balance sensitivity and stability in monitoring zero-inflated data. The research findings based on these criteria are summarized as follows.

When the observation was a ZIB (500, 0.01, 0.2) distribution, $ATA_0 = 370$, the TMECC was the most efficient process monitoring chart for the change at the 0.01, 0.05, 0.10, and 0.25 levels. While the TCC was the most efficient process monitoring chart for detecting changes at 0.50, 0.75, and 1.00, considering the results of VRL and MRL, it was consistent with ATA, as shown in Table 1.

Table 1. Performance comparison of Non-parametric Tukey-based control chart with $w=5$, $\alpha=0.05$ under the ZIB (500, 0.01, 0.2) distribution.

δ	TCC			TEMCC			TMECC		
	ATA	VRL	MRL	ATA	VRL	MRL	ATA	VRL	MRL
0.00	370.18	369.82	369	370.56	370.19	369	370.85	369.57	369
0.01	367.15	362.41	368	357.84	352.11	358	354.72	350.41	356
0.05	324.54	321.63	326	319.33	315.86	320	314.28	311.66	317
0.10	246.72	241.89	248	157.98	153.42	158	135.61	131.25	137
0.25	134.79	130.55	135	114.79	111.03	115	94.26	91.78	96
0.50	65.63	61.85	67	68.90	65.79	70	71.34	68.59	73
0.75	12.65	9.64	14	16.74	13.20	18	22.15	20.63	24
1.00	7.89	4.87	8	9.46	8.97	10	11.02	9.88	13

Bold is minimum ATA, VRL and MRL.

In Table 2, where the observation was a ZIB (500, 0.05, 0.2) distribution, $ATA_0 = 370$.

The performance measurement results are consistent.

Table 2. Performance comparison of Non-parametric Tukey-based control chart with $w=5$, $\alpha=0.05$ under the ZIB (500, 0.05, 0.2) distribution.

δ	TCC			TEMCC			TMECC		
	ATA	VRL	MRL	ATA	VRL	MRL	ATA	VRL	MRL
0.00	370.95	369.75	369	370.49	369.87	369	370.52	370.65	369
0.01	355.76	352.26	357	353.46	351.60	355	351.28	348.57	353
0.05	316.50	313.50	318	312.58	310.78	314	309.64	307.43	312
0.10	138.91	135.97	140	134.52	131.06	136	131.57	129.22	133

0.25	94.65	91.28	96	92.30	90.04	95	89.76	87.16	91
0.50	62.48	60.74	64	65.44	61.52	67	68.51	66.47	70
0.75	10.32	9.03	11	12.08	10.34	13	19.88	17.24	21
1.00	5.74	3.11	6	6.77	5.16	7	8.02	6.44	10

Bold is minimum ATA, VRL and MRL.

When the observation was a ZIP (1, 0.2) distribution, $ATA_0 = 370$, the TMECC was found to be the most advantageous process monitoring chart for detecting changes at the 0.01, 0.05, and 0.10 levels. While the TCC was

also the most efficient process monitoring chart for detecting the change at 0.25, 0.50, 0.75, and 1.00, when considering the results of VRL and MRL, it exhibited conformity to ATA, as shown in Table 3.

Table 3. Performance comparison of Non-parametric Tukey-based control chart with $w=5$, $\alpha=0.05$ under the ZIP (1, 0.2) distribution.

δ	TCC			TEMCC			TMECC		
	ATA	VRL	MRL	ATA	VRL	MRL	ATA	VRL	MRL
0.00	370.25	369.80	369	370.16	370.82	369	370.89	370.13	369
0.01	365.72	363.11	366	362.78	359.73	364	359.47	356.41	361
0.05	334.85	331.29	336	324.89	321.06	326	321.08	319.86	322
0.10	147.96	145.37	148	145.25	141.55	146	142.38	140.77	143
0.25	98.41	95.09	100	104.95	97.30	106	101.27	98.67	103
0.50	72.35	69.48	75	79.30	76.98	81	75.64	72.56	78
0.75	26.98	24.65	28	32.08	30.41	34	28.69	25.61	30
1.00	13.06	11.92	14	16.33	14.90	17	14.50	12.33	15

Bold is minimum ATA, VRL and MRL.

In addition, Table 4, where the observation is assumed to be a realization of a ZIP (5, 0.2) random variable, $ATA_0 = 370$,

shows consistent performance measurement results.

Table 4. Performance comparison of Non-parametric Tukey-based control chart with $w=5$, $\alpha=0.05$ under the ZIP (5, 0.2) distribution.

δ	TCC			TEMCC			TMECC		
	ATA	VRL	MRL	ATA	VRL	MRL	ATA	VRL	MRL
0.00	370.58	369.72	369	370.19	370.05	369	370.64	370.44	369
0.01	366.12	364.51	367	365.80	361.72	366	364.21	361.35	365
0.05	340.69	338.92	342	338.65	336.86	340	335.94	331.70	336
0.10	152.30	149.60	153	151.46	148.05	152	147.82	145.77	148
0.25	110.85	107.68	112	115.62	112.49	117	113.50	110.49	115
0.50	79.69	75.23	81	83.20	80.37	85	81.67	79.51	83
0.75	31.33	28.55	32	34.09	31.58	36	32.61	30.40	34
1.00	16.44	14.69	18	19.66	16.53	20	17.95	15.30	19

Bold is minimum ATA, VRL and MRL.

3.3 Illustrative example

The findings derived from the analysis of process monitoring chart efficiency are categorized into two distinct cases, which are subsequently presented:

3.3.1 Zero-inflated binomial distribution

To illustrate the practical application of the ZIB process monitoring chart, this section presents an illustrative case. The data used in

this investigation originate from a manufacturing process involved in the production of handbrake cables for specific automotive models [19]. In this procedure, individual cables (O_i) were systematically grouped into samples of $n=20$, inspected, and classified according to their conformity status.

A total of 100 observations were used to estimate the within-acceptable-limits (WAL) values of θ and p . By fitting the dataset to the ZIB distribution and applying Eqs. (2.2)-

(2.3). The parameters were estimated to ensure proper alignment with the underlying distribution characteristics. It was found that $\hat{\theta} = 0.762$ and $\hat{p} = 0.0197$. Additionally, the process monitoring chart parameters are defined as follows: the moving average window width is set to 3, ensuring a stable smoothing effect in the monitoring process, while the weighting parameter α is assigned a value of 0.05, allowing for gradual adjustments to new observations while maintaining sensitivity to process variations.

The performance analysis results of TCC, TEMCC, and TMECC can be explained as follows: Fig. 1 shows the detection of changes in TCC, revealing that no data points exceed the control bound. Next, Fig. 2 illustrates the detection of changes in the TEMCC, indicating that all data points remain within the upper and lower bounds of control. Lastly, Fig. 3 presents the detection of changes in the TMECC, where data points 73, 74, and 75 exceed the upper bound of control. Therefore, the TMECC is the most effective in detecting changes.

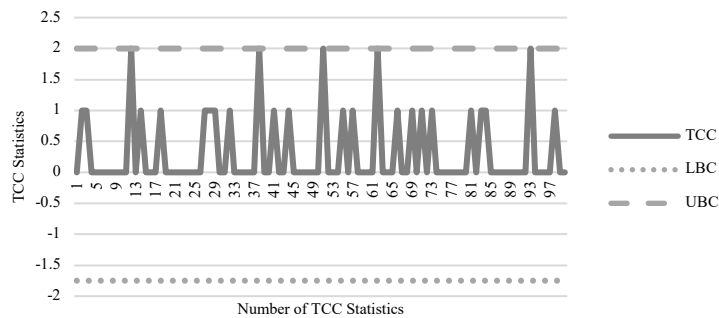


Fig. 1. TCC for ZIB process.

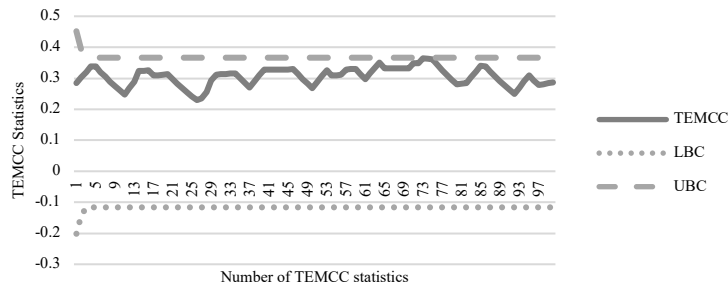


Fig. 2. TEMCC for ZIB process.

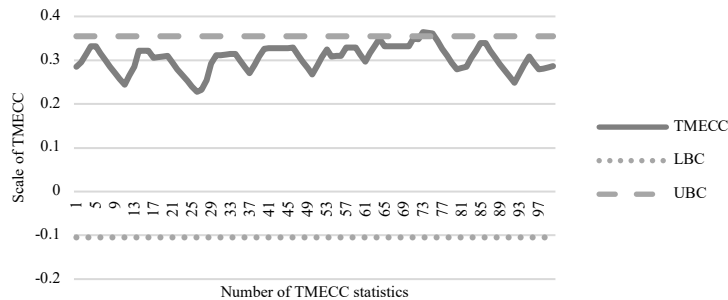


Fig. 3. TMECC for ZIB process.

3.3.2 Zero-inflated poisson distribution

A dataset of 90 observations (days) from which a subset was selected to estimate the acceptable limits (WAL) values parameters (π_0). The method of moments was applied for parameter estimation, following the formulations in Eqs. (2.7)-(2.8) to maintain consistency and accuracy in the control process analysis [20]. It was found that $\hat{\pi} = 0.52$ and $\hat{\lambda} = 2.15$. The process monitoring chart parameters are set with a moving average window width of 3 for smoothing and a weighting parameter α of 0.05 to balance sensitivity and stability in detecting process variations.

The evaluation of the performance results for TCC, TEMCC, and TMECC can be

explained in detail as follows. Fig. 4 illustrates the detection of changes in the TCC, where no data points exceed the upper or lower bounds of the controls, indicating a stable process without significant variations. Subsequently, Fig. 5 presents the detection of changes using the TEMCC, demonstrating that all data points remain within the defined bound of control, suggesting that the process is well-maintained within acceptable boundaries. Finally, Fig. 6 illustrates the detection of changes using the TMECC, revealing that data points 77, 78, and 79 exceed the control bound. Therefore, it can be concluded that TMECC detects changes in the data more quickly than TCC and TEMCC, demonstrating its higher sensitivity in identifying process shifts.

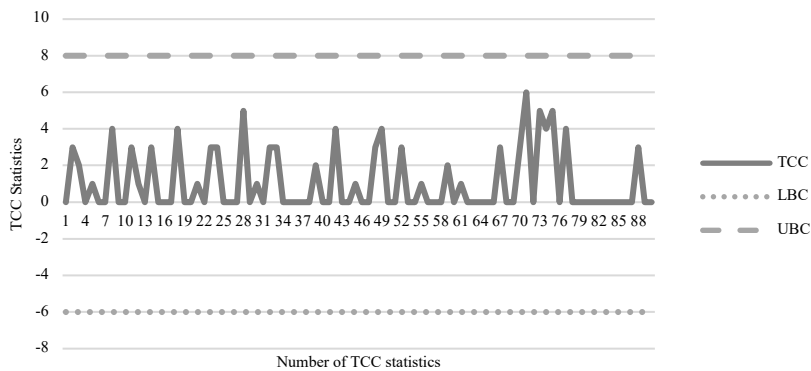


Fig. 4. TCC for ZIP process.

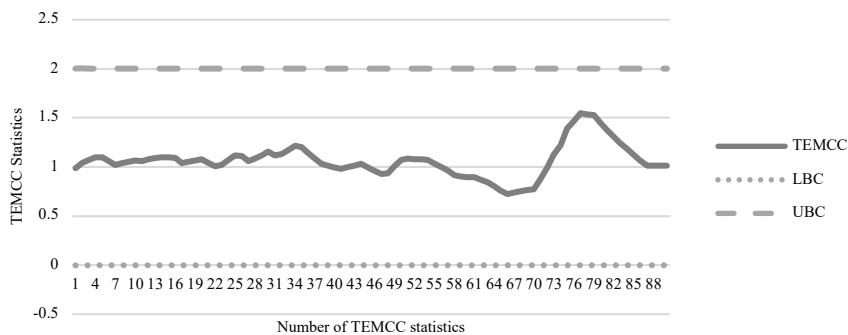


Fig. 5. TEMCC for ZIP process.

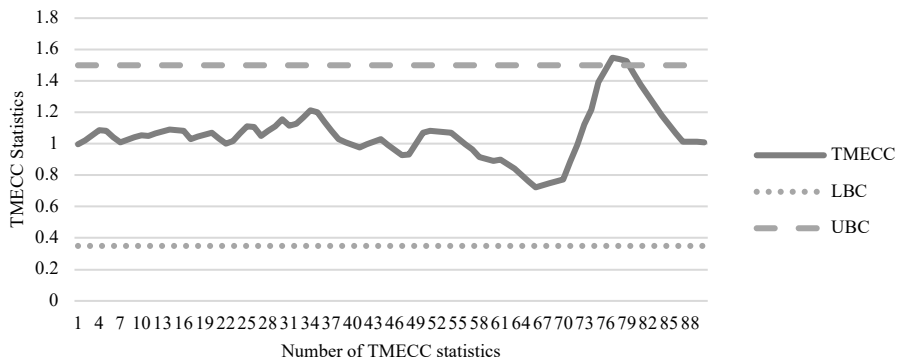


Fig. 6. TMECC for ZIP process.

4. Conclusions and Discussion

The performance comparison of the non-parametric Tukey-based control charts — Tukey control chart (TCC), Tukey-based exponentially weighted moving average – moving average control chart (TEMCC), and Tukey-based mixed exponentially weighted moving average – moving average control chart (TMECC)—was conducted using average time until an alarm (ATA), variability of run length (VRL), and middle run length (MRL) as key criteria for measuring efficiency. The results showed that when the process followed either a zero-inflated binomial (ZIB) or zero-inflated Poisson (ZIP) distribution, the average time until an alarm (ATA) was maintained at 370 under the within-acceptable-limits (WAL) condition, ensuring that false alarms remained at an acceptable level. When the process is out of control, the following conclusions can be drawn:

For the ZIB distribution, the TMECC demonstrated superior performance in detecting process changes when the shift magnitude ranged from 0.01 to 0.50. It could quickly identify minor to moderate process shifts more effectively than the other process monitoring charts. However, for more significant shifts of 0.75 and 1.00, the TCC exhibited better detection efficiency, suggesting that it was more effective in identifying substantial deviations in the process.

For the ZIP distribution, the TMECC was the most effective process monitoring chart for detecting process changes when the shift magnitude was between 0.01 and 0.10, showing that it responded well to small shifts in a zero-inflated Poisson process. However, when the shift magnitude increased to 0.25 and 1.00, the TCC outperformed the others, indicating that it was more efficient in detecting more significant shifts in zero-inflated Poisson-distributed data.

These findings suggest that the TMECC is particularly effective for detecting small to moderate shifts in both ZIB and ZIP distributions. The TCC performs better when detecting larger shifts, making it a suitable option for processes with more significant deviations.

This study's limitations include its focus solely on changes in the zero-inflated parameters of the zero-inflated binomial (ZIB) and zero-inflated Poisson (ZIP) distributions, as well as the specific sample sizes and process monitoring chart parameters considered. The findings indicate that the mixed non-parametric Tukey-based control chart effectively detects small process shifts but requires longer processing time than single control charts. Single control charts are more practical and convenient when dealing with large datasets.

For future research, it is recommended to explore changes in the mean of zero-inflated data across the entire process and to

incorporate additional performance evaluation methods. It would enhance the understanding and interpretation of zero-inflated data characteristics, improving the applicability of process monitoring charts in various scenarios.

Acknowledgements

The authors sincerely thank King Mongkut's University of Technology North Bangkok, Thailand, for supporting this research through a grant with contract number KMUTNB-68-Basic-34 and the Department of Applied Statistics for providing the supercomputer.

References

- [1] Xie M. He B.Goh TN. Zero-inflated Poisson model in statistical process control. *Comput. Stat.Data Anal.* 2001; 38(2): 191–201.
- [2] Sim CH. Lim MH. Attribute charts for zero-inflated processes. *Commun. Stat. Simul. Comput.* 2008; 37(7): 1440-52.
- [3] Phantu S. Areepong Y. Saowanit S. On Designing of Modified Exponentially Weighted Moving Average Control Chart based on Sign Rank for Zero-Inflated Data. *Sci. Technol. Asia.* 2025; 30(1): 158–76.
- [4] Shewhart WA. *Economic Control of Quality of Manufactured Product.* New York: D. Van Nostrand Company; 1931.
- [5] Roberts SW. Control chart tests based on geometric moving average. *Technometrics.* 1959; 42(1): 239-50.
- [6] Khoo MBC. A moving average control chart for monitoring the fraction non-conforming. *Qual. Reliab. Eng. Int.* 2004; 20(6): 617-35.
- [7] Ryan TP. *Statistical Methods for Quality Improvement* (2nd ed). New York Wiley; 2000.
- [8] Alemi F. Tukey's control chart. *Qual. Manag. HealthCare.* 2004; 13(4): 216-21.
- [9] Yang SF. Lin JS. Cheng SW. A new non-parametric EWMA sign control chart. *Expert Sys. Appl.* 2011; 38(5), 6239-43.
- [10] Sukparungsee S. Robustness of Tukey's control chart in detecting a changes of parameter of skew distributions. *Int. J. Appl. Phys. Math.* 2012; 2(5): 379-82.
- [11] Khaliq QUA. Riaz M. Robust Tukey-CUSUM control chart for process monitoring. *Qual. Reliab. Eng. Int.* 2016; 32(3): 933-48.
- [12] Khaliq QUA. Riaz M. Ahmad S. On designing a new Tukey- EWMA control chart for process monitoring. *Int. J. Adv. Manuf. Tech.* 2016; 82: 1-23.
- [13] Riaz M. Khaliq Q. Gul, S. Mixed Tukey EWMA- CUSUM control chart and its applications. *Qual. Technol. Quant. Manag.* 2017; 14(4): 378–411.
- [14] Mongkoltawat P. Sukparungsee S. Areepong Y. Exponentially weighted moving average Tukey's control chart for moving rage and rang. *The Journal of KMUTNB.* 2017; 27(4): 1-12.
- [15] Abbasi A. Aslam M. Saghir A. Amixed non-parametric control chart for efficient process monitoring. *Int. J. Adv. Manuf. Tech.* 2018; 99(9-12): 2549-61.
- [16] Chakraborti S. Graham MA. Non-parametric (distribution-free) control charts: An updated overview and some results. *Qual. Eng.* 2019; 31(1): 523-44.
- [17] Mabude K. Malela-Majika JC. Castagliola P. Shongwe SC. Distribution-free mixed GWMA- CUSUM and CUSUM- GWMA Mann Whitney charts to monitor unknown shifts in the process location. *Commun. Stat. Simul. Comput.* 2022; 51(1): 6667-90.
- [18] Taboran R. Sukparungsee S. Areepong Y. A New Non-parametric Tukey MA- EWMA Control Charts for Detecting Mean Shifts. *IEEE ACCESS*, 2020; 207249-59.

- [19] Joeques S. Barbosa, EP. An improved attribute control chart for monitoring non-conforming proportion in high-quality processes. *Control Eng. Prac.* 2013; 21(4): 407–12.
- [20] Fatahi AA. Noorossana R. Dokouhaki P. Moghaddam B. F. Zero inflated Poisson EWMA control chart for monitoring rare health-related events. *J. of Mech. Med. Biol.* 2012; 12(4): 1250065.