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Development and Performance Evaluation of a Mixed DEWMA–MA Control Chart for Detecting Shifts in Standard Deviation

Suganya Phantu[1], Saowanit Sukparungsee[2, *]

[1] *Faculty of Science, Energy, and Environment, King Mongkut's University of Technology North Bangkok, Rayong 21120, Thailand*
[2] *Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok 10800, Thailand*

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ABSTRACT

This study introduces a Mixed Double Exponentially Weighted Moving Average–Moving Average (DEWMA–MA_S) control chart for monitoring process variability, with a focus on detecting shifts in standard deviation. By combining the long-term memory of the $DEWMA_S$ chart with the short-term smoothing of the MA_S chart, the proposed method improves sensitivity to small and moderate variability changes. Simulation results, evaluated using ARL, SRL, and MRL metrics, show that larger moving average windows enhance detection of moderate shifts, while smaller λ values are more effective for small shifts. Compared with conventional S, MA_S, and $DEWMA_S$ charts, the DEWMA–MA_S consistently demonstrates superior performance, particularly for moderate shifts, and remains competitive for large shifts. A soft-drink filling case study further validates its practical advantages, confirming the DEWMA–MA_S chart as a more effective and reliable tool for modern industrial variability monitoring.

Keywords: Double Exponentially Weighted Moving Average (DEWMA); Detection; Dispersion; Moving Average (MA) chart; Performance

1. Introduction

The development of statistical process control (SPC) has long been a cornerstone of quality improvement in industrial processes. The pioneering work of Shewhart introduced the Shewhart control chart [1], which was primarily effective in detecting large process shifts. Although simple and widely adopted, Shewhart charts

***Corresponding author:** saowanit@sci.kmutnb.ac.th
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have limited power in identifying small and moderate shifts, which are often more critical in modern high-precision manufacturing environments. To address this limitation, Roberts [2] proposed the exponentially weighted moving average (EWMA) chart, which incorporates exponentially decreasing weights on all past data, thereby providing a memory of historical information. This design improves sensitivity to persistent small-to-moderate shifts in process parameters compared to Shewhart charts.

Building on the EWMA concept, Shamma and Shamma [3] introduced the Double EWMA (DEWMA) chart, which applies two layers of exponential smoothing. This extension provides additional responsiveness to subtle process changes, especially when early detection of small shifts is critical. Later, studies reinforced the utility of DEWMA. For example, Mahmoud and Woodall [4] conducted the first comprehensive evaluation of DEWMA, comparing it against EWMA and Shewhart charts using average run length (ARL) analysis. Their findings demonstrated that DEWMA performs better for small-to-moderate shifts but that its added complexity may not always provide significant gains over EWMA. Similarly, Alkahtani [5] examined the robustness of DEWMA relative to EWMA under non-normal distributions, showing that DEWMA charts generally outperform EWMA when the data deviate from normality, thereby expanding their applicability in real-world industrial environments.

While early research concentrated primarily on monitoring the process mean, modern industrial practice increasingly emphasizes the importance of monitoring process variability as well [6]. Variability, typically represented by the standard deviation (SD), plays a pivotal role in ensuring process capability and product reliability. Even when the mean remains stable, an increase in SD can lead to higher defect rates, product inconsistency, or even equipment malfunctions. Thus, variability monitoring has been recognized as a crucial dimension of SPC [7] and [8]. Several researchers have proposed adaptations of memory-type charts for variability detection. For instance, Maravelakis [7] developed an EWMA chart specifically for standard deviation, offering enhanced sensitivity to small changes in dispersion compared to the traditional Shewhart S-chart. In parallel, variants of DEWMA, such as Max-DEWMA and sum-of-squares DEWMA, have been introduced to jointly monitor process mean and variance, thereby broadening the applicability of DEWMA-based frameworks.

In addition to these developments, hybrid and modified forms of DEWMA have emerged to address practical challenges. Adeoti [9] introduced the repetitive-sampling DEWMA (RS-DEWMA) chart, which demonstrated faster detection of small shifts. Supharakonsakun and Areepong [10] extended DEWMA to processes with autocorrelated data, providing theoretical ARL derivations and empirical validation with time-series data. Similarly, other researchers have proposed distribution-free DEWMA variants [11] and coefficient of variation (CV)-based DEWMA charts [12], thereby strengthening the robustness of DEWMA in both parametric and nonparametric settings. These innovations collectively illustrate DEWMA's evolution from a mean-monitoring chart into a versatile framework capable of addressing diverse quality monitoring needs.

On the other hand, the Moving Average (MA) chart is a window-type chart that uses recent observations to smooth short-term fluctuations [13]. This makes MA charts particularly effective at reducing the impact of random variation, but their limitation lies in their "short memory," as they only consider a fixed number of past samples. To overcome the individual shortcomings of both memory-type and window-type charts, recent studies have proposed mixed or hybrid charts. For example, Sukparungsee et al. [14] introduced the mixed EWMA–MA chart, demonstrating its superior sensitivity to small-to-moderate shifts compared to traditional charts. Later, Raza et al. [15] refined this design by incorporating covariance structures between overlapping MA terms, improving ARL precision and robustness under both symmetric and skewed distributions. Recently, Raza et al. [16] extended the concept to a nonparametric setting, proposing a distribution-free mixed EWMA–MA chart based on the sign statistic, validated with real-world data from a gas turbine system.

Despite these advances, the integration of DEWMA with MA for standard deviation monitoring has not yet been fully developed. Given DEWMA's strength in detecting small shifts through its long memory and MA's ability to smooth local fluctuations, combining the two approaches offers a promising pathway to more effective variability monitoring. A mixed DEWMA–MA chart for SD would be expected to provide heightened sensitivity to subtle changes in process variability, reduced influence of random noise, and robustness under nonnormal and autocorrelated conditions. This hybrid framework can therefore extend the utility of control charts in modern manufacturing and service industries, where both mean stability and variability control are critical to quality assurance and process reliability.

2. Materials and Methods

This section presents the specific characteristics of four types of control charts for detecting changes in the standard deviation.

Let σ represent the process standard deviation from a set of subgroups assumed to follow a normal distribution $N(\mu, \sigma^2)$ [16]. For a subgroup i, the sample standard deviation is denoted as S_i. Establishing appropriate control limits is essential to determine whether the process variability remains in control.

Since the true value of σ is usually unknown, it must be estimated from historical data. The unbiased estimator of the average sample standard deviation, $\bar{S}$ is calculated from an initial reference dataset of subgroup standard deviations as follows:

$$\hat{\sigma} = \bar{S}/c_4, \quad \bar{S} = \frac{1}{k}\sum_{i=1}^{k} S_i,$$

where m is the number of subgroups and c_4 is a correction constant that depends on the subgroup sample size k. The following control charts are applied in this research, which can be classified as follows.

2.1 Control chart for standard deviation (S chart)

The S chart is a Shewhart-type chart designed to monitor the process standard deviation [17]. Control limits are defined as

$$UCL/LCL = \hat{\sigma} \pm B_1 \hat{\sigma}\sqrt{1 - c_4^2}, \quad (2.1)$$

where B_1 is the factor of control limit of the S chart. A sample point plot that deviates from the control limit indicates instability in the process. c_4 bias correction factor depending on sample size k.

2.2 Moving average for standard deviation control chart (MA$_S$ chart)

The MA$_S$ chart was proposed to improve detection of moderate shifts in variability by smoothing consecutive standard deviation values [13]. The MA$_S$ statistic is given by

$$MA_i = \begin{cases} \frac{S_i+S_{i-1}+S_{i-2}+\cdots+S_1}{i} & ; i < \omega, \\ \frac{S_i+S_{i-1}+\cdots+S_{i-\omega+1}}{\omega} & ; i \geq \omega, \end{cases} \quad (2.2)$$

where ω is the moving average window size. Its expected value is

$$E(MA_i) = c_4\hat{\sigma}, \quad (2.3)$$

and its variance can be expressed as

$$Var(MA_i) = \begin{cases} \frac{\hat{\sigma}^2(1-c_4^2)}{i} & ; i < \omega, \\ \frac{\hat{\sigma}^2(1-c_4^2)}{\omega} & ; i \geq \omega, \end{cases} \quad (2.4)$$

yielding control limits:

$$\frac{UCL}{LCL} = \begin{cases} c_4\hat{\sigma} + B_2\sqrt{\frac{\hat{\sigma}^2(1-c_4^2)}{i}} & ; i < \omega, \\ c_4\hat{\sigma} - B_2\sqrt{\frac{\hat{\sigma}^2(1-c_4^2)}{i}} & ; i \geq \omega, \end{cases} \quad (2.5)$$

where B_2 is the factor of control limit of the MA$_S$ chart.

2.3 Double exponentially weighted moving average control chart (DEWMA$_S$ chart)

The DEWMA$_S$ chart is an extension of the EWMA chart, defined as [4]:

$$D_t = \lambda E_t + (1-\lambda)D_{t-1}, \quad (2.6)$$

where $E_t = \lambda S_t + (1-\lambda)D_{t-1}$ is statistic of EWMA. The expectation of D_t is

$$E(D_t) = E(E_t) = E(S_t) = c_4\hat{\sigma}, \quad (2.7)$$

and the variance is

$$V(D) = \hat{\sigma}^2\left[\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}\right](1-c_4^2), \quad (2.8)$$

and the control limits are

$$\frac{UCL}{LCL} = c_4\hat{\sigma} \pm B_3\hat{\sigma}\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)(1-c_4^2)}{(2-\lambda)^3}}, \quad (2.9)$$

where B_3 the control limit coefficient.

2.4 Double Exponentially weighted moving average-moving average control chart (DEWMA-MA$_S$ chart)

The proposed DEWMA–MA$_S$ chart integrates the DEWMA$_S$ and MA$_S$ approaches. Here, the MA$_S$ statistic MA$_t$ replaces Et in the DEWMA$_S$ formulation, yielding [18-21]:

$$DEM_t = \lambda MA_t + (1-\lambda)DEM_{t-1} \quad (2.10)$$

where λ is the smoothing parameter and the starting value DEM$_0$ is set equal to the process target. The expected value of the mixed statistic remains the same as the MA$_S$ chart, while its variance incorporates contributions from both DEWMA$_S$ and MA$_S$. The expectation of DEM$_t$ is

$$E(DEM_t) = E(MA_t) = E(S_t) = c_4\hat{\sigma}, \quad (2.11)$$

and the variance is

$$V(DEM_t) = \hat{\sigma}_{MA}^2\left[\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}\right]. \quad (2.12)$$

The control limits are expressed as:

$$\frac{UCL}{LCL} = \mu_{MA} \pm B_4\sigma_{MA}\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}, \quad (2.13)$$

where B_4 is the control limit factor.

It is important to note that the moving average (MA) statistic involves overlapping observations, which induces serial dependence. When this dependent statistic is used within the DEWMA recursion, the resulting DEM□ statistic also inherits serial correlation. Because a fully explicit variance expression accounting for this dependence is analytically complex, the control limits are calibrated via Monte Carlo simulation ($ARL_0 \approx 370$), thereby preserving the exact stochastic dependence structure in all reported ARL, SRL, and MRL performance measures.

The hybrid design is intended to leverage the long-term memory of $\mathrm{DEWMA_S}$ with the short-term smoothing of $\mathrm{MA_S}$, thereby enhancing sensitivity to small and moderate shifts in process variability. For each combination of (λ, ω, k), the constant B_4 is determined via a bisection search such that the simulated in-control average run length satisfies $|ARL_0 - 370| < 1$, using 200,000 Monte Carlo replications. Representative B_4 are reported in Tables 1–6 to facilitate practical implementation. Since B_4 on λ, ω, k through the steady-state variance of the DEWMA–$\mathrm{MA_S}$ statistic, interpolation may be used for intermediate parameter settings.

3. Performance Evaluation Methods

The performance of control charts is evaluated using run length–based measures, which quantify how quickly a chart signals a process change.

The Run Length (RL) is defined as the number of samples plotted before a signal occurs. Since RL is a random variable, its distribution is used to assess chart efficiency. The three most widely used metrics are:

3.1 Average Run Length (ARL)

The expected number of samples until a signal is triggered. ARL_0 is in-control ARL (false alarm rate). ARL_1 is out-of-control ARL (detection speed).

$$ARL = \sum_{i=1}^{T} RL_i / T. \tag{3.1}$$

In this case, the sample being examined before the process surpasses the control limits for the first time is indicated by RL_i. T, which is set to 200,000, is the number of experiment repetitions in the simulation during round i. The simulation procedure preserves the dependence structure induced by the MA window and DEWMA recursion, ensuring that performance measures reflect the true stochastic behavior of the mixed statistic.

3.2 Standard Deviation of Run Length (SRL)

Measures variability in detection times around the ARL. Lower SRL implies more consistent detection.

$$SDRL = \sqrt{E(RL_i)^2 - ARL^2}. \tag{3.2}$$

3.3 Median Run Length (MRL)

The 50th percentile of the run length distribution, representing the typical detection time.

$$MRL = Median(RL_i). \tag{3.3}$$

These performance indices together provide a robust framework for chart evaluation. While ARL remains the standard criterion for design, incorporating SRL and MRL ensures a more comprehensive understanding, especially when RL distributions are skewed [7].

In this study, ARL, SRL, and MRL are used to evaluate the proposed mixed

DEWMA–MA chart in detecting shifts in standard deviation, and results are compared against the traditional S chart, MA_S chart, and DEWMA chart.

4. Numerical Results

This section presents the empirical results of the study. The analysis is divided into three parts in order to highlight both the internal performance of the proposed chart and its comparative advantage over existing methods.

First, Section 4.1 evaluates the performance of the proposed DEWMA–MA_S chart under different parameter settings by examining the Average Run Length (ARL) across a range of shift sizes. The analysis varies both the moving average window size ω and the smoothing parameter (λ) for subgroup sample sizes of 5, 10, and 15, respectively.

Next, Section 4.2 compares the DEWMA–MA_S chart with conventional alternatives, namely the S chart, the MA_S chart, and the $DEWMA_S$ chart. The comparison is conducted using ARL, SDRL, and MRL to provide a comprehensive picture of detection speed, stability, and reliability. Finally, Section 4.3 provides an illustration using a simulated sample to demonstrate the practical operation of the proposed chart.

4.1 The performance of DEWMA-MA chart

The performance of the proposed DEWMA–MA_S control chart was evaluated using the average run length criterion under various parameter settings. Tables 1–6 present the comparison of ARL values obtained for different combinations of the moving average window size (ω) and the smoothing parameter (λ). The comparison was carried out under the constraint that the in-control ARL (ARL_0) was maintained at approximately 370, ensuring a fair basis for performance assessment across charts. Simulated data were generated from a normal distribution with a mean of 0 and a variance of 1. The control-limit constant B_4 for each parameter combination is determined according to the calibration procedure described in Section 2.4 to maintain ARL_0 = 370.

The results are organized into two main parts. Tables 1–3 investigate the effect of changing the subgroup sample size (k = 5, 10, 15) while fixing the smoothing parameter at $\lambda = 0.05$. In this part, the moving average window size (ω) is varied to examine its influence on the Average Run Length (ARL). Tables 4–6, on the other hand, examine the effect of varying the smoothing parameter λ (0.05, 0.25, 0.50) while keeping the moving average window size fixed at $\omega = 2$. The analysis is conducted for subgroup sample sizes of k = 5,10, and 15, respectively.

Table 1 presents the ARL values of the DEWMA–MA_S chart when the subgroup sample size is $k = 5$ and the smoothing parameter is fixed at $\lambda = 0.05$. The in-control ARL (ARL_0) is maintained at 370. The results show that when the process is stable (δ = 1.00), the chart performs as expected. As the shift size increases slightly (δ = 1.01–1.05), the ARL_1 decreases gradually but remains relatively large (200–340), indicating slower detection of very small changes. For moderate shifts (δ = 1.10–1.20), the ARL_1 values drop sharply (22–115), suggesting faster responsiveness. For large shifts ($\delta \geq 1.5$), the ARL approaches values near 1–5, reflecting almost immediate detection. Thus, the DEWMA–MA_S chart with $k = 5$ and $\lambda = 0.05$ is effective for moderate shifts but shows limitations in detecting very small shifts.

Table 1. Average run length of DEWMA-MA_S chart when ARL_0 = 370, ω = 5 and λ = 0.05.

Shift size (δ)	ω=2 B_4=1.752	ω=5 B_4=1.745	ω=10 B_4=1.712	ω=15 B_4=1.742
1.00	370.8	370.12	370.12	370.27
1.01	**340.56**	344.97	345.82	349.04
1.02	**306.07**	326.54	314.85	323.94
1.03	**275.27**	300.01	287.97	302.72
1.04	**241.09**	272.99	263.01	269.66
1.05	**207.44**	243.25	236.09	244.71
1.06	**177.58**	208.96	209.02	209.05
1.07	**153.21**	183.85	180.72	182.32
1.08	**123.71**	158.68	150.34	155.11
1.09	**106.32**	135.64	122.38	130.42
1.10	**90.54**	115.68	104.26	107.49
1.20	30.61	34.57	25.96	**22.29**
1.50	5.68	5.2	3.8	**3.46**
2.00	1.92	1.85	1.62	**1.53**
2.50	1.30	1.31	1.22	**1.2**

Bold is minimum ARL.

Table 2. Average run length of DEWMA-MA_S chart when ARL_0 = 370, ω = 10 and λ = 0.05.

Shift size (δ)	ω=2 B_4=1.752	ω=5 B_4=1.745	ω=10 B_4=1.712	ω=15 B_4=1.742
1.00	370.45	370.16	370.95	370.45
1.01	**335.98**	344.94	347.08	345.90
1.02	**293.74**	314.30	312.77	312.55
1.03	**252.93**	282.88	274.08	269.10
1.04	**201.28**	245.18	235.77	228.01
1.05	**158.81**	202.10	193.15	182.17
1.06	**121.97**	164.40	153.01	143.15
1.07	**98.52**	135.82	124.36	110.81
1.08	**79.86**	110.55	95.28	86.63
1.09	**64.18**	88.03	73.46	64.18
1.10	**53.43**	74.87	57.84	**49.91**
1.20	16.83	16.26	11.04	**9.11**
1.50	2.60	2.48	1.95	**1.80**
2.00	1.18	1.20	1.15	**1.12**
2.50	1.03	1.04	1.03	**1.02**

Bold is minimum ARL.

Table 2 considers a subgroup sample sizes (k = 10) while keeping λ = 0.05. The results demonstrate that increasing the subgroup sample sizes enhances detection performance, particularly for small and moderate shifts. For instance, at δ = 1.02–1.05, the ARL_1 values are lower (158–293) compared with Table 1 (207–306). At, δ = 1.10, the ARL_1 reduces further to 49–75, showing significant improvement in sensitivity. Large shifts ($\delta \geq 1.5$) are again detected almost instantly, with ARL close to 2. Overall, increasing the subgroup sample sizes from 5 to 10 provides a more balanced sensitivity, especially for moderate shifts, without affecting the in-control ARL.

Table 3 extends the analysis by setting the subgroup sample sizes to k = 15 with λ = 0.05. The results reveal further improvement in sensitivity compared to Tables 1 and 2. At, δ =1.02–1.05, ARL_1 values decline more steeply (116–277), indicating faster detection of small-to-moderate shifts. By δ =1.10, ARL_1 values are between 29–50, showing very efficient detection. At larger shifts ($\delta \geq 1.5$), the ARL_1 drops to values very close to 1, almost immediate detection. These findings confirm that increasing the subgroup sample sizes enhances chart performance, making k = 15 particularly effective in identifying both small and moderate shifts.

Table 3. Average run length of DEWMA-MA_S chart when ARL_0 = 370, ω = 15 and λ = 0.05.

Shift size (δ)	ω=2 B_4=1.752	ω=5 B_4=1.745	ω=10 B_4=1.712	ω=15 B_4=1.742
1.00	370.44	370.56	370.78	370.28
1.01	**323.59**	346.52	335.82	340.00
1.02	**277.05**	315.71	298.94	300.74
1.03	**208.75**	272.26	250.23	253.79
1.04	**155.77**	220.63	198.34	196.59
1.05	**116.11**	167.21	152.69	148.30
1.06	**86.67**	132.59	114.39	108.71
1.07	**73.5**	104.32	84.36	74.92
1.08	**59**	82.40	60.64	**54.84**
1.09	**47**	61.70	45.79	**39.89**
1.10	**37.05**	49.93	35.52	**29.85**
1.20	9.39	9.48	6.37	**5.18**
1.50	1.72	1.73	1.44	**1.38**
2.00	1.05	1.07	1.04	**1.03**
2.50	1.00	1.01	1.00	**1.00**

Bold is minimum ARL.

Examination of Tables 1–3 indicates that the moving average window size influences detection performance in a shift-dependent manner. For very small shifts ($\delta \leq 1.05$), smaller window sizes (e.g., ω = 2) consistently yield the lowest ARL values. However, for moderate shifts ($\delta \geq 1.20$), larger window sizes (e.g., ω = 10 or 15) tend to produce smaller ARL values, reflecting improved smoothing of short-term variability. This behavior illustrates the classical bias–variance trade-off inherent in smoothing-based control charts.

Table 4 explores the effect of varying λ (0.05, 0.25, 0.50) while holding the subgroup sample sizes constant at $k = 5$. The results indicate that the choice of λ influences detection capability. When λ = 0.05, the chart performs better for small shifts, with relatively lower ARL_1 values at δ = 1.02–1.05 compared to larger λ. However, as λ increases to 0.25 and 0.50, detection of moderate-to-large shifts becomes slightly faster, although small shift detection worsens. For example, at δ = 1.05, ARL_1 is 207 for λ = 0.05, but increases to 244 for λ = 0.50. This demonstrates a tradeoff: smaller λ favors sensitivity to small changes, whereas larger λ accelerates detection of larger shifts.

Table 4. Average run length of DEWMA-MA$_S$ chart when ARL_0 = 370, k = 5 and ω = 2.

Shift size	λ=0.05	λ=0.25	λ=0.50
(δ)	B_4=1.1671	B_4=1.693	B_4=1.752
1.00	370.8	370.12	370.8
1.01	**340.56**	341.19	344.47
1.02	**306.07**	312.75	319.58
1.03	**275.27**	285.99	296.06
1.04	**241.09**	256.15	270.29
1.05	**207.44**	223.25	244.21
1.06	**177.58**	196.55	220.65
1.07	**153.21**	171.87	194.36
1.08	**123.71**	143.92	166.25
1.09	**106.32**	121.83	148.23
1.10	**90.54**	106.31	128.37
1.20	**30.61**	31.16	38.94
1.50	5.68	**5.56**	5.99
2.00	1.92	**1.88**	2.14
2.50	1.30	**1.27**	1.41

Bold is minimum ARL.

Table 5 repeats the analysis for k = 10 with λ = 0.05, 0.25, and 0.50. The findings align with Table 4 but highlight the effect of increasing k. At, δ = 1.05, ARL = 158 (λ = 0.05), 176 (λ = 0.25), and 214 (λ = 0.50). At, δ = 1.10, ARL_1 = 53, 58, and 80, respectively. These results suggest that for moderate shifts, λ=0.05 continues to provide superior detection performance. Higher λ values reduce detection ability for small shifts, although the difference narrows for larger shifts ($\delta \geq 1.5$). Thus, for k = 10, a smaller λ remains preferable when the objective is early detection of small or moderate shifts.

Table 5. Average run length of DEWMA-MA$_S$ chart when ARL_0 = 370, k = 10 and ω = 2.

Shift size	λ=0.05	λ=0.25	λ=0.50
(δ)	B_4=1.1605	B_4=1.6772	B_4=1.7365
1.00	370.46	370.16	370.45
1.01	**335.98**	336.22	351.13
1.02	**293.74**	303.57	321.89
1.03	**252.93**	261.01	283.08
1.04	**201.28**	217.70	251.42
1.05	**158.81**	176.89	213.6
1.06	**121.97**	145.15	174.22
1.07	**98.52**	116.16	146.15
1.08	**79.86**	90.39	120.34
1.09	**64.18**	73.94	98.38
1.10	**53.43**	57.96	80.1
1.20	**16.83**	15.45	18.5
1.50	**2.60**	**2.79**	2.87
2.00	**1.18**	**1.24**	1.28
2.50	**1.03**	**1.05**	1.07

Bold is minimum ARL.

Table 6 examines the performance for the largest subgroup sample sizes, k = 15, with λ = 0.05, 0.25, and 0.50. The results confirm the earlier patterns: smaller λ yields the lowest ARL_1 for small-to-moderate shifts. At, δ = 1.05, ARL_1 is 116 (λ = 0.05), compared with 142 (λ = 0.25) and 173 (λ = 0.50). At, δ = 1.10, ARL increases from 37 (λ = 0.05) to 54 (λ = 0.50). For large shifts ($\delta \geq 1.5$), the ARL_1 values converge near 1–2, showing negligible differences among λ values. The results suggest that λ has little effect when shifts are large but plays a critical role in the detection of small-to-moderate shifts, where λ = 0.05 is consistently optimal.

Collectively, Tables 1–6 show a clear interaction between the moving average window size ω and the smoothing parameter λ. Smaller window sizes (e.g., ω = 2) generally yield faster detection for very

Table 6. Average run length of DEWMA-MA_S chart when ARL_0 = 370, k = 15 and ω = 2.

Shift size	λ=0.05	λ=0.25	λ=0.50
(δ)	B_4=1.5833	B_4=1.6658	B_4=1.715
1.00	370.44	370.28	370.44
1.01	**323.59**	336.72	338.89
1.02	**277.05**	292.87	304.13
1.03	**208.75**	236	262.28
1.04	**155.77**	183.40	216.16
1.05	**116.11**	141.96	172.71
1.06	**86.67**	104.55	138.77
1.07	**68.30**	78.49	108.01
1.08	**54.56**	60.92	84.02
1.09	**44.80**	48.38	65.59
1.10	**37.05**	37.98	53.78
1.20	**9.39**	9.56	11.38
1.50	**1.72**	**1.89**	1.98
2.00	**1.05**	**1.08**	1.09
2.50	**1.00**	**1.01**	1.01

Bold is minimum ARL.

small shifts, whereas larger window sizes (e.g., $\omega \geq 5$) enhance sensitivity to moderate shifts by providing stronger short-term smoothing. In addition, smaller smoothing parameters (e.g., λ = 0.05) tend to improve responsiveness to small-to-moderate shifts, while larger λ values may be preferable when shifts are large. Therefore, the parameter choice should be guided by the expected shift magnitude: ω = 2 with λ = 0.05 is recommended when early detection of small shifts is prioritized, whereas $\omega \geq 5$ can be selected to strengthen detection of moderate variability increases.

4.2 The performance of DEWMA-MA chart

To further evaluate the performance of the proposed DEWMA–MA_S control chart, its detection capability was compared with three established alternatives: S chart, the MA_S chart, and the $DEWMA_S$ chart. Tables 7–9 present this comparison under the condition that the in-control average run length (ARL_0) is maintained at approximately 370, ensuring a fair basis of evaluation. The results are reported in terms of ARL (Average Run Length), SDRL (Standard Deviation of Run Length), and MRL (Median Run Length), thereby capturing not only the expected detection time but also the variability and typicality of detection performance.

Table 7 reports a comparative analysis of the S chart, MA_S chart, $DEWMA_S$ chart, and DEWMA–MA_S chart under the conditions of subgroup size k = 5, smoothing parameter λ=0.05, and moving average window ω=2. The performance measures considered are ARL, SDRL, and MRL, with the in-control ARL (ARL_0) maintained at approximately 370 across all charts. The results indicate that all methods exhibit relatively slow detection for very small shifts (δ = 1.01–1.05), with ARLs exceeding 200. However, the DEWMA–MA_S chart demonstrates superior performance by consistently producing lower ARL, SDRL, and MRL values across both small and moderate shifts. For moderate shifts (δ = 1.10–1.20), the advantages of the DEWMA–MA_S chart become more pronounced, yielding faster detection and more stable run length distributions compared with the alternative control schemes.

Table 8 presents the results for an increased subgroup size of k = 10, with the smoothing parameter fixed at λ=0.05 and the moving average window set to ω= 2. The findings indicate that enlarging the subgroup size enhances the ability of all charts to detect moderate shifts. Nevertheless, the DEWMA–MA_S chart consistently demonstrates superior performance, achieving the lowest ARL, SDRL, and MRL values across nearly all shift magnitudes. In particular, for small-to-moderate shifts (δ = 1.05-1.10), its ARL_1 is substantially reduced compared with the S, MA_S, and $DEWMA_S$ charts, highlighting its enhanced sensitivity in the region where early

Table 7. The comparison of control chart when ω = 2, λ = 0.05, k = 5 and ARL_0 = 370.

Shift	S			MA_S			DEWMA_S			DEWMA-MA_S		
size	B_1=1.987			B_2=3.337			B_3=4.068			B_4=1.1671		
(δ)	ASL	SDRL	MRL	ASL	SDRL	MRL	ASL	SDRL	MRL	ASL	SDRL	MRL
1.00	370.52	166.89	369	370.01	176.82	369	370.23	168.55	369	370.8	168.8	369
1.01	**350.24**	173.63	357	350.96	182.42	352.5	347.13	172.58	353.5	**340.56**	**171.09**	**352**
1.02	**334.74**	177.37	343	332.78	185.91	343.5	323.02	176.15	340.5	**306.07**	**176.94**	**324**
1.03	**316.53**	180.02	316.5	311.43	189.07	314.5	292.36	180.3	293	**275.27**	**168.6**	**293**
1.04	**298.03**	179.17	298	288.06	189.43	298.5	262.3	176.78	253	**241.09**	**175.43**	**245.5**
1.05	**277.46**	179.45	258	259.51	186.56	248	233.68	171.15	214.5	**207.44**	**169.49**	**203.5**
1.06	**256.92**	175.09	224.5	235.46	183.52	201.5	204.72	163.69	171	**177.58**	**160.86**	**188.5**
1.07	**336.06**	169.46	191	214.8	177.82	164	181.28	149.05	158.5	**153.21**	**145.96**	**150**
1.08	**217.18**	162.66	172.5	190.54	166.07	141.5	154.48	134.89	134	**123.71**	**134.26**	**125**
1.09	**202.52**	158.78	156	170.86	157.15	122	132.33	120.02	115	**106.32**	**131.37**	**105**
1.10	**180.73**	150.46	136.5	156.39	148.26	111	115.65	107.63	91	**90.54**	**111.09**	**88**
1.20	**67.11**	66.39	46	44.37	47.72	30	35.97	32.71	35	**30.61**	**30.12**	**26**
1.50	**10.21**	**9.71**	7	5.98	6.44	5	6.68	4.6	5	**5.68**	**4.35**	**4**
2.00	**2.99**	**2.41**	2	**1.79**	**1.40**	**1**	2.89	1.41	3	1.92	1.51	2
2.50	**1.73**	**1.11**	1	**1.29**	**0.75**	**1**	2.03	0.86	2	1.30	0.79	1

Bold is a minimum of ARL1, SDRL and MRL.

Table 8. The comparison of control chart when ω = 2, λ = 0.05, k = 10 and ARL_0 = 370.

Shift	S			MA_S			DEWMA_S			DEWMA-MA_S		
size	B_1=1.6877			B_2=3.27			B_3=4.65			B_4=1.605		
(δ)	ASL	SDRL	MRL	ASL	SDRL	MRL	ASL	SDRL	MRL	ASL	SDRL	MRL
1.00	370.52	189.95	370	370.76	181.18	369	370.45	181.88	370	370.45	175.25	369
1.01	**341.88**	177.08	348.5	345.96	176.76	348	351.02	173.48	348.5	**335.98**	**168.36**	**348.5**
1.02	**320.96**	178.29	334	315.45	191.55	334.5	319.11	180.02	335.5	**293.74**	**162.94**	**323**
1.03	**294.1**	179.27	301	283.58	190.67	288.5	275.8	179.76	260.5	**252.93**	**152.36**	**260.5**
1.04	**268.54**	175.73	241.5	252.94	185.77	228	236.57	170.75	219	**201.28**	**145.78**	**195**
1.05	**239.46**	169.67	192.5	221.46	174.9	185	199.3	158.54	173	**158.81**	**134.23**	**155.5**
1.06	**213.41**	161.26	169.5	186.93	160	147	164.46	138.6	130	**121.97**	**127.95**	**124.5**
1.07	**186.05**	150.69	149.5	162.59	148.11	118	132.78	120.06	107	**98.52**	**121.14**	**99**
1.08	**167.46**	139.99	129	132.72	127.55	95	109.2	100.92	88.5	**79.86**	**100.37**	**82**
1.09	**148.36**	129.54	109.5	110.8	111.52	78.5	89.22	83.7	72	**64.18**	**82.28**	**65**
1.10	**129.13**	116.81	97	91.34	93.22	64	73.56	69.08	57	**53.43**	**58.29**	**52.5**
1.20	**38.42**	38.59	26	19.43	22.22	12	17.59	14.7	13	**16.83**	**13.79**	**13**
1.50	**4.62**	**3.87**	3.5	**2.55**	**2.13**	**1**	3.7	1.82	3	**2.60**	**2.17**	**2**
2.00	**1.54**	**0.88**	1	**1.15**	**0.5**	**1**	1.92	0.67	2	1.18	0.61	1
2.50	**1.14**	**0.38**	1	**1.02**	**0.17**	**1**	1.42	0.53	1	1.03	0.27	1

Bold is a minimum of ARL1, SDRL and MRL.

detection is crucial. Furthermore, the reduced SDRL and MRL values confirm the robustness of the DEWMA–MA_S chart, as it provides more stable and reliable detection outcomes than the competing control schemes.

Table 9 reports the results for the largest subgroup size, k = 15, with λ=0.05 and ω=2. This setting highlights the influence of a broader smoothing horizon on chart performance. Among the competing methods, the DEWMA–MA_S chart exhibits the strongest detection capability, particularly for small-to-moderate shifts (δ = 1.02–1.10). For instance, at δ =1.05, its ARL decreases to 116, markedly lower than those of the other charts, while at δ=1.10, detection is achieved within an average of 37 samples, surpassing both the $DEWMA_S$ and MA_S charts. Furthermore, the SDRL and MRL values substantiate the robustness of the DEWMA–MA_S chart, indicating not only faster detection but also reduced variability and greater consistency in median performance.

In summary, the results presented in Tables 7–9 indicate that the DEWMA–MA_S chart consistently outperforms the S, MA_S, and $DEWMA_S$ charts across varying subgroup sizes and parameter settings. Its advantages are most evident for moderate shifts, where early detection is of

Table 9. The comparison of control chart when $\omega = 2$, $\lambda = 0.05$, $k = 15$ and $ARL_0 = 370$.

Shift	S			MA_S			DEWMA_S			DEWMA-MA_S		
size	B_1=1.5655			B_2=3.235			B_3=4.035			B_4=1.715		
(δ)	ASL	SDRL	MRL	ASL	SDRL	MRL	ASL	SDRL	MRL	ASL	SDRL	MRL
1.00	370.58	179.76	370	370.56	179.88	369	370.58	179.91	369	370.44	179.67	369
1.01	**345.17**	176.63	368	345.73	175.38	368.5	342.99	176.54	367.5	**323.59**	**172.56**	**367**
1.02	**316.79**	175.15	350.5	311.39	173.05	351	304.69	172.24	341	**277.05**	**171.82**	**338.5**
1.03	**288.31**	172.04	335	276.12	172.06	336.5	257.71	171.23	235.5	**208.75**	**169.77**	**235**
1.04	**252.9**	168.29	226.5	233.25	168.62	200	207.27	167.49	175	**155.77**	**167.4**	**168**
1.05	**225.4**	150.55	177.5	187.15	150.87	147.5	158.9	149.43	126.5	**116.11**	**148.41**	**120**
1.06	**196.55**	127.55	144.5	151.06	126.54	104	126.73	126.71	101	**86.67**	**126.18**	**91**
1.07	**172.51**	107.24	123.5	120.73	103.84	80	96.93	104.85	68.5	**73.5**	**103.21**	**68.5**
1.08	**145.72**	91.59	102.5	94.97	91.96	60	75.06	91.36	54.5	**59**	**82.64**	**54.5**
1.09	**122.94**	66.68	88	77.3	66.72	50	58.78	64.85	42	**47**	**63.85**	**44.5**
1.10	**104.54**	53.35	72	62.18	57.43	41	47.56	55.55	38	**37.05**	**52.38**	**36**
1.20	**26.16**	16.69	18	11.66	12.62	8	11.16	18.73	8	**9.39**	**10.25**	**8**
1.50	**2.96**	**2.45**	2	**1.69**	**1.07**	**1**	2.77	1.09	2	**1.72**	**1.34**	**1**
2.00	**1.18**	**0.45**	1	**1.04**	**0.22**	**1**	1.57	0.55	2	1.05	0.31	1
2.50	**1.03**	**0.17**	1	**1**	**0.03**	**1**	1.16	0.37	1	1.00	0.13	1

Bold is a minimum of ARL1, SDRL and MRL.

critical importance, while for large shifts all charts demonstrate convergence toward near-immediate detection. The consistent improvements in ARL, SDRL, and MRL achieved by the DEWMA–MA_S chart highlight its combined strengths in detection efficiency and stability, thereby confirming the effectiveness of the proposed hybrid design as a superior monitoring tool relative to traditional alternatives.

4.3 Illustration example

To demonstrate the performance of the proposed Mixed DEWMA–MA_S chart, its results are compared with three established charts: S chart, MA_S chart, and $DEWMA_S$ chart. The comparison is conducted using a real-world–inspired case study of soft-drink beverage filling, where maintaining consistency in bottle fill volume is a critical quality requirement [17].

To assess the performance more quantitatively, the detection time for each chart was analyzed by measuring how many samples it took to signal a shift. The detection delay for the DEWMA–MA_S chart, compared to the traditional charts, was found to be significantly lower for moderate shifts but remained slower for very small shifts. The fill volume of beverage bottles is measured by placing a gauge over the crown and comparing the liquid level in the bottle neck against a coded scale. On this scale, a reading of zero indicates the target fill height. Deviations in fill level are therefore reflective of process variability, with excessive variation leading to underfilled or overfilled bottles, both of which are unacceptable in practice.

For illustration, fifteen samples of subgroup size $k = 15$ were analyzed. The goal is to detect shifts in the process standard deviation using four control charts:

1) S Chart.
2) MA_S Chart with window size $\omega = 3$.
3) $DEWMA_S$ Chart with smoothing parameter $\lambda = 0.05$.
4) Mixed DEWMA–MA_S chart combining $DEWMA_S$ Charts with MA_S ($\omega = 3$, $\lambda = 0.05$).

Fig. 1 illustrates the detection performance of the S control chart in monitoring changes in process standard deviation. The results indicate that the S chart fails to detect such changes, as all observations remain within the control limits.

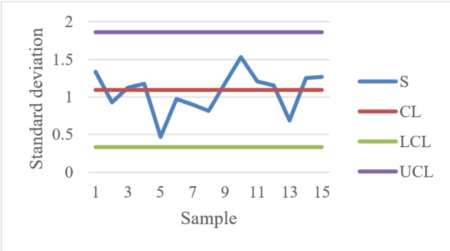

Fig. 1. S chart.

Fig. 2 presents the performance of the MA_S control chart, which incorporates a moving average of the data to enhance detection capability. However, the results similarly demonstrate that the MA_S chart does not identify shifts in the standard deviation, with no out-of-control signals observed.

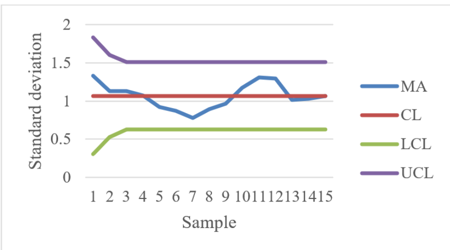

Fig. 2. MA chart.

Fig. 3 depicts the performance of the $DEWMA_S$ control chart, which incorporates double exponential weighting of both past and current data to enhance sensitivity to shifts in the standard deviation. Despite this extended memory property, the results show that all plotted points remain within the control limits, and no out-of-control signals are detected.

Fig. 4 presents the performance of the mixed DEWMA–MA_S control chart, which combines double exponential weighting of past and current observations with a moving average scheme to improve the detection of shifts in standard deviation. In contrast to the preceding charts, the DEWMA–MA_S chart successfully detects changes in process variability, as evidenced by out-of-control signals at samples 8 and 9 that exceed the control limits.

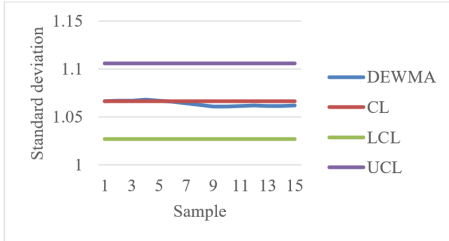

Fig. 3. DEWMA chart.

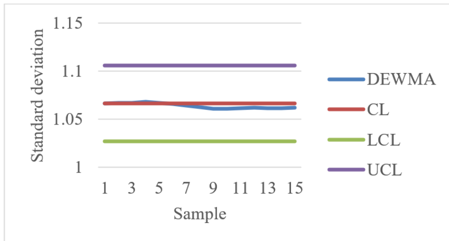

Fig. 4. DEWMA-MA chart.

The soft-drink filling case study (Figs. 1–4) serves as an illustrative example to demonstrate the practical operation of the DEWMA–MA_S chart, showcasing its sensitivity to moderate shifts in process variability. This example is provided to give a real-world context but is not a comprehensive validation, as no specific change points were predefined in the analysis.

The results presented in Figs. 1–4 reveal distinct differences in the detection capabilities of the four control charts for monitoring changes in process standard deviation. The S chart (Fig. 1), MA_S chart (Fig. 2), and $DEWMA_S$ chart (Fig. 3) failed to detect variability shifts, as all plotted statistics remained within the control limits and no out-of-control signals were observed. In contrast, the mixed DEWMA–MA_S chart (Fig. 4) successfully identified changes in process variability, with signals occurring at samples 8 and 9. These findings demonstrate that the DEWMA–MA_S chart exhibits superior sensitivity to moderate shifts in standard deviation relative to the traditional S, MA_S, and $DEWMA_S$ charts.

5. Conclusions and Discussion

The findings of this study underscore the advantages of the proposed DEWMA–MA_S control chart relative to conventional approaches for monitoring process variability. The simulation results (Tables 1–6) demonstrate that detection performance is jointly influenced by the moving average window size and the smoothing parameter. Specifically, the moving average window size ω controls a trade-off between early detection of small shifts and enhanced sensitivity to moderate shifts: smaller ω (e.g., $\omega = 2$) tends to react faster to very small changes, whereas larger ω (e.g., ω) provides stronger smoothing that improves detection for moderate increases in standard deviation. Consistent with memory-type chart behavior, smaller λ values (e.g., $\lambda = 0.05$) generally offer improved sensitivity to small-to-moderate shifts without sacrificing in-control performance. This observation is consistent with prior research on EWMA and DEWMA charts, which emphasizes the critical role of parameter selection in achieving an appropriate balance between detection speed and false alarm control [4, 5].

The comparative results presented in Tables 7–9 further reinforce the superiority of the DEWMA–MA_S chart. Whereas the S and MA_S charts demonstrate effectiveness primarily for large shifts, and the $DEWMA_S$ chart shows improved detection for small-to-moderate shifts, the DEWMA–MA_S hybrid consistently achieves the lowest ARL, SDRL, and MRL values across a broad range of shift magnitudes.

This consistent performance indicates that the DEWMA–MA_S chart not only facilitates faster detection but also ensures greater stability, thereby enhancing its practical reliability as a variability monitoring tool.

The illustrative soft-drink filling case study (Figs. 1–4) further substantiates these findings. The S, MA_S, and $DEWMA_S$ charts failed to signal variability shifts, with no out-of-control points detected within the control limits. In contrast, the DEWMA–MA_S chart signaled process changes at samples 8 and 9, thereby demonstrating its practical effectiveness in identifying moderate shifts in variability under real-world conditions. Such timely detection is particularly critical in industrial applications, where even modest increases in process standard deviation can lead to substantial quality losses and elevated production costs. Despite the promising performance of the DEWMA–MA_S chart, several limitations warrant consideration. First, the case study, while illustrative of the DEWMA–MA_S chart's operational capability, has several constraints.

It focuses only on univariate process monitoring, does not include a known predefined change point, and assumes normally distributed process data. In real-world industrial settings, process data may exhibit skewness, heavy tails, or autocorrelation, which could affect detection performance. Therefore, additional validation with larger datasets and predefined shifts is needed to fully assess the method's robustness. Second, the present study focused exclusively on univariate variability monitoring, whereas modern manufacturing systems often involve multivariate processes that may require more sophisticated extensions of the proposed method. Third, the selection of parameters (λ and w) was determined based on simulation settings, and the optimal configuration may vary depending on specific industry contexts and process dynamics.

Finally, the empirical validation was restricted to simulated and illustrative datasets; additional testing with large-scale industrial data would be necessary to confirm the practical robustness and generalizability of the DEWMA–MA_S chart.

This method demonstrates clear improvements in detecting moderate shifts in process variability, as shown by the case study. The DEWMA–MA_S chart demonstrates clear improvements in detecting moderate shifts in process variability, as shown by the case study. However, the case study is illustrative, and additional testing with larger datasets and predefined change points would be necessary to confirm its full applicability and performance in real-world scenarios.

Future studies may address these limitations by extending the DEWMA–MA_S framework to non-normal and autocorrelated processes, where robust or distribution-free approaches could further enhance performance. Another promising direction is the development of multivariate DEWMA–MA_S charts for monitoring variability across correlated quality characteristics. Incorporating adaptive parameter selection methods or optimization techniques (e.g., genetic algorithms, Bayesian updating) could make the chart more flexible for real-time monitoring. Finally, applying and validating the DEWMA–MA_S chart in diverse industrial case studies, such as pharmaceuticals, electronics, or energy systems would provide deeper insights into its effectiveness and encourage broader adoption in practice.

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