



Confidence Interval Methods for Parameter Estimation of the Odoma Distribution with Engineering Application

Panicha Kaskasamkul¹, Wararit Panichkitkosolkul^{2,3,*}

¹*Department of Mathematics, Faculty of Science, Naresuan University, Phitsanulok 65000, Thailand*

²*Department of Mathematics and Statistics, Faculty of Science and Technology, Thammasat University, Phatum Thani 12120, Thailand*

³*Thammasat University Research Unit in Mathematical Sciences and Applications, Phatum Thani 12120, Thailand*

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ABSTRACT

Four confidence intervals (CIs) for parameter estimation in the Odoma distribution, which is used in lifetime data analysis, were developed and evaluated in this study. These included the likelihood-based, Wald-type, bootstrap-t, and bias-corrected and accelerated (BCa) bootstrap methods. We used a Monte Carlo simulation study and a real data set to compare the CIs. The criteria in CI comparison are based on their empirical coverage probability (ECP) and average length (AL) in various situations. The explicit formula of the Wald-type CI was derived in this study. The simulation results indicated that the ECPs of the likelihood-based and Wald-type CIs approached the nominal confidence level of 0.95 in almost all cases. In cases of small sample sizes ($n = 10, 20$, or 30), the bootstrap-t and BCa bootstrap CIs yielded ECPs below 0.95. With the increase of sample sizes, the ECPs of the bootstrap-t and BCa bootstrap CIs converged to the nominal confidence level, while the parameter values also influenced the ECP. For low parameter values, the ECPs of the likelihood-based and Wald-type CIs approached the nominal confidence level of 0.95. On the other hand, the ECPs for the bootstrap-t and BCa bootstrap CIs showed low coverage at high parameter values and small sample sizes. The performance of these proposed CIs has been confirmed by their application to the strength data of aircraft window glass, with results corresponding with those from the simulation study.

Keywords: Bootstrap method; Interval estimation; Likelihood function; Lifetime distribution; Wald-type method

1. Introduction

Lifetime data analysis is the method of estimating the time until a specific event occurs, such as claim occurrence, system breakdown, or component replacement [1]. Lifetime data are typically non-negative and often exhibit right-skewed behavior like the Weibull, exponential, gamma, log-normal, or Lindley. This skewness arises because failure times tend to cluster near the origin with a long tail extending toward larger values. Such characteristics cannot be adequately captured by symmetric distributions.

The exponential and Lindley distributions commonly are constrained by their inflexibility and varying hazard rates. Moreover, the exponential distribution assumes a constant hazard rate and exhibits a memoryless property [2], which is often unrealistic in practical reliability data. The Lindley distribution is better than a model based on exponential distribution [3]. In contrast, the lognormal distribution allows for skewness but lacks a simple hazard structure and may not adequately capture early-life failures.

However, these three distributions are less effective in modeling data with non-constant hazard rates or more complicated failure time distributions. Therefore, development of new, flexible probability distributions is crucial to enhance data fitting accuracy.

Several methods have been employed to introduce new probability distributions or improve the performance of classical probability distributions. Some methods for constructing probability distributions use additional factors which improve flexibility, such as the two-parameter Lindley distribution [4], the three-parameter Lindley distribution [5], the exponentiated Shanker distribution [6], and the two-parameter Shanker distribution [7]. However, a more complex distribution with several parameters is harder to comprehend, even though the flexibility of the probability distributions is increased. This can result in overfitting, necessitating additional

data to achieve precise results and complicating the process of identifying the appropriate values. Adding several parameters also increases the computational effort necessary for practical applications.

Many authors have proposed new mixture probability distributions which do not require additional parameters including Shanker [8], Aradhana [9], Gharaibeh [10], Iwueze [11], Juchez [12], and Ola [13]. The mixture probability distributions are particularly useful when the underlying data are generated from heterogeneous populations or multiple failure mechanisms. In such cases, a classical distribution may fail to adequately capture the complexity of the data. For example, in engineering applications, failures may arise from different sources such as material defects, fatigue, or environmental factors. A mixture distribution can model these multiple sources more effectively than a single-component model.

Odom and Ijomah [14] introduced the Odoma distribution as a mixture of exponential and gamma distributions, which naturally accommodates right-skewed data. The exponential component captures early failures, while the gamma components provide additional flexibility to model longer lifetimes and varying hazard rates. Therefore, the Odoma distribution is theoretically appropriate for modeling lifetime data characterized by right-skewness and heterogeneous failure mechanisms. Furthermore, the Odoma distribution overcomes these limitations by combining exponential and gamma components, allowing it to model both early failures and long-term survival behavior within a unified framework.

The confidence interval (CI) is an important tool for estimating the distribution parameter in statistical inference. However, a review of the current literature found no studies to construct CIs for the parameter of the Odoma distribution. Therefore, this study focuses on the estimation of CIs for the parameter of the Odoma distribution using the likelihood-based CI, Wald-type CI, bootstrap-

t CI, and bias-corrected and accelerated (BCa) bootstrap CI. Monte Carlo simulation experiments were also conducted, and a real dataset was analyzed to evaluate the performances of these four CI estimation methods.

2. Methods

This section describes the Odoma distribution, point parameter estimation, and CIs.

2.1 The Odoma distribution and point parameter estimation

The Odoma distribution is derived by combining the exponential and two gamma distributions using appropriate mixing probabilities. The gamma distribution has a fixed scale parameter θ and shape parameters 3 and 5. Let X be a random variable following the Odoma distribution with parameter θ . The probability density function (pdf) of the Odoma distribution, denoted by $f(x; \theta)$, can be derived using a mixture distribution with three components and their respective mixing probabilities as in Eq. (2.1):

$$f(x; \theta) = p_1 f_{\text{Exp}}(x; \theta) + p_2 f_{\text{Gam}}(x; \theta, 3) + p_3 f_{\text{Gam}}(x; \theta, 5), \quad (2.1)$$

where $f_{\text{Exp}}(x; \theta)$ is the pdf of the exponential distribution with rate parameter θ , and $f_{\text{Gam}}(x; \theta, 3)$ and $f_{\text{Gam}}(x; \theta, 5)$ are the pdfs of the gamma distribution with shape parameters 3 and 5, respectively, and rate parameter θ . The mixing proportions, p_1, p_2 and p_3 ($p_1 + p_2 + p_3 = 1$), are given by

$$p_1 = \frac{\theta^5}{\theta^5 + \theta^3 + 24}, \quad p_2 = \frac{\theta^3}{\theta^5 + \theta^3 + 24},$$

and

$$p_3 = \frac{24}{\theta^5 + \theta^3 + 24}.$$

The pdf of the Odoma distribution is given in Eq. (2.2):

$$f(x; \theta) = \frac{\theta^5}{2(\theta^5 + \theta^3 + 24)} (2x^4 + \theta x^2 + 2\theta) e^{-\theta x}, \quad (2.2)$$

where $x > 0, \theta > 0$. The pdf plot of the Odoma distribution for several parameter values is shown in Fig. 1.

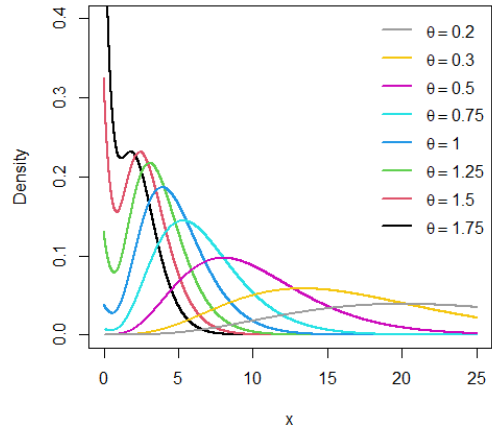


Fig. 1. The pdf plot of the Odoma distribution for several parameter values.

The mean ($\mu_X = E(X)$), the r^{th} non-central moment (μ'_r), and the moment generating function of X can be represented as:

$$E(X) = \frac{\theta^5 + 3\theta^3 + 120}{\theta(\theta^5 + \theta^3 + 24)},$$

$$\mu'_r = \left(\frac{2(r+4)! + \theta^3(r+2)! + 2\theta^5 r!}{2\theta^r(\theta^5 + \theta^3 + 24)} \right),$$

where $r = 1, 2, 3, \dots$ and

$$M_X(t) = \frac{\theta^5}{2(\theta^5 + \theta^3 + 24)} \left(\frac{24}{(\theta - t)^5} + \frac{\theta}{(\theta - t)^3} + \frac{\theta}{(\theta - t)} \right),$$

where $t < \theta$.

2.2 Point parameter estimation

The point parameter estimates for θ of the Odoma distribution can be obtained by the maximum likelihood (ML) method by the following four steps:

Step 1: Define the likelihood function

The likelihood function $L(\theta; x_i)$ is a fundamental concept in statistics used to estimate unknown parameters of a probability distribution. It is the joint probability of observing the random sample from the Odoma distribution, which is defined as in Eq. (2.3):

$$L(\theta; x_i) = \left(\frac{\theta^5}{2(\theta^5 + \theta^3 + 24)} \right)^n \cdot \prod_{i=1}^n (2x_i^4 + \theta x_i^2 + 2\theta) \cdot e^{-\theta \sum_{i=1}^n x_i}. \quad (2.3)$$

Step 2: Find the log-likelihood function

We find the log-likelihood function because the likelihood function is difficult to differentiate. It is the natural logarithm of the likelihood function, which is defined as in Eq. (2.4):

$$\log L(\theta; x_i) = n \log \left(\frac{\theta^5}{2(\theta^5 + \theta^3 + 24)} \right) + \sum_{i=1}^n \log(2x_i^4 + \theta x_i^2 + 2\theta) - \theta \sum_{i=1}^n x_i, \quad (2.4)$$

where $\log L(\theta; x_i)$ denotes the log-likelihood function.

Step 3: Differentiate the log-likelihood function

We take the derivative of $\log L(\theta; x_i)$ with respect to θ to find the value of θ that maximizes the log-likelihood function. The score function $S(\theta; x_i)$ are obtained in Eq. (2.5):

$$S(\theta; x_i) = \frac{\partial}{\partial \theta} \log L(\theta; x_i)$$

$$= \frac{5n}{\theta} - \frac{n(5\theta^4 + 3\theta^2)}{\theta^5 + \theta^3 + 24} + \sum_{i=1}^n \frac{x_i^2 + 2}{2x_i^4 + \theta x_i^2 + 2\theta} - \sum_{i=1}^n x_i. \quad (2.5)$$

Step 4: Set the derivative equal to zero and solve for the ML estimator

Solve for θ by setting the score function to zero: $S(\theta; x_i) \stackrel{\text{set}}{=} 0$. This gives the critical points which could be potential maximum points for θ . Since an exact mathematical expression for the ML estimator of the parameter θ is unavailable, numerical iteration methods are used to solve the related non-linear problem [15]. This study used ML estimation with the Newton-Raphson method, as executed in the maxLik package [16] within an R statistical computing environment.

2.3 Confidence intervals

This section provides the details of the proposed CIs: the likelihood-based, Wald-type, bootstrap-t, and BCa bootstrap CIs.

2.3.1 Likelihood-based confidence interval

The likelihood-based CI is one of the parameter interval estimations. This estimation considers the likelihood function, which quantifies the probability of observing a given dataset under a specified statistical distribution and its associated parameters. The fundamental concept of this method is to identify the range of parameter values within which the likelihood function achieves its maximum value while maintaining the nominal confidence level. The log-likelihood function is then optimized with respect to the parameter of interest.

The ML estimator of θ is obtained after solving $S(\theta; x_i) \stackrel{\text{set}}{=} 0$; this estimate is the most likely given the observed data. The likelihood-based CI is then constructed around the ML estimator. The likelihood-based method begins by defining a likelihood ratio $\lambda(\theta)$ as $\lambda(\theta) = L(\theta; x) / L(\hat{\theta}; x)$. Under regular

conditions, Wilks' theorem states that $-2\log \lambda(\theta)$ follows a chi-square distribution, where the degrees of freedom are equal to the number of parameters being estimated. Therefore, the likelihood-based CI for θ at $(1-\alpha)100\%$ confidence level is given by

$$\left\{ \theta \left| -2 \log \frac{L(\theta; x)}{L(\hat{\theta}; x)} \leq \chi_{1-\alpha, 1}^2 \right. \right\}$$

$$\left\{ \theta \left| -2 \log \left[\frac{\frac{\theta^{5n} (\hat{\theta}^5 + \hat{\theta}^3 + 24)^n}{\hat{\theta}^{5n} (\theta^5 + \theta^3 + 24)^n} \cdot \frac{\prod_{i=1}^n (2x_i^4 + \theta x_i^2 + 2\theta)}{\prod_{i=1}^n (2x_i^4 + \hat{\theta} x_i^2 + 2\hat{\theta})} \cdot \exp \left(-\theta \sum_{i=1}^n x_i + \hat{\theta} \sum_{i=1}^n x_i \right) \right] \leq \chi_{1-\alpha, 1}^2 \right. \right\},$$

where $\chi_{1-\alpha, 1}^2$ is $(1-\alpha)$ quantile of the chi-square distribution with one degree of freedom [17]. When the Odoma distribution is considered, the likelihood ratio test (LRT) becomes more complicated due to its composite structure. To address this issue, the numerical optimization methods are used to estimate the likelihood-based CI.

The plot of $-2\log \lambda(\theta)$ versus θ (solid blue line), $\chi_{0.95, 1}^2$ (dashed red line), and 95% likelihood-based CI (solid green line) for a random sample size of 20 from the Odoma distribution with $\theta = 1$ is shown in Fig. 2.

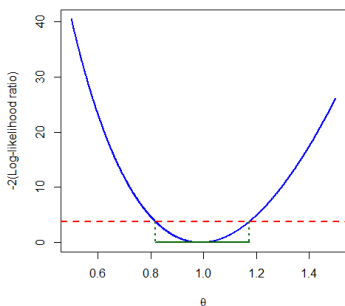


Fig. 2. The plot of $-2\log \lambda(\theta)$ versus θ .

2.3.2 Wald-type confidence interval

The Wald-type CI for the parameter of the Odoma distribution relies on the parameter's ML estimate. It is constructed using a quadratic approximation of the log-likelihood function, denoted as $L(\theta; x)$, which can be further extended using a Taylor series around $\hat{\theta}$. The Wald statistic approximates the log-likelihood ratio by expanding the second-order component around the ML estimate, with the first-order term being zero:

$$\log L(\theta; x_i) \approx \log L(\hat{\theta}; x_i) + (\theta - \hat{\theta}) \frac{\partial}{\partial \theta} \log L(\theta; x_i) \Big|_{\theta=\hat{\theta}}$$

$$+ \frac{1}{2} (\theta - \hat{\theta})^2 \frac{\partial^2}{\partial \theta^2} \log L(\theta; x_i) \Big|_{\theta=\hat{\theta}},$$

$$\log \frac{L(\theta; x_i)}{L(\hat{\theta}; x_i)} \approx \frac{1}{2} (\theta - \hat{\theta})^2 \frac{\partial^2}{\partial \theta^2} \log L(\theta; x_i) \Big|_{\theta=\hat{\theta}},$$

$$-2 \log \frac{L(\theta; x_i)}{L(\hat{\theta}; x_i)} \approx (\theta - \hat{\theta})^2 I(\hat{\theta}),$$

where $I(\hat{\theta})$ is the observed Fisher information. When the sample size is sufficiently large for the asymptotic properties to hold, the Wald statistic can be used to approximate the LRT statistic, resulting in a quadratic approximation of the log-likelihood ratio [18]. The first and second derivatives of the log-likelihood function for the Odoma distribution are as follows:

$$\frac{\partial}{\partial \theta} \log L(\theta; x_i) = \frac{5n}{\theta} - \frac{n(5\theta^4 + 3\theta^2)}{\theta^5 + \theta^3 + 24}$$

$$+ \sum_{i=1}^n \left(\frac{x_i^2 + 2}{2x_i^4 + \theta x_i^2 + 2\theta} \right) - \sum_{i=1}^n x_i,$$

$$\frac{\partial^2}{\partial \theta^2} \log L(\theta; x_i) = -\frac{5n}{\theta^2} - \frac{n(20\theta^3 + 6\theta)}{(\theta^5 + \theta^3 + 24)^2}$$

$$+ \frac{n(5\theta^4 + 3\theta^2)^2}{(\theta^5 + \theta^3 + 24)^2} - \sum_{i=1}^n \left[\frac{(x_i^2 + 2)^2}{(2x_i^4 + \theta x_i^2 + 2\theta)^2} \right]$$

Therefore, the estimated Fisher information is as follows in Eq. (2.6):

$$I(\hat{\theta}) = -\frac{\partial^2}{\partial \theta^2} \log L(\theta; x_i) | \hat{\theta}$$

$$= \frac{5n}{\hat{\theta}^2} + \frac{n(20\hat{\theta}^3 + 6\hat{\theta})}{(\hat{\theta}^5 + \hat{\theta}^3 + 24)} - \frac{n(5\hat{\theta}^4 + 3\hat{\theta}^2)^2}{(\hat{\theta}^5 + \hat{\theta}^3 + 24)^2}$$

$$+ \sum_{i=1}^n \left[\frac{x_i^4 + 4x_i^2 + 4}{(4x_i^8 + 4\hat{\theta}x_i^6 + (8\hat{\theta} + \hat{\theta}^2)x_i^4 + 4\hat{\theta}^2x_i^2 + 4\hat{\theta}^2)} \right]. \quad (2.6)$$

The Wald-type CI for θ at a confidence level of $(1-\alpha)100\%$ is given in Eq. (2.7):

$$\left(\hat{\theta} - z_{1-(\alpha/2)} \sqrt{I^{-1}(\hat{\theta})}, \hat{\theta} + z_{1-(\alpha/2)} \sqrt{I^{-1}(\hat{\theta})} \right), \quad (2.7)$$

where $z_{1-(\alpha/2)}$ denotes the $(1-(\alpha/2))^{\text{th}}$ quantile of the standard normal distribution, and $I^{-1}(\hat{\theta})$ is the inverse function of $I(\hat{\theta})$ given in (6) by replacing with the bootstrap estimator $\hat{\theta}^*$.

2.3.3 Bootstrap-t confidence interval

The bootstrap-t CI estimates the CI for a parameter by combining the principles of the bootstrap method and the student's t-distribution. This method also considers the standard error of the estimator. The bootstrap-t CI improves the accuracy and reliability of the percentile bootstrap method, especially in cases where the sample size is small or the estimators do not follow a normal distribution [19]. The procedure for constructing a bootstrap-t CI is as follows:

1. The procedure begins by selecting a random sample $X_1, \dots, X_n$ from the population. The parameter of interest is estimated.

2. Generate B bootstrap samples, $X_1^*, \dots, X_n^*$, by randomly sampling with replacement from the original dataset.

3. For each bootstrap sample, calculate the bootstrap replicate of the estimator, denoted as $\hat{\theta}^*$.

4. Construct the bootstrap-t statistic for each replicate as follows:

$$t^*(X, \hat{\theta}, \hat{\theta}^*) = \frac{\hat{\theta}^* - \hat{\theta}}{\sqrt{I^{-1}(\hat{\theta}^*)}},$$

where $I^{-1}(\hat{\theta}^*)$ is the inverse function of $I(\hat{\theta}^*)$ given in (6) by replacing with the bootstrap estimator $\hat{\theta}^*$.

5. Repeating this procedure B times results in an empirical distribution of the estimator, which is used to approximate the distribution of the pivotal quantity.

6. Construct the empirical distribution of the bootstrap-t statistics using the set of B bootstrap replicates.

7. Find the quantile values, $t_{(\alpha/2)}^*$ and $t_{(1-(\alpha/2))}^*$, which correspond to the $(\alpha/2)^{\text{th}}$ and $(1-(\alpha/2))^{\text{th}}$ quantiles of the empirical bootstrap-t distribution,

$$\frac{\#(t^*(X, \hat{\theta}, \hat{\theta}^*) \leq t_{(\alpha/2)}^*)}{B} = \alpha/2$$

and $\frac{\#(t^*(X, \hat{\theta}, \hat{\theta}^*) \leq t_{(1-(\alpha/2))}^*)}{B} = 1 - (\alpha/2),$

where $\#(\cdot)$ denotes the number of times the condition is true.

8. The bootstrap-t CI can be represented in Eq. (2.8):

$$\left(\hat{\theta} + t_{(\alpha/2)}^* \sqrt{I^{-1}(\hat{\theta})}, \hat{\theta} + t_{(1-(\alpha/2))}^* \sqrt{I^{-1}(\hat{\theta})} \right). \quad (2.8)$$

2.3.4 Bias-corrected and accelerated (BCa) bootstrap confidence interval

The BCa bootstrap CI is the method that improves on the basic bootstrap method by

addressing bias and skewness in the bootstrap distribution [20]. This method is conducted by first calculating a bias correction factor, which adjusts for the proportion of bootstrap estimates below the observed statistic, and then incorporating an acceleration parameter, which accounts for the asymmetry in the distribution. This adjustment ensures that the CIs are more accurate and reliable, especially for small-sample or skewed data, providing a robust estimation without assuming a specific underlying distribution for the statistic [21]. The procedure is described below:

1. The procedure begins with a random sample $X_1, \dots, X_n$ selected from the population. Then, we estimate the parameter of interest from the original sample.

2. Generate B bootstrap samples, $X_1^*, \dots, X_n^*$, by resampling with replacement from the original data.

3. For each bootstrap sample, compute the bootstrap estimate $\hat{\theta}_b^*$, for $b = 1, 2, \dots, B$.

4. Determine the proportion of bootstrap estimates that are less than the original estimate $\hat{\theta}$, denoted p . The bias correction factor z_0 is the quantile of the standard normal distribution corresponding to p .

5. Find the acceleration value a that accounts for the estimator's distributional asymmetry. This is often estimated using the jackknife method or other methods that quantify the skewness of the sampling distribution.

6. Transform the bias-corrected normal deviates to adjust the percentiles for constructing the CI. The adjusted percentiles are given by

$$p_L^* = \Phi \left(z_0 + \frac{z_0 + z_{\alpha/2}}{1 - a(z_0 + z_{\alpha/2})} \right)$$

and

$$p_U^* = \Phi \left(z_0 + \frac{z_0 + z_{1-(\alpha/2)}}{1 - a(z_0 + z_{1-(\alpha/2)})} \right),$$

where Φ is the standard normal cumulative distribution function, and $z_{\alpha/2}$ and $z_{1-\alpha/2}$ are the $(\alpha/2)^{\text{th}}$ and $(1-(\alpha/2))^{\text{th}}$ quantiles of the standard normal distribution, respectively.

7. The BCa bootstrap CI is constructed using the percentiles p_L^* and p_U^* to extract the corresponding quantiles from the bootstrap distribution of $\hat{\theta}_b^*$. The BCa bootstrap CI is as follows in Eq. (2.9):

$$\left(\hat{\theta}_{(p_L^*)}^*, \hat{\theta}_{(p_U^*)}^* \right), \quad (2.9)$$

where $\hat{\theta}_{(p_L^*)}^*$ and $\hat{\theta}_{(p_U^*)}^*$ are $(p_L^*)^{\text{th}}$ and $(p_U^*)^{\text{th}}$ quantiles of the bootstrap estimates $\hat{\theta}_b^*$.

3. Simulation Studies and Results

The objective of this simulation study is to evaluate and compare the performance of different CI methods for the parameter of the Odoma distribution. Specifically, the methods are assessed in terms of empirical coverage probability (ECP) and average length (AL) under various sample sizes and parameter settings. This paper proposed the 95% two-sided CIs for the parameter of the Odoma distribution. The CIs were constructed using four different methods as likelihood-based, Wald-type, bootstrap-t, and BCa bootstrap. The Monte Carlo simulations in this study were conducted using RStudio, with 2,000 replications performed for each configuration. The number of bootstrap resamples (B) was set to 2,000.

Samples of size ($n = 20, 30, 50, 100, 200$, and 500) are generated from the Odoma distribution for selected parameter values ($\theta = 0.20, 0.30, 0.50, 0.75, 1.00, 1.50, 2.00$, and 2.50). The parameter values examined in this study are chosen to illustrate a range of distributional shapes, from highly right-skewed to moderately dispersed cases.

The simulation results reported in Tables 1-2 and Figs. 3-6 provide a comparison of the ECP and AL of four CIs for the

parameter of the Odoma distribution. The results indicate that the likelihood-based and Wald-type CIs generally had the highest ECPs, close to the nominal confidence level of 0.95 in many situations. The ECPs of these two CIs stayed stable at the nominal level as the sample size increased as expected. For small sample sizes, particularly when $n = 10, 20$ or 30 the bootstrap-t and BCa bootstrap CIs had lower ECPs, often falling below 0.95. This indicated that these bootstrap-based methods provided a less reliable coverage than the likelihood-based and Wald-type CIs when the sample size was small. However, as the sample size increased to $n = 100, 200$, and 500 the ECPs of all four CIs converged toward the nominal confidence level, suggesting that sample size had a significant impact on the performance of all CIs.

In terms of the impact of parameter values, the likelihood-based and Wald-type CIs maintained a consistent ECP close to 0.95 for low parameter values such as 0.20, 0.30, and 0.50. As the parameter values increased to 1.00, 1.50, 2.00, and 2.50 the ECPs of the

bootstrap-t and BCa bootstrap CIs noticeably decreased, particularly for small sample sizes. For example, when $n = 10$ and the parameter value was 2.00, the ECP of the bootstrap-t and BCa bootstrap CIs dropped to 0.8917 and 0.9180, respectively, while the likelihood-based and Wald-type CIs achieved ECPs of 0.9520 and 0.9723, respectively. This demonstrated the sensitivity of bootstrap-based CIs to high parameter values, particularly when the sample size was not sufficiently large. The likelihood-based and Wald-type CIs were more robust and less influenced by variations in parameter values, maintaining ECPs close to the nominal level in most situations. As the parameter values increased, the AL of all four CIs also increased. The bootstrap-t and BCa bootstrap CIs typically had shorter average lengths than the likelihood-based and Wald-type CIs, indicating that these bootstrap-based methods offered more precise intervals. However, this resulted in a decreased probability of coverage, particularly in situations involving high parameter values and small sample sizes.

Table 1. Empirical coverage probability and average length of the 95% CIs for the parameter of the Odoma distribution when $\theta = 0.20, 0.30, 0.50$, and 0.75 .

θ	n	Empirical Coverage Probability				Average Length			
		Likelihood	Wald	Boot-t	BCa	Likelihood	Wald	Boot-t	BCa
0.20	10	0.9510	0.9527	0.8987	0.8960	0.1130	0.1128	0.1011	0.1059
	20	0.9413	0.9433	0.9153	0.9167	0.0793	0.0792	0.0751	0.0766
	30	0.9520	0.9523	0.9363	0.9363	0.0644	0.0643	0.0621	0.0629
	50	0.9507	0.9527	0.9397	0.9420	0.0497	0.0497	0.0484	0.0490
	100	0.9490	0.9487	0.9417	0.9430	0.0352	0.0352	0.0347	0.0350
	200	0.9520	0.9517	0.9430	0.9473	0.0248	0.0248	0.0245	0.0247
	500	0.9473	0.9477	0.9457	0.9470	0.0157	0.0157	0.0156	0.0157
0.30	10	0.9473	0.9483	0.8987	0.8947	0.1696	0.1693	0.1539	0.1611
	20	0.9490	0.9530	0.9233	0.9203	0.1189	0.1188	0.1127	0.1149
	30	0.9507	0.9500	0.9340	0.9343	0.0968	0.0967	0.0934	0.0947
	50	0.9460	0.9477	0.9307	0.9330	0.0747	0.0747	0.0728	0.0735
	100	0.9500	0.9500	0.9437	0.9443	0.0526	0.0526	0.0519	0.0524
	200	0.9470	0.9463	0.9427	0.9447	0.0372	0.0371	0.0368	0.0371
	500	0.9557	0.9557	0.9533	0.9540	0.0235	0.0235	0.0233	0.0235
0.50	10	0.9500	0.9553	0.8990	0.8977	0.2803	0.2800	0.2535	0.2631
	20	0.9480	0.9463	0.9220	0.9257	0.1963	0.1962	0.1857	0.1888
	30	0.9483	0.9493	0.9310	0.9333	0.1601	0.1601	0.1552	0.1571
	50	0.9573	0.9563	0.9420	0.9427	0.1235	0.1235	0.1207	0.1220
	100	0.9500	0.9513	0.9413	0.9463	0.0871	0.0871	0.0860	0.0868
	200	0.9433	0.9450	0.9400	0.9423	0.0616	0.0616	0.0612	0.0616
	500	0.9423	0.9433	0.9410	0.9397	0.0390	0.0390	0.0387	0.0390
0.75	10	0.9553	0.9580	0.9020	0.8960	0.4074	0.4085	0.3698	0.3777
	20	0.9533	0.9540	0.9263	0.9273	0.2870	0.2874	0.2719	0.2742
	30	0.9423	0.9423	0.9220	0.9283	0.2337	0.2340	0.2257	0.2272
	50	0.9520	0.9527	0.9390	0.9453	0.1810	0.1811	0.1769	0.1781
	100	0.9510	0.9517	0.9457	0.9473	0.1281	0.1282	0.1265	0.1273
	200	0.9517	0.9530	0.9493	0.9490	0.0904	0.0905	0.0895	0.0902
	500	0.9513	0.9510	0.9437	0.9453	0.0571	0.0572	0.0566	0.0570

Table 2. Empirical coverage probability and average length of the 95% CIs for the parameter of the Odoma distribution when $\theta = 1.00, 1.50, 2.00$, and 2.50 .

θ	n	Empirical Coverage Probability				Average Length			
		Likelihood	Wald	Boot-t	BCa	Likelihood	Wald	Boot-t	BCa
1.00	10	0.9493	0.9470	0.8927	0.8923	0.5059	0.5091	0.4581	0.4646
	20	0.9567	0.9540	0.9223	0.9203	0.3585	0.3603	0.3402	0.3411
	30	0.9537	0.9537	0.9307	0.9317	0.2934	0.2947	0.2846	0.2855
	50	0.9480	0.9503	0.9393	0.9367	0.2276	0.2284	0.2225	0.2235
	100	0.9453	0.9470	0.9400	0.9373	0.1608	0.1614	0.1588	0.1598
	200	0.9487	0.9480	0.9410	0.9453	0.1136	0.1140	0.1125	0.1134
	500	0.9520	0.9527	0.9487	0.9500	0.0719	0.0721	0.0716	0.0721
1.50	10	0.9460	0.9537	0.8897	0.8987	0.6626	0.6668	0.5897	0.6355
	20	0.9463	0.9560	0.9270	0.9327	0.4636	0.4682	0.4403	0.4562
	30	0.9487	0.9537	0.9340	0.9373	0.3769	0.3811	0.3618	0.3708
	50	0.9503	0.9523	0.9393	0.9420	0.2907	0.2942	0.2838	0.2890
	100	0.9437	0.9467	0.9347	0.9380	0.2052	0.2079	0.2027	0.2053
	200	0.9490	0.9517	0.9430	0.9447	0.1449	0.1468	0.1437	0.1451
	500	0.9360	0.9397	0.9347	0.9340	0.0915	0.0927	0.0909	0.0917
2.00	10	0.9520	0.9723	0.8917	0.9180	0.9559	0.9771	0.8236	0.9931
	20	0.9507	0.9647	0.9147	0.9377	0.6441	0.6609	0.5988	0.6498
	30	0.9487	0.9587	0.9263	0.9400	0.5184	0.5333	0.4919	0.5187
	50	0.9560	0.9653	0.9383	0.9513	0.3980	0.4103	0.3852	0.3986
	100	0.9407	0.9510	0.9367	0.9410	0.2784	0.2874	0.2722	0.2783
	200	0.9523	0.9620	0.9453	0.9473	0.1970	0.2036	0.1948	0.1975
	500	0.9503	0.9583	0.9473	0.9513	0.1241	0.1283	0.1233	0.1247
2.50	10	0.9647	0.9647	0.9190	0.8880	1.4983	1.6478	1.3193	1.6384
	20	0.9527	0.9740	0.9377	0.9310	1.0150	1.0895	0.9288	1.0266
	30	0.9500	0.9710	0.9250	0.9400	0.8062	0.8654	0.7528	0.8086
	50	0.9523	0.9690	0.9280	0.9413	0.6114	0.6567	0.5821	0.6114
	100	0.9500	0.9647	0.9343	0.9450	0.4252	0.4569	0.4138	0.4254
	200	0.9507	0.9680	0.9440	0.9463	0.2994	0.3220	0.2942	0.2996
	500	0.9543	0.9680	0.9460	0.9530	0.1879	0.2021	0.1860	0.1883

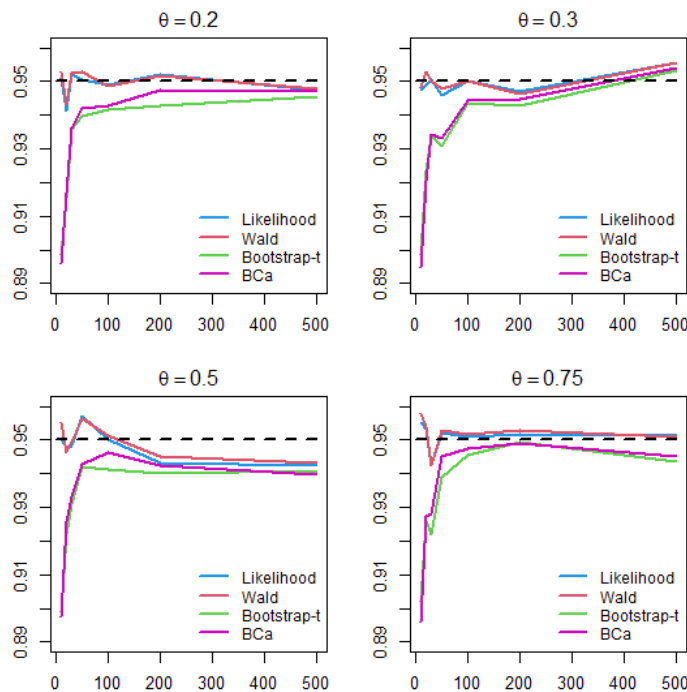


Fig. 3. Plots of the ECPs of the CIs for the parameter of the Odoma distribution when $\theta = 0.20, 0.30, 0.50$, and 0.75 .

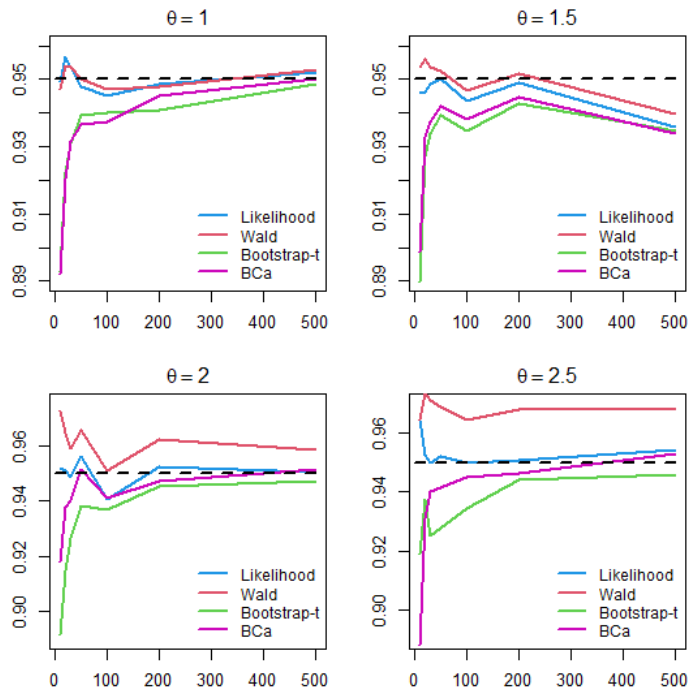


Fig. 4. Plots of the ECPs of the CIs for the parameter of the Odoma distribution when $\theta = 1.00, 1.50, 2.00$, and 2.50 .

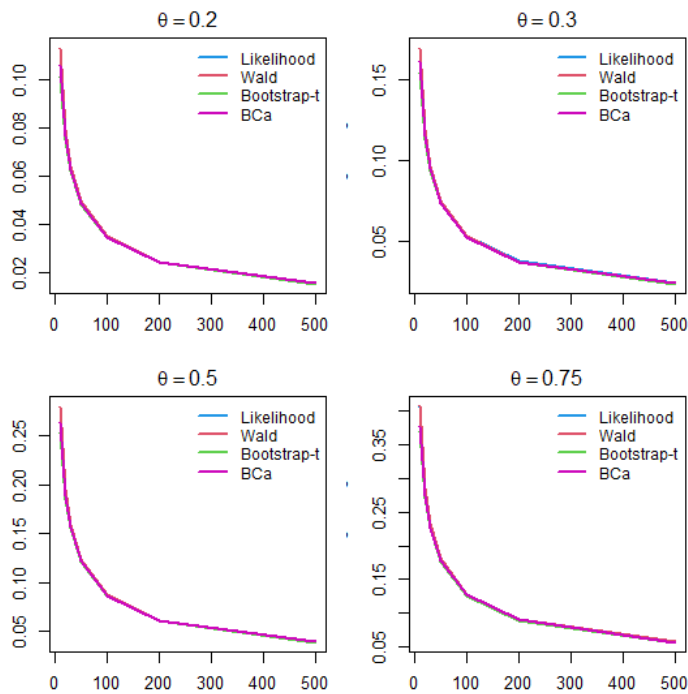


Fig. 5. Plots of the ALs of the CIs for the parameter of the Odoma distribution when $\theta = 0.20, 0.30, 0.50$, and 0.75 .

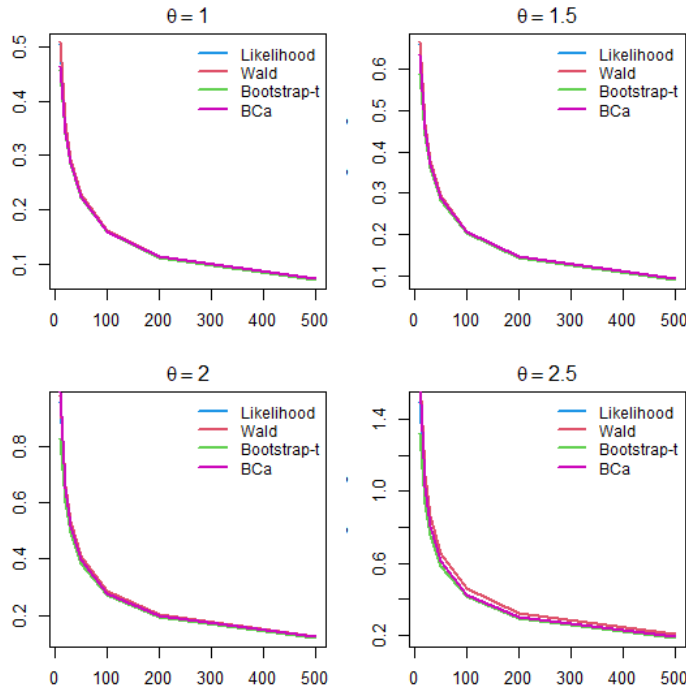


Fig. 6. Plots of the ALs of the CIs for the parameter of the Odoma distribution when $\theta = 1.00, 1.50, 2.00$, and 2.50 .

4. Application to Real Data

The CIs for the parameter of the Odoma distribution were applied in a real-world situation. The performance of the Odoma distribution was compared with the following alternative distributions. All these pdfs are defined for $x > 0$ and are characterized by a parameter $\theta > 0$:

1) The Pratibha distribution [22]

$$f(x; \theta) = \frac{\theta^3}{\theta^3 + \theta + 2} (\theta + x + x^2) e^{-\theta x},$$

2) The Komal distribution [23]

$$f(x; \theta) = \frac{\theta^2}{\theta^2 + \theta + 1} (1 + \theta + x) e^{-\theta x},$$

3) The Chris-Jerry distribution [24]

$$f(x; \theta) = \frac{\theta^2}{\theta + 2} (1 + \theta x^2) e^{-\theta x},$$

4) The Kpenadidum distribution [25]

$$f(x; \theta) = \frac{\theta^4}{2(\theta^4 + \theta + 6)} (2x^3 + x^2 + 2\theta) e^{-\theta x},$$

5) The Juchez distribution [12]

$$f(x; \theta) = \frac{\theta^4}{\theta^3 + \theta^2 + 6} (1 + x + x^3) e^{-\theta x},$$

6) The Adya distribution [26]

$$f(x; \theta) = \frac{\theta^3}{\theta^4 + 2\theta^2 + 2} (\theta + x)^2 e^{-\theta x},$$

7) The Pranav distribution [27]

$$f(x; \theta) = \frac{\theta^4}{\theta^4 + 6} (\theta + x^3) e^{-\theta x},$$

8) The Ishita distribution [28]

$$f(x; \theta) = \frac{\theta^3}{\theta^3 + 2} (\theta + x^2) e^{-\theta x},$$

9) The Rama distribution [29]

$$f(x; \theta) = \frac{\theta^4}{\theta^3 + 6} (1 + x^3) e^{-\theta x},$$

10) The Sujatha distribution [30]

$$f(x; \theta) = \frac{\theta^3}{\theta^2 + \theta + 2} (1 + x + x^2) e^{-\theta x},$$

11) The Garima distribution [31]

$$f(x; \theta) = \frac{\theta}{\theta + 2} (1 + \theta + \theta x) e^{-\theta x},$$

12) The Shanker distribution [8]

$$f(x; \theta) = \frac{\theta^2}{\theta^2 + 1} (\theta + x) e^{-\theta x},$$

13) The Akash distribution [32]

$$f(x; \theta) = \frac{\theta^3}{\theta^2 + 2} (1 + x^2) e^{-\theta x},$$

14) The Lindley distribution [3]

$$f(x; \theta) = \frac{\theta^2}{\theta + 1} (1 + x) e^{-\theta x},$$

15) The exponential distribution

$$f(x; \theta) = \theta e^{-\theta x},$$

Fuller et al. [33] recorded the strength data of aircraft window glass. The sample sizes of this real dataset ($n = 31$) fall within the range of sample sizes considered in the simulation study. Table 3 presents the descriptive statistics for this dataset, while Fig. 7 presents a histogram, box and whisker plot, kernel density plot, and violin plot of this dataset, illustrating its positive skewness.

The strength data of aircraft window glass exhibit a slight positive skewness (skewness = 0.405), indicating a mildly right-skewed distribution. Furthermore, the kurtosis value (2.287), being less than 3, suggests a platykurtic distribution, implying that the data are more dispersed and flatter compared to the normal distribution.

Table 3. Descriptive statistics for the strength data of aircraft window glass.

Descriptive statistics	Values
Sample size	31
Min	18.83
Q1	25.51
Median	29.90
Mean	30.81
Q3	35.83
Max	45.38
SD	7.25
Skewness	0.405
Kurtosis	2.287

A comprehensive evaluation was conducted using various criteria including the log-likelihood ($\log(\hat{L})$), Akaike information criterion (AIC), and Bayesian information criterion (BIC), also known as the Schwarz information criterion, to compare the goodness

of fit of different distributions. The AIC and BIC are defined as $AIC = 2k - 2\log \hat{L}$ and $BIC = 2k \log(n) - 2\log \hat{L}$, where k is the number of estimated parameters in the model and $\hat{L}$ is the maximized value of the likelihood function. A distribution with lower values of the AIC and BIC statistics is considered to be preferable. Table 4 presents the parameter estimates along with their corresponding standard errors (SEs) and goodness-of-fit measures for the dataset under investigation.

Empirical evidence from this dataset further supports the suitability of the Odoma distribution, as it provides lower AIC and BIC values compared to classical distributions such as the Lindley and exponential distributions. The Kolmogorov-Smirnov (K-S) statistic [34] was used to perform a goodness-of-fit test to evaluate whether the strength data of aircraft window glass follow the Odoma distribution. The K-S statistic is equal to 0.2232 with a p-value of 0.0774. Since the p-value is greater than the 0.05 significance level, there is insufficient evidence to reject the null hypothesis, indicating that the Odoma distribution provides an adequate fit to the data.

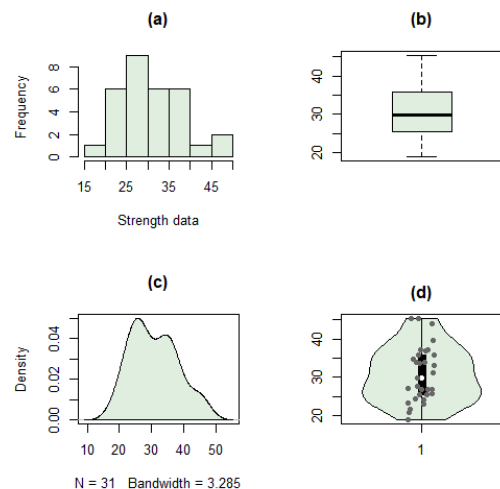


Fig. 7. (a) Histogram, (b) Box and Whisker plot, (c) Kernel density plot, and (d) Violin plot of the strength data of aircraft window glass.

Table 4. Analysis of model fit for several distributions applied to the strength data of aircraft window glass, with models ranked in ascending order based on their AIC values.

Distribution	Estimate (SE)	Log(L)	AIC	Rank(AIC)	BIC	Rank(BIC)
Odoma	0.1623 (0.01304)	-113.6276	229.2551	1	230.6891	1
Pranav	0.1298 (0.01183)	-116.3876	234.7752	2	236.2092	2
Rama	0.1298 (0.01183)	-116.3962	234.7924	3	236.2263	3
Juchez	0.1298 (0.01183)	-116.4447	234.8894	4	236.3234	4
Kpenadidum	0.1291 (0.01000)	-116.5236	235.0472	5	236.4812	5
Ishita	0.0973 (0.01000)	-120.2435	242.4871	6	243.921	6
Adya	0.0970 (0.01000)	-120.3198	242.6396	7	244.0736	7
Akash	0.0971 (0.01000)	-120.3409	242.6818	8	244.1157	8
Pratibha	0.0958 (0.01000)	-120.6626	243.3252	9	244.7592	9
Sujatha	0.0956 (0.01000)	-120.7516	243.5031	10	244.9371	10
Chris-Jerry	0.0954 (0.00010)	-121.2989	244.5977	11	246.0317	11
Shanker	0.0647 (0.00837)	-126.1765	254.3530	12	255.7870	12
Lindley	0.0647 (0.00837)	-127.0170	256.0339	13	257.4679	13
Komal	0.0628 (0.00775)	-127.0450	256.0901	14	257.5240	14
Garima	0.0516 (0.00775)	-133.6390	269.2780	15	270.7120	15
Exponential	0.0325 (0.00548)	-137.2644	276.5289	16	277.9629	16

Table 5. Comparative analysis of CI lengths for five best-fitting distributions in the strength data of aircraft window glass.

Distribution	Point Estimate	Method	CI	Length
Odoma	0.1623	Likelihood	(0.1380, 0.1892)	0.0512
		Wald	(0.1367, 0.1878)	0.0511
		Bootstrap-t	(0.1491, 0.1756)	0.0265
		BCa	(0.1497, 0.1759)	0.0262
Pranav	0.1298	Likelihood	(0.1083, 0.1540)	0.0457
		Wald	(0.1070, 0.1526)	0.0456
		Bootstrap-t	(0.1196, 0.1411)	0.0215
		BCa	(0.1188, 0.1400)	0.0212
Rama	0.1298	Likelihood	(0.1083, 0.1540)	0.0457
		Wald	(0.1069, 0.1526)	0.0457
		Bootstrap-t	(0.1190, 0.1402)	0.0212
		BCa	(0.1197, 0.1415)	0.0218
Juchez	0.1298	Likelihood	(0.1083, 0.1540)	0.0457
		Wald	(0.1070, 0.1526)	0.0456
		Bootstrap-t	(0.1197, 0.1405)	0.0208
		BCa	(0.1193, 0.1405)	0.0212
Kpenadidum	0.1291	Likelihood	(0.1077, 0.1532)	0.0455
		Wald	(0.1064, 0.1519)	0.0455
		Bootstrap-t	(0.1193, 0.1393)	0.0200
		BCa	(0.1185, 0.1399)	0.0214

Table 5 presents the point estimates and corresponding CIs obtained using likelihood-based, Wald-type, and bootstrap-based methods for five best-fitting distributions. Based on the ML method, the point estimator and its standard error (SE) for the Odoma distribution parameter were 0.1623 and 0.01304, respectively. Overall, the results clearly demonstrate that the bootstrap-based methods (Bootstrap-t and BCa) consistently provide shorter CIs compared to the likelihood and Wald methods across all distributions. This indicates that bootstrap methods provide higher precision in interval estimation.

For the Odoma distribution, the lengths of the likelihood-based and Wald-type CIs are relatively large (0.0512 and 0.0511, respectively), whereas the bootstrap-based methods substantially reduce the interval length to 0.0265 (Bootstrap-t) and 0.0262 (BCa). A similar pattern is observed for the Pranav, Rama, Juchez, and Kpenadidum distributions, where the likelihood-based and Wald-type CIs are consistently around 0.0455-0.0457, while the bootstrap-based CIs are significantly narrower, ranging approximately from 0.0200 to 0.0218.

The findings from the simulation study are consistent with the results obtained from the real datasets. In particular, the bootstrap-based methods (BCa and bootstrap-t) provide relatively narrower CIs, reflecting higher precision, whereas the likelihood-based method yields more stable and reliable interval estimates. This consistency suggests that the performance patterns observed under controlled simulation settings are also applicable in practical scenarios, thereby supporting the robustness of the proposed methods.

5. Conclusions and Discussions

This paper developed and assessed four CIs for parameter estimation in the Odoma distribution including likelihood-based, Wald-type, bootstrap-t, and bias-corrected and accelerated (BCa) bootstrap. The Wald-type CI was formulated and presented as an explicit equation. All the CIs were assessed through Monte Carlo simulation studies based on empirical coverage probability (ECP) and the average length (AL) of the intervals. The simulation results indicated that likelihood-based and Wald-type CIs were superior for attaining ECPs at the nominal confidence level of 0.95, providing consistent performance across various sample sizes and parameter values. The bootstrap-t and BCa bootstrap CIs, while providing shorter average lengths and thus more precise estimates, exhibited lower ECPs, especially when the sample size was small or the parameter value was high. As the sample size increased, the performance of the bootstrap-t and BCa bootstrap CIs improved, with ECPs approaching the nominal confidence level of 0.95 and ALs becoming comparable to the likelihood-based and Wald-type CIs. Therefore, for applications requiring high ECP, the likelihood-based and Wald-type CIs are recommended, particularly when dealing with small sample sizes or high parameter values. Conversely, the bootstrap-t and BCa bootstrap CIs, with their shorter average lengths, may be more suitable when a more precise interval estimate is required that

can accept a slight deviation in ECP. Selection of the approach depends on the specific requirements of the investigation, including the preferred balance between empirical coverage probability and interval precision.

Previous research on related distributions, such as the mixture distributions (e.g., Fav-Jerry [35], Rama [36], Iwueze [37], and other distributions), has consistently reported that bootstrap-based methods, particularly the BCa bootstrap and bootstrap-t methods, tend to produce narrower CIs, indicating higher precision. However, these methods may exhibit under-coverage in small samples due to their reliance on resampling variability. On the other hand, a likelihood-based method is often found to provide more stable coverage probabilities, especially in small to moderate sample sizes, as it is based on likelihood profiling rather than asymptotic normal approximations.

The results of the present study are consistent with these findings. Specifically, the bootstrap-based methods (BCa bootstrap and bootstrap-t) yield relatively shorter interval lengths, while the likelihood-based method demonstrates more stable and reliable coverage performance across different sample sizes. This agreement confirms that the general behavior of CI methods observed in the literature also holds for the Odoma distribution.

The computational requirements of bootstrap techniques, particularly the bootstrap-t and BCa bootstrap CIs, can pose a challenge to limited computational resources. To facilitate the computation of bootstrap CIs in RStudio, several packages are accessible, with the ‘boot’ package (Canty and Ripley, 2024) and the ‘bootstrap’ package (Kostyshak, 2024) being notable examples. Future research could study hypothesis testing for the parameter of the Odoma distribution. This topic represents valuable opportunities for future studies.

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