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A Bi-Objective Two-Level Capacitated Facility Location Model with Primary–Backup Assignments under Depot Disruptions

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ABSTRACT

This study develops a bi-objective two-level capacitated facility location model with predetermined primary-backup assignments under depot disruptions. Each customer is assigned to primary and backup depots through the same satellite to maintain service when the primary depot becomes unavailable. The model minimizes expected total system cost and expected total delivery time under capacity restrictions. The nonlinear formulation is reformulated as an equivalent mixed-integer linear programming model, and a normalized weighted-sum approach is used to examine the objective trade-off. Computational experiments on networks containing 20-70 nodes reveal a clear cost-delivery-time trade-off. All evaluated configurations with up to 35 nodes are solved with zero optimality gap, whereas larger networks require greater computational effort and may terminate with positive gaps. The results highlight the importance of balancing economic performance and delivery responsiveness in disruption-aware distribution network design.

Keywords: Depot disruption; Facility disruption; Primary-backup assignment; Multi-objective optimization; Two-level capacitated facility location

1. Introduction

Facility location decisions play an important role in supply chain and distribution network design by determining facility locations and customer allocations. Classical models include the Uncapacitated Facility Location Problem (UFLP) and the Capacitated Facility Location Prob-

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lem (CFLP) [1–3]. In multi-level distribution systems, products pass through intermediate facilities before reaching customers. Aardal et al. [4] studied the two-level uncapacitated facility location problem, while Tragantalerngsak et al. [5, 6] investigated the two-echelon, single-source, capacitated facility location problem. Facility capacities are particularly important under disruptions because surviving facilities must accommodate demand transferred from unavailable facilities.

Facility disruptions can affect service continuity and increase distribution costs. Early studies addressed location decisions involving unreliable facilities [7] and the expected costs of facility failures [8]. Snyder [9] reviewed facility location models under uncertainty, while Cui et al. [10] and Li et al. [11] investigated reliable facility location under disruptions. Afify et al. [12] incorporated heterogeneous failure probabilities, backup assignments, capacity restrictions, and facility fortification into a reliable capacitated facility location problem. Subsequent studies considered capacity failures with correlated demand uncertainty [13], two-stage stochastic facility location with disruptions and restricted shortages [14], and protection and backup decisions under decision-dependent uncertainty [15]. These studies demonstrate different approaches to incorporating disruption risks and recovery decisions into facility location models.

Disruption-aware network design has also been examined from resilience and multi-objective perspectives. Aldrighetti et al. [16] reviewed the costs of resilience and disruptions in supply chain network design, while Fugaro and Sgalambro [17] surveyed multi-objective facility location problems. Medal et al. [18] developed a multi-objective facility location-hardening model examining trade-offs between maximum distances before and after disruptions. In a different application, Titkanna et al. [19] proposed a hybrid metaheuristic for location-allocation optimization in agricultural supply chains exposed to insurgency-related risks. These studies highlight the relevance of resilience and multiple objectives in location decisions, although their network structures, disruption mechanisms, and performance measures differ.

Primary-backup assignments provide a means of maintaining service when a facility becomes unavailable [8, 11, 12]. In two-level distribution networks, disruptions at upper-level facilities may require demand to be transferred to surviving facilities while maintaining downstream service. Ramshani et al. [20] investigated a two-level uncapacitated facility location problem with disruptions and alternative service arrangements. Harnnarong and Boonperm [21] considered a two-level capacitated facility location problem under depot disruptions and incorporated depot fortification. These studies provide relevant foundations for disruption-aware two-level networks but differ in their treatment of capacities, recovery decisions, and optimization objectives. Table 1 compares selected studies with the proposed model.

Fig. 1 illustrates the primary-backup mechanism considered in this study. Each customer is normally served through a satellite supplied by its primary depot. If the primary depot becomes unavailable, supply is transferred to a predetermined backup depot while the satellite-to-customer assignment remains unchanged. The backup depot must have sufficient capacity to accommodate both its regular demand and the transferred demand.

Although previous studies have

Table 1. Comparison of related facility location studies.

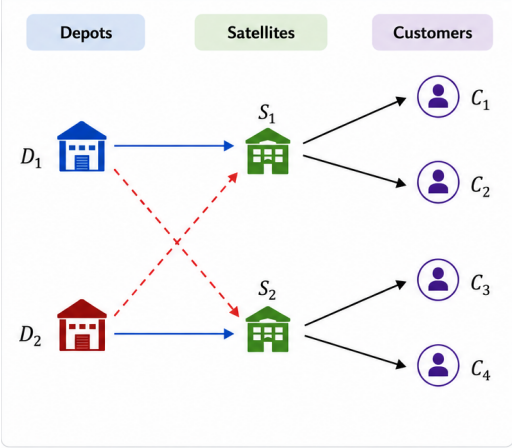
Study	Problem / method	Objectives	Disruption / uncertainty	Key distinction
Snyder and Daskin [8]	Reliable facility location	Expected system cost	Facility failures	Expected failure-cost modeling
Afify et al. [12]	Capacitated FLP; linearization	Cost minimization	Heterogeneous facility failures	Backup assignments, capacity, and fortification
Medal et al. [18]	Multi-objective location-hardening	Pre- and post-disruption maximum distances	Facility disruptions	Trade-offs involving facility hardening
Ramshani et al. [20]	Two-level uncapacitated FLP	Cost minimization	Facility disruptions	Alternative service paths without facility capacities
Harnnarong and Boonperm [21]	Two-level capacitated FLP	Cost minimization	Depot disruptions	Depot fortification in a capacitated two-level network
Hu and Tang [15]	Robust facility location	Cost-oriented optimization	Decision-dependent uncertainty	Protection and backup decisions
This study	**Bi-objective TCFLP; MILP**	**Expected cost and delivery time**	**Depot disruption probabilities**	**Primary-backup assignments, contingency capacities, and fixed satellite assignments**

investigated backup arrangements, depot fortification, and multi-objective facility location, limited attention has been given to their integration in capacitated two-level networks with predetermined primary-backup assignments, fixed satellite-to-customer relationships, and expected cost and delivery-time objectives. A backup depot offering lower transportation costs may not provide the shortest delivery time. Moreover, surviving depots must have sufficient capacity to accommodate their regular demand, and demand transferred from a disrupted primary depot. These considerations motivate a bi-objective formulation integrating economic and delivery-time performance with contingency-specific capacity requirements.

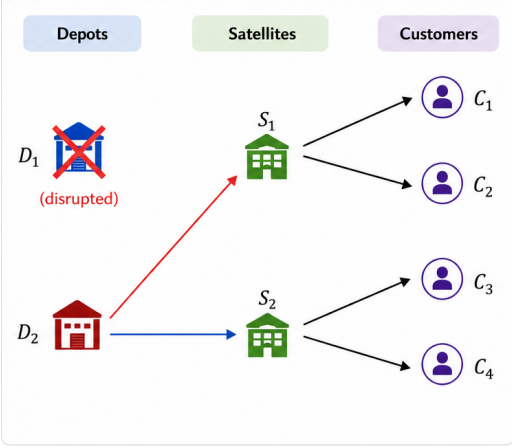
Unlike Harnnarong and Boonperm [21], whose capacitated two-level model incorporates depot fortification, the present study focuses on predetermined primary-backup assignments without fortification. Each customer is assigned primary and backup depots through the same satellite, and the backup path is activated when the primary depot becomes unavailable. The proposed model jointly minimizes expected total system cost and expected total delivery time while enforcing depot and satellite capacities under normal operation and individual primary-depot contingencies.

The contributions of this study are threefold. First, a bi-objective two-level capacitated facility location problem is formulated to integrate predetermined primary-backup assignments, a same-satellite recovery policy, and expected cost and delivery-time objectives. Second, contingency-specific capacity constraints ensure that surviving depots can accommodate their regular demand and demand transferred from a disrupted primary depot. Third, the nonlinear formulation is converted into an equivalent mixed-integer linear programming (MILP) model using standard binary-product linearization, and a normalized weighted-sum approach is applied to examine cost-delivery-time trade-offs and computational performance. The contribution lies in the integration of these modeling features and their computational analysis rather than in developing new linearization or multi-objective optimization algorithms.

The remainder of this paper is orga-



(a) Planned assignments before disruption.



(b) Assignments after disruption of D_1.



Fig. 1. Primary and backup assignments in a two-level distribution network under depot disruption.

nized as follows. Section 2 presents the problem description and mathematical formulation. Section 3 describes the multi-objective solution approach. Section 4 presents the computational experiments and results, and Section 5 concludes the paper.

2. Problem Description and Mathematical Formulation

Consider a two-level capacitated distribution network consisting of candidate depots, candidate satellites, and customers. Products are transported from opened depots to opened satellites and subsequently delivered to customers. Both facility levels are subject to capacity restrictions, and direct shipments from depots to customers are not allowed.

To maintain service under depot disruptions, each customer is assigned a primary depot and a predetermined backup depot. Both service paths use the same satellite. Under normal operation, the satellite receives the customer's demand from its primary depot. If the primary depot becomes unavailable, supply is transferred to the predetermined backup depot while the satellite-to-customer assignment remains unchanged. Thus, a depot disruption affects only the depot-to-satellite portion of the service path.

Each depot i has a depot-specific disruption probability p_i. For a customer assigned to depot i as its primary depot, the primary service path is used with probability $1 - p_i$, whereas the predetermined backup path is activated with probability p_i. These probabilities describe the operational state of the assigned primary depot rather than mutually exclusive system-wide disruption scenarios. The model considers individual primary-depot contingencies; simultaneous disruptions involving multiple depots, including the concurrent unavailability of a primary depot and its backup depot, are outside its scope.

The model jointly determines facility-opening and primary-backup assignment decisions subject to facility capacities while minimizing expected total system cost and expected total delivery

time. Depot disruption is the sole source of uncertainty considered, allowing its effects on network design and recovery decisions to be examined separately from other sources of operational uncertainty. The nonlinear formulation is presented first, followed by an equivalent MILP reformulation used in the computational experiments.

The following assumptions are adopted:

- Customer locations and demands are known, and each customer's demand must be fully satisfied without demand splitting.
- Candidate depots and satellites have known capacities and fixed opening costs.
- Each customer is assigned to exactly one primary service path and one predetermined backup service path. The two paths use the same satellite but different depots.
- Primary and backup assignments are determined before disruptions occur, and the backup path is activated when the corresponding primary depot becomes unavailable.
- Disruptions occur only at the depot level. Satellites and their connections to customers remain operational following a depot disruption.
- Each depot has a known depot-specific disruption probability. The model considers individual primary-depot contingencies and does not account for simultaneous disruptions involving multiple depots.
- Transportation costs, travel distances, and average transportation speed are known and treated as deterministic planning parameters.

Let I, J, and K denote the sets of candidate depots, candidate satellites, and customers, respectively. The sets, parameters, and decision variables used in the mathematical formulation are summarized in Table 2.

For each combination (i, j, k), the transportation parameters c_{ijk}, l_{ijk}, and t_{ijk} refer to the complete service path from depot i through satellite j to customer k, rather than to an individual network arc. The distance l_{ijk} represents the total travel distance along this path, and the corresponding delivery time is calculated as $t_{ijk} = 60l_{ijk}/v$.

2.1 Mathematical Formulation

The problem is formulated as a bi-objective integer nonlinear programming model that minimizes expected total system cost and expected total delivery time. Both objectives use the primary-depot contingency mechanism described in the preceding section: the primary path is weighted by the probability that its assigned depot remains operational, while the predetermined backup path is weighted by the disruption probability of that depot. Fixed facility-opening costs are incurred independently of depot disruptions. The corresponding service states and capacity requirements are summarized in Table 3.

The formulation distinguishes between expected performance and capacity feasibility. Disruption probabilities are used to evaluate the objectives, while capacity constraints ensure that surviving depots can accommodate their primary demand and any demand transferred from a disrupted depot.

Table 2. Notation used in the mathematical formulation.

Symbol	Definition
Sets and indices	
I	Set of candidate depots, indexed by i and r.
J	Set of candidate satellites, indexed by j and o.
K	Set of customers, indexed by k.
Parameters	
c_{ijk}	Transportation cost of serving customer k along the complete path from depot i through satellite j to customer k.
l_{ijk}	Total travel distance along the complete path from depot i through satellite j to customer k.
t_{ijk}	Delivery time along the complete path from depot i through satellite j to customer k, where $t_{ijk} = 60l_{ijk}/v$.
F_i	Fixed opening cost of depot i.
M_j	Fixed opening cost of satellite j.
Q_i	Capacity of depot i.
Q'_j	Capacity of satellite j.
d_k	Demand of customer k.
p_i	Disruption probability associated with depot i.
v	Average transportation speed.
Decision variables	
x_i	$\begin{cases}1, & \text{if depot } i \text{ is opened,}\\ 0, & \text{otherwise.}\end{cases}$
y_j	$\begin{cases}1, & \text{if satellite } j \text{ is opened,}\\ 0, & \text{otherwise.}\end{cases}$
w_{ijk}	$\begin{cases}1, & \text{if } (i,j) \text{ is the primary path for customer } k,\\ 0, & \text{otherwise.}\end{cases}$
w'_{ijk}	$\begin{cases}1, & \text{if } (i,j) \text{ is the backup path for customer } k,\\ 0, & \text{otherwise.}\end{cases}$

Table 3. Primary-depot service states.

State	Probability	Path	Capacity
Normal	$1 - p_r$	Primary	Primary-depot capacity
Disrupted	p_r	Backup	Backup capacity including transferred demand

The first objective minimizes expected total system cost and is formulated as

$$\min Z_1 = \sum_{i\in I} F_i x_i + \sum_{j\in J} M_j y_j + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} c_{ijk} w_{ijk}(1-p_i) + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{\substack{r\in I\\ r\neq i}}\sum_{o\in J} c_{ijk} w'_{ijk} w_{rok} p_r. \tag{2.1}$$

The first two terms in Eq. (2.1) represent the fixed opening costs of depots and satellites. The third term represents the expected transportation cost of primary service paths, weighted by the probability that the corresponding primary depot remains operational. The final term represents the expected transportation cost of backup service paths. Specifically, $w'_{ijk} w_{rok} = 1$ when customer k is assigned to depot r through satellite o as its primary path and to depot i through satellite j as its backup path. The corresponding backup transportation cost is weighted by p_r, the disruption probability of the assigned primary depot.

The second objective minimizes expected total delivery time and is formulated as

$$\min Z_2 = \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} t_{ijk} w_{ijk}(1-p_i) + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{\substack{r\in I\\ r\neq i}}\sum_{o\in J} t_{ijk} w'_{ijk} w_{rok} p_r. \tag{2.2}$$

The first term in Eq. (2.2) represents

the expected delivery time through primary service paths, whereas the second represents the expected delivery time through predetermined backup paths. The delivery-time objective is customer-oriented rather than demand-weighted: each customer contributes one route delivery time regardless of its demand volume, while customer demand affects network feasibility through the depot and satellite capacity constraints. Thus, the two objectives evaluate the same disruption mechanism in terms of economic performance and delivery responsiveness.

The network design must satisfy facility-opening, assignment, and capacity requirements under normal operation and under each primary-depot contingency considered by the model. The corresponding constraints are given below.

$$w_{ijk} \leq x_i \qquad \forall i \in I,\ j \in J,\ k \in K \qquad (2.3)$$

$$w'_{ijk} \leq x_i \qquad \forall i \in I,\ j \in J,\ k \in K \qquad (2.4)$$

$$w_{ijk} \leq y_j \qquad \forall i \in I,\ j \in J,\ k \in K \qquad (2.5)$$

$$w'_{ijk} \leq y_j \qquad \forall i \in I,\ j \in J,\ k \in K \qquad (2.6)$$

$$\sum_{i \in I} \sum_{j \in J} w_{ijk} = 1 \qquad \forall k \in K \qquad (2.7)$$

$$\sum_{i \in I} \sum_{j \in J} w'_{ijk} = 1 \qquad \forall k \in K \qquad (2.8)$$

$$\sum_{j \in J} \left(w_{ijk} + w'_{ijk} \right) \leq 1 \qquad \forall i \in I,\ k \in K \qquad (2.9)$$

$$\sum_{i \in I} w_{ijk} = \sum_{i \in I} w'_{ijk} \qquad \forall j \in J,\ k \in K \qquad (2.10)$$

$$\sum_{j \in J} \sum_{k \in K} d_k w_{ijk} \leq Q_i x_i \qquad \forall i \in I \qquad (2.11)$$

$$\sum_{j \in J} \sum_{k \in K} d_k w_{ijk} + \sum_{j \in J} \sum_{k \in K} \sum_{o \in J} d_k w'_{ijk} w_{rok} \leq Q_i x_i \qquad \forall i, r \in I,\ i \neq r \qquad (2.12)$$

$$\sum_{i \in I} \sum_{k \in K} d_k w_{ijk} \leq Q'_j y_j \qquad \forall j \in J \qquad (2.13)$$

$$x_i \in \{0,1\} \qquad \forall i \in I \qquad (2.14)$$

$$y_j \in \{0,1\} \qquad \forall j \in J \qquad (2.15)$$

$$w_{ijk},\ w'_{ijk} \in \{0,1\} \qquad \forall i \in I,\ j \in J,\ k \in K. \qquad (2.16)$$

Constraints Eqs. (2.3)-(2.6) link primary and backup assignments to facility-opening decisions, ensuring that a depot or satellite can participate in a service path only if the corresponding facility is opened. Constraints Eqs. (2.7)-(2.8) require each customer to have exactly one primary service path and one predetermined backup

service path, respectively.

Constraint Eq. (2.9) requires the primary and backup depots assigned to the same customer to be different. Constraint Eq. (2.10) requires both service paths to use the same satellite. Consequently, if the primary depot becomes unavailable, supply is transferred to the predetermined backup depot without changing the satellite-to-customer assignment. This policy assumes that disruptions affect only the upstream depot layer and preserves the established downstream service relationship during recovery.

Constraints Eqs. (2.11)-(2.13) enforce capacity feasibility. Constraint Eq. (2.11) limits the primary demand assigned to each depot under normal operation. For each contingency in which depot r becomes unavailable, Constraint Eq. (2.12) requires every surviving depot i to accommodate both its own primary demand and the demand transferred from depot r for which it serves as the designated backup. The transferred demand is counted only when the corresponding primary and backup assignment variables both equal one. Thus, the reserve capacity required for backup service is determined by the facility-opening and assignment decisions rather than specified as a separate fixed parameter. This assumes that the unused capacity of a surviving depot is available to accommodate transferred demand when a disruption occurs. Disruption probabilities are used to evaluate expected objective values; they do not reduce physical facility capacities or relax the contingency-specific capacity requirements.

Constraint Eq. (2.13) limits the total demand handled by each satellite. Because the primary and backup service paths of a customer use the same satellite, a primary-depot disruption changes the supplying depot but does not change the demand handled by the satellite. Finally, Constraints Eqs. (2.14)-(2.16) specify the binary domains of the facility-opening and assignment variables.

The formulation combines expected performance evaluation with capacity feasibility under each primary-depot contingency. Its nonlinear terms arise from products of primary and backup assignment variables in the objective functions and depot-capacity constraints. These products are reformulated in the next subsection to obtain an equivalent MILP.

2.2 Model Linearization

The nonlinear terms in the proposed formulation arise from products of binary assignment variables of the form $w'_{ijk}w_{rok}$. To obtain an equivalent mixed-integer linear programming (MILP) formulation, an auxiliary binary variable u_{ijrok} is introduced for all $i, r \in I$, $j, o \in J$, and $k \in K$ with $i \neq r$, such that

$$u_{ijrok} = w'_{ijk}w_{rok}. \tag{2.17}$$

The variable u_{ijrok} equals one when customer k is assigned to depot r through satellite o as its primary service path and to depot i through satellite j as its backup service path, and zero otherwise. Since both assignment variables are binary, their product can be represented exactly by the following linear constraints:

$$u_{ijrok} \leq w'_{ijk}, \tag{2.18}$$

$$u_{ijrok} \leq w_{rok}, \tag{2.19}$$

$$u_{ijrok} \geq w'_{ijk} + w_{rok} - 1, \tag{2.20}$$

for all $i, r \in I$, $j, o \in J$, and $k \in K$ with $i \neq r$. Constraints Eqs. (2.18)-(2.20) enforce $u_{ijrok} = 1$ if and only if both assignment variables equal one.

Replacing the nonlinear products with the corresponding auxiliary variables

yields the following linear objective functions:

$$\min Z_1 = \sum_{i\in I} F_i x_i + \sum_{j\in J} M_j y_j + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} c_{ijk} w_{ijk}(1-p_i) + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{\substack{r\in I\\ r\neq i}}\sum_{o\in J} c_{ijk} u_{ijrok} p_r. \tag{2.21}$$

$$\min Z_2 = \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} t_{ijk} w_{ijk}(1-p_i) + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{\substack{r\in I\\ r\neq i}}\sum_{o\in J} t_{ijk} u_{ijrok} p_r. \tag{2.22}$$

Similarly, for each $i, r \in I$ with $i \neq r$, the nonlinear depot-capacity constraint Eq. (2.12) is reformulated as

$$\sum_{j\in J}\sum_{k\in K} d_k w_{ijk} + \sum_{j\in J}\sum_{k\in K}\sum_{o\in J} d_k u_{ijrok} \leq Q_i x_i. \tag{2.23}$$

All remaining constraints, Eqs. (2.3)-(2.11) and Eqs. (2.13)-(2.16), remain unchanged. For all $i, r \in I$ with $i \neq r$, $j, o \in J$, and $k \in K$, the auxiliary variables are restricted to be binary:

$$u_{ijrok} \in \{0, 1\}. \tag{2.24}$$

The linearization is exact. If either assignment variable is zero, the upper-bound constraints force $u_{ijrok} = 0$. If both assignment variables are one, the lower-bound constraint forces $u_{ijrok} = 1$. Hence, $u_{ijrok} = w'_{ijk} w_{rok}$ holds for every feasible solution. Conversely, any feasible solution of the original formulation can be extended to the linearized formulation by setting each auxiliary variable equal to the corresponding binary product. Therefore, the two formulations have the same feasible solutions in the original decision variables and identical objective values. The resulting bi-objective MILP preserves the primary-backup assignment structure and contingency-specific capacity requirements without introducing any approximation. This formulation is used in the subsequent multi-objective solution procedure and computational experiments.

3. Multi-Objective Solution Approach

The proposed model considers two conflicting objectives: expected total system cost and expected total delivery time. To examine their trade-off, a normalized weighted-sum approach is employed [22]. Normalization is required because Z_1 and Z_2 are measured in different units and may differ substantially in magnitude.

The normalization reference values are obtained from the two single-objective solutions. Let $Z_1^{\min}$ and $Z_2^{\min}$ denote the minimum values of Z_1 and Z_2, respectively. Let $Z_1^{\max}$ denote the value of Z_1 at the minimum-time solution, and $Z_2^{\max}$ the value of Z_2 at the minimum-cost solution. The normalized objectives are

$$\bar{Z}_1 = \frac{Z_1 - Z_1^{\min}}{Z_1^{\max} - Z_1^{\min}}, \tag{3.1}$$

and

$$\bar{Z}_2 = \frac{Z_2 - Z_2^{\min}}{Z_2^{\max} - Z_2^{\min}}. \tag{3.2}$$

The normalized bi-objective problem is

$$\min Z(\lambda) = \lambda \bar{Z}_1 + (1-\lambda)\bar{Z}_2, \qquad 0 \leq \lambda \leq 1, \tag{3.3}$$

where λ represents the weight assigned to expected total system cost. Thus, $\lambda = 1$ gives the minimum-cost solution, whereas $\lambda = 0$ gives the minimum-time solution.

Varying λ generates alternative cost-delivery-time trade-off solutions. Because the problem is discrete, different weights may produce the same network configuration, and the weighted-sum method may not identify unsupported nondominated solutions. The method is therefore used to examine alternative objective preferences rather than to enumerate the complete nondominated frontier. The solutions are reported using the original values of Z_1 and Z_2 for direct interpretation.

4. Computational Experiments and Results

This section evaluates the proposed bi-objective TCFLP in terms of the cost-delivery-time trade-off and the computational performance of the linearized formulation.

4.1 Experimental setup

The test networks are synthetically generated within a geographical region representing Bangkok and its surrounding area. The parameter ranges are based on Harnnarong and Boonperm [21], with adjustments to satellite opening costs, facility capacities, and customer demand. Parameter values are randomly generated within the specified ranges without setting a fixed random seed. The geographical bounds and parameter settings are reported in Table 4.

The experiments consider networks containing 20-70 nodes, where the total network size is $|I|+|J|+|K|$. Different compositions of depots, satellites, and customers are examined in increments of five. Instances with the same total number of nodes may differ in computational difficulty because their numbers of assignment and auxiliary variables depend on the composition of the three network levels.

Let $m = |I|$, $n = |J|$, and $l = |K|$. The number of binary variables is $N_{\text{binary}} = m + n + 2mnl + m(m-1)n^2l$. The formulation-size ranges in Table 5 are calculated over all possible compositions, including those not evaluated computationally.

For example, the 20-node case includes $(5, 5, 10)$, $(5, 10, 5)$, and $(10, 5, 5)$, representing the numbers of depots, satellites, and customers, respectively. Of the 363 possible compositions, 124 are evaluated. All compositions are evaluated for networks containing 20-35 nodes, while subsets are considered for larger networks. Table 5 reports the composition counts and formulation-size ranges over all possible compositions.

All models are implemented in Python and solved using the Gurobi Optimizer on a 64-bit AMD Ryzen 7 3700U computer with 8 GB of RAM running Windows 10 Home. Each optimization run has a time limit of 3,600 seconds. If the limit is reached, the best feasible solution and its reported optimality gap are recorded.

4.2 Illustrative 20-node instance

A 20-node instance with 5 candidate depots, 5 candidate satellites, and 10 customers is used to illustrate the cost-delivery-time trade-off. Table 6 reports the single-objective solutions used as normalization reference points.

Relative to the minimum-cost solution, the minimum-time solution reduces expected total delivery time by 34.4% but increases expected total system cost by approximately 290.8%, demonstrating a pro-

Table 4. Parameter settings used in the computational experiments.

Parameter	Range or value
Depot opening cost, F_i	$[2000, 4000]$
Satellite opening cost, M_j	$[2000, 4000]$
Depot capacity, Q_i	$[700, 2000]$
Satellite capacity, Q'_j	$[700, 2000]$
Customer demand, d_k	$[10, 30]$
Depot disruption probability, p_i	$[0.10, 0.20]$
Average transportation speed, v	60 km/h
Latitude	$[13.556748, 14.079317]$
Longitude	$[100.311785, 100.837157]$

Table 5. Network compositions and formulation-size ranges.

Nodes	Possible	Evaluated	Binary variables	Linear constraints
20	3	3	5,510-11,765	10,580-23,075
25	6	6	8,260-46,020	15,865-91,080
30	10	10	11,010-106,525	21,150-211,610
35	15	15	13,760-238,530	26,435-474,865
40	21	20	16,510-477,030	31,720-949,700
45	28	18	19,260-861,035	37,005-1,716,255
50	36	17	22,010-1,528,040	42,290-3,048,260
55	45	14	24,760-2,410,045	47,575-4,810,315
60	55	11	27,510-3,762,550	52,860-7,512,820
65	66	7	30,260-5,643,800	58,145-11,269,205
70	78	3	33,010-8,178,805	63,430-16,335,535

Table 6. Single-objective results for the illustrative 20-node instance.

Objective	Cost	Time (min)
Minimum cost	7,455.15	887.53
Minimum time	29,136.64	582.27

nounced conflict between the two objectives.

The normalized weighted-sum model is then solved for $\lambda = 0.1, 0.2, \ldots, 0.9$. The resulting trade-off solutions are reported in Table 7.

Table 7. Trade-off solutions for the illustrative 20-node instance.

λ	Expected cost	Expected time (min)
0.9	7,591.15	794.40
0.8	8,826.26	683.64
0.7	8,826.26	683.64
0.6	8,826.26	683.64
0.5	11,470.63	641.44
0.4	13,854.95	616.03
0.3	16,970.13	594.17
0.2	19,465.69	583.99
0.1	19,472.50	583.99

At $\lambda = 0.8$, expected cost increases by 18.4% relative to the minimum-cost solution, while expected delivery time decreases by 23.0%. At $\lambda = 0.2$, expected delivery time is 583.99 minutes, only 1.72 minutes above the minimum-time value, while expected cost remains considerably lower. These results illustrate how intermediate solutions can improve delivery responsiveness without incurring the cost of the minimum-time extreme.

Fig. 2 illustrates the trade-off. The identical objective values obtained for $\lambda = 0.6$, 0.7, and 0.8 are consistent with the discrete nature of facility-opening and assignment decisions.

4.3 Computational performance

The computational performance is examined for networks containing 20-70 nodes. For each evaluated composition, the minimum-cost and minimum-time models are solved separately, each with a 3,600-second time limit. The total runtime in Table 8 is the sum of these two runs; values ap-

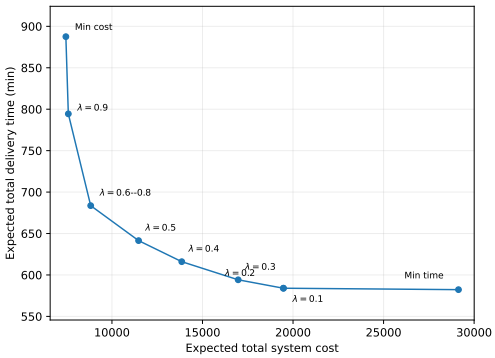


Fig. 2. Cost-delivery-time trade-off for the illustrative 20-node instance.

proaching 7,200 seconds therefore indicate that both runs approached their respective limits.

Table 8. Computational performance for different network sizes.

Nodes	Evaluated	Avg. total time (s)	Avg. gap (%)
			Cost / Time
20	3	4.89	0.00 / 0.00
25	6	49.17	0.00 / 0.00
30	10	235.84	0.00 / 0.00
35	15	1032.56	0.00 / 0.00
40	20	2503.86	2.14 / 8.35
45	18	2719.64	3.67 / 3.75
50	17	4913.19	12.40 / 12.77
55	14	5916.68	27.76 / 22.62
60	11	6179.25	30.06 / 25.64
65	7	$\approx 7,200$	41.68 / 33.60
70	3	$\approx 7,200$	51.51 / 21.81

For all evaluated configurations containing 20-35 nodes, both single-objective models terminate with zero optimality gap, although average total runtime increases from 4.89 to 1,032.56 seconds. From 40 nodes onward, some runs reach the time limit before optimality is established, resulting in positive average gaps. For the evaluated 65- and 70-node configurations, average combined runtimes approach 7,200 seconds. At 70 nodes, the average gaps are 51.51% and 21.81% for the cost and time objectives, respectively.

Figs. 3 and 4 illustrate the increasing computational effort. Since only subsets of possible compositions are evaluated for larger networks, the reported averages characterize the tested configurations rather than all networks of the corresponding size. Increasing runtimes and optimality gaps indicate that larger instances are more computationally demanding, motivating more scalable solution approaches.

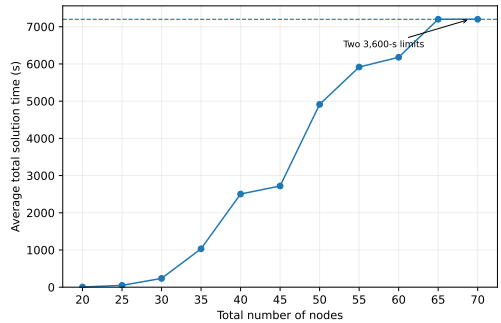


Fig. 3. Average total solution time for the two single-objective models.

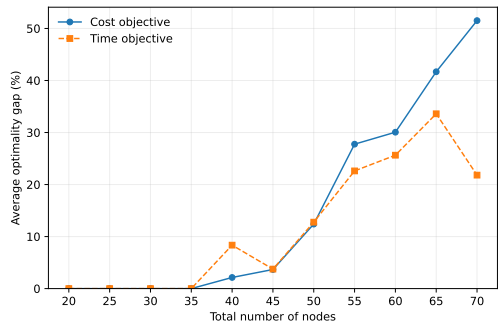


Fig. 4. Average optimality gaps for the single-objective models.

4.4 Multi-objective analysis

The multi-objective experiments examine the trade-off between expected total system cost and expected total delivery time. For 20-30 nodes, all evaluated compositions are solved using $\lambda = 0.1, 0.2, \ldots, 0.9$. For 35-50 nodes, $\lambda =$

$0.2, 0.4, 0.6, 0.8$ is applied to 15 compositions each at 35, 40, and 45 nodes and 8 compositions at 50 nodes. For 55 and 60 nodes, $\lambda = 0.25, 0.5, 0.75$ is applied to 3 and 2 compositions, respectively. No multi-objective experiments are reported for 65-70 nodes.

Across the tested instances, increasing λ generally favors lower expected cost and higher delivery time, while decreasing λ produces the opposite pattern. Neighboring weights may yield identical objective values because facility-opening and assignment decisions are discrete.

The results illustrate alternative cost-delivery-time outcomes. For larger instances with nonzero optimality gaps, the reported solutions are not necessarily proven Pareto-optimal. Moreover, the weighted-sum method may miss unsupported nondominated solutions. Thus, the reported trade-offs do not constitute a complete nondominated frontier.

5. Conclusions

This study developed a bi-objective two-level capacitated facility location model with predetermined primary-backup assignments under depot disruptions. When a primary depot becomes unavailable, supply is transferred to its predetermined backup depot while the assigned satellite remains unchanged. The model minimizes expected total system cost and delivery time. Its nonlinear formulation was converted into an equivalent MILP and solved using a normalized weighted-sum approach.

The computational results reveal a trade-off between cost and delivery time: improving delivery responsiveness may require substantially higher system costs, while intermediate solutions offer alternative compromises. All evaluated 20-35-node configurations achieved zero optimality gaps for both single-objective models, whereas some larger instances terminated with positive gaps. In practice, the model can support facility selection, primary-backup assignments, and capacity planning, subject to validation against actual operating conditions.

The model assumes deterministic demand, transportation costs, distances, capacities, and speed, and excludes simultaneous depot disruptions. Uncertainty in these parameters may affect network decisions and cost-delivery-time trade-offs. The weighted-sum approach may miss unsupported nondominated solutions, and multi-objective performance indicators were not evaluated. Future research could compare alternative backup strategies and multi-objective methods, including the ε-constraint approach, using appropriate indicators. Decomposition methods and heuristics could also improve scalability for larger networks.

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