Estimation of Power Consumption by Centrifugal Pump with Reduced Impeller Size

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Abstract

Energy saving in centrifugal pumps by impeller size reduction is an efficient method. It is popular to estimate energy saving of this method by using the affinity laws. However, using the affinity laws alone to estimate energy saving without considering the characteristics of the individual pump and the pumping system may yield erroneous results. This paper is aimed at presenting a better method by using actual pump performance curves and incorporating the pumping system into the analysis. It is shown that both head and efficiency can be approximated by quadratic functions of flow rate. Together with the assumption that the system curve is also a quadratic function, the presented method is capable of computing pump power input at given values of flow rate and static head. Results from this method suggest a simple and more accurate formula for estimating energy saving in centrifugal pumps by impeller size reduction.

Key Words: pump characteristics, energy efficiency, impeller trimming, affinity laws

1. Introduction

Centrifugal pumps are essential machinery in virtually all factories and commercial buildings. Energy consumption by centrifugal pumps accounts for a substantial fraction of energy use by a factory or a building. Therefore, any effort to improve energy efficiency cannot ignore measures to improve energy efficiency of centrifugal pumps. In order for a pump to operate most efficiently, it must have the maximum efficiency at the operation point. However, it is difficult to ensure that this objective is achieved in practice because the operation of a pump also depends on the pumping system, in which some uncertainty exists. It is a standard practice to specify an oversized pump during the design stage to allow for either future expansions or unforeseen losses in the pumping system. Consequently, the oversized pump may deliver too much flow rate and require the use of a throttle valve to reduce flow rate. Such a practice results in frictional losses in the throttle valve. Furthermore, the actual operation point is different from the design operation point, which means that the actual pump efficiency is lower than the maximum efficiency.

Two well-known energy-efficient methods to reduce pump flow rate are variable-speed control and impeller diameter reduction. Compared with variable-speed control, impeller diameter reduction is cheaper and easier to implement [1]. Pump manufacturers usually supply various impeller sizes. If the required impeller size is not available, the impeller may be trimmed. A pump performance curve showing relationship between head and flow rate depends on impeller diameter. At a given flow rate, the head is reduced with the reduction of impeller diameter because the fluid velocity at discharge is lower for a

smaller impeller. Using a smaller impeller to reduce flow rate is energy efficient because the pump will operate at or near its maximum efficiency. This means that using a smaller impeller will result in energy saving compared with using a throttle vale to reduce flow rate.

Estimation of energy saving by reducing impeller size is usually done by using the affinity laws, which state that pump power input varies with the cube of impeller diameter, and flow rate varies with impeller diameter [2]. Therefore, the straightforward application of the affinity laws implies that pump power input varies with the cube of flow rate. However, the validity of this formula is questionable due to the uncertainty of the effect of impeller diameter on the pump performance curve. Furthermore, the effect of the pumping system must not be ignored. The main objective of this paper is to investigate how pump power input varies with flow rate in 6 commercial centrifugal pumps by taking into account actual pump performance data and the effect of pumping system. Results of the investigation are then used to develop a formula estimating energy saving from impeller size reduction.

2. Affinity Laws

Head *(H)* and power *(P)* of a centrifugal pump are functions of several parameters. If effects of viscosity and fluid compressibility are assumed to be negligible, the remaining parameters are flow rate *(Q)*, pump speed *(N)*, impeller outer diameter *(D)*, impeller inner diameter *(d)*, impeller width *(b)*, fluid density (ρ), and gravitational acceleration *(g)*. Dimensional analysis results in 3 dimensionless parameters: flow coefficient *(q)*, head coefficient *(h)*, and power coefficient *(p)*. Expressions of these dimensionless parameters are

$$q = \frac{Q}{\pi b d N D} \tag{1}$$

$$h = \frac{gH}{N^2 D^2} \tag{2}$$

$$p = \frac{P}{\rho \pi N^3 b d D^3} \tag{3}$$

Functional relationships among these parameters may be written as

$$h = f_2(q) \tag{4}$$

$$p = f_2(q) \tag{5}$$

Note that impeller trimming reduces D, but not d and b. Consider 2 centrifugal pumps with 2 geometrically similar impellers. Both impellers have the same inner diameter and width, but their outer diameters are D_1 and D_2. Assume that both pumps operate at the same speed. Equations (1)-(5) imply that

$$\frac{Q_1}{Q_2} = \frac{D_1}{D_2} \tag{6}$$

$$\frac{H_1}{H_2} = \left(\frac{D_1}{D_2}\right)^2 \tag{7}$$

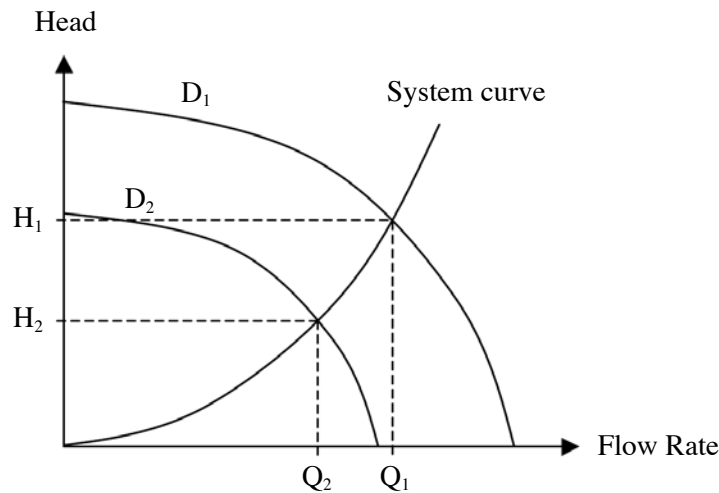$$\frac{P_1}{P_2} = \left(\frac{D_1}{D_2}\right)^3 \tag{8}$$

Fig. 1. Intersection between system curve with no static head and pump performance curves of two impeller diameters at two points.

These are known as the affinity laws for centrifugal pumps having different impeller sizes [2]. Performance curves of several pump manufactures follow these laws.

Although Eq. (8) is widely used to estimate energy saving from impeller size reduction, this estimation is good only when there is no static head in the system curve. Figure 1 shows that the operation points for cases of 2 impeller sizes are the intersection at (Q_1, H_1) between the system curve and the performance curve of the larger impeller and the intersection at (Q_2, H_2) between the system curve and the performance curve of the smaller impeller. Provided that Q_1, Q_2, D_1, H_1, and P_1 are known, it can be shown that D_2, H_2, and P_2 are correctly computed from Eqs. (6)-(8), respectively, if the system curve is a parabola. However, an actual pumping system may have a static head due to the need to transport fluid to a higher elevation or the requirement of the differential pressure setting at zero flow rate [3]. Static head will cause an error in determining D_2, H_2, and P_2 by using the affinity laws alone. In this case, a more detailed analysis considering the effect of the system curve must be performed, which will result in larger values of D_2, H_2, and P_2 than the values obtained from Eqs. (6)-(8).

Furthermore, there is some uncertainty as to whether Eqs. (6) and (8) are applicable for all centrifugal pumps. Recently, Savar et al. [4] performed an experiment on impeller trimming, in which pump heads and flow rates of a centrifugal pump were measured for each of 7 impeller diameters, and used the results to draw a performance curve for each impeller diameter. It can be shown that if each data point (Q, H) in performance curves is converted to $(Q/D^2, H/D^2)$, the curve fitting of all converted data points results in a parabola. This suggests that, for the pump used in this experiment, the flow coefficient in Eq. (1) should be revised as follows.

$$q = \frac{Q}{\pi b N D^2} \tag{9}$$

As a result, Eqs. (6) and (8) should be revised to, respectively

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^2 \tag{10}$$

and

$$\frac{P_1}{P_2} = \left(\frac{D_1}{D_2}\right)^4 \tag{11}$$

Therefore, it is evident that there is no unique formula for computing energy saving from impeller size reduction. Sufficient details of characteristics of the pump and the pumping system are required for the derivation of such a formula. Since there are many parameters involved in the pump performance and the pumping system, one may expect a formula to be quite complicated. However, it will be shown in subsequent sections that, with some assumptions, a simple formula can be derived.

3.   Pump Performance Curves

Pump manufacturers have tested their pumps extensively, and used test results to construct pump performance curves similar to Fig. 2. These curves show how head varies with flow rate for different impeller diameters. Also shown are iso-efficiency lines for different impeller diameters. Pump performance curves are available either on-line or upon request. For the purpose of this case study, pump performance curves from the following 5 manufacturers are chosen because of their on-line availability.

- Aurora (http://www.aurorapump.com)
- Bell & Gossett (http://www.bellgossett.com)
- Goulds Pumps (http://www.gouldspumps.com)
- KSB (http://www.ksb.com)
- GEA Tuchenhagen (http://www.tuchenhagen.com)

In addition, pump performance curves can be constructed from the experimental data obtained by Savar et al. [4], and are also included in this study. Table 1 gives details of the 6 centrifugal pumps. It should be noted that Savar et al. [4] do not give the pump speed explicitly. The value of 2900 rpm can be inferred from their data. Although more performance curves of other impeller diameters may be available from the manufacturers and Savar et al. [4], but only performance curves of impeller diameters listed in Table 1 are chosen for this study because it has been suggested that trimming should be limited to about 75% of the maximum impeller diameter [5].
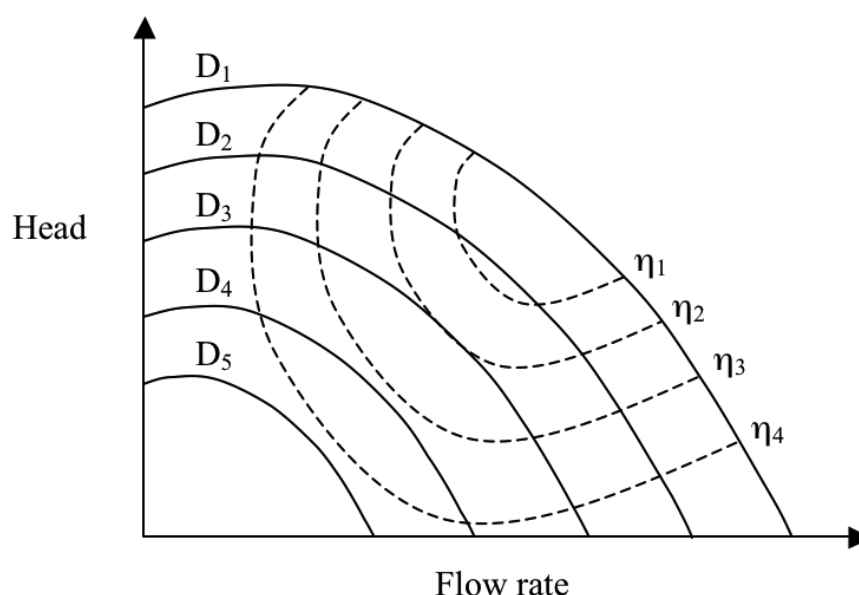

Fig. 2. Typical pump performance curves supplied by pump manufacturers.

Table 1. Details of 6 centrifugal pumps chosen for this study.

Pump Number	Source	Model	Impeller Sizes (m)	Speed (rpm)
1	Aurora	410 Size 2x2 1/2x10	0.241, 0.216, 0.191	2880
2	Bell & Gossett	4x6x10M HSC	0.305, 0.287, 0.269, 0.251, 0.234	3565
3	Goulds Pumps	3656/3756 S Group	0.151, 0.143, 0.130, 0.121	2900
4	KSB	Elite E32-13	0.139, 0.130, 0.120	2900
5	GEA Tuchenhagen	TP 1020	0.130, 0.120, 0.110	2900
6	Savar et al. [4]	N/A	0.190, 0.180, 0.170, 0.160	2900

The analysis of energy saving from impeller size reduction is simplified if it is assumed that the affinity laws are obeyed by all pumps in Table 1. However, it is uncertain whether Eq. (6) or Eq. (10) should be used. Therefore, it is proposed that this affinity law should be revised as follows.

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^k \tag{12}$$

where the affinity-law exponent (k) is between 1 and 2. The experiment by Savar et al. [4] revealed that $k = 2$ for the pump under testing, whereas most pump manufacturers claim that $k = 1$ for their pumps. It is interesting to note that k is also equal to 1 in the affinity laws for centrifugal fans. Recently, Chunxi et al. [6] carried out an experiment to investigate the performance of a centrifugal fan with 3 impeller diameters. They found that k was approximately equal to 1 when the impeller diameter was increased by 10% and 20%.

The procedure for determining the suitable value of affinity-law exponent of each pump in Table 1 begins with the extraction of data points from each pump performance curve, as illustrated in Fig. 3. Let D_1 be the largest impeller diameter for each pump in Table 1. Each data point (Q, H) is converted to ($Q(D_1/D)^k$, $H(D_1/D)^2$), and placed on a new plot. Benier and Lemire [3], Chantasiriwan [7], and Ulanicki et al. [8] suggest that the variation of head with flow rate may be approximated as a quadratic function. Their suggestion is followed here.
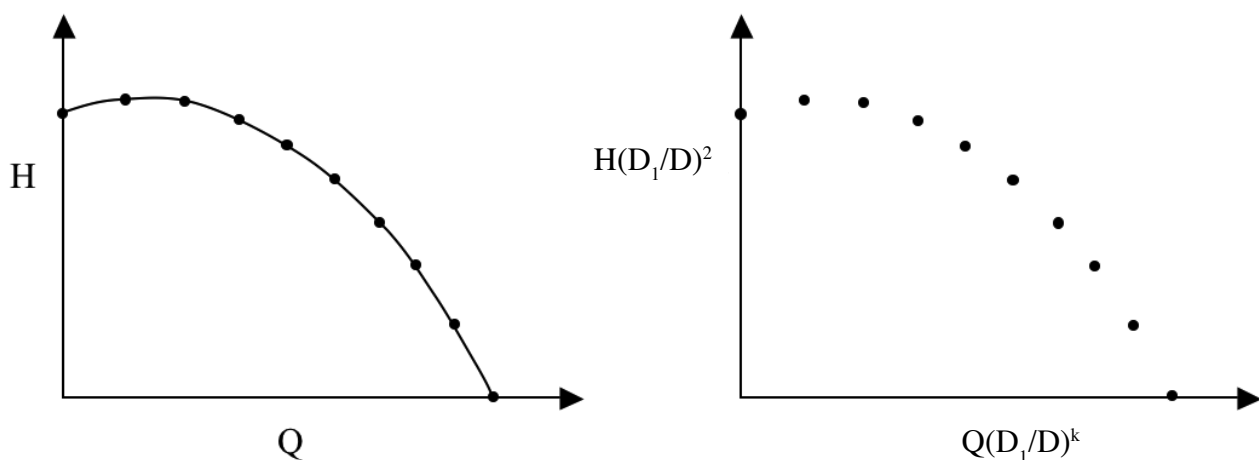

Fig. 3. Data extraction of pump performance curves.

Table 2. Values of coefficients in Eq. (14) from curve fitting.

Pump Number	k	a_1	a_2	a_3	r
1	1.5	-0.0074	0.3498	69.35	0.989
2	1.0	-0.0008	0.0736	186.5	0.992
3	1.5	-0.0198	0.2158	30.24	0.988
4	1.5	-0.0246	0.1822	24.57	0.993
5	1.0	-0.0444	0.1482	23.01	0.995
6	2.0	-0.0067	-0.1998	40.80	0.992

$$H\left(\frac{D_1}{D_2}\right)^2 = a_1 Q^2\left(\frac{D_1}{D}\right)^{2k} + a_2 Q\left(\frac{D_1}{D}\right)^k + a_3 \tag{13}$$

Numerical values of a_1, a_2 and a_3 in Eq. (13) can be determined by curve fitting of all converted data points if k is given. Although it is possible to search for the optimum value of k that yields the maximum correlation coefficient (r) using several values of k, it is decided that there is no need for this. Only three values of k are considered: 1, 1.5, and 2. It is found that k equals 1.5 for pumps #1, #3 and #4; k equals 1 for pump #2 and #5; and k equals 2 for pump #6. Table 2 lists values of k, a_1, a_2 and a_3 for the 6 pumps along with correlation coefficients.

The assumption that the affinity laws are obeyed by all pumps in Table 1 also implies that there exist a functional relationship between efficiency (η) and $Q(D_1/D)^k$. Chantasiriwan [7] and Ulanicki et al. [8] suggest that efficiency should be approximated as a cubic function of flow rate because efficiency must be zero at zero flow rate and maximum flow rate in addition to being maximum at the optimum or design flow rate. However, the range of pump efficiency of interest is usually limited to the range near the maximum efficiency. It can be shown that, in this range, the quadratic approximation is quite satisfactory. Therefore, it can be assumed that

$$\eta = b_1 Q^2\left(\frac{D_1}{D_2}\right)^{2k} + b_2 Q\left(\frac{D_1}{D}\right)^k + b_3 \tag{14}$$

Once affinity-law exponents have been found, the procedure used to find a_1, a_2 and a_3 can also be used to find b_1, b_2 and b_3. Data points are extracted from iso-efficiency curves of impeller diameter D. Each data point (Q, η) is then converted to $(Q(D_1/D)^k, \eta)$ with k fixed at the value obtained from Table 2, and placed on a new plot as shown in Fig. 4. Curve fitting is then performed for each pump to determine b_1, b_2 and b_3. Table 3 lists values of b_1, b_2 and b_3 for the 6 pumps along with correlation coefficients.

Table 3. Values of coefficients in Eq. (15) from curve fitting.

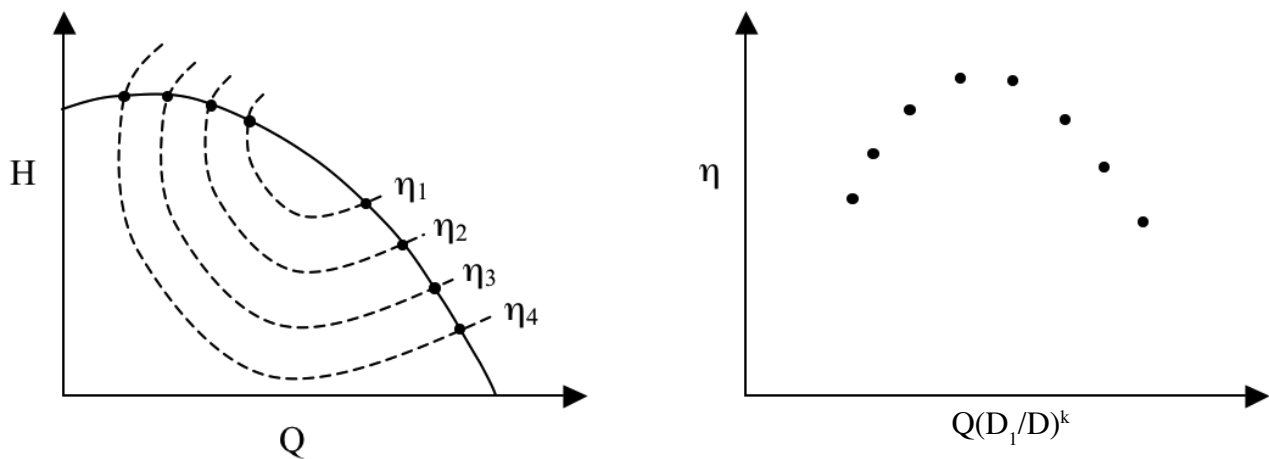| Pump Number	b_1	b_2	b_3	r
1	-0.0169	2.0838	6.636	0.990
2	-0.0009	0.4654	23.06	0.978
3	-0.1195	5.552	4.637	0.966
4	-0.1411	5.159	7.573	0.926
5	-0.3110	7.446	4.068	0.948
6	-0.0512	3.225	7.237	0.982

Fig. 4. Data extraction of iso-efficiency curves.

4. Model of Pump Power Input at Reduced Impeller Size

It is assumed that, originally, a pump is designed to operate with the largest impeller (D_1) at the design point (Q_D, H_D), where efficiency (η_D) is maximum. The expression for Q_D can be found by maximizing η in Eq. (14).

$$Q_D = \frac{-b^2}{2b_1} \tag{15}$$

Next, H_D and η_D are determined by inserting Q_D into Eqs. (13) and (14), respectively.

$$H_D = a_1 Q_D^2 + a_2 Q_D + a_3 \tag{16}$$

$$\eta_D = b_1 Q_D^2 + b_2 Q_D + b_3 \tag{17}$$

The power input at the design point can be computed from

$$P_D = \frac{\rho g Q_D H_D}{\eta_D} \tag{18}$$

where ρ is the fluid density. Numerical values of Q_D, H_D, η_D and P_D for the 6 pumps are shown in Table 4.

Table 4. Flow rate, head, efficiency, and power requirement for 6 pumps at their design operation points.

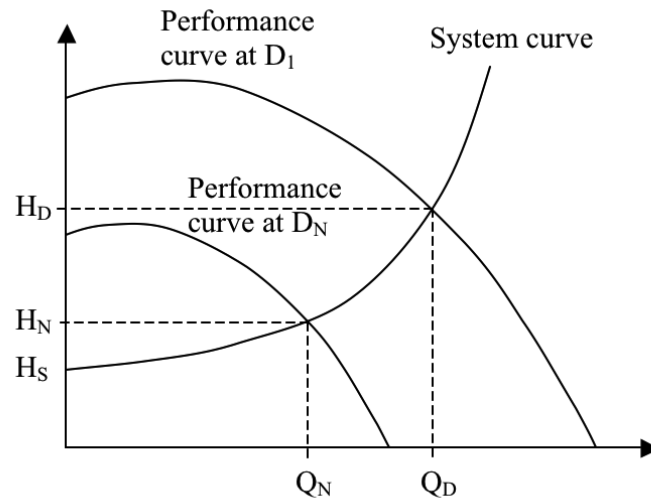| Pump Number	Q_D (m³/h)	H_D (m)	η_D (%)	P_D (kW)
1	61.65	62.79	70.87	14.88
2	258.6	152.0	83.23	128.7
3	23.23	24.57	69.12	2.250
4	18.28	19.68	54.73	1.791
5	11.97	18.42	48.64	1.236
6	31.49	27.86	58.02	4.121

Fig. 5. Using reduced impeller size to decrease the design flow rate (Q_D) to a new flow rate (Q_N).

The design flow rate is now assumed to be too high, and it is desired to reduce flow rate from Q_D to Q_N by using a smaller impeller. This situation is illustrated in Fig. 5. In order to find the impeller size that gives the desired flow rate, it is necessary to make an assumption about the system curve. It has been suggested that the system curve should be approximated by a parabola. This suggestion is followed in this paper, and the system equation is assumed to be

$$H = KQ^2 + H_S \tag{19}$$

where K depends on the amount of friction in the system, and H_S is the static head of the system. The static head is considered to be a free parameter, whereas the K can be determined from the fact that the design point (Q_D, H_D) must lie on the system curve.

$$K = \frac{(H_D - H_s)}{Q_D^2} \tag{20}$$

Given the desired flow rate (Q_N), pump head at the new operation point can be computed from Eq. (19).

$$H_N = KQ_N^2 + H_s \tag{21}$$

Replacing Q, H and D in Eq. (14) by Q_N, H_N and D_N, respectively, results in an equation that can be solved for D_N.

$$H_N \left(\frac{D_1}{D_N}\right)^2 = a_1 Q_N^2 \left(\frac{D_1}{D_N}\right)^{2k} + a_2 Q_N \left(\frac{D_1}{D_N}\right)^k + a_3 \tag{22}$$

It should be noted that if $k = 1$, Eq. (22) becomes a quadratic equation, and the formula for finding its roots is well known. However, Eq. (22) becomes a nonlinear equation without a known formula for finding its roots if $k = 1.5$ or 2, and an iterative method must be used to find the solution. First a guess of the value of D_N/D_1 (which is less than 1) must be made. Then the updated value of D_N/D_1 is computed from the rearranged expression from Eq. (22).

$$\frac{D_N}{D_1} = \sqrt{\frac{[H_N - a_1 Q_N^2 (D_N/D_1)^{2-2k} - a_2 Q_N (D_N/D_1)^{2-k}]}{a_3}} \tag{23}$$

The iterative process is then repeated until there is a negligible change in the value of D_N/D_1. Once D_N has been found, pump power input at the new operation point can be computed from

$$P_N = \frac{PgQ_NH_N}{\eta_N} \tag{24}$$

where

$$\eta_N = b_1 Q_N^2 \left(\frac{D_1}{D_N}\right)^{2k} + b_2 Q_N \left(\frac{D_1}{D_N}\right)^{k} + b_3 \tag{25}$$

For each pump in Table 1, computation of D_N/D_1 and P_N/P_D can now be carried out if values of Q_N/Q_D and H_S/H_D are given. However, D_N/D_1 is restricted to be larger than or equal to the ratio of the minimum impeller diameter and the maximum impeller diameter shown in Table 1 because results corresponding to smaller D_N/D_1 are not supported by available pump performance data.

5.   Results and Discussion

For pump #1, the maximum and minimum impeller diameters in Table 1 are 0.241 m and 0.191 m. Therefore, the range of D_N/D_1 for which the affinity laws are applicable is expected to be between 0.79 and 1.00. Figure 6 plots variations of D_N/D_1 with Q_N/Q_D for 6 values of H_S/H_D within this range. It can be seen that D_N/D_1 decreases monotonically with Q_N/Q_D and H_S/H_D. Further inspection of Fig. 6 reveals that D_N/D_1 varies linearly with Q_N/Q_D if H_S/H_D is held constant, and varies linearly with H_S/H_D if Q_N/Q_D is held constant. Therefore, there should be the following bi-linear relationship:

$$\frac{D_N}{D_1} = c_1 \left(\frac{Q_N}{Q_D}\right)\left(\frac{H_s}{H_D}\right) + c_2 \left(\frac{Q_N}{Q_D}\right) + c_3 \left(\frac{H_s}{H_D}\right) + c_4 \tag{26}$$

where c_1, c_2, c_3 and c_4 are determined by curve fitting procedure. Similar results can be obtained for pumps #2 – #6. Values of c_1, c_2, c_3 and c_4 are shown in Table 5. It can be seen that the affinity law from Eq. (6) is applicable to pumps #2 and #5 only when $H_S = 0$.
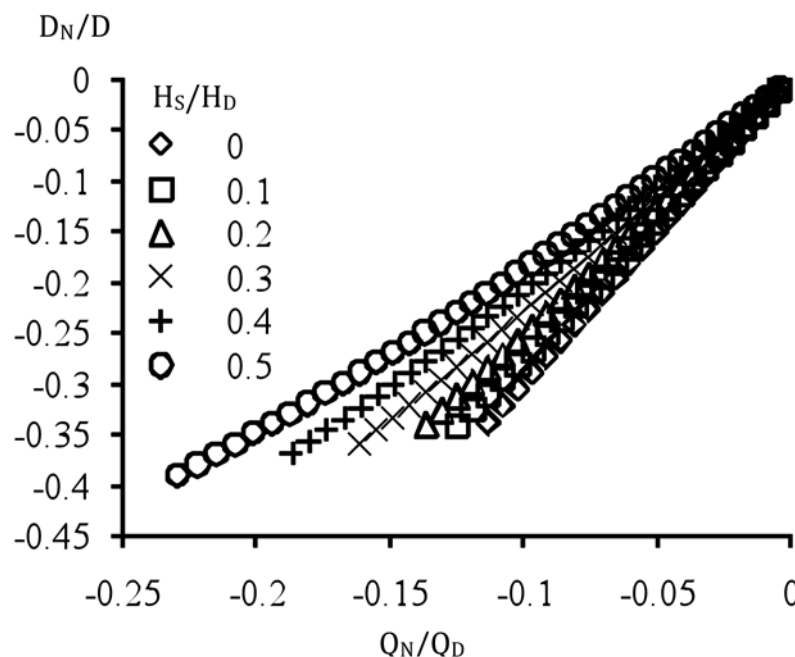

Fig. 6. Variations of impeller diameter of pump #1 with flow rate for 6 values of H_S/H_D.

Table 5. Coefficients of the bi-linear relationship in Eq. (27) for 6 pumps.

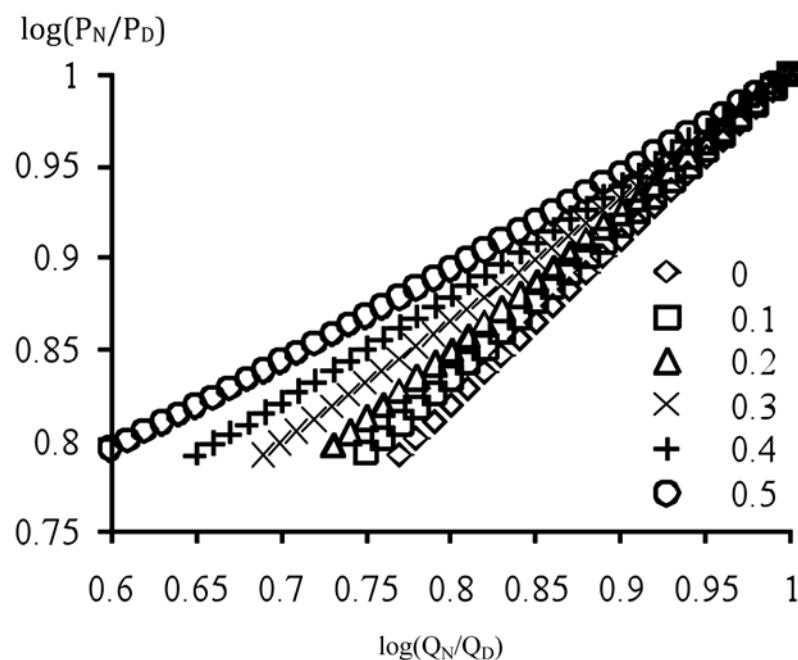| Pump Number	c_1	c_2	c_3	c_4
1	-0.783	0.906	0.778	0.094
2	-0.917	1.000	0.901	0
3	-0.729	0.894	0.725	0.107
4	-0.703	0.891	0.703	0.109
5	-0.843	1.000	0.840	0
6	-0.604	0.794	0.601	0.207

Fig. 7. Variations of power input of pump #1 with flow rate for 6 values of H_S/H_D.

The affinity laws suggest that there is a functional relationship between $\log (P_N/P_D)$ and $\log (Q_N/Q_D)$. However, this relationship must also depend on H_S in an actual pumping system. Figure 7 plots variations of D_N/D_1 with Q_N/Q_D for 6 values of H_S/H_D for pump #1. Similar results can be obtained for the other 5 pumps. Inspection of Fig. 7 reveals that, at a given value of H_S/H_D, the curve fitting of the data points results in a parabola. Hence,

$$\log\left(\frac{P_N}{P_D}\right) = d_1\left[\log\left(\frac{Q_N}{Q_D}\right)\right]^2 + d_2\log\left(\frac{Q_N}{Q_D}\right) \tag{27}$$

where d_1 and d_2 depend on H_S/H_D. It is found that there is a linear relationship between d_2 and H_S/H_D.

$$d_2 = 3\text{-}2\left(\frac{H_S}{H_D}\right)^2 \tag{28}$$

Table 6. Coefficients of the quadratic relationship in Eq. (30) for 6 pumps.

Pump Number	e_1	e_2	e_3
1	-3.004	3.635	0.260
2	-6.076	5.962	0
3	-3.305	3.776	0.238
4	-3.200	3.681	0.222
5	-4.537	5.360	0
6	-1.982	1.901	0.693

However, the relationship between d_1 and H_S/H_D turns out to be quadratic.

$$d_1 = e_1 \left(\frac{H_S}{H_D}\right)^2 + e_2 \left(\frac{H_S}{H_D}\right) + e_3 \tag{29}$$

Values e_1, e_2 and e_3 are determined by curve fitting procedure, and shown in Table 6 for the 6 pumps.

Equation (27) is quite a complicated formula. Its use is inconvenient due to the requirement of many coefficients. A simpler formula is obtained when linear curve fitting is performed on curves in Fig. 7. The resulting formula is

$$\left(\frac{P_N}{P_D}\right) = \left(\frac{Q_N}{Q_D}\right)^{3-\beta(H_S/H_D)} \tag{30}$$

where β is a parameter that depends on pump. Table 7 gives the values of β for the 6 pumps. It can be seen that Eq. (31) gives a lower estimate of energy saving by impeller size reduction than the straightforward application of the affinity laws. Comparison between P_N/P_D from Eq. (30) and P_N/P_D obtained from the computational procedure in Section 4 indicates that the difference is less 3% for each pump. Therefore, Eq. (30) should be considered as a good alternative formula for estimating energy saving by impeller size

Table 7. Coefficients of the linear relationship in Eq. (32) for 6 pumps.

Pump Number	β
1	2.428
2	2.520
3	2.391
4	2.246
5	2.290
6	2.302

reduction.

6.   Conclusion

A method for determining power input for a centrifugal pump having an arbitrary impeller size and operating in a system with a given static head is presented. Head and efficiency are assumed to be quadratic functions of flow rate. These assumptions are supported by actual pump performance curves of 6 pumps. Results from this method is used to develop a simple formula for estimating energy saving from impeller size reduction that is more accurate than a formula derived from the affinity laws without considering individual pump and system characteristics. Although results presented in this paper are limited for the 6 selected pumps and some selected values of static head, application of this method to another pump in a system with an arbitrary static head is possible. In fact, this method is recommended to pump manufacturers in constructing additional charts or formulas showing how much energy saving can be expected from using smaller impeller diameters in their pumps. Such charts or formulas will give a more realistic estimate of energy saving than assuming that pump power input varies with the cube of flow rate.

References

[1]   Singh, G., and Mitchell, J. W., Energy Savings from Pump Impeller Trimming, ASHRAE Journal, Vol. 40, pp. 60-63, 1998.

[2]   Karassik, I. J., Messina, P., Cooper, and P., Heald, C. C. (Eds.), Pump Handbook, McGraw-Hill, New York, 2001.

[3]   Bernier, M., and Lemire, N., Non-dimensional Pumping Power Curves for Water Loop Heat Pump Systems, ASHRAE Transactions, Vol. 105 Part 2, pp. 1226-1232, 1999.

[4]   Savar, M., Kozmar, H., Sutlovic, I., Improving Centrifugal Pump Efficiency by Impeller Trimming, Desalination, Vol. 249, pp. 654-659, 2009.

[5]   Nelik, L., Centrifugal and Rotary Pumps: Fundamentals with Applications, CRC Press, Boca Raton, 1999.

[6]   Chunxi, L., Ling, W. S., Yakui, J., The Performance of a Centrifugal Fan with Enlarged Impeller, Energy Conversion and Management, Vol. 52, pp. 2902-2910, 2011.

[7]   Chantasiriwan, S., A More Accurate Method of Estimating Energy Saving by Variablespeed Control of Centrifugal Pump, KMUTT Research and Development Journal, Vol. 32, pp. 203-210, 2009.

[8]   Ulanicki, B., Kahler, J., and Coulbeck, B., Modeling the Efficiency and Power Characteristics of a Pump Group, Journal of Water Resources Planning and Management, Vol. 134, pp. 88- 93, 2008.