Work Stress Effects on MHD Natural Convection Flow Along a Sphere

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Abstract

In this analysis, the pressure work and viscous dissipation effects with magneto- hydrodynamic (MHD) natural convection flow along a sphere have been described. The laminar natural convection flow from a sphere immersed in a viscous incompressible fluid in the presence of a magnetic field has been investigated. The governing boundary layer equations are first transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations are then solved numerically using a very efficient finite-difference method with the Keller-box scheme. Here we have focused our attention on the evaluation of the shear stress in terms of local skin friction and the rate of heat transfer in terms of local Nusselt number, velocity profiles as well as temperature profiles for some selected values of parameters set consisting of magnetic parameter M, pressure work parameter Ge, viscous dissipation parameter N and the Prandlt number Pr.

Keywords: Natural convection, viscous dissipation, Pressure work and MHD.

1. Introduction

A study of the flow of electrically conducting fluid in the presence of a magnetic field is important from a technical point of view and such types of problems have received much attention by many researchers. The specific problem selected for study is the flow and heat transfer in an electrically conducting fluid adjacent to the surface on a sphere. The surface is maintained at a uniform temperature T_w, which may either exceed the ambient temperature T_∞ or may be less then T_∞. When $T_w > T_\infty$, an upward flow is established along the surface due to free convection; while when $T_w < T_\infty$, there is a down flow. Additionally, a magnetic field of strength β_0 acts normal to the surface. The interaction of the magnetic field and the moving electric charge carried by the flowing fluid induces a force, which tends to oppose the fluid motioning edge. The velocity is very small so that the magnetic force, which is proportional to the magnitude of the longitudinal velocity and acts in the opposite direction, is also very small. Consequently, the influence of the magnetic field on the boundary layer is exerted only through induced forces within the boundary layer itself, with no additional effects arising from the free stream pressure gradient. The influence and importance of viscous sworic tress work effects in laminar flows have been examined by Gebhart [1] and Gebhart and Mollendorf [2]. In both of the investigations, special flows over semi-infinite flat surfaces parallel to the direction of body force were considered. Gebhart [1] considered the flow generated by the plate surface temperature, which varies as powers of ξ (the distance along the plate surface from the leading edge), Gebhart and Mollendorf [2] considered the plate temperature varying exponentially with ξ. Ackroyd [3] studied the stress work effects in laminar flat plate natural convection flow. Joshi and Gebhart [4] investigated the effect of pressure work stress and viscous dissipation in some natural convection flows. Kuiken [5] studied the problem of magneto-hydrodynamic free convection in a strong cross-field. Also the effect of magnetic field on the free convection heat transfer has been studied by Sparrow and Cess [6]. MHD free convection flows of visco-elastic fluid past an infinite porous plate have been investigated by Chowdhury and Islam [7]. Raptis and Kafousias [8] have investigated the problem of magnetohydrodynamic free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Elbashbeshy [9] also discussed the effect of free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of a magnetic field. Hossain [10] introduced the viscous and Joule heating effects on MHD-free convection flow with variable plate temperature. Moreover, Hossain *et al.* [11], Hossain and Ahmed [12] and Hossain *et al.* [13] discussed the forced and free convection boundary layer flow of an electrically conducting fluid in the presence of a magnetic field. The problems of free convection boundary layer flow over or on bodies of various shapes has been discussed by many researchers. Amongst them Nazar *et al.* [14], considered the free convection boundary layer on an isothermal sphere and on an isothermal horizontal circular cylinder in a micropolar fluid. To our knowledge, stress work effects on magnetohydrodynamics free convection flow from an isothermal sphere has not been studied yet, and the present work demonstrates this issue.

The present work considers the natural convection boundary layer flow on a sphere of an electrically conducting and steady viscous in compressible fluid in the presence of a strong magnetic field and the pressure work stress. The governing partial differential equations are reduced to locally non-similar partial differential forms by adopting appropriate transformations. The transformed boundary layer equations are solved numerically using an implicit finite difference scheme together with the Keller box technique. Here we have focused our attention on the evolution of the surface shear stress in terms of local skin friction and the rate of heat transfer in terms of local Nusselt number, velocity distribution as well as temperature distribution, for a selection of parameters consisting of viscous dissipation parameter N, the pressure work parameter Ge, the magnetic parameter M and the Prandtl number Pr.

2. Formulation of the problems

Natural convection boundary layer flow on a sphere of an electrically conducting and steady two-dimensional viscous incompressible fluid in the presence of a strong magnetic field is considered. It is assumed that the surface temperature of the sphere is T_w, where $T_w > T_\infty$, T_∞ is the ambient temperature of the fluid. Under the usual Boussinesq and boundary layer approximation, the governing equations are:

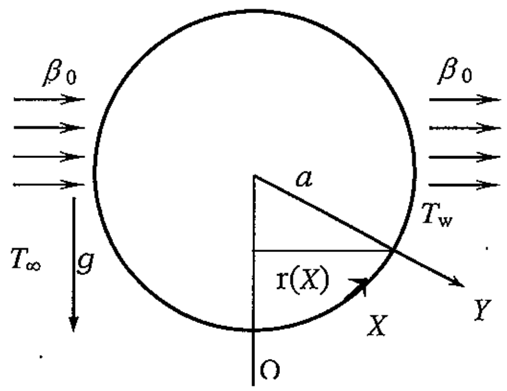

Fig 1. Physical model and coordinate system

$$\frac{\partial}{\partial X}(rU)+\frac{\partial}{\partial Y}(rV)=0 \quad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\nu\frac{\partial^2 U}{\partial Y^2} +g\beta(T-T_\infty)\sin\left(\frac{X}{a}\right)-\frac{\sigma_0\beta_0^2}{\rho}U \quad (2)$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial Y^2} +\frac{T\beta}{\rho C_P}U\frac{\partial p}{\partial X}+\frac{\nu}{\rho C_P}\left(\frac{\partial U}{\partial Y}\right)^2 \quad (3)$$

We know for hydrostatic pressure, $\partial P/\partial X=\rho g$. The boundary conditions for equations (2) to (3) are:

$$U=0, V=0, T=T_w \text{ at } Y=0$$
$$U=0,\ T=T_\infty \text{ as } Y\to\infty \quad (4)$$

$$\text{where } r=a\sin\left(\frac{X}{a}\right), \text{ where } r=r(X) \quad (5)$$

$r(X)$ is the radial distance from the symmetrical axis to the surface of the sphere, g is the acceleration due to gravity, β is the coefficient of thermal expansion, ν is the kinematics viscosity, T is the local temperature, and C_P is the specific heat at constant pressure. To transform the above equations into non-dimensional equations, the following dimensionless variables are introduced:

$$x=\frac{X}{a},\ y=G_r^{1/4}\frac{Y}{a},\ u=\frac{a}{\nu}G_r^{-1/2}U,$$
$$v=\frac{a}{\nu}G_r^{-1/4}V;\ \theta=\frac{T-T_\infty}{T_w-T_\infty} \quad (6)$$

Where $G_r=g\beta(T_w-T_\infty)a^3/\nu^2$ is the Grashof number and θ is the non dimensional temperature.

Thus (5) becomes $r(x)=a\sin x$ (7)

Using the above values, the equations (1) to (3) take the following form:

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial y}(rv)=0 \quad (8)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+\theta\sin x-\frac{\sigma_0\beta^2 a^2}{\rho\nu Gr^{1/2}}u \quad (9)$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+\frac{Gr}{a^2C_P(T_w-T_\infty)}\left(\frac{\partial u}{\partial y}\right)^2-\left(\frac{T_\infty}{T_w-T_\infty}+\theta\right)\frac{g\beta a}{C_P}u \quad (10)$$

Where, $M=\left(\sigma_0\beta^2a^2/\rho\nu Gr^{1/2}\right)$ is the magnetic parameter, $Gr/a^2C_P(T_w-T_\infty)=N$, is the viscous dissipation parameter, $Ge=(g\beta a)/C_P$, which is the pressure work parameter. Therefore momentum and energy equations (9) and (10) can be written as:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2} +\theta\sin x-Mu \quad (11)$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{p_r}\frac{\partial^2\theta}{\partial y^2} +N\left(\frac{\partial u}{\partial y}\right)^2-Ge\left(\frac{T_\infty}{T_w-T_\infty}+\theta\right)u \quad (12)$$

The boundary conditions associated with equations (4) are:

$$u=v=0,\ \theta=1 \text{ at } y=0$$
$$u\to 0,\ \theta\to 0 \text{ as } y\to\infty \quad (13)$$

To solve equations (11) and (12) subject to the boundary conditions (13), we assume the following variables u and v where $\psi(x,y)=xr(x)f(x,y)$, $\psi(x,y)$ is a non-dimensional stream function, which is related to the velocity components in the usual way as:

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y} \quad \text{and} \quad v=-\frac{1}{r}\frac{\partial\psi}{\partial x} \tag{14}$$

$$\begin{aligned} &u=x(\partial f/\partial y),\\ &(\partial^2 u/\partial y^2)=x(\partial^3 f/\partial y^3)\\ &v=-\begin{bmatrix}(1+x\cos x/\sin x)f(x,y)\\ +x(\partial f/\partial x)\end{bmatrix}\\ &(\partial u/\partial x)=(\partial f/\partial y)+x(\partial^2 f/\partial x\partial y)\end{aligned} \tag{15}$$

Using the above transformed values in equations (11) and (12) and simplifying, we have the following:

$$\begin{aligned} &\frac{\partial^3 f}{\partial y^3}+\left(1+\frac{x}{\sin x}\cos x\right)f\frac{\partial^2 f}{\partial y^2}\\ &-\left(\frac{\partial f}{\partial y}\right)^2+\frac{\theta}{x}\sin x- M\frac{\partial f}{\partial y}\\ &=x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y\partial x}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right)\end{aligned} \tag{16}$$

$$\begin{aligned} &\frac{1}{p_r}\frac{\partial^2\theta}{\partial y^2}+\left(1+\frac{x}{\sin x}\cos x\right)f\frac{\partial\theta}{\partial y}\\ &+Nx^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2-Ge\left(\frac{T_\infty}{T_w-T_\infty}+\theta\right)\frac{\partial f}{\partial y}\\ &=x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}\right)\end{aligned} \tag{17}$$

The corresponding boundary conditions are:

$$\begin{aligned} &f=\frac{\partial f}{\partial y}=0\ ,\ \theta=1 \text{ at } y=0\\ &\frac{\partial f}{\partial y}\to 0\ ,\ \theta\to 0 \quad \text{as } y\to\infty\end{aligned} \tag{18}$$

It has been found that near the lower stagnation point of the sphere i.e. $x \approx 0$ or considering the limiting value as $x \to 0$, equations (16) and (17) reduce to the following ordinary differential equations:

$$\begin{aligned} &\frac{d^3 f}{dy^3}+2f\frac{d^2 f}{dy^2}-\left(\frac{df}{dy}\right)^2\\ &+\theta-M\frac{df}{dy}=0\end{aligned} \tag{19}$$

and

$$\begin{aligned} &\frac{1}{p_r}\frac{d^2\theta}{dy^2}+2f\frac{d\theta}{dy}-\\ &Ge\left(\frac{T_\infty}{T_w-T_\infty}+\theta\right)\frac{\partial f}{\partial y}=0\end{aligned} \tag{20}$$

with the boundary conditions:

$$\left.\begin{aligned} &f=\frac{\partial f}{\partial y}=0,\ \theta=1\ at\ y=0\\ &\frac{\partial f}{\partial y}\to 0,\ \theta\to 0\ as\ y\to\infty\end{aligned}\right\} \tag{21}$$

In practical applications, the physical quantities of principal interest are the heat transfer and the skin- friction coefficient, which can be written in non-dimensional form as:

$$\begin{aligned} &Nu_X=\frac{aGr^{-1/4}}{k(T_w-T_\infty)}q_w \text{ and}\\ &C_{fX}=\frac{Gr^{-3/4}a^2}{\mu\nu}\tau_w\end{aligned} \tag{22}$$

Where $q_w=-k\left(\frac{\partial T}{\partial Y}\right)_{Y=0}$ and $\tau_w=\mu\left(\frac{\partial U}{\partial Y}\right)_{Y=0}$, k being the thermal conductivity of the fluid. Using the new variables (6), we have:

$$Nu_x = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \qquad (23)$$

$$C_{fx} = x\left(\frac{\partial^2 f}{\partial y^2}\right)_{y=0} \qquad (24)$$

3. Results and Discussion

Here we have investigated the effects of pressure work and viscous dissipation with magnetohydrodynamic natural convection flow on a sphere. Solutions are obtained for the fluid having Prandtl number Pr = 1.00, 1.74, 2.00, 3.00, viscous dissipation parameter N = 0.10, 0.30, 0.50, 0.70, 1.00, pressure work parameter Ge = 0.10, 0.40, 0.70, 0.90 against y at any position of x and for a wide range of values of magnetic parameter M. Also the results for local skin friction coefficient and local rate of heat transfer have been obtained for fluids having Prandtl number Pr = 1.00, 1.74, 2.00 3.00 and pressure work parameter Ge = 0.10, 0.40, 0.70, 0.90 at different positions of x for a wide range of values of magnetic parameter M. Here it is found that from Fig. 2(a), velocity increases as the values of viscous dissipation parameter N increases in the region $y \in$ [0, 12], but near the surface of the sphere velocity increases significantly and then decreases slowly and finally approaches zero.

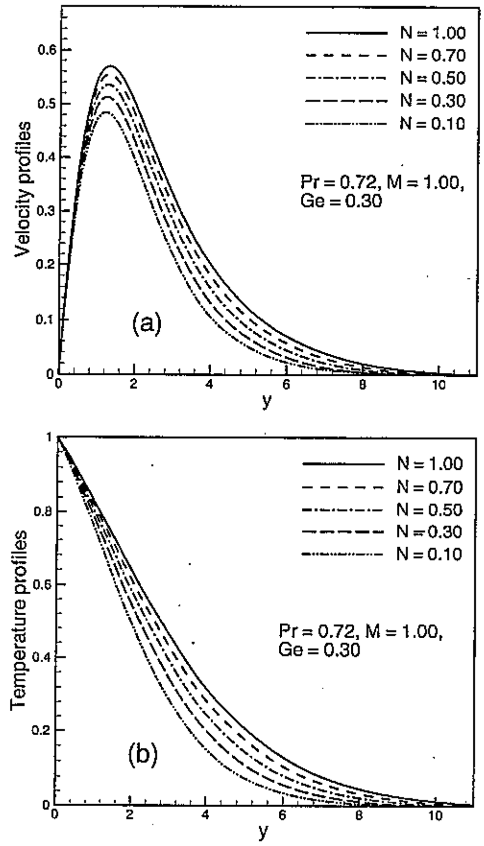

Fig 2. (a) Velocity and (b) temperature profiles for different values of viscous dissipation parameter N with fixed values of other parameters.

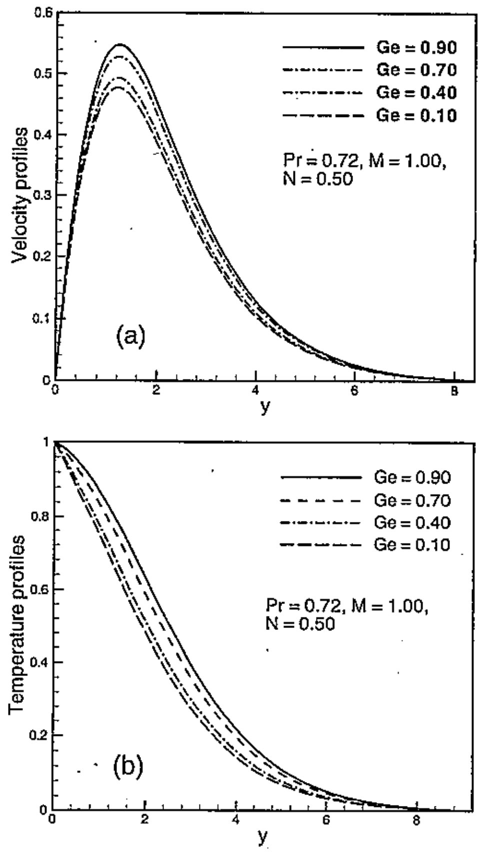

Fig 3. (a) Velocity and (b) temperature profiles for different values of pressure work parameter Ge with fixed values of other parameters.

The maximum values of the velocity are 0.48450, 0.51282, 0.53527, 0.55384 and 0.56949 for N = 0.10, 0.30, 0.50, 0.70 and 1.00, respectively, which occur at y = 1.23788 for first, second and third maximum values, y = 1.30254 for fourth and fifth maximum values. Here it is observed that the velocity increases by 17.54 % as N increases from 0.10 to1.00. From Fig. 2(b), it is seen that when the values of viscous dissipation parameter N increases in the region $y \in$ [0, 12], the temperature also increases.

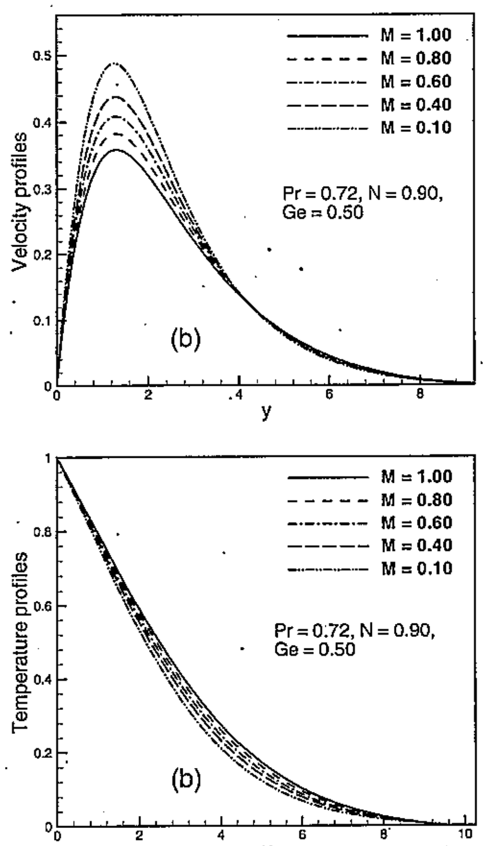

Fig 4. (a) Velocity and (b) temperature profiles for different values of magnetic parameter M.

We also observed that the maximum temperature has been found at the surface on the sphere. Here it is found from Fig. 3(a) that the velocity increases slightly as the pressure work parameter Ge increases in the region $y \in$ [0, 9], but near the surface of the sphere velocity increases and becomes maximum and then decreases, and at a certain point velocity profiles coincide and finally approach zero. The maximum values of the velocity are 0.43056, 0.44332, 0.47589 and 0.53463 for Ge = 0.10, 0.40, 0.70 and 0.90, respectively, which occur at y = 1.17520 for first, second and third maximum values and at y = 1.23788 for the last maximum values. Here we see that the velocity increases by 24.17 % as Ge increases from 0.10 to 0.90.

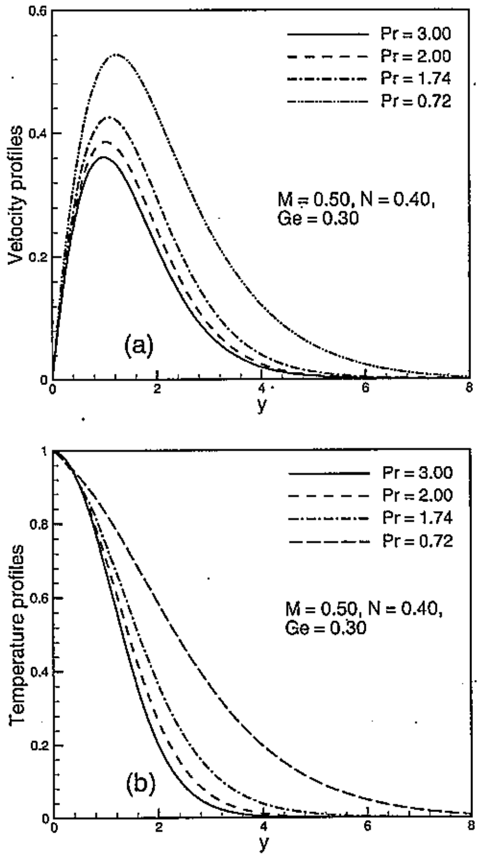

Fig 5. (a) Velocity and (b) temperature profiles for different values of Prandtl number Pr.

From Fig. 3(b), it is seen that when the values of pressure work parameter Ge increase in the region $y \in$ [0, 9], the temperature also increases. Near the surface of the sphere temperature is maximum and then decreases away from the surface and finally takes an asymptotic value. The effects of magnetic parameter M for Pr = 0.72, N = 0.90 and Ge = 0.50 on the velocity and temperature profiles are shown in Figures 4(a) and 4(b). From these figures, it is seen that the velocity decreases and the temperature increases with increasing values of the magnetic parameter M respectively.

Figs. 5(a) and 5(b) indicate the effects of the Prandtl number Pr with $M = 0.50$, $N = 0.40$ and $Ge = 0.30$ on the velocity profiles and the temperature profiles. From Figure 5(a) it is observed that the increasing values of Prandtl number Pr leads to the decrease in the values of velocity. The maximum values of the velocity are 0.52815, 0.42524, 0.38592 and 0.36155 for Pr = 0.72, 1.74, 2.00 and 3.00, respectively, which occur at y = 1.23788 for the first maximum value and $y = 0.99806$ for the second, third and fourth maximum values. Here it is depicted that the velocity decreases by 31.54 % as Pr increases from 1.00 to 3.00. Again from Fig. 5(b) it is observed that the temperature decreases with the increasing values of Prandtl number Pr. Near the surface of the sphere, temperature is maximum and then decreases away from the surface and finally take an asymptotic value.

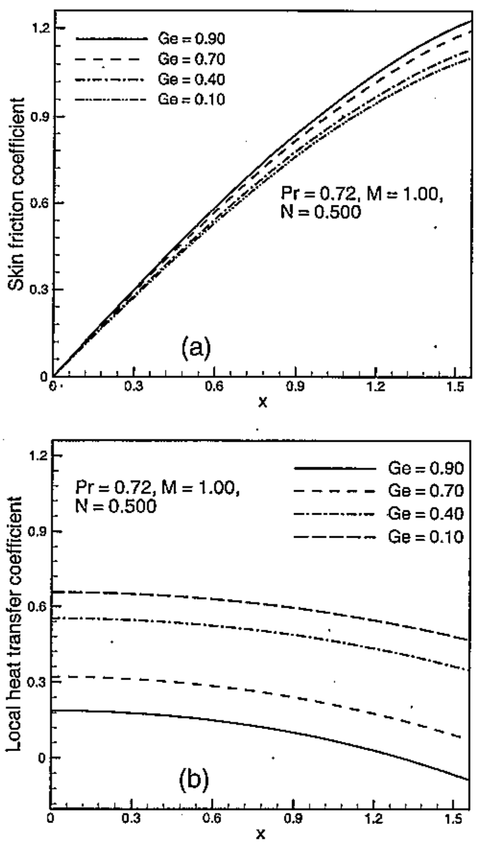

Fig 6. (a) Skin friction coefficient and (b) local heat transfer coefficient for different values of pressure work parameter Ge.

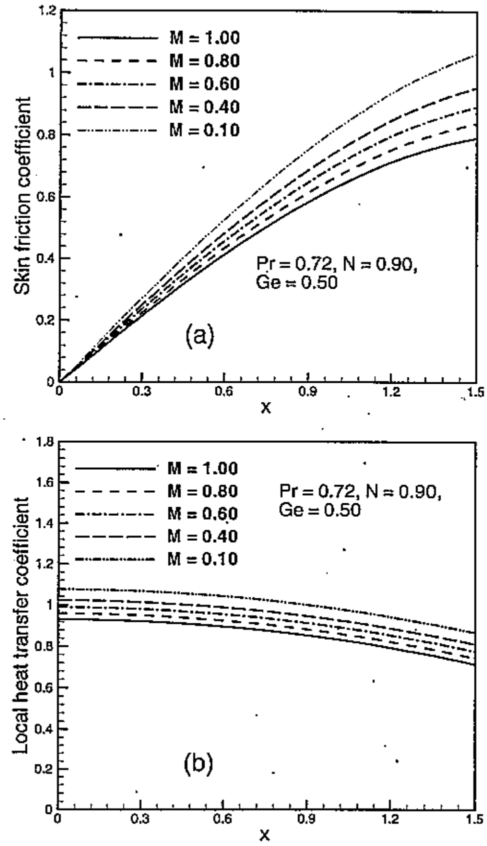

Fig 7. (a) Skin friction coefficient and (b) local heat transfer coefficient for different values of magnetic parameter M.

In Figures 6(a) and 6(b), we observed the effects for different values of pressure work parameter Ge for the magnetic parameter M =1.00, viscous dissipation parameter $N = 0.50$ and Prandtl number $Pr = 0.72$ on the reduced local skin friction coefficient C_{fX} and local rate of heat transfer Nu_X. The skin friction coefficient C_{fX} and heat transfer coefficient Nu_X increase with the increasing values of pressure work parameter Ge.

Table 1. Skin friction coefficient against x for different values of magnetic parameter M while $Pr = 0.72$, $N = 0.90$ and $Ge = 0.50$.

	$M = 0.40$	$M = 0.80$	$M = 1.00$
x	C_{fX}	C_{fX}	C_{fX}
0.00000	0.00000	0.00000	0.00000
0.10472	0.08795	0.07933	0.07569
0.20944	0.17522	0.15800	0.15073
0.40143	0.33132	0.29842	0.28455
0.50615	0.41318	0.37181	0.35437
0.80285	0.62615	0.56121	0.53401
1.01229	0.75407	0.67310	0.63938
1.20428	0.85072	0.75569	0.71638
1.30900	0.89399	0.79161	0.74945
1.50098	0.95406	0.83900	0.79206
1.57080	0.96930	0.84993	0.80142

Table 2. Local heat transfer coefficient against x for different values of magnetic parameter M while $Pr = 0.72$, $N = 0.90$ and $Ge = 0.50$.

	$M = 0.40$	$M = 0.80$	$M = 1.00$
x	Nu_X	Nu_X	Nu_X
0.00000	1.02248	0.96003	0.93160
0.10472	1.02141	0.95893	0.93045
0.20944	1.01838	0.95582	0.92731
0.40143	1.00768	0.94485	0.91625
0.50615	0.99902	0.93597	0.90730
0.80285	0.96344	0.89952	0.87053
1.01229	0.92818	0.86343	0.83415
1.20428	0.88812	0.82249	0.79291
1.30900	0.86301	0.79685	0.76711
1.50098	0.81068	0.74355	0.71350
1.57080	0.78954	0.72206	0.69192

The effects of magnetic parameter M for $Pr = 0.72$, $N = 0.90$ and $Ge = 0.50$ on the skin friction coefficient C_{fX} and the coefficient of heat transfer Nu_X are shown in Figures 7(a) to 7(b). From Figures 7(a) and 7(b) it is observed that the increasing values of magnetic parameter M leads to the decrease of the skin friction co-efficient C_{fX} and the local heat transfer co-efficient Nu_X.

In Table 1 and Table 2 are given the tabular values of the local skin friction coefficient C_{fX} and local Nusselt number Nu_X for different values of magnetic parameter M while $Pr = 0.72$, $N = 0.90$ and $Ge = 0.50$. Here we found that the values of local skin friction coefficient C_{fX} decrease at different position of x for magnetic parameter $M = 0.40, 0.80, 1.00$. The rate of the local skin friction coefficient C_{fX} decreases by 14.72% as the magnetic parameter M changes from 0.400 to 1.00 and $x = 0.80285$. Furthermore, it is seen that the numerical values of the local Nusselt number Nu_X decrease for increasing values of magnetic parameter M. The rate of decrease of the local Nusselt number Nu_X is 9.64% at position $x = 0.80285$ as the magnetic parameter M changes from 0.40 to 1.00.

4. Conclusions

The effects of viscous dissipation and pressure work on natural convection flow on a sphere in the presence of a magnetic field has been investigated for different values of relevant physical parameters. The governing boundary layer equations of motion are transformed into a non-dimensional form and the resulting non-linear systems of partial differential equations are reduced to local non-similarity boundary layer equations, which are solved numerically by using an aimplicit finite difference method together with the Keller-box scheme. From the present investigation the following conclusions may be drawn:

- Significant effects of magnetic parameter M on velocity and temperature profiles as well as on skin friction coefficient C_{fX} and the rate of heat transfer Nu_X have been found in this investigation. The effects of magnetic parameter M on rate of heat transfer is more significant.
- An increase in the values of magnetic parameter M leads to both the local skin friction coefficient C_{fX} and the local rate of heat transfer Nu_X to decrease. The velocity decreases, but the temperature increases with an increasc in the values of Mo.
- The velocity, temperature and the local skin friction coefficient C_{fX} increase significantly when the values of pressure work parameter Ge increase, but the local rate of heat transfer Nu_X decreases for increasing values of pressure work parameter Ge.
- As viscous dissipation parameter N increases, both the velocity and the temperature also increase significantly.

- Increasing values of Prandtl number Pr leads to a decrease in the velocity and the temperature.

Nomenclature

C_p : Specific heat at constant pressure.
C_{fx} : Local skin friction coefficient.
f : Dimensionless stream function
g : Acceleration due to gravity.
Ge : The pressure work parameter.
Gr : The local Grashof number.
M : The Magnetic parameter.
N : Viscous dissipation parameter
Nu_X : The local Nusselt number coefficient.
Pr : Prandtl number.
P : Fluid pressure.
q_w : Surface heat flux.
T : Temperature of the fluid.
T_w : Temperature at the surface.
T_∞ : Temperature of the ambient fluid.
U : Velocity component in the X-direction.
V : Velocity component in the Y-direction.
X : Measured from the leading edge.
Y : Distance normal to the surface.
x : The dimensionless cordinate.
y : The pseudo-similarity variable and the dimensionless cordinate.

Greek symbols

β : Co-efficient of volume expansion
β_0 : The magnetic field strength.
ν : Kinematic viscosity
μ : Viscosity of the fluid
θ : Dimensionless temperature
ρ : Density of the fluid inside the boundary layer.
ψ : Stream function
κ : Thermal conductivity of the fluid.

Subscripts

w : Condition at wall
∞ : Condition at infinity

Superscript

$'$: Differentiation with respect to y

5. References

[1] Gebhart, B., Effects of Viscous Dissipation in Natural Convection, *J. Fluid Mech.*, Vol. 14, pp. 225-232, 1962.

[2] Gebhart, B., and Mollendorf, J., Viscous Dissipation in External Natural Convection Flows. *J. Fluid Mech.*, Vol.38, pp. 97-107, 1969.

[3] Ackroyd, J.A.D., Stress Work Effects in Laminar Flat-plate Natural Convection, *J. Fluid Mech.*, Vol. 62, pp.677-695, 1974.

[4] Joshi,Y. and Gebhart, B., Effect of Pressure Stress Work and Viscous Dissipation in Some Natural Convection Flows, *Int. J. Heat Mass Transfer*, Vol. 24, No. 10, pp. 1377-1388, 1981.

[5] Kuiken, H. K., Magneto-hydrodynamic Free Convection in a Strong Cross Field, *Journal of Fluid Mechanics*, Vol. 4, No. 1, pp.21-38, 1970.

[6] Sparrow, E. M. and Cess, R. D., The Effect of a Magnetic Field on Free Convection Heat Transfer, *Int. J. Heat Mass Transfer*, Vol. 3, pp.267-274, 1961.

[7] Chowdhury, M. K. and Islam, M. N., MHD Free Convection flow of Visco-elastic Fluid Past an Infinite Porous Plate, *Heat and Mass Transfer*, Vol. 36, pp. 439-447, 2000.

[8] Raptis, A. and Kafousias, N. G., Magneto-Hydrodynamic Free Convection flow and Mass Transfer Through a Porous Medium Bounded by an Infinite Vertical Porous Plate With Constant Heat Flux, *Canadian Journal of Physics*, Vol. 60, No.12, pp.1725-1729, 1982.

[9] Elbashbeshy, E. M. A., Free Convection Flow with Variable Viscosity and Thermal Diffusivity Along a Vertical Plate in the Presence of Magnetic Field, *Int. J. Engineering Science*, Vol.38, No. 2, pp. 207-213, 2000.

[10] Hossain, M .A., Viscous and Joule Heating Effects on MHD-free Convection flow with Variable Plate Temperature, *Int. J Heat Mass Transfer*, Vol.35, No. 12, pp 3485-3487, 1992.

[11] Hossain, M. A., Das, S. K. and Pop, I., Heat Transfer Response of MHD Free Convection flow Along a Vertical Plate to Surface Temperature Oscillation, Int. J. Non-Linear Mechanics, Vol.33, No. 3, pp 541-553, 1998.

[12] Hossain, M. A. and Ahmed, M., MHD Forced and Free Convection Boundary Layer flow Near the Leading Edge, *Int. J. Heat Mass Transfer*, Vol.33, No. 3, pp 571-575, 1990.

[13] Hossain, M. A., Alam, K.C.A. and Rees, D.A.S., MHD Forced and Free Convection Boundary Layer Flow Along a Vertical Porous Plate, *Applied Mechanics and Engineering*, Vol. 2, No.1, pp33-51, 1997.

[14] Nazar, R., Amin, N., Grosan, T. and Pop, I., Free Convection Boundary Layer on an Isothermal Sphere in a Micropolar Fluid, *Int. Comm. Heat Mass Transfer,* Vol.29, No.3, pp. 377-386, 2002.