Unsteady Flow About a Stagnation Point on a Stretching Sheet in the Presence of Variable Free Stream

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Abstract

Aim of the paper is to investigate unsteady two-dimensional flow of a viscous incompressible fluid about a stagnation point on a stretching sheet in the presence of time dependent free stream. The governing equations of motion and energy are transformed into non-linear ordinary differential equations and solved numerically using shooting method. The velocity and temperature distributions are discussed numerically and presented through graphs. Skin-friction coefficient and the Nusselt number at the sheet are derived and discussed numerically and shown through graphs.

Keywords: Unsteady, stagnation point flow, stretching sheet, shooting method, skin-friction coefficient and Nusselt number.

1. Introduction

Flow and heat transfer of an incompressible viscous fluid over a stretching sheet may find applications in manufacturing processes such as the cooling of metallic plates, nuclear reactors, extrusion of polymers, etc. In all such applications the final product depends on the rate of cooling and boundary layer flow near the stretching surface.

Flow in the neighbourhood of a stagnation point in a plane was initiated by Hiemenz [1]. Flow of a viscous incompressible fluid in the neighbourhood of a stagnation point in a plane has been presented by Pai [2], Schlichting [3], Bansal [4]. Crane [5] presented the flow over a a stretching sheet and obtained similarity solution in closed analytical form. Wirz [6] analysed unsteady combined free and forced convection in boundary layer stagnation point flow. Soundalgekar *et.al.* [7] studied heat transfer in MHD unsteady stagnation point flow with variable surface temperature. Mahapatra and Gupta[8] investigated the magnetohydrodynamic stagnation point flow towards an isothermal stretching sheet and pointed out that velocity decreases/increases with the increase in magnetic field intensity when free stream velocity is smaller/greater than the stretching velocity. Mahapatra and Gupta [9] studied heat transfer in stagnation point flow toward a stretching sheet with viscous dissipation effect. Pop *et.al.* [10] discussed the flow over a stretching sheet near a stagnation point taking into account radiation effect. Elbashbeshy *et.al.* [11] studied heat transfer on an unsteady stretching sheet. Ishak *et.al.* [12] have analysed mixed convection near a stagnation point on a vertical stretching sheet. Yurusoy [13] has discussed the unsteady flow of thin fluid film of a power law fluid due to stretching of the surface. Sharidan *et.al.* [14] have presented a similarity solution for the unsteady boundary layer and heat transfer due to a stretching sheet.

There are many situations, when the flow field and heat transfer are unsteady due to sudden stretching of a sheet. When the sheet is stretched impulsively [11, 14] in the presence of unsteady free stream [6], initially unsteady flow develops and it becomes fully developed after some time. Thus the aim of the paper is to investigate two-dimensional unsteady flow of a viscous incompressible fluid about a stagnation point on a stretching sheet in the presence of time dependent free stream.

2. Formulation of the Problem

Consider unsteady two-dimensional flow of a viscous incompressible fluid in the vicinity of a stagnation point on a stretching sheet in the presence of time dependent free stream. The stretching sheet has temperature T_w, velocity $u_w(x,t)$ and the free stream velocity is $U(x,t)$. A stretching sheet is placed in the plane y = 0, and the x-axis is taken along the sheet as shown in Figure 1. The fluid occupies the half plane (y > 0).

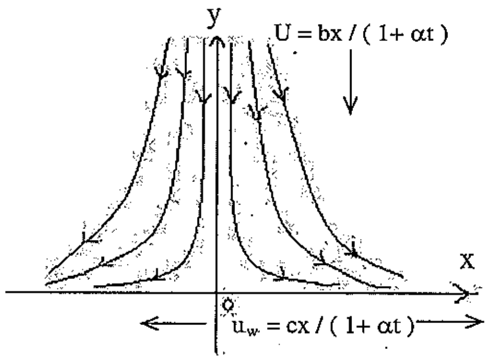

Fig 1. Physical model.

The governing equations of continuity, momentum and energy are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\partial U}{\partial x}+\frac{\partial U}{\partial t}+v\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are velocity components along x- and y-axes, respectively; T the fluid temperature, κ the thermal conductivity, $\nu(=\mu/\rho)$ the kinematic viscosity and ρ the density of fluid. Here the viscous dissipation term is neglected.

The boundary conditions are:

$$y=0:\ u=u_w(x,t)=\frac{cx}{1+\alpha t}, v=0, T=T_w,$$

$$y\to\infty: u=U(x,t)=\frac{bx}{1+\alpha t}, T=T_\infty; \tag{4}$$

where b, c are positive constants and α is also a constant.

3. Method of Solution

Introducing the stream function $\psi(x, y)$ as defined by:

$$u=\frac{\partial\psi}{\partial y}\ and\ v=-\frac{\partial\psi}{\partial x}, \tag{5}$$

the similarity variables:

$$\left.\begin{aligned}
&\eta(y,t)=\left[\frac{c}{\nu(1+\alpha t)}\right]^{1/2} y,\\
&\psi(x,y,t)=\left[\frac{\nu c}{1+\alpha t}\right]^{1/2} xf(\eta)\\
&\text{and}\quad \theta=\frac{T-T_\infty}{T_w-T_\infty},\\
&\text{where}\\
&T_w-T_\infty=\frac{1}{(1+\alpha t)^2};
\end{aligned}\right\} \tag{6}$$

into the equations (1) to (3), we get:

$$f'''+\left(f+\frac{1}{2}\eta\alpha^*\right)f''+(\alpha^*-f')f' +\lambda^2-\lambda\alpha^*=0 \tag{7}$$

and

$$\theta''+\Pr\left(f+\frac{1}{2}\eta\alpha^*\right)\theta'+2\Pr\alpha^*\theta=0, \tag{8}$$

where $\lambda\left(=\frac{b}{c}\right)$ is the ratio of free stream velocity parameter and stretching parameter,

$\Pr\left(=\frac{\mu C_p}{\kappa}\right)$ is the Prandtl number and $\alpha^*\left(=\frac{\alpha}{c}\right)$ is a parameter. It is noted that eq.(1) is identically satisfied.

The corresponding boundary conditions are reduced to:

$$f(0)=0, f'(0)=1, \theta(0)=1, f'(\infty)=\lambda \text{ and } \theta(\infty)=0 \quad (9)$$

The governing boundary layer equations (7) and (8) with the boundary conditions (9) are solved using the Runge-Kutta fourth order technique alongwith the shooting method (Conte and Boor [15]). First of all, higher order non–linear differential equations (7) and (8) are converted into simultaneous linear differential equations of first order and then further transformed into an initial value problem applying the shooting technique. Once the problem is reduced to an initial value problem, then it is solved using the Runge-Kutta fourth order technique.

4. Skin-friction

The Skin-friction coefficient at the sheet is given by:

$$C_f = \mathrm{Re}^{-1/2} f''(0), \quad (10)$$

where $\mathrm{Re}=\frac{u_w x}{\nu}$ is the local Reynolds number.

5. Nusselt Number

The rate of heat transfer in terms of the Nusselt number at the sheet is given by:

$$Nu = -\mathrm{Re}^{1/2}\theta'(0). \quad (11)$$

6. Particular Case

When $\alpha^* = 0$ i.e. the flow is steady, the results of the present paper are reduced to those obtained by Pop, Grosan, Pop [10] in the absence of a radiation effect, and Mahapatra and Gupta [9] in the absence of viscous dissipation.

7. Results and Discussion

Equations (7) and (8) are solved using the Runge-Kutta fourth order method for different values of λ and α^* taking step size 0.01.

It is observed from Table-1 that the numerical results of f ''(0) of the present paper when $\alpha^* = 0$ are in good agreement with the results obtained by Pop, Grosan and Pop [10] and Mahapatra and Gupta[9].

Table 1. Values of $f''(0)$ for different values of λ and $\alpha^* = 0$ are compared with the results obtained by Pop, Grosan and Pop[10], and Mahapatra and Gupta[9].

λ	$f''(0)$		
	Pop et.al.[10]	Mahapatra and Gupta [9]	Present results
0.1	-0.9694	-0.9694	-0.969386
0.2	-0.9181	-0.9181	-0.9181069
0.5	-0.6673	-0.6673	-0.667263
2.0	2.0174	2.0175	2.01749079
3.0	4.7290	4.7293	4.72922695

The numerical values of $f'(\eta)$ and $\theta(\eta)$ are obtained for various values of α^* $(0.0 \le \alpha^* \le 1.8)$(Wirz [6]) and λ are presented in Table 2.

Table 2. Values of $f''(0)$ and $-\theta(0)$ for different values of λ and α^*.

α^*	$\lambda = 0.1$	
	$f''(0)$	$-\theta(0)$
0.0	-0.969386	0.486896
0.2	-0.911573	0.171984
0.4	-0.852887	-0.480279
0.6	-0.793453	-4.596674
0.8	-0.733368	3.150040
1.0	-0.672706	1.546025
1.2	-0.611530	1.066158
1.4	-0.549890	0.792840
1.6	-0.487827	0.593172
1.8	-0.425378	0.427924
	$\lambda = 0.5$	
	$f''(0)$	$-\theta(0)$
0.0	-0.667263	0.577963
0.2	-0.641147	0.366833
0.4	-0.614562	0.090973
0.6	-0.587520	-0.305452
0.8	-0.560031	-0.971086

1.0	-0.532107	-2.477025
1.2	-0.503761	-10.99785
1.4	-0.475005	8.990691
1.6	-0.445851	3.773154
1.8	-0.416311	2.518568
	$\lambda = 2.0$	
	$f''(0)$	$-\theta(0)$
0.0	2.017502	0.841904
0.2	1.982756	0.717741
0.4	1.947660	0.582709
0.6	1.912209	0.434570
0.8	1.876399	0.270401
1.0	1.840227	0.086308
1.2	1.803687	-0.123014
1.4	1.766775	-0.364989
1.6	1.729488	-0.650326
1.8	1.691822	-0.995078

Figures 2 and 3 represent velocity profiles of fluid for $\lambda = 0.1$ and 0.5, respectively. It is seen that with the increase in α^*, the fluid velocity increases. The velocity profile for higher value of α^* has higher value of $-f''(\eta)$ and hence tends to reach boundary condition earlier. Due to this fact, the velocity profile, which is higher for initial value of η, becomes lower thereafter when a certain value of η is reached (Shown by dotted line). of At $\lambda = 1.0$, there would be no formation of the boundary layer, as the fluid velocity is equal to surface velocity (Pai [2]). For $\lambda = 2.0$, it is observed from Figure 4 that with increase in α^*, fluid velocity decreases, which is due to the fact that an inverted boundary is formed in the case of $\lambda > 1$. The boundary layer thickness decreases with the increase in α^* and λ. Figures 5 and 6 represent fluid temperature profiles for $\lambda = 0.1$ and 0.5, respectively; which show that with the increase in α^*, the fluid temperature profiles first increase then start decreasing, and again start increasing, but for $\lambda = 2.0$ the temperature profiles increase with the increase in α^*. The thermal boundary layer thickness decreases with the increase in α^* and λ. It is observed from Figure 8 that skin-friction coefficient increases with the increase in α^* for $\lambda = 0.1$ and 0.5, but for $\lambda = 2.0$ it decreases due to increase in α^*. Figure 9 depicts that for $\lambda = 0.1$ and 0.5 with the increase in α^*, Nusselt number first decreases then increases, and again start decreasing, however in the case when $\lambda = 2.0$, a monotonic decrease in Nusselt number is observed.

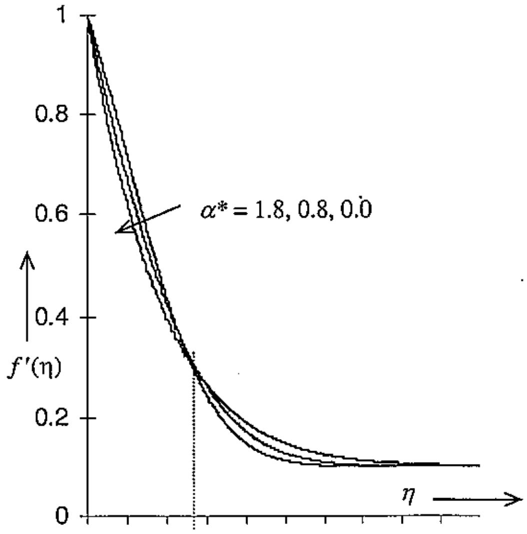

Fig 2. Velocity distribution versus η when $\lambda = 0.1$.

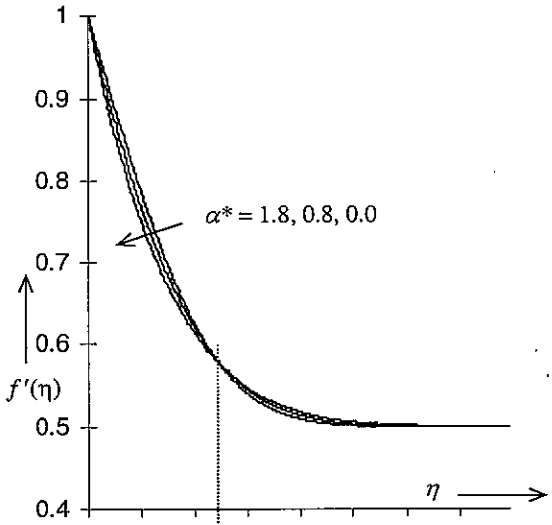

Fig 3. Velocity distribution versus η when $\lambda = 0.5$.

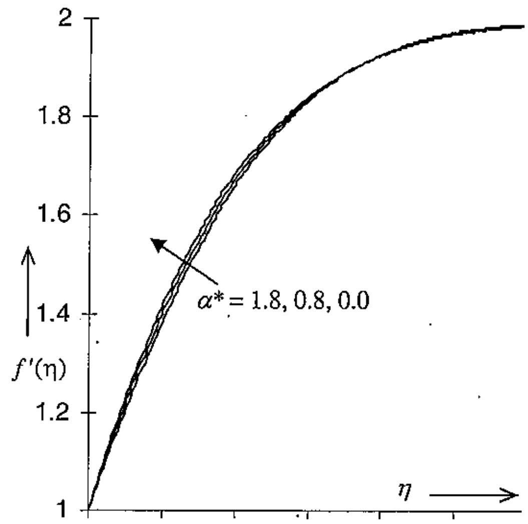

Fig 4. Velocity distribution versus η when $\lambda = 2.0$.

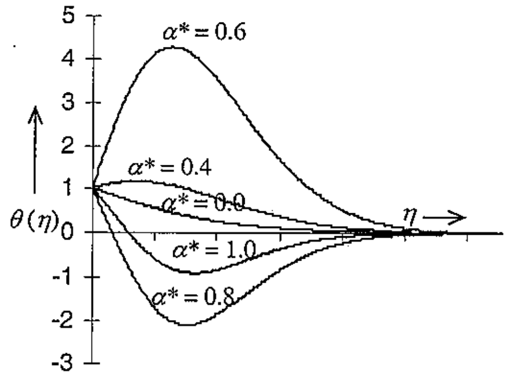

Fig 5. Temperature distribution versus η when $\lambda = 0.1$ and Pr = 0.72.

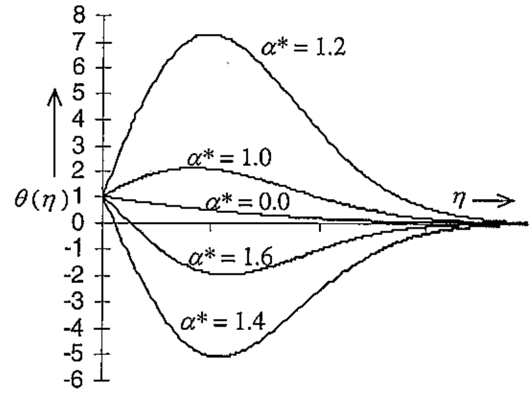

Fig 6. Temperature distribution versus η when $\lambda = 0.5$ and Pr = 0.72.

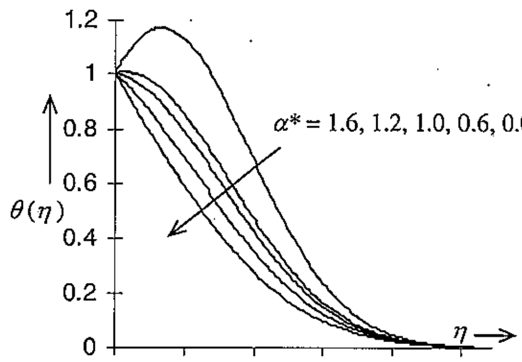

Fig 7. Temperature distribution versus η when $\lambda = 2.0$ and Pr = 0.72.

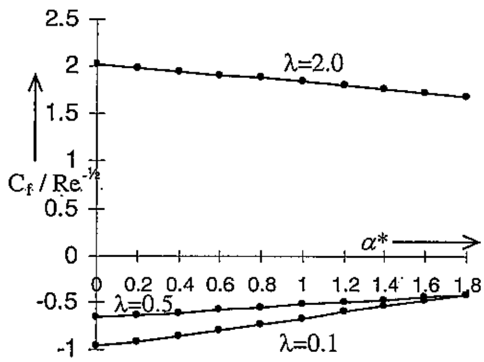

Fig 8. Skin-friction Coefficient (C_f / $Re^{-1/2}$) versus α^*.

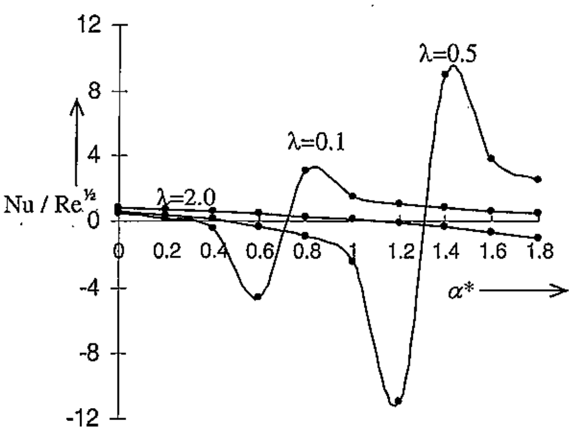

Fig 9. Nusselt Number ($Nu / Re^{½}$) versus α^*.

8. Conclusions

1. Velocity profiles for λ = 0.1 and 0.5 increase with the increase in α^*. Velocity profile which is higher for initial value of η becomes lower when a certain value of η is reached. For λ = 2.0 velocity, profiles decrease with the increase in α^*.
2. At λ = 1.0, there is no formation of a boundary layer.
3. Boundary layer thickness decreases with the increase in α^* and λ.
4. Temperature profiles for λ = 0.1 and 0.5 increase with the increase in α^* up to a value, after which profiles start decreasing and again start increasing. For λ = 2.0 temperature profiles increase with the increase in α^*.
5. Thermal boundary layer thickness decreases with the increase in α^*.
6. Skin-friction coefficient increases with the increase in α^* for λ = 0.1 and 0.5, and decreases for λ = 2.0 with the increase in α^*.
7. Nusselt number first decreases, then increases and again starts increasing with the increase in α^* for λ = 0.1 and 0.5 ; but for λ = 2.0, it continuously decreases with the increase in α^*.

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