Optimal Design of Fibrous Concrete Beams Through Simulated Annealing

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Abstract

In this paper, a formulation and solution technique using Simulated Annealing for optimizing the moment capacity of steel fiber reinforced concrete beams, with random orientated steel fibers, is presented along with identification of design variables, objective function and constraints. Steel fibers form an expensive constituent of steel fiber concrete and therefore it is important to determine ways and means of using these fibers in a efficient way with care consistent with economy for achieving the desired benefits. The most important factors which influence the ultimate load carrying capacity of Fiber Reinforced Concrete (FRC) are the volume percentage of the fibers, their aspect ratios and bond characteristics. Hence an attempt has been made to analyze the effective contribution of fibers to bending of FRC beams. Equations are derived to predict the ultimate strength in flexure of SFRC beams with uniformly dispersed and randomly oriented steel fibers. Predicted strengths using the derived expressions have been compared with the experimental data. A reasonable agreement (within the range of ± 20 percent!) was evident with different types of steel fibers, aspect ratio, and material characteristics. Computer coding has been developed based on the formulations and the influence of various parameters on the ultimate flexural strength. A computer algorithm that conducts a random search in the space of four variables- beam width, beam depth, fiber content and aspect ratio to yield an optimum solution for a given objective function (ultimate moment (M_u)), is presented. The outlined methods provide a simple and effective tool to assess the optimum flexural strength of steel fiber reinforced concrete beams. Using the results obtained, the influence of various parameters on the ultimate strength are discussed. Particular attentions are given to the construction practice as well as the reduction of searching space. It has been shown that within a reasonable and finite number of searches the developed algorithm is able to yield optimum solutions for the given objective function.

Keywords: optimization, fiber reinforced concrete, flexure, Shear, Simulated Annealing,

1. Introduction

In the area of Structural Engineering, the method of optimization has been steadily applied to various structural problems. Distinguishable linear and non linear optimization techniques have been successfully developed for finding an optimum set of the material, topology, geometry or cross-sectional dimensions of different types of structures subject to particular loading systems. In the field of linear programming and non linear programming techniques, refined algorithms have branched out in order to take into account the discrete nature of structures, and fabricated standardized structural components Although great success has been achieved during the past decades in structural optimization, these techniques generally have difficulties in avoiding local minima and results are sometimes dependent upon the choice of the initial values in the design space. With recent advances of computer technology, combinatorial optimization techniques have emerged. Genetic algorithm (GA) and simulated annealing (SA) are quite popular among them and they can efficiently solve optimization problems with higher probability.

SA can be characterized by finding a global minimum of an objective function by combining gradient decent with a random process. It is also capable of escaping local minima in addition to added probability in converging to a local minimum. Design variables constituting a set of configurations can be any type – real or discrete and consequently the searching nature of SA does not require continuity or derivative functions.

2. Research Significance

Several investigations have shown that the presence of steel fibers in beams reinforced with high strength deformed bars increases the ultimate strength [11].To achieve efficiency in performance and economy, optimization techniques can be used. The main objective of this study is to accommodate the usefulness of SA in practically optimizing reinforced steel fibrous concrete beams with due considerations given to construction practice. Realizations of much of the code provisions in regard to strength requirements as well as structural constraints have been considered.

3. Literature Review

In spite of performing various optimizational techniques for reinforced concrete structures by various researchers, the algorithms developed using SA for structural optimization of fiber reinforced concrete structural elements are much limited. Optimization techniques for the element level of reinforced concrete structures have been presented by different researchers [2]. These methods were based on sequential linear programming, continuum- type optimality criteria, and nonlinear programming such as Powell's algorithm. Recently, the discrete optimization of structures has been performed using Genetic Algorithms [14]. Very little literature is available in the field of fiber reinforced concrete structural optimization because design methods for FRC are yet to be fully developed, though some guidelines are available for its applications to airfield pavements and some hydraulic structures [5]. This is the case with standard test procedures to be adopted for testing and evaluation of the performance of FRC elements. Ezeldin and Hsu [8] optimized reinforced fibrous concrete beams using a direct search technique. The algorithm conducts a systematic search in the space of four variables: beam width, beam depth, fiber content, and aspect ratio of fibers, to yield an optimum solution for a given objective function. It is therefore the main objective of this research to develop an algorithm using SA that performs the optimum design of reinforced fibrous concrete beams. Unlike other optimal design problems here, the objective function is considered as the maximization of ultimate moment capacity rather than the cost. The algorithm is developed for the optimum design using "C" language and satisfies the specifications provided in the ACI code [1]. The flow chart for the simple SA is given in Fig1.

4. Flexural Analysis of Fiber-Reinforced Concrete Beams

The analysis is based on the following assumptions:

1. Plane sections remain in-plane after bending.
2. The compressive force equals the tensile force.
3. The internal moment equals the applied bending moment.

It has been customary to neglect the tensile resistance of concrete in calculating the ultimate flexural capacity of concrete beams. Kukreja. et al. [11] proved that fiber reinforced concrete greatly increases the tensile capacity of concrete. So, the contribution of fibers must be taken into account in the flexural analysis of beams. The analysis presented in this paper is based on the conventional compatibility and equilibrium conditions used for normal reinforced concrete, except that the effects of steel strain hardening and contribution of the steel fibers in the tension zone are recognized (APPENDIX A). A Comparison of analytical results of the authors with other researchers is shown in Table 1. The analysis is based on the compression stress blocks in ACI Code. The actual and assumed stress and strain distributions at failure are shown in Fig. 2. The maximum usable strain at the extreme concrete compression fiber is taken as 0.0035 mm/mm. The tensile contribution of the steel fibers is represented by a tensile stress block equal to the force required to develop the interfacial bond stress (τ) between fiber and matrix that are effective in that portion of the beam cross section (Fig. 2). In evaluating the tensile stress in the fiber concrete in the tension, zone correction factors are introduced to take account of the three-dimensional random orientation of the fibers, the length of the fiber, and the bond efficiency of the fiber. In this analysis orientation factor [13] α_0 =0.41; and bond efficiency factor[9] $\alpha_B = 1.0$; are used. The length correction factor α_l is calculated using the equation proposed by Cox[7]

Details of evaluating σ_{cu}, the ultimate strength of the fiber concrete in tension at failure is given in APPENDIX A, Equation A3.

5. Simulated Annealing

5.1 Background

The simulated annealing algorithm was derived from statistical mechanics. Kirkpatric et.al [10] proposed an algorithm which is based on the analogy between the annealing of solids and the problem of solving combinatorial optimization problems. Annealing is the physical process of heating up a solid and then cooling it down slowly until it crystallizes. The atoms in the material have high energies at high temperature and have more freedom to arrange themselves. As the temperature is reduced the atomic energy decreases. A crystal with regular structure is obtained at the state where the system has minimum energy. If the cooling is carried out very quickly, which is known as rapid quenching, wide spread irregularities and defects are seen in the crystal structure. The system does not reach the minimum energy state and ends in a poly crystalline state, which has higher energy. At a given temperature the probability distribution of system energy is determined by the Boltzmann probability:

$$P(E) \alpha e^{[-E/(kT)]} \qquad (1)$$

where E is system energy, k is Boltzmann's constant, T is temperature and $P(E)$ is the probability that the system is in a state of energy E.

At high temperatures, E converges to 1 for all energy states according to Eqn(1).

It can also be seen that there exists a small probability that the system might have high energy even at low temperatures. Therefore, the statistical distribution of energies allows the system to escape from local minimum.

5.2 Basic Elements

In the analogy between a combinatorial optimization problem and the annealing process, the states of solid represent feasible solutions of the optimization problem, the energies of the states correspond to the values of the objective function computed at those solutions, the minimum energy state corresponds to the optimal solution to the problem and rapid quenching can be viewed as local optimization.

The algorithm consists of a sequence of iterations. Each iteration consists of randomly changing the current solution to create a new solution in the neighborhood of the current solution. The neighborhood is defined by the choice of the generation mechanism. Once a new solution is created, the corresponding change in the cost function is computed to decide whether the newly produced solution can be accepted as the current solution. If the change in the cost function is negative the newly produced solution is taken as negative. Otherwise, it is accepted according to Metropoli's criterion, based on Boltzmann's probability.

According to Metropoli's criterion, if the difference between the cost function values of the current and the newly produced solutions is equal to or larger than zero, a random number (δ) in [0,1] is generated from a uniform distribution and if:

$$(\delta) \leq e^{(-\Delta E/T)} \quad (2)$$

then the newly produced solution is accepted as the current solution. If not, the current solution is unchanged in Equation (2), ΔE is the difference between the cost function values of the two solutions.

The flow chart for the simulated annealing algorithm is presented in Fig. 1. In order to implement the algorithm for a problem, there are four principle choices that must be made. These are :

- Representation of solutions
- Representation of cost function
- Defining of the generation mechanism for the neighbors
- Designing a cooling schedule

Solution representation and cost function definitions are as for GAs. Various generation mechanisms could be developed that again could be borrowed from GAs, for example mutation and inversion.

In designing the cooling schedule for a simulated annealing algorithm, four parameters must be specified. These are an initial temperature, a temperature update rule, the number of iterations to be performed at each temperature step, and a stopping criterion for the search. There are several cooling schedules presented in the literature. These employ different temperature updating schemes. Of these, stepwise, continuous, and non monotonic temperature reduction schemes include very simple cooling strategies. One example is the geometric cooling rule. This rule updates the temperature by the following formula

$$T_{i+1} = cT_i \quad i = 0,1.... \quad (3)$$

where 'c' is a temperature factor which is a constant smaller than 1 but close to 1.

6. Formulation of Objective Function

6.1 Transformation of Constrained Optimization to Unconstrained Optimization

SA is ideally suited for unconstrained minimization optimization problems. As the present problem is a constrained maximization problem, we need to make two transformations. The first transformation transforms the original constrained problem into an unconstrained problem, using the concept of penalty function. A formulation based on the application of penalty, whenever there is a violation of specified constraints, is used in the present study for transformation. Here, if the design variable set violates the constraint, then a lower value of 1.0 will be assigned, and if not, a higher value of 10.0 is assigned as violation parameter. The violation coefficient 'ϕ' is computed as follows:

$$\phi = \sum_{i=1}^{m} \phi_i \quad (4)$$

$$\phi_i = a \times g_i(x) \quad (5)$$

$$\text{if } g_i(x) < 0 \quad a = 10; \text{ and if, } g_i(x) \geq 0 \quad a = 1.0 \quad (6)$$

Where

m = number of constraints
a = penalty parameter and
$g_i(x)$ = constraint function.

The modified objective function is given as:

$$Z_m(x) = Z(x) + \phi \quad (7)$$

Where 'Z(x)' is the objective function subject to:

$$X_l^{(l)} \leq X_i \leq X_l^{(u)}, i = 1,2,...,n \quad (8)$$

The objective function for the present study is the maximization of moment capacity of an SFRC beam. The equation for this is derived in APPENDIX A [Eqn. [A9]].

The second transformation is to transform the minimization problem into a maximization problem. The maximization of a function *Z(x)* is equivalent to the minimization of the negative of the same function. For example, the objective function Minimize [*Z(x)*], is equivalent to

Maximize $[Z'(x)] = -Z_m(x)$ (9)

7. Formulation of Constraints

7.1 Strength of Fiber Reinforced Concrete in Uni-Axial Tension

According to Cooper [6], the fracture of a composite under tension can be classified into two categories; single fractures and multiple fractures. A composite fails by a single fracture if both the components are fractured at the same point along their axis, and in this case the composite will immediately break into two pieces. Multiple fracture occurs only if only one of the components is fractured while the other is intact. In this case, as the applied stress is increased, more and more cracks will appear in the damaged component, until finally the other components is also fractured. In this case it is physically obvious that during the interval between the appearance of the first crack in the weaker component and the final fracture, the applied stress will produce a larger strain than that produced in the perfect composite. In other words, during this interval the composite will behave in a partially plastic fashion. Thus, the behavior of the composite will be similar to that of a ductile metal, and the stress at which the first crack appears in the weaker component can be considered to be the yield stress of the composite.

Following Cooper [6], we can derive simple criteria for the occurrence of single and multiple fractures in two-component composites. Let us consider a two–component cylindrical composite under a tension along its axis. We shall assume that both the components are parallel to each other and to the axis of the cylindrical composite. The two components will be identified by the subscripts '*m*' and '*f*'. In the case of fiber composite, for example, one of the components will be the fibers and the other will be the matrix.

Let $C_{f,m}$ and $E_{f,m}$ denote the volume fraction ($C_f = V_f / V$; $C_m = V_m / V$) and the Young's modulus respectively, of the fibers and matrix.

Suppose that a longitudinal stress 'σ' produces uniform strain 'ε' in the composite. So long as ε is small, so that both the components and, hence the composite are in stage I or the elastic zone, up to a certain value of the stress, the strain will be proportional to the stress, provided the cross sectional area remains constant. We can write the following equation with the help of the law of mixtures:

$$\sigma = C_f\, E_f\, \varepsilon + C_m\, E_m\, \varepsilon \qquad (10)$$

Let ε_2 denote the strain at which the matrix fails. Assuming elastic behavior throughout the region $0 \le \varepsilon \le \varepsilon_2$, the value of σ at $\varepsilon = \varepsilon_2$ is given by:

$$\sigma_{c'} = C_f\, E_f\, \varepsilon_2 + C_m\, E_m\, \varepsilon_2 \qquad (11)$$

$$\sigma_{c'} = C_f\, E_f\, \varepsilon_2 + C_m\, \sigma_2 \qquad (12)$$

$$\sigma_{c'} = C_f\, \sigma_{f'} + C_m\, \sigma_2 \qquad (13)$$

Where $\sigma_2 = E_m\, \varepsilon_2$ and we have also assumed that 'ε_1', the failure strain of fiber component, is larger than 'ε_2'. The stress in fibers at the strain level, corresponding to the matrix cracking strain, is given as '$\sigma_{f'}$' where $\sigma_{f'} = E_f\, \varepsilon_2$.

$\sigma_{c'}$ as given by Eqn (13), is the stress in the composite at which the matrix will fail. Thus, the conditions for single and multiple fractures can be written as:

Single fracture:

$$C_f\, \sigma_{fu} < \sigma_{c'} \qquad (14)$$

Multiple fracture:

$$C_f\, \sigma_{fu} > \sigma_{c'} \qquad (15)$$

Where $\sigma_{fu} = E_f\, \varepsilon_1$ is the stress in the fiber at its failure strain 'ε_1' or in other words, it is the ultimate fracture strength of fibers.

In the case of a multiple fracture, i.e. when Eqn. (15) is satisfied, the critical stress at which the whole composite will fail is given by:

$$\sigma_{cu} = C_f\, \sigma_{fu}\,. \qquad (16)$$

In this case as discussed earlier, the composite will behave like a ductile metal even if both the components (fiber and matrix) are brittle.

7.2 Constraints on critical volume

The tensile response of fiber–cement composites significantly depends on the volume fraction of the fibers used. The response of fiber reinforced composite up to the cracking strength of the matrix can be described using Eqn (13). Beyond this point, assuming that the matrix does not contribute any further, the strength of the composite is a function of the strength and volume fraction of the fiber (Eqn (16)). At a fiber loading up to the critical volume, the release of a matrix cracking load onto the fibers causes the exhaustion of fiber strength; failure is thus by the formation a single crack. For high volume fractions, the fibers are able to carry load in excess of the matrix cracking load; thus the ultimate strength of the composite is higher than the matrix strength, and distributed cracking can exist. The transition from single cracking to multiple cracking takes place above a certain fiber volume fraction, referred to as the critical volume fraction 'C_{cr}'. It has been derived by equating Eqn (13) and (16) as given below:

$$C_f\sigma_{f'} + C_m\sigma_2 = C_f\sigma_{fu} \qquad (17)$$

Since

$$C_f + C_m = 1 \qquad (18)$$

$$C_f\sigma_{f'} + (1 - C_f)\sigma_2 = C_f\sigma_{fu} \qquad (19)$$

Where $\sigma_{f'}$, σ_2 and σ_{fu} are already defined in the previous section. Solving for 'C_f' and taking $C_f = C_{cr}$ the following equation is obtained.

$$C_{cr} = \frac{\sigma_2}{\sigma_{fu} + (\sigma_2 - \sigma_{f'})} \qquad (20)$$

From the above Equation we can conclude that to ensure multiple cracking in the composite the volume fraction of fibers incorporated in the matrix must be greater than or equal to the critical volume 'C_{cr}'.

7.3 Constraints on Aspect Ratio

To utilize the fracture strength of fibers, there should be excellent bonding between fiber and the matrix. If a fiber of diameter 'd' and length 'l' is to fracture at its mid–length, the bond strength developed over the length $\frac{l}{2}$ must be greater than the fracture strength, i.e.,

$$\frac{\tau\,\pi\,d\,l}{2} \geq \frac{\pi d^2}{4}\sigma_{fu}$$

$$\frac{l}{d} \geq \frac{\sigma_{fu}}{2\tau}$$

$$A_{sp} \geq \frac{\sigma_{fu}}{2\tau} \qquad (21)$$

Equation (21) states that for the fibers to fail by fracture, their aspect ratio ($A_{sp} = \frac{l}{d}$) must be equal to or greater than, the critical value given by the right- hand side.

7.4 Constraints on ultimate moment

To ensure safety, the ultimate moment must be greater than or equal to the applied moment.

$$M_u \geq M \qquad (22)$$

7.5 Constraints on tension steel

Providing excessive reinforcement in beams can result in congestion, thereby adversely affecting the proper placement and compaction of concrete. Excess steel reinforcement also results in over reinforced sections. To provide ductile failure, the member should be designed for A_{st} less than 0.75 A_{sb} where A_{sb} = area of steel required for balanced condition. It may be found by applying the equilibrium and strain compatibility conditions in APPENDIX B, Equation B3:

$$A_{st\,max} = 0.75\,A_{sb} \qquad (23)$$

Where

$$A_{sb} = \frac{bD}{\sigma_y}\left[(0.7225 f_{c'} + \sigma_{cu})\left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y}\right) - \frac{\sigma_{cu}D}{d}\right] \qquad (24)$$

Where fc' = Cylinder compressive strength of concrete; ε_{cu} = Usable compressive strain in concrete; ε_y = yield strain of steel reinforcing bar; σ_y = yield stress of reinforcing bar; b = width of the beam; d = effective depth of the beam; D = overall depth of the beam and other variables defined earlier.

A minimum area of tension reinforcing steel is required in flexural members not only to resist possible load effects, but also to control cracking in concrete due to shrinkage and temperature variations. It can be obtained by equating the cracking moment of the section (using the modulus of rupture of fiber concrete) to the strength computed as a reinforced fiber concrete section as in APPENDIX B, Equation B3. As recommended in Ref. 8, this value is taken as:

$$A_{stmin} = bd\left[\frac{400}{\sigma_y}\right] \qquad (25)$$

7.6 Constraints on shear strength

Several studies have shown that steel fibers are particularly effective in providing reinforcement against shear stresses in conventionally reinforced concrete[12]. It is evident from the test results of various authors that stirrups and fibers can be used effectively in combination.

The equation proposed by Narayanan and Darwish [12] has been used in this study to predict the ultimate shear strength of fiber reinforced concrete beams.

$$V_{fs} = e\left(0.24 f_t' + 80\rho\, d/a\right) + 4.56F \;\; N/mm^2 \qquad (26)$$

Where

a/d = effective shear span to depth ratio

e = 1.0 when a/d 2.8 , and 2.8 (d/a) When $a/d \le 2.8$

$f_{t'}$ = Splitting cylinder strength of fiber concrete

ρ = Percentage of area of tensile steel to area of concrete

F = fiber factor (= $((\frac{l}{d})C_f))f$) where f = 0.5 for round fibers, 0.75 for crimped fibers, and 1.0 for fibers with deformed ends.

The method proposed in ACI Code [1] is used for calculating the contribution of stirrups (V_s) to the shear capacity, to which is added the resisting force of concrete from the added fibers V_{fs} obtained from Equation (26):

$$V_s = A_{sv}\,\sigma_{st}\,\frac{d}{S} \qquad (27)$$

d = Effective depth of the beam

S = Spacing of stirrups

σ_{st} = Yield stress of stirrups

The constraint to check the safety of the concrete against shear is given as:

$$V_u \le V_{un} \qquad (28)$$

where V_{un} is ultimate shear strength of fiber concrete and V_u is ultimate shear force applied.

7.7 Constraints on Area and Spacing of Stirrups

The minimum and maximum stirrup area is taken as proposed in ACI Code [1]:

$$A_{svmin} = \frac{50\,b\,S}{\sigma_{st}} \qquad (29)$$

and

$$A_{svmax} = 5\left(A_{svmin}\right) \qquad (30)$$

The constraint on stirrup spacing is formulated based on the assumption that the stirrup spacing 'S' varies from $\frac{d}{2}$ down to $\frac{d}{4}$.

The code objective in recommending such minimum shear reinforcement is to prevent sudden formation of an inclined crack in an unreinforced (or very lightly reinforced) web, possibly leading to an abrupt failure. Further, the provision of nominal web reinforcement restrains the growth of inclined shear cracks, improves the dowel action of the longitudinal tension bars, introduces ductility in shear, and provides a warning of the impending failure.

7.8 Constraint for limiting span/depth ratio for deflection control

Excessive deflections in beams are generally undesirable as they cause psychological discomfort to the occupants of the building, and also lead to excessive crack-widths and subsequent loss of durability. The selection of cross sectional sizes of flexural members is often provided by the need to control deflections under service loads.

In the present study to formulate the constraint for limiting span/depth ratio for deflection control, limits for serviceability limit state of deflection as set out in BS 8110:Part 2 clause 3.2.1 [2], are used. It is stated in the code that the deflection is noticeable if it exceeds span/250. The code states that the basic span to effective depth ratios for rectangular and flanged beams is so determined as to limit the total deflection to span/250. For simply supported beams, spans up to 10 m of the basic value is taken as 20. If the span exceeds 10 m, the basic value is multiplied by 10/span in meters. Depending on the area and the stress of steel for tension reinforcement, the basic value shall be modified by multiplying with the modification factor given by the formula in the code:

$$ModificationFactor = 0.55 + \frac{477 - f_s}{120(0.9 + M_u/bd^2)} \leq 2.0 \qquad (31)$$

Note that the amount of tension reinforcement present is measured by the term M_u/bd^2

The service stress is estimated from the equation:

$$f_s = \frac{5\sigma_y A_{s,req}}{8A_{s,prov}} \frac{1}{\beta_b} \qquad (32)$$

Where $A_{s,req}$ is the area of tension steel required at mid–span to support ultimate loads, $A_{s,prov}$ is the area of tension steel provided at mid -span and:

$$\beta_b = \frac{moment\ after\ redistribution}{moment\ before\ redistribution} \qquad (33)$$

For simply supported beams $\beta_b = 1$;
To satisfy the deflection limits:

$$\frac{Actual\ span}{d} \leq \frac{Allowable\ span}{d} \qquad (34)$$

7.9 Constraints on design variables

The design constraints are formulated as:

$$b_{\min} \leq b \leq b_{\max}$$
$$C_{f\min} \leq C_f \leq C_{f\max}$$
$$D_{\min} \leq D \leq D_{\max}$$
$$A_{sp\min} \leq A_{sp} \leq A_{sp\max}$$

where $b_{\min,\max}$; $C_{f\min,\max}$; $D_{\min,\max}$; $A_{sp\min,sp\max}$ represents the lower, upper bound values of variables, width(b), Volume fraction (C_f), Overall depth (D), and aspect ratio (A_{sp}) respectively.

8. Input Parameters (Assumed Values)

The following are the assumed values of various parameters used in the present study.

Effective span(l_{eff}) = 3.23 m; Live load = 10 KN/m ; Cube strength of concrete (f_{ck}) = 20 N/mm^2 ; Cylinder Compressive strength of concrete (fc') = 0.85, (f_{ck}) = 17N/mm^2 ; Interfacial bond stress between fiber and the matrix (τ) = 7.0 N/mm^2 ; Shear span (a) = 0.84 m ; σ_y = *415.0* N/mm^2 ; split tensile strength (f_t) = 0.1 fc' ; Stress value corresponding to failure strain of matrix (σ_2) = 1.412 f_t ; $E_f = 2\times10^5$ N/mm^2. The method of calculation of stress in fiber corresponding to the failure strain of matrix, is given below.

In the pre-cracking stage, the influence of matrix cracking in the composite is neglected. It is assumed that the steel fiber bonds perfectly with the concrete matrix and no slippage occurs at the fiber–matrix interface. Since steel fibers have a minor effect on compressive strength, the compressive model for plain concrete as recommended in BS 8110-1985:Pt.1 [2] is adopted for SFR concrete. The initial tangent modulus of concrete in compression, E_{oc} as given in the above recommendation is as follows:

$$E_{oc} = 5.5\left[\frac{f_{cu}}{\gamma_m}\right]^{0.5} \quad (35)$$

Where γ_m is the partial factor of safety for the material. Here this value is assumed as 1.5 for concrete in flexure.

The strain value corresponding to the matrix cracking stress (σ_2) can be calculated as follows:

$$\varepsilon_{cr} = \frac{\sigma_2}{E_{oc}} \quad (36)$$

Assuming a perfect bond between the matrix and fiber and uniform strain throughout through out the composite, the stress in fiber at the strain level corresponding to the matrix cracking strain is given as '$\sigma_{f'}$'

where

$$\sigma_{f'} = E_f \varepsilon_{cr} \quad (37)$$

9. Working Procedure of the Algorithm

The simulated annealing algorithm starts with a "high" temperature, T_o. A sequence of design vectors is then generated randomly until equilibrium is reached; that is the average value of 'Z_m' reaches a stable value as 'i' increases. The best point reached is recorded as X_{opt}. Once thermal equilibrium is reached, the temperature T, is reduced and a new sequence of moves is made and continued until a sufficiently low temperature is reached, at which stage no more improvement in the objective function value can be expected. The basic algorithm is shown as a flow diagram in Fig. 3.

Starting from an initial vector X_1, the algorithm generates successively improved points randomly $X_2, X_3, ...$ moving toward the global maximum solution. If X_i denotes the current point, random moves are made along each coordinate direction, in turn. The new coordinate values are uniformly distributed around the corresponding coordinate of X_i. One half of these intervals along the coordinates are stored as the step vector S_i. If the point falls outside the range given in Equation (8), a new point satisfying Equation (8) is found. A candidate design vector X is accepted or rejected according to criteria known as the Metropolis criterion. (Refer to Eqn(2)).

The various SA parameters used in the present study is as follows:

Starting value of Temperature (T_0)		500
No. of iterations	(niter)	100
Temperature reducing rate	(c)	0.99
No. of variables	(nvar)	4

The lower and upper bound values of the above variables used in the present study are given in Table 2.The parametric study results of SA is shown in Table 3. It is seen that the final values of the objective function improves from the initial stage to the final one.

Fig 4 shows the performance of the algorithm. At the initial stage, the variation of the objective is too high. As the temperature parameter reduces, the variation is also reduced and convergence is achieved. At the culminating stage, the objective function converges to a steady and maximum value.

The objective function for optimization is, maximization of ultimate moment for reinforced fiber concrete beams, subjected to bending and shear. It can be formulated as given in APPENDIX A, Equation A9. The program is designed to read the required data-limiting values of variables, and SA parameters. The program searches for a maximum for the objective function in the space of four variables, namely the Volume fraction of the fiber (C_f), the Width of the beam (b), the Depth of the beam (D), and the Aspect ratio of the fiber (A_{sp}). The stirrup spacing varies from $\frac{d}{2}$ down to $\frac{d}{4}$ while the stirrup area increases from the minimum allowed by the ACI Code[1] ($A_{sv\min} = \frac{50\, b\, S}{\sigma_{st}}$) up to five times this value. The maximum of the objective function is recorded if all the constraints are satisfied. If any of the constraints is violated, the penalty for violation is given, and the objective function is modified. The modified objective function incorporating the constraint violation is given in Equation (7).

10. Conclusions

Based on the formulation for Simulated Annealing based optimal design of Reinforced Concrete Beams along with identification of design variables, the objective function and constraints, the following conclusions are arrived at:

1. The developed SA successfully led the randomly distributed initial design points in the design space to the local optimum design point.
2. The overall effect of fiber addition on ultimate strength was studied. Predicted strengths using the derived expressions were compared with experimental data. [17, 4, 20]. The ultimate moment values found using the derived expression is on the conservative side. One possible reason for these conservative estimates is the uncertainty in the values of τ. Experimental studies undertaken by many investigators show a wide disparity of fiber matrix interfacial bond stress value in SFR concrete[16, 21]. The values depend on the response stage, concrete properties, fiber type and other characteristics.
3. The outlined methods provide a simple and effective tool to assess the optimum flexural strength of steel fiber reinforced concrete beams with randomly distributed steel fibers.
4. The optimum design results can well be controlled by the designer by specifying various design requirements.
5. In the present approach, the number of the variables used in SA is considerably reduced as the quantity of tensile reinforcement, area of stirrups, and spacing of stirrups are not considered as variables. They are represented in the algorithm as implicit variables.
6. It must be emphasized that toughness improvement, cracking, and deflection control which are other reasons for use of fibers, were not considered. Future studies should combine the optimum use of fibers for strength and serviceability criteria.
7. The efficiency of the algorithm suggests its immediate application to other design optimization problems.

11. References

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APPENDIX A
DERIVATION FOR ULTIMATE MOMENT CAPACITY OF REINFORCED STEEL FIBROUS CONCRETE BEAMS

The strength of the fiber reinforced concrete in uniaxial tension as explained in section 7.1 is given as follows:

$$\sigma_{cu} = C_f \; \sigma_{fu} \qquad \text{(A1)}$$

The fiber strength σ_{fu} may be derived from bonding characteristics of fibers as follows.

$$\sigma_{fu} = 2\tau\left(\frac{l_f}{d_f}\right) \qquad \text{(A2)}$$

in which τ = interfacial bond stress between fiber and matrix.

Since the orientation, length, and bonding characteristics of fibers will influence the strength of fiber–reinforced concrete, these parameters must be incorporated in Eqn. A2. Incorporating all these factors, Eqn. A1 can be rewritten as:

$$\sigma_{cu} = \alpha_0 \; \alpha_1 \; \alpha_b \, 2\tau\left(\frac{l_f}{d_f}\right)C_f \qquad \text{(A3)}$$

in which , α_0, α_1 and α_b are orientation factor, length - efficiency factor, and bond efficiency factor of fibers, respectively.

The orientation factor α_0 is known to be about 0.41 for uniformly distributed fiber-reinforced concrete, and the bond efficiency factor α_b is about 1.0 for straight fibers. The present study exploits Cox's [7] results for length- efficiency factor as follows.

$$\alpha_l = 1 - \frac{\tanh\left(\frac{\beta\, l_f}{2}\right)}{\left(\frac{\beta\, l_f}{2}\right)} \qquad \text{(A4)}$$

$$\beta = \sqrt{\frac{2\pi\, G_m}{E_f \;\; A_f l_n\left(\frac{S}{r_f}\right)}} \qquad \text{(A5)}$$

$$S = 25\sqrt{\frac{d_f}{C_f\ l_f}} \qquad \text{(A6)}$$

in which G_m = shear modulus of concrete matrix; E_f = elastic modulus of fiber; A_f = cross sectional area of fiber; S = average spacing of fiber; r_f = radius of fiber ; d_f = diameter of fiber; l_f Length of fiber and C_f = volume ratio of fiber. Eqn. (A3) of fiber–reinforced composite may now be employed to derive the flexural capacity of concrete beams containing steel fibers. The strain profile as shown in Fig.2 has been assumed for a cracked section in pure bending. The concrete has reached its ultimate compressive strain ε_{cu}. The stress block in the compression zone is the one commonly assumed in ultimate strength calculations. It has been adopted under the assumption that the behaviour of the fiber-reinforced compression zone is similar to that of one without fiber-reinforcement.

Equating forces in Fig. 2.:

$$C = T_F + T_{ST} \qquad \text{(A7)}$$

Where and $T_F = \sigma_{cu} bD(D - k_1 D)$; Hence k_1 can be obtained from equilibrium conditions:

$$k_l = \frac{\sigma_{cu} bD + \sigma_y A_{st}}{bD(0.7225\, fc' + \sigma_{cu})} \qquad \text{(A8)}$$

in which f_c' = Cylinder compressive strength of concrete; b = Width of the beam; D = Overall depth of the beam; C_f = Volume fraction of the fibers; and $\sigma_{cu} = 2\ \alpha_0\ \alpha_1\ \alpha_b \tau (l_f / d_f) C_f$ = Ultimate strength of composite incorporating orientation, length and bond efficiency factor of fibers. σ_y = yield strength of tensile steel; A_{st} = Area of tensile steel. Equating moments about C, we obtain the theoretical moment strength as below:

$$M_u = \sigma_{cu} b(D - k_1 D)\frac{[(D + 0.15k_1 D)]}{2} + \sigma_y A_{st}(d - 0.425k_1 D)$$

(A9)

APPENDIX B
DETERMINATION OF TENSION STEEL FOR BALANCED STRAIN CONDITION

At the balance conditions (Ref : Fig. 2):

$$\frac{(k_1 D)_{Bal}}{d} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y} \qquad \text{(B1)}$$

From Force equilibrium:

$$A_{sb}\sigma_y + \sigma_{cu} b((D - k_1 D)) = b(k_1 D)_{Bal}\, 0.7225 f_{c'}$$

(B2)

From B1 and B2:

$$A_{sb} = \frac{bd}{\sigma_y}\left[(0.7225 f_{c'} + \sigma_{cu})\left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y}\right) - \frac{\sigma_{cu} D}{d}\right]$$

(B3)

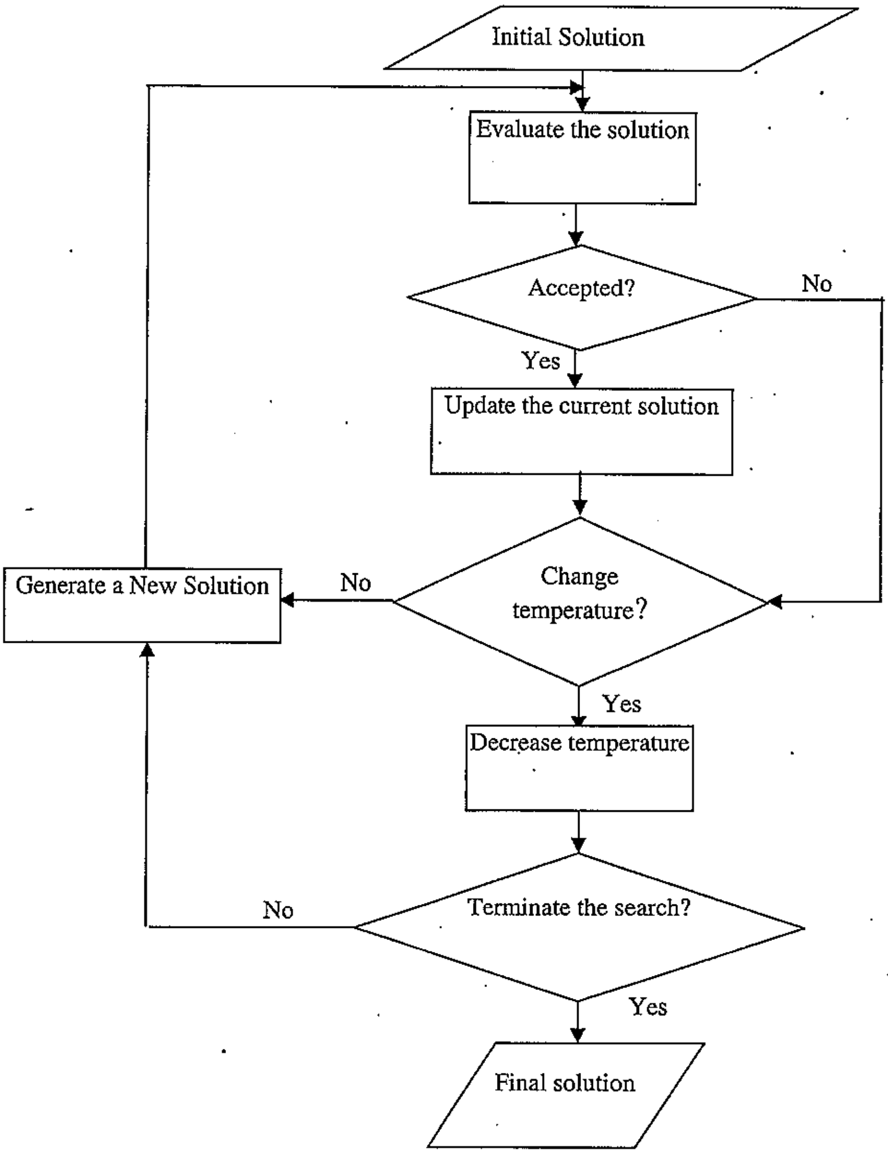

Fig 1. Flow Chart of a Simple Simulated Annealing Algorithm

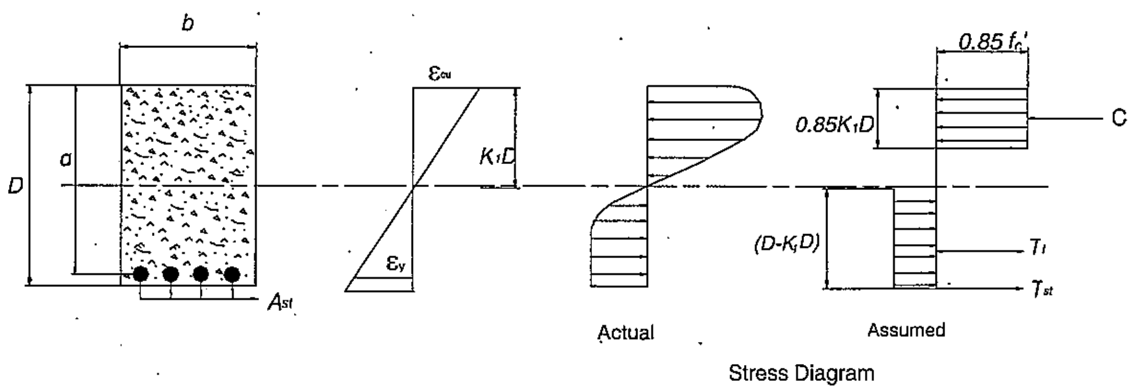

Fig 2. Strain and Stress Distribution at Cross Section of SFRC Beam

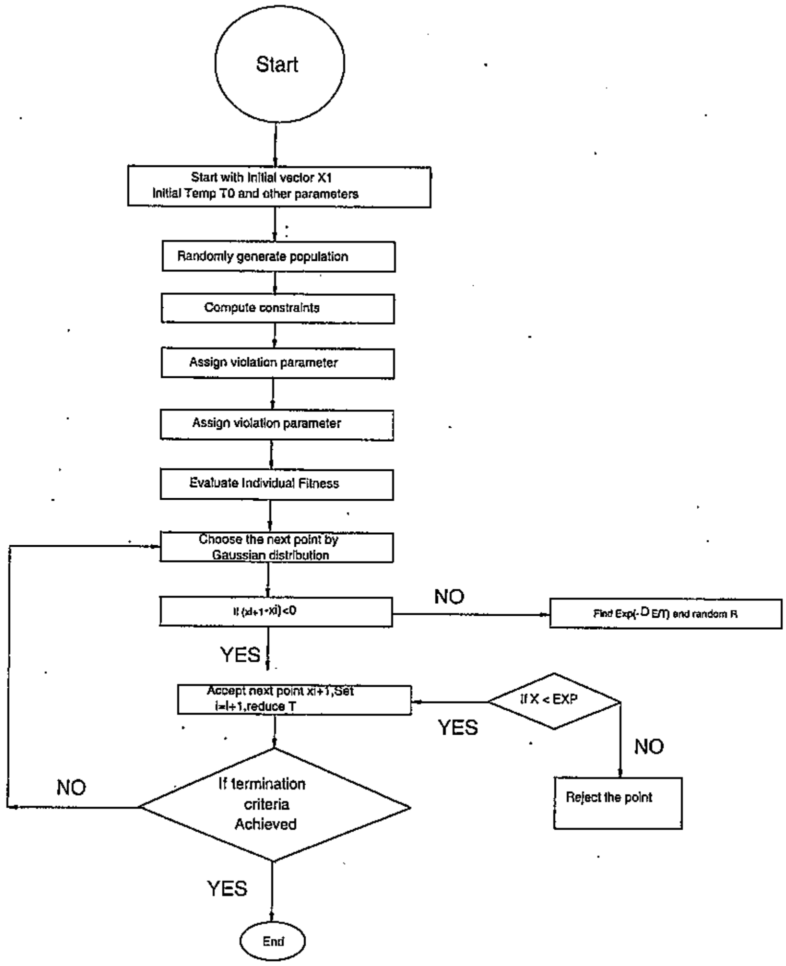

Fig 3. Flow Chart for the Proposed Algorithm

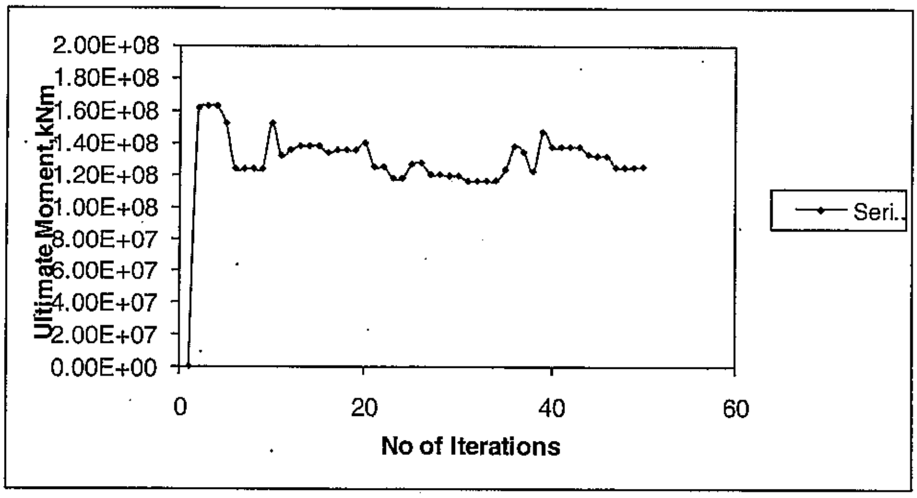

Fig 4. Performance of SA

Table. 1 Comparison of test results with analytical model

Sl. No.		1						2				3	
Investigator		Swamy And Al-Ta' An [17]						Byung [4]				Veerendra Kumar et al [20]	
Beam No.		DR11	DR12	DR21	DR22	DR31	DR32	S1V1	S1V2	S2V1	S2V2	Series2 Set2	Series3 Set2
Size		$b=130mm,\ D=203mm,\ d=176mm$						$b=120mm,\ D=180mm,\ d=140mm$				$b=150mm,\ D=150mm,\ d=120mm$	
Fiber aspect ratio, $\frac{l_f}{d_f}$		100	100	100	100	100	100	57	57	57	57	75	75
Fiber percent by volume, C_f **%**		0.5	1.0	0.5	1.0	0.5	1.0	1.0	2.0	1.0	2.0	1.0	1.0
Tension steel	**Area,** A_{st}, in^2. (mm^2)	0.35 (226)	0.35 (226)	0.622 (402)	0.622 (402)	0.35 (226)	0.35 (226)	0.69 (253)	0.39 (253)	0.62 (397)	0.62 (397)	0.24 (157)	0.35 (226)
	Yield strength σ_y, Ksi (kN/mm^2)	66.76 (0.460)	66.76 (0.460)	66.76 (0.460)	66.76 (0.460)	89.55 (0.617)	89.55 (0.617)	60.95 (0.420)	60.95 (0.420)	60.95 (0.420)	60.95 (0.420)	76.92 (0.530)	81.28 (0.560)
Interfacial bond stress, τ **ksi(kN/mm^2)**		0.374 (0.0026)	0.374 (0.0026)	0.374 (0.0026)	0.374 (0.0026)	0.374 (0.0026)	0.374 (0.0026)	0.374 (0.0026)	0.374 (0.0026)	0.374 (0.0026)	0.374 (0.0026)	0.155 (0.0011)	0.332 (0.0023)
$M_{uAuthor}$, $\times 10^6$ **lb-in. (kNm)**		0.166 (18.71)	0.189 (21.45)	0.279 (31.56)	0.295 (33.35)	0.226 (25.6)	0.249 (28.15)	0.130 (14.7)	0.115 (13.0)	0.181 (20.4)	0.177 (20.0)	0.108 (12.28)	0.118 (13.3)
M_{uExpt}, $\times 10^6$ **lb-in. (kNm)**		0.206 (23.25)	0.211 (23.81)	0.298 (33.68)	0.310 (35.06)	0.253 (28.62)	0.273 (30.83)	0.135 (15.229)	0.159 (17.963)	0.200 (22.638)	0.207 (23.373)	0.115 (12.94)	0.131 (14.75)
$\frac{M_{uExpt}}{M_{uAuthor}}$		1.24	1.11	1.07	1.05	1.12	1.1	1.03	1.38	1.10	1.16	1.05	1.10

Table. 2 Limiting values of variables assumed

S. No.	Limiting values	Volume fraction (%), C_f	Width of the beam, b,(mm)	Overall depth of the beam, D,(mm)	Aspect ratio of fibers, $A_{sp} = \frac{l_f}{d_f}$
1	Lower bound values	0.005	230	300	50
2	Upper bound values	0.03	300	450	100

Table. 3 Optimal Results Using SA

Iter. No.	Fiber percent by volum, C_f (%)	Width of beam, b, in. (mm)	Depth of beam, D, in. (mm)	Aspect Ratio, $\frac{l_f}{d_f}$	Spacing of stirrup, S, in. (mm)	Area of Stirrup, A_{sv} in.^{2} (mm^2)	Area of tension steel, A_{st}, in.^{2} (mm^2)	Ultimate moment, M_u x 10^6 lb-in. (kNm)	Ultimate shear, V_{ut}, kips (kN)
10	(0.03)	11.3 (287)	15.55 (395)	53.96	3.72 (94.63)	0.035 (22.57)	0.367 (237.03)	1.64 (185.83)	340.28 (1514.25)
20	(0.03)	9.96 (253)	15.51 (394)	53.96	3.72 (94.63)	0.030 (19.88)	0.324 (208.84)	1.91 (215.83)	339.28 (1509.79)
30	(0.03)	11.14 (283)	17.51 (445)	78.57	4.22 (107.43)	0.04 (25.27)	0.410 (264.26)	2.28 (257.56)	555.55 (2472.20)
40	(0.03)	11.14 (283)	17.51 (445)	78.57	4.23 (107.45)	0.04 (25.27)	0.410 (264.26)	2.28 (257.56)	555.55 (2472.20)
50	(0.03)	11.3 (287)	15.55 (395)	53.96	3.72 (94.63)	0.035 (22.57)	0.367 (237.03)	1.64 (185.83)	340.28 (1514.25)
60	(0.03)	11.3 (287)	15.55 (395)	53.96	3.72 (94.63)	0.035 (22.57)	0.367 (237.03)	1.64 (185.83)	340.28 (1514.25)
70	(0.03)	11.22 (285)	17.24 (438)	78.00	4.16 (105.80)	0.038 (25.00)	0.406 (262)	2.51 (283.54)	523.83 (2331.07)
80	(0.03)	11.22 (285)	17.24 (438)	78.00	4.16 (105.80)	0.038 (25.00)	0.406 (262)	2.51 (283.54)	523.83 (2331.07)
90	(0.03)	11.22 (285)	17.24 (438)	78.00	4.16 (105.80)	0.038 (25.00)	0.406 (262)	2.51 (283.54)	523.83 (2331.07)
100	(0.03)	11.22 (285)	17.24 (438)	78.00	4.16 (105.80)	0.038 (25.00)	0.406 (262)	2.51 (283.54)	523.83 (2331.07)